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FAST TRACK COMMUNICATION

Composite fermion wavefunctions derived by conformal field theory

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Online at stacks.iop.org/JPhysA/46/012001**Abstract**

The Jain theory of hierarchical Hall states is reconsidered in the light of recent analyses that have found exact relations between projected Jain wavefunctions and conformal field theory correlators. We show that the underlying conformal theory is precisely given by the W -infinity minimal models introduced earlier. This theory involves a reduction of the multicomponent Abelian theory that is similar to the projection to the lowest Landau level in the Jain approach. The analysis closely parallels the bosonic conformal theory description of the Pfaffian and Read–Rezayi states.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The search for fractional exchange statistics in quasi-particle excitations of the quantum Hall effect is actively pursued both theoretically [1, 2] and experimentally [3, 4]. The observation of non-Abelian, i.e. multidimensional, statistics, first suggested in the Pfaffian state [5], would be particularly interesting as it could find application to quantum computation [6].

The methods of two-dimensional conformal field theory (CFT) [7] have been extremely useful in this domain. Their description is twofold: they model the dynamics of massless chiral excitations at the edge of the Hall droplet [8], and express analytic wavefunctions for electrons in the lowest Landau level. In particular, the CFT wavefunctions include non-analytic prefactors that make explicit the fractional statistics of excitations. All these methods have been fully developed in the case of Pfaffian and Read–Rezayi [9] states, where they have been instrumental for obtaining several physical results.

The studies of hierarchical Hall states have somehow remained behind, in spite of their experimental relevance stemming from their richness and higher stability. On one side, the Jain theory of the composite fermion [10] predicts very accurate ground-state wavefunctions and

has been confirmed by many experiments, but it does not provide full understanding of quasi-particle statistics. On the other side, the CFT description was based on the multicomponent Abelian theory (multicomponent Luttinger liquid) [11]. This approach has had some problems, such as the prediction of several distinct types of electrons¹.

The precise relation between these two approaches has remained rather unclear until recent results that form the basis of this work. In a series of papers, Hansson *et al* [13] have found *exact* CFT expressions for the Jain wavefunctions projected to the lowest Landau level. These functions have been written in terms of the expected multicomponent Abelian CFT plus some additional requirements to be fully interpreted. Other works [14] have studied the short-distance and pairing (clustering) properties of Jain wavefunctions and compared them to those of the Pfaffian and other non-Abelian Hall states.

In this communication, we shall relate these results with another theory of the hierarchical Hall states that has been formulated independently of the composite fermion picture, by Cappelli *et al* [16]. Its main physical input is the incompressibility of the electron fluid and its symmetry under *area-preserving diffeomorphisms* of the plane, also called $W_{1+\infty}$ symmetry. The implementation of this symmetry in the dynamics of edge excitations and the use of results of representation theory lead to a general analysis of conformal theories suitable for spin-polarized Hall states. While they are generally found to be equivalent to the multicomponent Abelian theories, there are special cases with enhanced $SU(n)$ symmetry, where a projection of degrees of freedom is needed to obtain irreducible representations, i.e. elementary excitations. This is achieved through a coset construction [7], as follows:

$$\widehat{U(1)}^n \longrightarrow \widehat{U(1)} \times \widehat{SU(n)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(n)}_1}{SU(n)}. \quad (1)$$

The conformal theories obtained in this way were called $W_{1+\infty}$ minimal models [17]; they were found to be in one-to-one correspondence with the Jain states, but to differ from the standard multicomponent Abelian theory owing to the projection of the $SU(n)$ symmetry which, among other things, leads to a unique type of electron and to non-Abelian fusion rules. This projection was also described by introducing a term in the edge Hamiltonian [18]. However, this theory was not yet applied to describe wavefunctions: in this communication, we complete the analysis and establish a direct relation with the Jain composite fermion theory. Our results are as follows.

