Methods

Table 1: Linear Model Simulation: data generating models, parameter values, and mis-specifications.

	Data Generating Model	Parameters	Data Fitting Model
Correct	$X_i \sim N(0,1)$ $\mu_{i,j} = X_i\beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\beta = (4, -5)$ $\sigma_y = 1$	$X_i \sim N(0,1)$ $\mu_{i,j} = X_i\beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$
Mis-specified	$X_i \sim N(0,1)$ $\mu_{i,j} = X_i\beta$ $y'_{i,j} \sim N(\mu_{i,j}, exp(\sigma_y))$		$X_i \sim N(0,1)$ $\mu_{i,j} = X_i\beta$ $y'_{i,j} \sim N(\mu_{i,j}, \sigma_y)$

Table 2: Mixed Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Data Fitting Model	Data Generating Model	Parameters	Data Fitting Model
Linear Mixed Model			Generalized Linear Mixed Model		
Correct			Correct		
$X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = X_i\beta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\beta = (4, -8)$ $\sigma_u = 2$ $\sigma_y = 0.5$	$X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = X_i\beta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$	$\beta = log(2)$ $size = 1$ $\sigma_u = 1$	$u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$
Mis-specified			Mis-specified		
Correct $X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = X_i\beta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$		Missing Random Effect $X_i \sim Unif(-0.5, 0.5)$ $\mu_{i,j} = X_i\beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$ Missing Covariate $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = \beta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	Correct $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$		Missing Random Effect $\mu_{i,j} = exp(\beta)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$ Mis-specified Distribution (Poisson) $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim Poisson(\mu_{i,j})$
Lognormal Random Effect $X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$ $\mu'_{i,j} = X_i\beta + e^{u_j}$ $y'_{i,j} \sim N(\mu'_{i,j}, \sigma_y)$		Correct $X_i \sim Unif(-0.5, 0.5)$ $\mu_{i,j} = X_i\beta + u_j$ $y'_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	Gamma Random Effect $u_j \sim Gamma(1, 1)$ $\mu'_{i,j} = exp(\beta + u_j)$ $y'_{i,j} \sim NBinom(\mu_{i,j}, size)$		Correct $u_j \sim N(0, \sigma_u)$ $\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$

Table 3: Linear Mixed Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Data Fitting Model	Data Generating Model	Parameters	Data Fitting Model
Linear Mixed Model			Generalized Linear Mixed Model		
Correct			Correct		
$\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$	$a = 2$ $u[1] = 0$ $\sigma_u = 1$ $\sigma_y = 1$	$\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$	$\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$	$a = .01$ $u[1] = 0$ $\sigma_u = 0.05$ $CV = 0.5$	$\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$
Mis-specified			Mis-specified		
Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$		Missing Random Effect $y_i \sim N(a(1:n), \sigma_y)$ Missing Drift Term $\mu_i = u_{i-1}$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$	Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$		Mising Random Effect $y_i \sim Gamma(\frac{1}{CV}^2, e^{a} CV^2)$ Missing Drift Term $\mu_i = u_{i-1}$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$ Mis-specified Distribution Normal $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$
Heteroscedasticity $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $\sigma_y = \sqrt{(1:\frac{n}{2}}^{1.3}$ $y_i^{'} \sim N(u_i, \sigma_y)$		Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i^{'} \sim N(u_i, \sigma_y)$			
Lognormal Random Effect $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i^{'} \sim N(e^{u_i}, \sigma_y)$		Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i^{'} \sim N(u_i, \sigma_y)$	Gamma Random Effect $u_i = u_{i-1} + Gamma(0.5, 20)$ $y_i^{'} \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$		Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i} CV^2)$

