Coulomb blockade in quantum Hall droplets

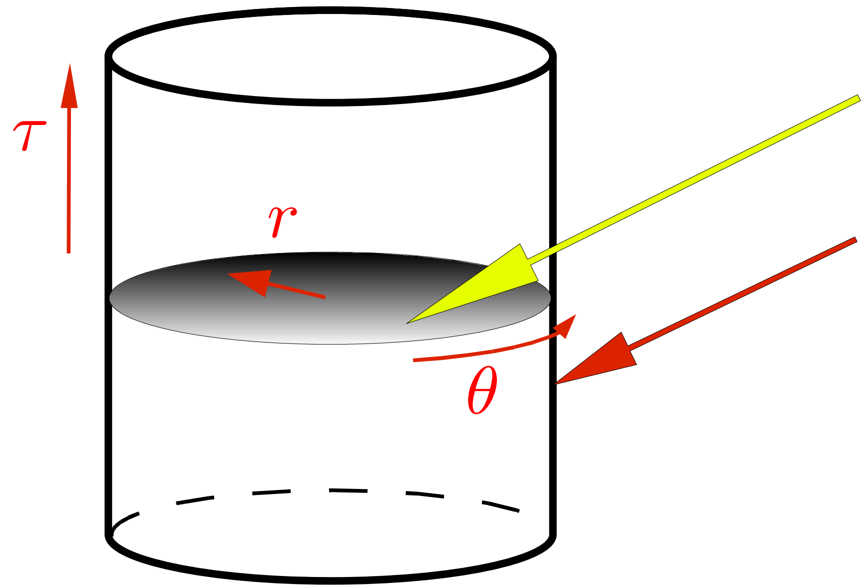
Andrea Cappelli
(INFN and Physics Dept., Florence)

Outline

- Introduction: Coulomb blockade conductance peaks
- Partition function of edge excitations
- Conductance peaks in Hierarchical and Read-Rezayi states
- Abelian and non-Abelian features

work with: L. Georgiev (Sofia), G. Zemba (Buenos Aires), G. Viola (Florence)

CFT descriptions of QHE



$z = re^{i\theta}$ plane (bulk excit)

$\zeta = e^{\tau} e^{i\theta}$ cylinder (edge excit)

correlator $\langle \, \phi_1 \quad \phi_2 \, \rangle_{CFT}$

$(z_1 - z_2)^{\alpha^2}$ anyon wave function

$(\zeta_1 - \zeta_2)^{\alpha^2}$ edge-excit. correlator

- same function by analytic continuation from the circle:
  - both equivalent to Chern-Simons theory in 2+1 dim
- spectrum of Luttinger CFT proofs Laughlin's fractional Q and $\frac{\theta}{\pi}$
  - wave functions: spectrum of anyons and braiding
  - edge correlators: conduction experiments (low V and small I)

Non-Abelian fractional statistics

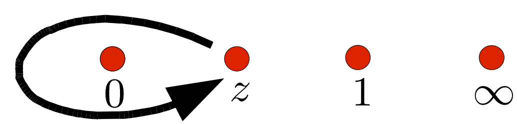
- $\nu = \frac{5}{2}$ described by Moore-Read "Pfaffian state" ~ Ising CFT x boson
- Ising fields: I identity, ψ Majorana = electron, σ spin = anyon
- fusion rules:
  - $\psi \cdot \psi = I$ 2 electrons fuse into bosonic bound state
  - $\boxed{\sigma \cdot \sigma = I + \psi}$ 2 channels of fusion = 2 conformal blocks

$$\langle \sigma(0)\sigma(z)\sigma(1)\sigma(\infty)\rangle = a_1 F_1(z) + a_2 F_2(z)$$ Hypergeometric

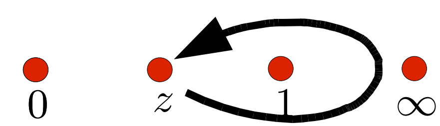
$\Longrightarrow$ state of 4 anyons is two-fold degenerate (Moore, Read '91)

- statistics of anyons ~ analytic continuation $\Longrightarrow$ 2x2 matrix

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (z e^{i2\pi}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (z)$$