- (i) We show that the Hansson *et al* expression of Jain wavefunctions can be completely *derived* within the $W_{1+\infty}$ minimal models. Furthermore, the projection of the multicomponent Abelian theory realized in the minimal models can be related to the Jain projection to the lowest Landau level.
- (ii) We remark that the Jain wavefunctions in the Hansson *et al* form are remarkably similar to the Pfaffian and Read–Rezayi wavefunctions expressed in terms of another projection of Abelian theories [19]; the analogies between the two approaches are emphasized.
- (iii) We argue that quasi-holes excitations over the Jain states could possess non-Abelian fractional statistics as predicted by the $W_{1+\infty}$ minimal models; the mechanism for non-Abelian statistics is similar to that of the Pfaffian state and results from the identification of the independent components (effective Landau levels) down to a single one.

¹ An alternative construction without multiple electrons has been given in [12].

2. Hansson *et al* results

The Jain theory is based on a correspondence between integer, $1/\nu^* = 1/n$, and fractional, $1/\nu = p + 1/n$, Hall effects (p even); this is realized by introducing the ground-state wavefunction [10]:

$$\Psi_\nu = \mathcal{P}_{LLL} \prod_{i < j}^N (z_i - z_j)^p \Psi_{\nu^*=n}. \quad (2)$$

The term $\Psi_{\nu^*=n}$ is the Slater determinant for N electrons completely filling the first n Landau levels, say putting N/n of them in each level; \mathcal{P}_{LLL} expresses the projection to the lowest Landau level. Note that electrons originally placed in the j th level, $j = 1, 2, \dots$, have angular momentum of one-particle states shifted by $(1 - j)$.

In their careful study of quasi-particle excitations over the Laughlin and other states, Hansson *et al* [13] obtained an exact rewriting of (2). Using non-trivial determinant identities, they found the following formula:

$$\Psi_\nu = \mathcal{A} \left[\prod_{i < j}^{N/2} w_{ij}^{p+1} \prod_{k=1}^{N/2} \partial_{z_k} \prod_{i < j}^{N/2} z_{ij}^{p+1} \prod_{i,j}^{N/2} (z_i - w_j)^p \right]. \quad (3)$$

In this expression, we considered the $n = 2$ case for simplicity ($\nu = 2/(2p + 1)$), and put $N_1 = N_2 = N/2$ electrons in the first and second levels, respectively (the result can be extended to $N_1 > N_2$ and also to $n > 2$ [13]). Electron coordinates in the first and second levels were denoted by w_i and z_j , $i, j = 1, \dots, N/2$, respectively, and their differences by $w_{ij} = w_i - w_j$ and $z_{ij} = z_i - z_j$. The symbol \mathcal{A} indicates antisymmetrization with respect to all N electron coordinates.

The remarkable expression (3) admits a simple description in terms of CFT correlators. From the Jastrow factors inside square brackets, one can recognize the wavefunction of the standard two-component Abelian CFT description of hierarchical states [11] with the so-called K matrix equal to

$$K = \begin{pmatrix} p+1 & p \\ p & p+1 \end{pmatrix}. \quad (4)$$

This wavefunction corresponds to the correlator of two kinds of conformal fields (vertex operators), called V_+ and V_- , as follows:

$$\Psi_\nu = \mathcal{A}[(\partial_{z_1} V_+) \cdots (\partial_{z_{N/2}} V_+) V_- \cdots V_-]. \quad (5)$$

The vertex operators can be written in terms of charged ϕ and neutral φ scalar conformal fields as $V_\pm = e^{i\sqrt{p+\frac{1}{2}}\phi} e^{\pm i\frac{1}{\sqrt{2}}\varphi}$ (see [17] for details).

The expressions (3) and (5) can be interpreted as describing electrons belonging to the first two Landau levels, with the derivatives realizing the shift of angular momentum for the second level and the antisymmetrization ensuring identical electrons.

From the CFT point of view, derivatives operators usually describe local perturbations of states, i.e. excited (descendant) states. However, in the present case expressions with less than $N/2$ derivatives vanish under antisymmetrization (as checked in some examples and proven later), and thus derivative fields can represent a ground-state wavefunction.