Table 4: Spatial Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Data Fitting Model	Data Generating Model	Parameters	Data Fitting Model
Linear Mixed Model			Generalized Linear Mixed Model		
Correct			Correct		
$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + \omega_i$ $y \sim N(\eta, \sigma_y)$	$\beta_0 = 4$ $\sigma_y^2 = 1$ $\phi = 50$ $\kappa = \sqrt{8}/\phi$ $\sigma_\omega^2 = 1$	$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + \omega_i$ $y \sim N(\eta, \sigma_y)$	$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y \sim Pois(\eta, \sigma_y)$	$\beta = 0.5$ $\phi = 50$ $\kappa = \sqrt{8}/\phi$ $\sigma_\omega^2 = 0.25$	$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y \sim Pois(\eta, \sigma_y)$
Mis-specified			Mis-specified		
Correct $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + \omega_i$ $y \sim N(\eta, \sigma_y)$		Missing Random Effect $\eta_i = \beta_0$ $y \sim N(\eta, \sigma_y)$ Lognormal Error $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + \omega_i$ $\sigma_y = exp(N(0,1))$ $y \sim N(\eta, \sigma_y)$	Correct $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y \sim Pois(\eta)$		Missing Random Effect $\eta_i = exp(\beta_0)$ $y \sim Pois(\eta)$
Lognormal Random Effect $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + exp(\omega_i)$ $y^{'} \sim N(\eta, \sigma_y)$		Correct $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = \beta_0 + \omega_i$ $y^{'} \sim N(\eta, \sigma_y)$	Zero-Inflated Poisson $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y^{'} \sim B(1, 0.7) * Pois(\eta)$		Correct $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y \sim Pois(\eta)$
			Lognormal Random Effect $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + exp(\omega_i))$ $y^{'} \sim Pois(\eta)$		Correct $\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$ $\eta_i = exp(\beta_0 + \omega_i)$ $y^{'} \sim Pois(\eta)$

Results

Linear Model

Mixed Model

Table 5: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

		Type I Error	Power		
test	method	Correct	A: Missing RE	B: Missing X	C: Misp RE
AOV	Pearson	0.045	1.000	0.000	1.000
AOV	one-step Generic	0.066	1.000	0.000	0.059
AOV	one-step Gaussian	0.066	1.000	0.000	0.059
AOV	full Gaussian	0.066	1.000	0.000	0.059
AOV	cdf	0.066	1.000	0.000	0.059
AOV	MCMC	0.036	1.000	0.000	0.054
AOV	Unconditional ecdf, Rotated	0.066	1.000	0.000	0.048
AOV	Unconditional ecdf, Not Rotated	1.000	1.000	0.964	0.991
AOV	Conditional ecdf, Rotated	0.057	1.000	0.000	1.000
AOV	Conditional ecdf, Not Rotated	0.048	1.000	0.000	1.000
EqVar	Pearson	0.027	0.027	0.000	0.027
EqVar	one-step Generic	0.018	0.850	0.000	0.027
EqVar	one-step Gaussian	0.018	0.027	0.000	0.027
EqVar	full Gaussian	0.018	0.027	0.000	0.027
EqVar	cdf	0.018	0.166	0.000	0.027
EqVar	MCMC	0.027	0.238	0.000	0.027
EqVar	Unconditional ecdf, Rotated	0.043	0.782	0.000	0.032
EqVar	Unconditional ecdf, Not Rotated	0.050	0.825	0.127	0.045
EqVar	Conditional ecdf, Rotated	0.025	0.791	0.000	0.810
EqVar	Conditional ecdf, Not Rotated	0.027	0.821	0.000	0.820
GOF.ad	Pearson	0.036	1.000	1.000	0.998
GOF.ad	one-step Generic	0.043	1.000	1.000	0.036
GOF.ad	one-step Gaussian	0.043	1.000	1.000	0.036
GOF.ad	full Gaussian	0.043	1.000	1.000	0.036
GOF.ad	cdf	0.045	1.000	1.000	0.041
GOF.ad	MCMC	0.039	1.000	1.000	0.063
GOF.ad	Unconditional ecdf, Rotated	0.088	1.000	1.000	0.088
GOF.ad	Unconditional ecdf, Not Rotated	0.946	1.000	0.914	1.000
GOF.ad	Conditional ecdf, Rotated	0.077	1.000	1.000	1.000
GOF.ad	Conditional ecdf, Not Rotated	0.039	1.000	1.000	0.998
GOF.ks	Pearson	0.045	1.000	1.000	0.995
GOF.ks	one-step Generic	0.041	1.000	1.000	0.045
GOF.ks	one-step Gaussian	0.041	1.000	1.000	0.045
GOF.ks	full Gaussian	0.041	1.000	1.000	0.045
GOF.ks	cdf	0.043	1.000	1.000	0.048
GOF.ks	MCMC	0.032	1.000	1.000	0.054
GOF.ks	Unconditional ecdf, Rotated	0.052	1.000	1.000	0.057
GOF.ks	Unconditional ecdf, Not Rotated	0.905	1.000	0.821	1.000
GOF.ks	Conditional ecdf, Rotated	0.041	1.000	1.000	0.998
GOF.ks	Conditional ecdf, Not Rotated	0.029	1.000	1.000	0.995