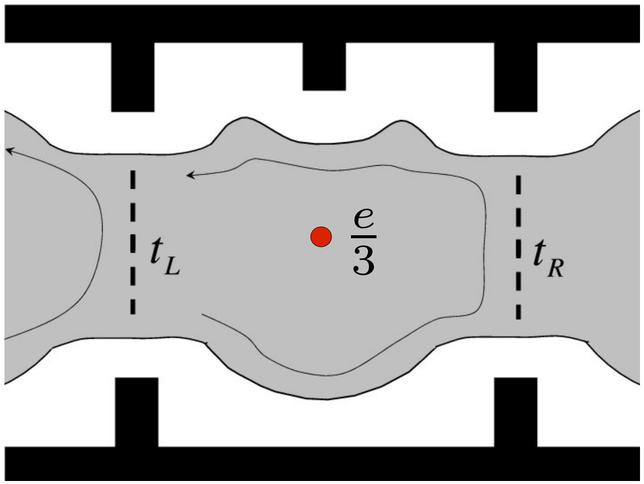


$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} ((z-1) e^{i2\pi}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (z)$$

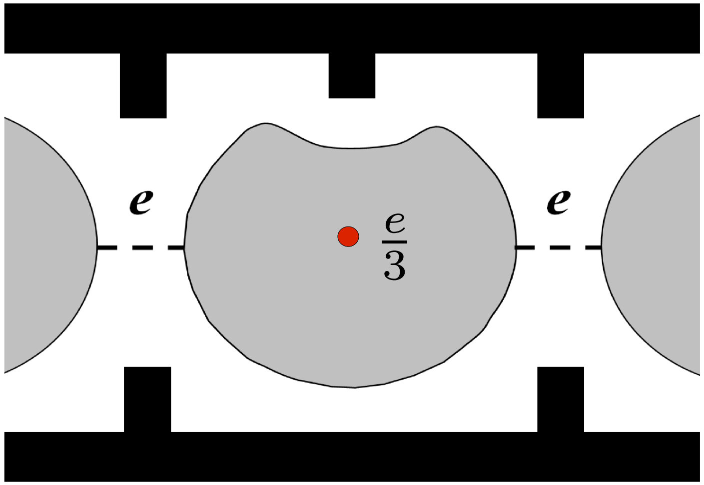


(all CFT tech redone: M. Freedman, Kitaev, Nayak, Slingerland, Wang, etc.)

Current experimental tests of CFT



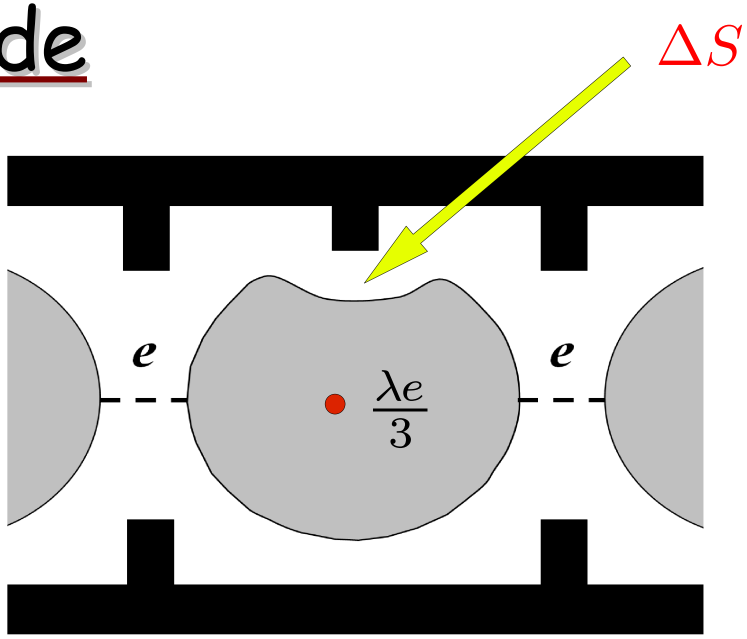
(a)



(b) (Ilan et al. '08)

- (a) interference of edge waves (Chamon et al. '97; Kitaev et al. 06)
  - ➡ Aharonov-Bohm phase, checks fractional statistics
  - – experiment is being done: instabilities (Goldman et al. '05; Willet et al. '08)
- (b) electron tunneling into the droplet (Stern, Halperin '06)
  - ➡ Coulomb blockade conductance peaks (Ilan, Grosfeld, Schoutens, Stern '08)
  - – check fusion rules and Hilbert space (A.C., Georgiev, Zemba '09)