3. Analogies with the Pfaffian state

Hansson *et al* expression (3) is very similar to the Pfaffian state in the two-component Abelian CFT description developed in [19], which reads ($1/\nu = M + 1$, M odd):

$$\Psi_{\text{Pfaff}} = \mathcal{A} \left[\prod_{i < j}^{N/2} w_{ij}^{M+2} \prod_{i < j}^{N/2} z_{ij}^{M+2} \prod_{i,j}^{N/2} (z_i - w_j)^M \right], \quad (6)$$

corresponding to the matrix

$$K = \begin{pmatrix} M+2 & M \\ M & M+2 \end{pmatrix}. \quad (7)$$

After antisymmetrization, the expression (6) can be shown to be identical to the standard form [5]:

$$\Psi_{\text{Pfaff}} = \prod z_{ij}^{M+1} \text{Pf} \left(\frac{1}{z_{ij}} \right). \quad (8)$$

The explanation for such Abelian representation of the Pfaffian state rests in the physics of electron pairing. Before antisymmetrization, the state (6) describes distinguishable electrons, possessing a twofold quantum number, say isospin. Their wavefunction does not vanish (for $M = 0$) when two electrons with different isospin meet at the same point $z_1 = z_2 = z$, indicating their pairing; when a third electron approaches, it necessarily vanishes as $\Psi \sim (z_3 - z)^2$. After (anti)symmetrization, all electrons become identical, i.e. the theory only contains isospin singlets, but the pairing property is retained, that is characteristic of the Pfaffian state.

The Abelian representation of the Pfaffian state extends easily to quasi-hole excitations [19]: before projection, these are of two kinds, each one coupling to its electron: after projection, they become identical and yield a multidimensional representation of the braid group, i.e. to non-Abelian statistics [20]. This is a nice way to understand non-Abelian statistics within the standard Abelian setting: it is just a consequence of the projection to identical electrons.

The similarity of the expressions (3) and (6) for the Jain and Pfaffian states brings in a series of results that will be relevant for the following discussion. A first observation is that the Jain states possess the same vanishing behaviour as that of the Pfaffian and Read–Rezayi states. When three or more particles approach the same point, equation (3) behaves as follows:

$$\Psi_{n=2} \sim z_{12}^{p-1} (z_{13}^{p+1} z_{14}^{p+1} \dots), \quad \frac{1}{\nu} = p + \frac{1}{2}, \quad (9)$$

$$\Psi_{n=3} \sim (z_{12} z_{13} z_{23})^{p-1} (z_{14} z_{15} \dots)^{p+1}, \quad \frac{1}{\nu} = p + \frac{1}{3}. \quad (10)$$

In the second expression, we also reported the $n = 3$ case to be compared with the \mathbb{Z}_3 parafermionic Read–Rezayi state [9, 19]. The result (9) can be easily proven by using a graphical representation for the action of derivatives in the expression (3) [22].

Therefore, we see that the $M = 0$ Pfaffian and the $p = 1$ Jain states have the same pairing properties. The difference between them is that the Pfaffian (Read–Rezayi) state is the lowest order polynomial, i.e. lowest angular momentum state, obeying that type of pairing (clustering), while the Jain state has higher angular momentum. Another way to put this fact is that the Pfaffian is the lowest momentum zero-energy state of a three-body short-distance potential, while Jain state is an excited state. A short-distance potential that could similarly single out the Jain state has not been found [14]. Another closely related state is the so-called

Gaffnian [15], that has the same angular momentum of the Jain state but higher order three-body vanishing behaviour: $\Psi \sim z_{13}^3$ rather than $\Psi \sim z_{13}^2$ ($p = 1$). The relations between Pfaffian, Jain and Gaffnian states will be further discussed in [22].

Let us remark a difference between the expressions (3) and (6) for the Jain and Pfaffian states: in the former case, the two layers are not exactly equivalent owing to the presence of derivative fields. We shall discuss this point in the following sections.