Table 6: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

		Type I Error	Power		
test	method	Correct	A: Missing RE	B: Missing X	C: Misp RE
AOV	Pearson	0.000	1.000	0.000	0.000
AOV	one-step Generic	0.066	1.000	0.000	0.059
AOV	one-step Gaussian	0.066	1.000	0.000	0.059
AOV	full Gaussian	0.066	1.000	0.000	0.059
AOV	cdf	0.066	1.000	0.000	0.059
AOV	MCMC	0.045	1.000	0.039	0.043
AOV	Unconditional ecdf, Rotated	0.073	1.000	0.000	0.061
AOV	Unconditional ecdf, Not Rotated	1.000	1.000	0.968	0.991
AOV	Conditional ecdf, Rotated	0.000	1.000	0.000	0.000
AOV	Conditional ecdf, Not Rotated	0.000	1.000	0.000	0.000
EqVar	Pearson	0.029	0.029	0.000	0.029
EqVar	one-step Generic	0.027	0.029	0.000	0.029
EqVar	one-step Gaussian	0.027	0.029	0.000	0.029
EqVar	full Gaussian	0.027	0.029	0.000	0.029
EqVar	cdf	0.027	0.029	0.000	0.029
EqVar	MCMC	0.029	0.029	0.000	0.029
EqVar	Unconditional ecdf, Rotated	0.039	0.202	0.000	0.034
EqVar	Unconditional ecdf, Not Rotated	0.041	0.027	0.000	0.029
EqVar	Conditional ecdf, Rotated	0.032	0.220	0.000	0.029
EqVar	Conditional ecdf, Not Rotated	0.027	0.023	0.000	0.027
GOF.ad	Pearson	0.020	0.059	0.063	0.027
GOF.ad	one-step Generic	0.050	0.079	0.093	0.041
GOF.ad	one-step Gaussian	0.039	0.050	0.088	0.041
GOF.ad	full Gaussian	0.052	0.091	0.077	0.036
GOF.ad	cdf	0.041	0.059	0.086	0.032
GOF.ad	MCMC	0.050	0.073	0.068	0.043
GOF.ad	Unconditional ecdf, Rotated	0.150	0.075	0.138	0.145
GOF.ad	Unconditional ecdf, Not Rotated	0.073	0.059	0.029	0.070
GOF.ad	Conditional ecdf, Rotated	0.116	0.095	0.063	0.079
GOF.ad	Conditional ecdf, Not Rotated	0.079	0.066	0.043	0.039
GOF.ks	Pearson	0.000	0.449	0.159	0.000
GOF.ks	one-step Generic	0.032	0.449	0.324	0.016
GOF.ks	one-step Gaussian	0.032	0.449	0.324	0.016
GOF.ks	full Gaussian	0.032	0.449	0.324	0.016
GOF.ks	cdf	0.032	0.449	0.324	0.016
GOF.ks	MCMC	0.039	0.449	0.186	0.034
GOF.ks	Unconditional ecdf, Rotated	0.034	0.372	0.315	0.016
GOF.ks	Unconditional ecdf, Not Rotated	0.456	0.460	0.002	0.163
GOF.ks	Conditional ecdf, Rotated	0.002	0.381	0.093	0.002
GOF.ks	Conditional ecdf, Not Rotated	0.000	0.442	0.159	0.000