Coulomb blockade



- Droplet capacity stops the electron

$$\Delta E(n) = -neV + \frac{(ne)^2}{2C} \ , \qquad Q = -ne$$

- quantized V values; one e at the time

$$\boxed{E(n+1) = E(n)}$$

  - peaks in the current

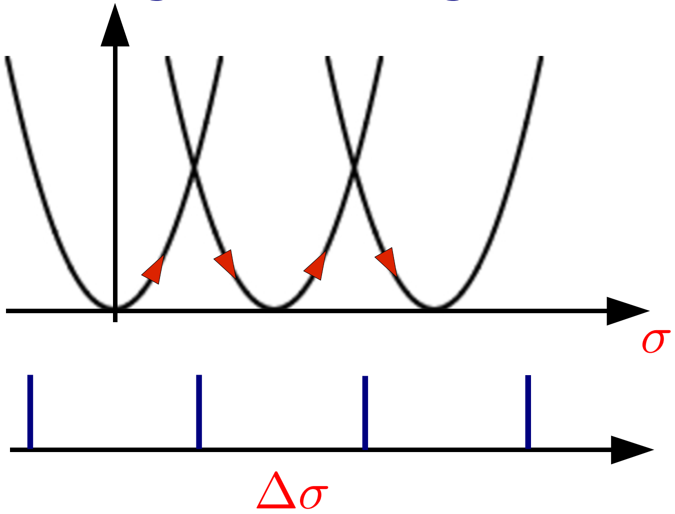
→ spectroscopy of edge states

- Laughlin $\nu = \frac{1}{3}$: three sectors $\qquad Q = \frac{\lambda}{3} + n \ , \quad \lambda = 0, 1, 2 \pmod 3$

- energy deformation by ΔS , i.e. varying the background charge

$$E(n) = \frac{v}{R} \frac{(\lambda + 3n - \sigma)^2}{6} \propto (Q - Q_{\text{bkg}})^2$$

$$\Delta\sigma = \frac{B\Delta S}{\Phi_o} = 3 = \frac{1}{\nu} \ , \qquad \Delta S = \frac{e}{n_o}$$



→ equidistant peaks

→ the same in the 3 sectors

- other plateaux:
  - hierarchical Jain states

$$\nu = \frac{m}{mp \pm 1}, \qquad \begin{cases} m = 2, 3, \ldots \\ p = 2, 4, \ldots \end{cases} \qquad U(1) \times SU(m)_1$$

(Read '90; Fröhlich, Zee '91; Wen, Zee '93)

  - Pfaffian and Read-Rezayi states

$$\nu = 2 + \frac{k}{kM + 2}, \quad \begin{cases} k = 2, 3, \ldots \\ M = 1, 3, \ldots \end{cases} \qquad U(1) \times \frac{SU(2)_k}{U(1)}$$

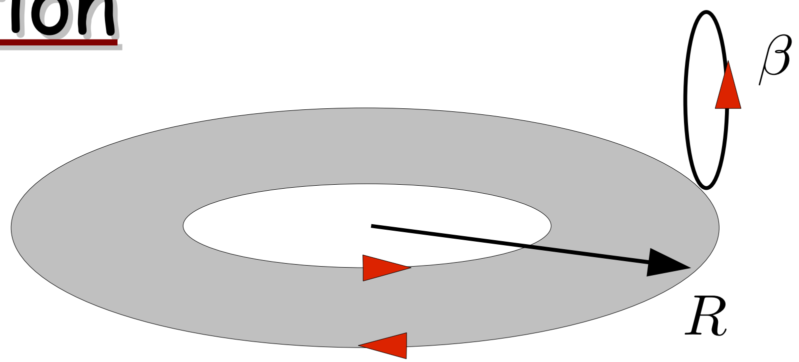
(Moore, Read '90; Read-Rezayi '99)

- non-trivial neutral excitations
- sectors given by fusion rules; specific pattern of peaks; multiplicities
- check qualitative features of CFT Hilbert space

$\Rightarrow$ $\boxed{\text{use partition function}}$

- modular invariant $\Rightarrow$ fusion rules built-in
- complete inventory of states
- already known *(AC, Zemba '97; AC, Georgiev, Todorov '01)*

Annulus partition function



$$2\pi \mathrm{Im}\tau = \beta \frac{v}{R} \qquad i2\pi\zeta = \beta(-V_o + i\mu)$$

$$Z_{\mathrm{ann.}} = \sum_{\lambda=1}^{p} |\theta_\lambda(\tau,\zeta)|^2, \quad \theta_\lambda(\tau,\zeta) = \mathrm{Tr}_{\mathcal{H}^{(\lambda)}} \left[e^{i2\pi\tau(L_0 - c/24) + i2\pi\zeta Q} \right], \quad \nu = \frac{1}{p}$$

Modular invariance building conditions

All geometrical properties have physical meaning:

$T^2: \ Z(\tau+2,\zeta) = Z(\tau,\zeta), \qquad L_0 - \overline{L}_0 = \frac{n}{2}$ half-integer spin excitations globally