4. Derivation of Jain wavefunctions from $W_{1+\infty}$ minimal models

We start from the two-component Abelian theory and describe the projection of the $SU(2)$ symmetry (1) on wavefunctions. Its physical meaning is that of allowing singlet excitations only, that are symmetric with respect to the exchange of the two layers. Symmetric edge excitations are irreducible (elementary) particle–hole transitions of the incompressible fluid made of two identical indistinguishable layers, that is richer than a single Laughlin fluid [17].

The transition between the two-component Abelian theory and the $W_{1+\infty}$ minimal model has been described by adding a relevant (non-local) interaction in the Luttinger Hamiltonian [18]. This splits symmetric and antisymmetric excitations and project the latter ones out in the limit of infinite coupling (infrared limit). The similar interplay between the Abelian 331 state and the Pfaffian state has been described in [19, 21].

We now implement the symmetrization on bulk degrees of freedom. The neutral parts of the vertex operators V_{\pm} in (5) have associated isospins values $t_z = \pm 1/2$ that should be identified; the resulting field is only characterized by its Virasoro dimension $h = t^2/2$, according to the coset projection (1):

$$\widehat{U(1)} \times \frac{\widehat{SU(2)}_1}{\widehat{SU(2)}} \sim \widehat{U(1)} \times \text{Vir}, \quad (11)$$

where Vir is the Virasoro minimal model in the $c \rightarrow 1$ limit.

A convenient way to perform the projection is by using the Dotsenko–Fateev screening operators Q_{\pm} [7]. These have vanishing scale dimension but non-vanishing isospin $t_z = \pm 1$, and can relate the two electron fields:

$$V_- \sim V_+ = Q_+ V_-, \quad Q_+ = J_0^+ = \oint du J^+(u). \quad (12)$$

Note that in this $c = 1$ theory, the screening operators are the zero modes J_0^{\pm} of the $\widehat{SU(2)}_1$ affine algebra.

The electrons should be represented by $\widehat{U(1)} \times \text{Vir}$ primary fields. Using the standard Dotsenko–Fateev procedure, the correlator of, say, four electrons can be expressed in terms of four vertex operators and two screening charges, as required by the vanishing of total isospin, as follows:

$$\Psi = \left\langle \oint_{C_1} J^+ \oint_{C_2} J^+ V_-(z_1) V_-(z_2) V_-(z_3) V_-(z_4) \right\rangle. \quad (13)$$

The next step is to choose the contours C_1, C_2 . In principle, there are several choices, corresponding to different intermediate states in the fusion of two electrons. Actually, after projection the fields in the Virasoro theory acquire non-Abelian fusion rules, corresponding to the addition of isospins, $\{1/2\} \times \{1/2\} = \{0\} + \{1\}$ (these are nothing else than the $c \rightarrow 1$ limit of the fusion rules of minimal Virasoro models).

The ground-state wavefunction should be completely (anti)symmetric with respect to all electrons; this requires the contours to encircle all of them as shown in figure 1. Unfortunately, this choice yields a vanishing result (explicitly checked in [18]) because the contours can be deformed at infinity where they correspond to acting on the invariant right vacuum, $\langle 0 | J_0^- = 0$.

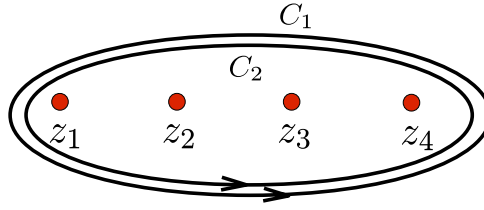


Figure 1. The contours for the screening charges.