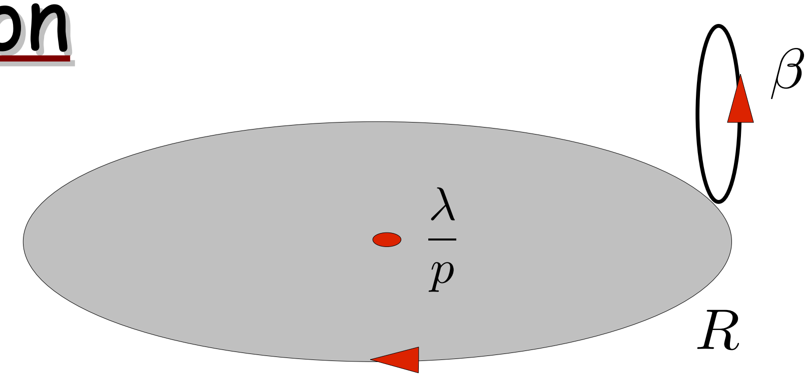
$S: \ Z\left(\frac{-1}{\tau}, \frac{-\zeta}{\tau}\right) = Z(\tau,\zeta),$ completeness $\quad \theta_\lambda\left(\frac{-1}{\tau}\right) = \sum_{\lambda'} S_{\lambda\lambda'}\theta_{\lambda'}(\tau)$ S matrix, fusion rules

$U: \quad Z(\tau,\zeta+1) = Z(\tau,\zeta), \qquad Q - \overline{Q} = n$ integer charge excitations globally

$V: \quad Z(\tau,\zeta+\tau) = Z(\tau,\zeta), \qquad \Delta Q = \nu$ add one flux: spectral flow

$$\theta_\lambda(\zeta+\tau) \sim \theta_{\lambda+1}(\tau)$$

Disk partition function



Annulus -> Disk (w. bulk q-hole $\bar{Q} = \frac{\lambda}{p}$)

$$Z_{\text{ann.}} \;\rightarrow\; \theta_\lambda(\tau,\zeta)$$

$$\theta_\lambda(\tau,\zeta) = K_\lambda(\tau,\zeta;p) = \sum_n e^{i2\pi\left[\tau\frac{(np+\lambda-\sigma)^2}{2p}+\zeta\frac{np+\lambda}{p}\right]}, \qquad \nu = \frac{1}{p}, \quad c=1$$

- sectors with charge $Q(n) = \frac{\lambda}{p} + n$ owing to U condition
- # sectors = p = $dim\,(S_{\lambda\lambda'})$ Wen's topological order
- vary S, compare $E(n;\sigma) = E(n+1;\sigma)$ as before: equidistant peaks
- vary B ~ add q-h in the bulk: $\Delta\Phi = \Phi_0, \quad \Delta\overline{Q} = \nu = \frac{1}{p}$
  - edge sector changes to keep integer charge: spectral flow

$$\theta_\lambda(\tau,\zeta) \;\rightarrow\; \theta_\lambda(\tau,\zeta+\tau) \sim \theta_{\lambda+1}(\tau,\zeta)$$

  - same peak pattern in any λ sector

CB peaks in hierarchical states

$$\nu = \frac{m}{mp \pm 1} \qquad \begin{cases} m = 2, 3, \ldots \\ p = 2, 4, \ldots \end{cases} \qquad U(1) \times SU(m)_1 \quad c = m$$

- specific charge lattice of m bosons with SU(m) symmetry
- charge vector and SU(m) weights are not orthogonal
- topological order $q = mp \pm 1$; solution of T^2, S, U, V conditions is:

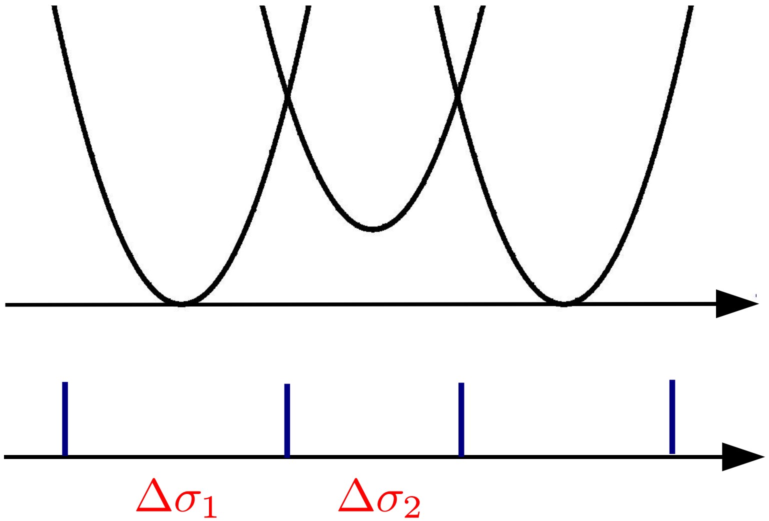
$$\boxed{\theta_a(\tau, \zeta) = \sum_{\beta=1}^{m} K_{ma+\beta q}(\tau, m\zeta; mq)\ \chi_\beta(\tau, 0)} \qquad a = 0, \ldots, q-1 \qquad \text{(A.C., Zemba '97)}$$