The solution of this puzzle is to identify the two-electron fields through another $\widehat{SU(2)}_1$ generator with same $t_z = 1$ isospin but leading to non-vanishing results. The natural replacement is

$$J_0^+ \rightarrow J_{-1}^+. \quad (14)$$

Acting on vertex operators, J_{-1}^+ yields the first descendent in the tower of states of the affine representation,

$$J_{-1}^+ V_- = L_{-1}^{(n)} J_0^+ V_- \propto \partial_z V_+, \quad (15)$$

where $L_{-1}^{(n)}$ is a Virasoro generator for the neutral CFT part. Equation (15) can be easily checked by using the vertex operator algebra. Note that the operators $\{J_{-1}^+, J_1^-, 1/2 - J_0^3\}$ form another $SU(2)$ algebra contained in the affine algebra $\widehat{SU(2)}_1$ [7], that is equivalent for realizing the projection (11).

Thus, we consider the following expression for the four-electron wavefunction:

$$\Psi' = \langle J_{-1}^+ V_-(z_1) J_{-1}^+ V_-(z_2) V_-(z_3) V_-(z_4) \rangle + \text{perm}. \quad (16)$$

This is actually equal to the Hansson *et al* expression (5), owing to (15).

Therefore, we have obtained a derivation of the Jain composite fermion wavefunction entirely from symmetry arguments, through the construction of the $W_{1+\infty}$ minimal models and their application to describing wavefunctions. The additional input with respect to the two-component Abelian theory has been the identification of the electrons in the two layers and the requirement of Fermi statistics through the choice of contours in figure 1.

Equation (16) is the main result of this paper. It is rewarding insofar as it supports the universality and robustness of the Jain theory; furthermore, it can be used to obtain a consistent description of excitations, as discussed in the next section.

Let us remark that the outcome (16) is not a simple symmetrization owing to the descendant fields needed to obtain a nonvanishing result. It is a singlet of the $SU(2)$ symmetry, but it involves a short-distance deformation operated by the Virasoro operators L_{-1} in (15). Also note that this last equation strictly holds for the neutral part of the vertex operators V_{\pm} : for the derivative to act on the whole electron field, as in (5), one needs to modify the $SU(2)$ generator for including a charged part:

$$J_{-1}^+ \rightarrow J_{-1}^+ + L_{-1}^{(c)} J_0^+ = L_{-1}^{(\text{tot})} J_0^+, \quad (17)$$

where $L_n^{(c)}$ are the Virasoro generators for the charged part. Upon varying the relative coefficient between the charged and neutral terms in (17), it is also possible to obtain a family of wavefunctions, the Jain wavefunction being one of them. It can be shown that this ambiguity is due to the normal ordering of fields in the Hansson *et al* hierarchical construction of Jain states [22]. Being a short-distance cutoff modification, it does not change the pairing property (9) of the wavefunction.

Finally, we remark that the present derivation of Jain wavefunctions by $W_{1+\infty}$ symmetry clearly extends to more than two Landau levels [22]: for $n = 3$, two $SU(3)$ generators $J_{-1}^{a_i}$ are employed, whose action on vertex operators leads to single and double derivatives in agreement with Hansson *et al*'s results [13].

5. Quasi-hole excitations and their statistics

In this section, we implement the projection (11), i.e. the symmetrization of the two layers, on quasi-hole excitations. We shall be rather brief and defer to [22] for a complete discussion and the corresponding analysis of quasi-particles. The procedure is similar to that of the Pfaffian state [20] in its Abelian description [19].

In the two-component Abelian theory of Jain states, there are two quasi-hole excitations of smallest charge with isospin $t_z = \pm 1/2$ [18]. They are represented by fields H_{\pm} that couple to the respective electron fields V_{\pm} , owing to the fusion rules $H_{\pm}V_{\mp} \sim I$. In the Jain description, each quasi-hole is made in one effective Landau level, and Hansson *et al* have verified that the insertion of H_{\pm} in the CFT correlator (5) reproduces the corresponding Jain wavefunction [13].

After the projection (11), the two quasi-holes should be identified, $H_- \sim H_+ = J_0^+ H_-$. As a consequence, the isospin component t_z is no longer a good quantum number and is not conserved in the fusion of quasi-holes. The total isospin t is still conserved and leads to non-Abelian fusion rules by union of Abelian channels, as follows:

$$H_{\pm}H_{\pm} \sim H_{s=1}, \quad H_+H_- \sim H_{s=0} = I \longrightarrow HH \sim I + H_{s=1}. \quad (18)$$

The first interesting case is for four quasi-holes, where three different amplitudes can be associated with the same physical excitation. The first one is

$$\Psi_{(12,34)} = \mathcal{A}_{z_i} \left[[H_+(\eta_1)H_+(\eta_2)H_-(\eta_3)H_-(\eta_4) + (+ \leftrightarrow -)] \prod V_e(z_i) \right], \quad (19)$$

where we collectively denoted electron fields by V_e and antisymmetrization acts on electron coordinates only. The other two amplitudes, $\Psi_{(13,24)}$ and $\Psi_{(14,23)}$, correspond to different assignments of (\pm) .

These three amplitudes give rise to a multidimensional representation of the braid group \mathcal{B}_4 , i.e. to non-Abelian statistics. Inspection of the four- and six-hole states shows that there are no degeneracies among the possible amplitudes, in contrast with the Pfaffian case [20]; thus, the number of $2k$ -hole states grows like $d_k = 2k!/2(k!)^2 \sim 2^{2k}/\sqrt{k}$. The same multiplicities can be inferred by tensoring isospin one-half representations as predicted by CFT.

In conclusion, the description of quasiholes over the Jain state in the $SU(2)$ projected theory, i.e. in $W_{1+\infty}$ minimal models, naturally suggests their non-Abelian fractional statistics.

We remark that the argument for non-Abelian statistics presented here is far from being a proof, lacking an understanding of several important issues. Let us briefly mention them [22].

- *Energetics.* In the case of the Pfaffian state [9], the three-body short-distance electron potential identifies the exact zero-energy subspace in which the non-Abelian excitations have the desired degeneracy. Moreover, the transition from the Abelian 331 state to the Pfaffian, corresponding to layer symmetrization, can also be understood as a deformation of the bulk Hamiltonian [21], where non-symmetric excitations have energies sent to infinity.

The corresponding analysis on the Jain state is missing: its exact pseudo-potential is not known and a comprehensive study of many quasi-holes states has not been performed to our knowledge [10]. Hopefully, these could be made by developing the works [14] and numerical analyses of edge excitations [23] and entanglement spectrum [24]. In this

respect, the realization of non-Abelian statistics requires that antisymmetric states have higher energy than symmetric ones and that the latter possess the degeneracies for braiding. A related problem is understanding the Gaffnian state, that has large overlap with the Jain state and some of the properties outlined in this paper, although it is believed to be gapless in the thermodynamic limit [15].

- *Shift.* In contrast to the Pfaffian case, the two layers in Jain theory are not exactly equivalent owing to the derivative fields, reproducing the familiar shift of orbital momentum of electrons in higher Landau levels. The quasi-holes are represented by primary fields H_{\pm} , but couple differently to electrons V_{-} , ∂V_{+} , such that a layer symmetrization is necessary in (19).

As said before, the shift is a short-distance deformation, implemented by Virasoro operators (15) and (17), commuting with the $SU(2)$ symmetry. There is the hope that this deformation does not affect the long-distance effects, such as fractional statistics and edge physics (which we started from).

- *CFT.* The Pfaffian state can be described by conformal field theory after antisymmetrization of (6), namely by the Majorana fermion leading to (8). Moreover, the space of states of this CFT matches the zero-energy subspace of the three-body pseudopotential [21, 9]. The corresponding conformal theory after antisymmetrization of the Jain state (5) is not yet understood [22].

On the positive side, we remark that the description of quasi-particles and the hierarchical construction of Jain states using Abelian conformal fields by Hansson *et al* [13] is completely consistent with the symmetry projection discussed in this paper. In particular, the quasi-particle that condense in the n th level Jain state and builds the $(n + 1)$ -th level is a singlet of $SU(n)$ [22]. Finally, notwithstanding the theoretical uncertainties, it would be interesting to extend the present experimental tests of non-Abelian statistics with interferometry [3] and thermal transport [4] to Jain states.

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