  - neutral $SU(m)_1$ characters χ_β, $\beta = 1, \ldots, m$
  - ex. $\nu = \frac{2}{5}$, $m = 2$, $q = 5$

$$\chi_1, \chi_2; K_1, \ldots, K_{10} \ \rightarrow \ \theta_0,\ \theta_{\pm 1},\ \theta_{\pm 2}$$

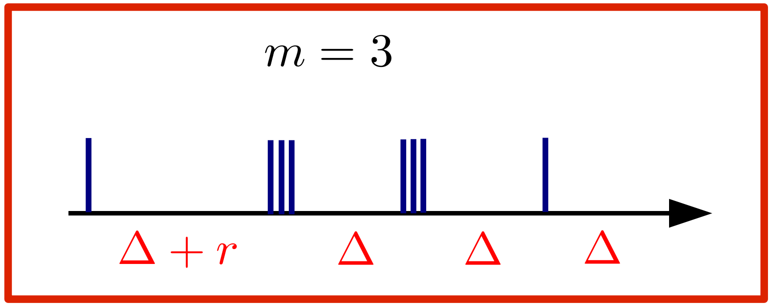
$$\theta_0 = \underbrace{K_0(\tau, 2\zeta; 10)}_{Q = 2n}\ \chi_0 + \underbrace{K_5(\tau, 2\zeta; 10)}_{Q = 2n+1}\ \chi_1$$

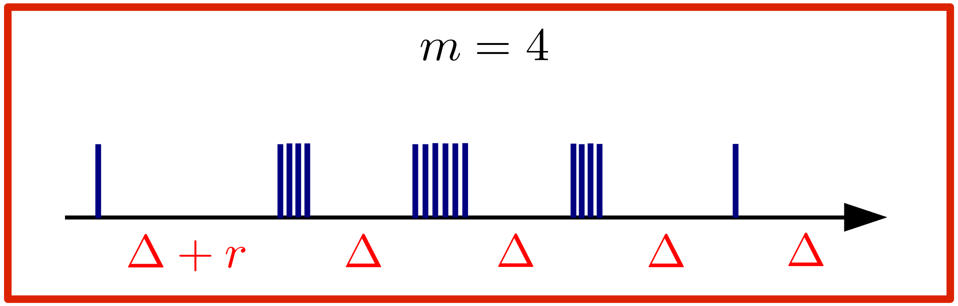
  - compare energies w. neutral parts



- neutral energy contribution -> modulated peak distances
- neutral character: $\chi_\beta \sim \binom{m}{\beta} \exp\left(i 2\pi\tau h_\beta\right)$ -> peak multiplicity

- peak pattern: groups of m peaks, $\Delta \sim \frac{1}{\nu}$, more distant of $r = \dfrac{v_n}{v_c} \sim 0.1$





- Characteristic features

- multiplicities check m-component edge theory (lots of literature)
  - no multiplicity theories:

    W_∞ minimal models (A.C., Trugenberger, Zemba '95)

    3-component theory (Fradkin, Lopez '99)

- peak pattern independent of a sector, i.e. of bulk quasi-holes

- no bulk-edge relaxation of neutral excitations

Pfaffian & Read-Rezayi states

$$\nu = 2 + \frac{k}{k+2}, \quad \left\{ \begin{array}{l} k = 2, 3, \ldots \\ M = 1 \end{array} \right. \qquad U(1)_{k+2} \times \frac{SU(2)_k}{U(1)_{2k}}$$

- Z_k parafermion fields and characters:

$$\chi^\ell_m, \quad \ell = 0, 1, \ldots, k, \quad m \bmod 2k, \quad \ell = m \bmod 2$$

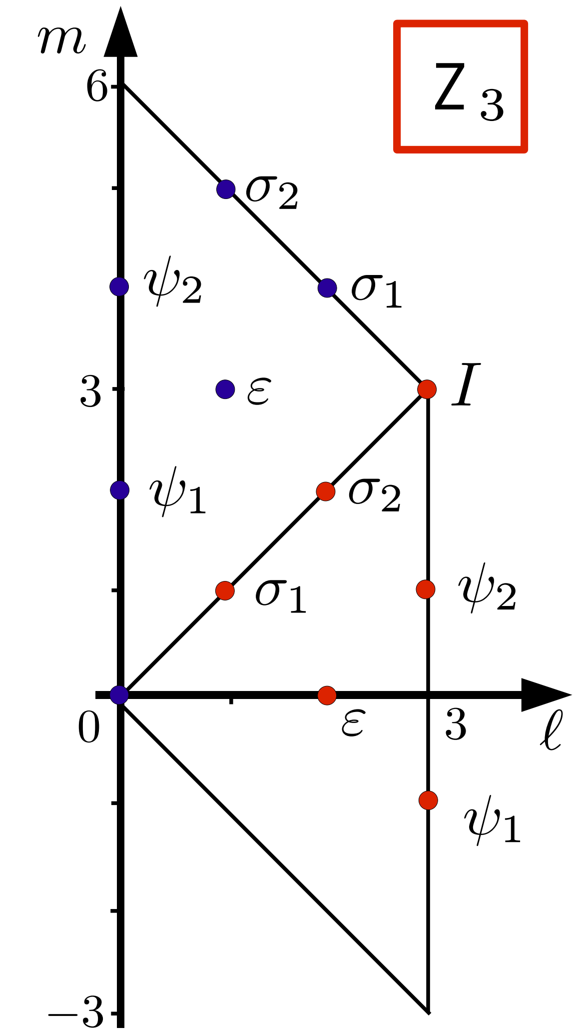
$$\chi^\ell_m = \chi^{k-l}_{m\pm k},$$

- Charged and neutral parts coupled by

  parity rule $\quad \lambda = m \bmod k$

- topological order: $(k+2) \times \frac{k(k+1)}{2} \times \frac{1}{k} = \frac{(k+2)(k+1)}{2}$

$$\boxed{\theta^\ell_a = \sum_{\beta=0}^{k-1} K_{a+\beta(k+2)} \, \chi^\ell_{a+2\beta}}$$



Z_3

- sectors labeled by (a, ℓ)

$$a = 0, \ldots, k+1, \quad \ell = 0, \ldots, k, \quad a = \ell \bmod 2$$

(A.C., Georgiev, Todorov '01)

CB peaks in Pfaffian & Read-Rezayi states

$$\theta_a^\ell = \sum_{\beta=0}^{k-1} K_{a+\beta(k+2)}\ \chi_{a+2\beta}^\ell \qquad a=0,\dots,k+1,\quad \ell=0,\dots,k$$

- pattern of peaks depends on ℓ sector:

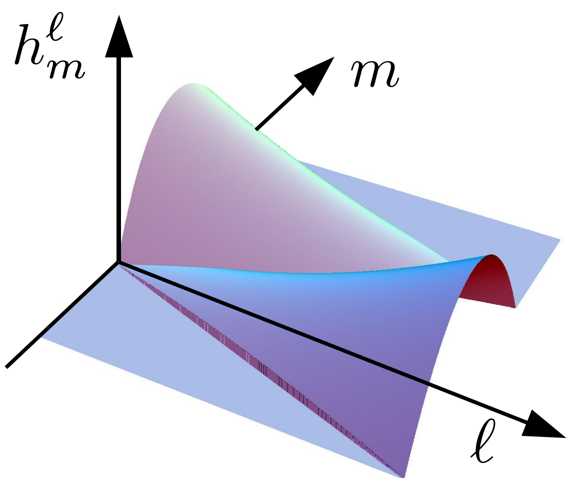
  ℓ = # basic quasi-holes in the bulk $\sigma_1,\quad (\ell,m)=(1,1)$

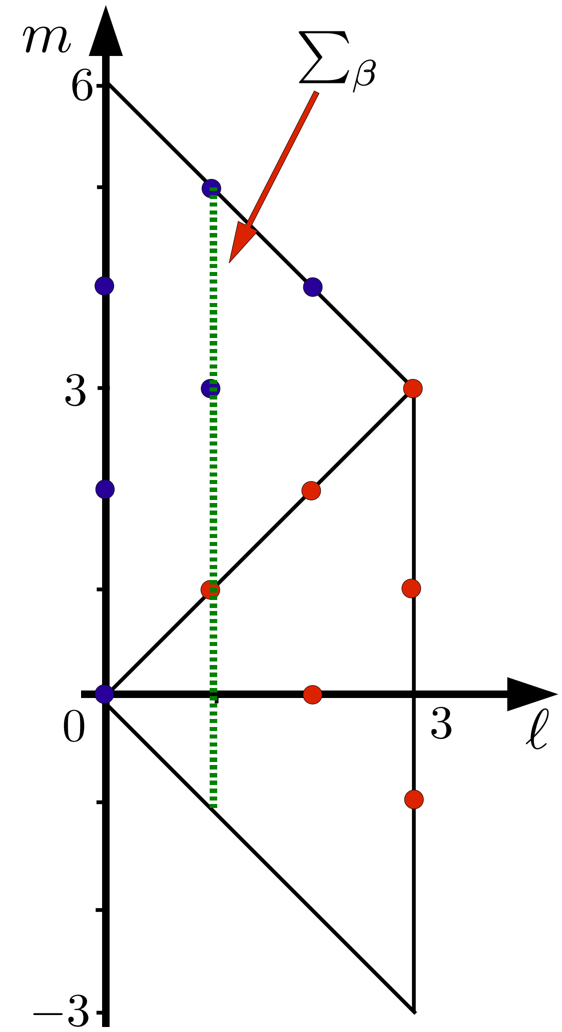
  $$\Delta\sigma_m = \Delta + \left(h^\ell_{m+4} - 2h^\ell_{m+2} + h^\ell_m\right)$$

  – discontinuous h^ℓ_m on diagonals ➡ modulation

$\ell = 0, k \quad : \ \Delta\sigma_n = (\Delta + 2r, \Delta, \dots, \Delta)\,,\quad r = v_n/v_c \qquad (k)$ groups

$\ell = 1,..,k{-}1 : \ \Delta\sigma_n = (\Delta + r, \Delta, \dots, \Delta + r, \dots, \Delta)\,,\quad (\ell),(k-\ell)$ groups





(Ilan, Grosfeld, Schoutens, Stern '08)

Bulk-edge relaxation

- neutral parts on edge can (slowly) recombine with bulk ones and lower the energy:

  no charge flow $(\Delta\ell, \Delta m) = (\pm 2, 0)$

  – modified peak pattern:

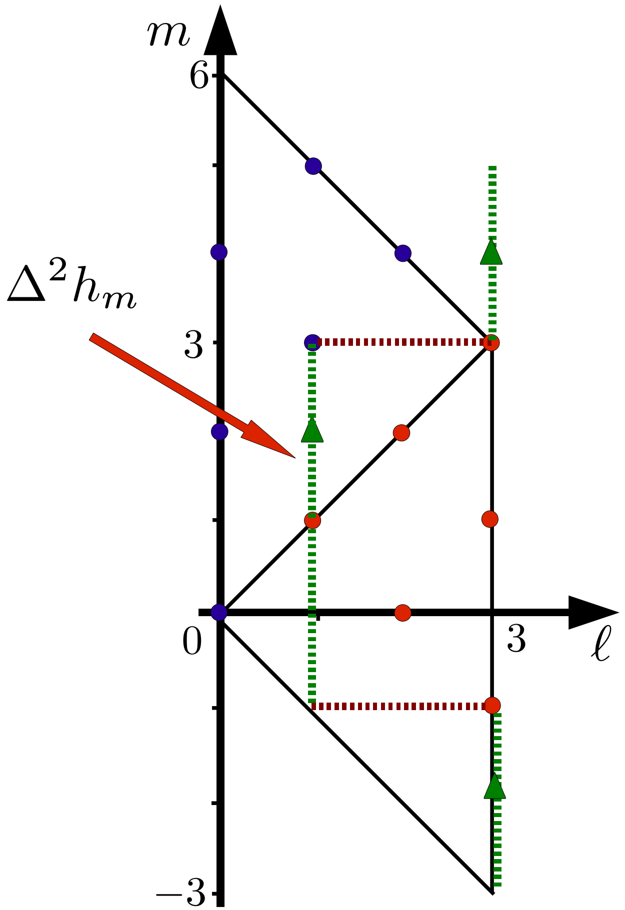
k even, any ℓ : $\binom{\frac{k}{2}}{}$ $\binom{\frac{k}{2}}{}$ groups

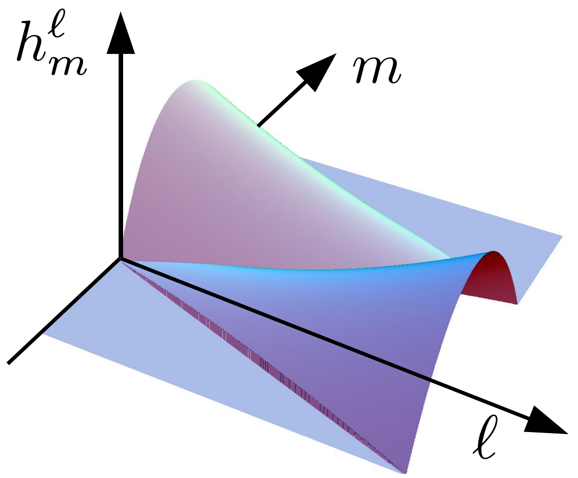
k odd, any ℓ : $\binom{\frac{k+1}{2}}{}$ $\binom{\frac{k-1}{2}}{}$ groups

  – ℓ dependence wiped off

- relaxation only possible w. non-Abelian fusion rules: e.g. $\epsilon \cdot \epsilon = 1 + \epsilon$

→ non-unique charged-neutral pairing





Pattern in (B,S) plane

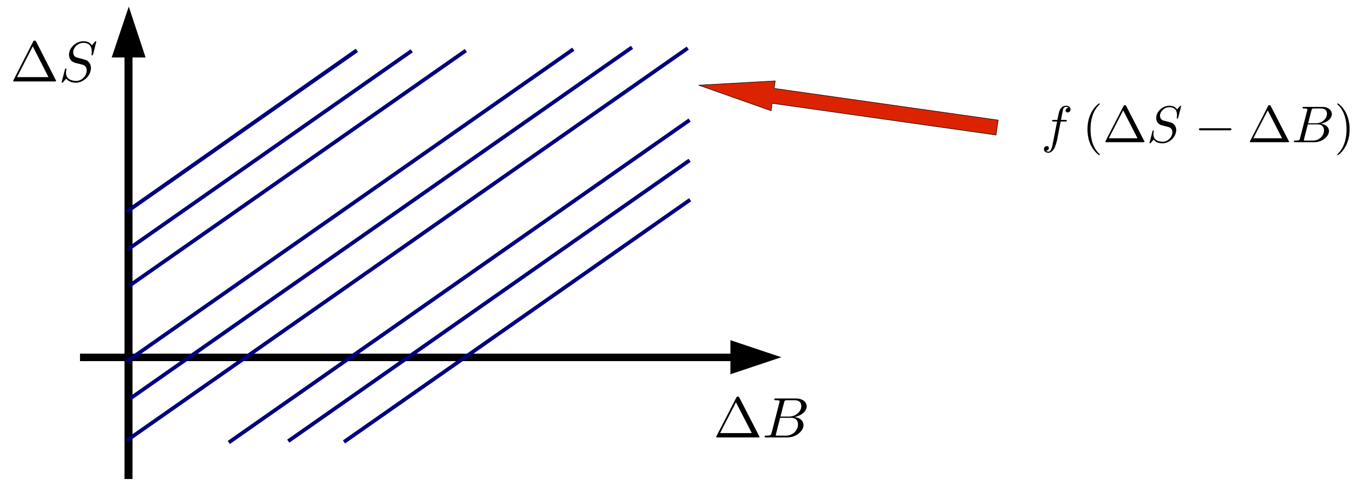
$$\theta_a^\ell = \sum_{\beta=0}^{k-1} K_{a+\beta(k+2)}\ \chi_{a+2\beta}^\ell \qquad a = 0,\dots,k+1, \quad \ell = 0,\dots,k$$

- spectral flow of charged part: for $\Delta\Phi = S\Delta B = \Phi_o$

$$K_\lambda(\tau, k\zeta; q) \ \rightarrow\ K_{\lambda+k}(\tau, k\zeta; q), \qquad \theta_a^\ell \ \rightarrow\ \theta_{a+k}^{k-\ell} = \theta_{a-2}^\ell$$

  - same neutral sectors ~ same peak pattern, only translated

$\boxed{Z_3, \ell = 0}$



  - cannot induce one σ_1 in the bulk at the time $(\Delta\ell, \Delta m) = (1,1)$

    would need an antidot in the bulk

Conclusions

- Coulomb blockade peaks: spectroscopy of edge states
  - tests fusion rules, neutral sectors, multiplicities
  - clean signal, but not-too-characteristic patterns
  - tests the multicomponent theory of hierarchical Jain states
- partition function of the disk from modular invariant annulus
  - it is useful (among other things.....it defines the CFT)
  - also good for the Topological Entanglement Entropy
  - further model building