Randomwalk

Table 7: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

		Type I Error	Power			
test	method	Correct	A: Missing RE	B: Heterosced.	D: Misp RE	NA
Auto	Pearson	0.050	1.000	0.000	1.000	1.000
Auto	one-step Generic	0.052	1.000	0.000	0.741	0.107
Auto	one-step Gaussian	0.052	1.000	0.000	0.955	0.100
Auto	full Gaussian	0.052	1.000	0.000	0.955	0.100
Auto	cdf	0.050	1.000	0.000	0.435	0.100
Auto	MCMC	0.041	1.000	0.000	0.054	0.073
Auto	Unconditional ecdf, Rotated	0.048	0.991	0.000	0.286	0.150
Auto	Unconditional ecdf, Not Rotated	1.000	0.984	0.463	0.955	1.000
Auto	Conditional ecdf, Rotated	0.070	0.991	0.000	0.989	0.986
Auto	Conditional ecdf, Not Rotated	0.054	0.989	0.000	0.991	0.989
GOF.ad	Pearson	0.061	1.000	1.000	1.000	1.000
GOF.ad	one-step Generic	0.041	1.000	1.000	1.000	1.000
GOF.ad	one-step Gaussian	0.041	1.000	1.000	1.000	1.000
GOF.ad	full Gaussian	0.041	1.000	1.000	1.000	1.000
GOF.ad	cdf	0.043	1.000	1.000	1.000	1.000
GOF.ad	MCMC	0.045	1.000	1.000	1.000	0.048
GOF.ad	Unconditional ecdf, Rotated	0.088	1.000	1.000	1.000	1.000
GOF.ad	Unconditional ecdf, Not Rotated	0.993	1.000	0.814	1.000	1.000
GOF.ad	Conditional ecdf, Rotated	0.095	1.000	1.000	1.000	1.000
GOF.ad	Conditional ecdf, Not Rotated	0.073	1.000	1.000	1.000	1.000
GOF.ks	Pearson	0.059	1.000	1.000	1.000	1.000
GOF.ks	one-step Generic	0.036	1.000	1.000	1.000	1.000
GOF.ks	one-step Gaussian	0.036	1.000	1.000	1.000	1.000
GOF.ks	full Gaussian	0.036	1.000	1.000	1.000	1.000
GOF.ks	cdf	0.034	1.000	1.000	1.000	1.000
GOF.ks	MCMC	0.039	1.000	1.000	1.000	0.039
GOF.ks	Unconditional ecdf, Rotated	0.045	1.000	1.000	1.000	1.000
GOF.ks	Unconditional ecdf, Not Rotated	0.989	1.000	0.764	1.000	1.000
GOF.ks	Conditional ecdf, Rotated	0.061	1.000	1.000	1.000	1.000
GOF.ks	Conditional ecdf, Not Rotated	0.057	1.000	1.000	1.000	1.000

Table 8: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

		Type I Error	Power			
test	method	Correct	A: Missing RE	B: Heterosced.	D: Misp RE	NA
Auto	Pearson	0.000	1	0.000	0.000	0.000
Auto	one-step Generic	0.002	1	0.000	0.023	0.000
Auto	one-step Gaussian	0.002	1	0.000	0.028	0.000
Auto	full Gaussian	0.002	1	0.000	0.037	0.000
Auto	cdf	0.002	1	0.000	0.035	0.002
Auto	MCMC	0.026	1	0.392	0.044	0.060
Auto	Unconditional ecdf, Rotated	0.002	1	0.000	0.014	0.000
Auto	Unconditional ecdf, Not Rotated	1.000	1	0.173	1.000	1.000
Auto	Conditional ecdf, Rotated	0.000	1	0.003	0.044	0.039
Auto	Conditional ecdf, Not Rotated	0.000	1	0.000	0.053	0.065
GOF.ad	Pearson	0.002	1	0.027	0.000	0.000
GOF.ad	one-step Generic	0.026	1	0.008	0.039	0.995
GOF.ad	one-step Gaussian	0.035	1	0.021	0.118	0.988
GOF.ad	full Gaussian	0.030	1	0.011	0.143	0.993
GOF.ad	cdf	0.028	1	0.016	0.845	1.000
GOF.ad	MCMC	0.039	1	0.451	0.044	0.039
GOF.ad	Unconditional ecdf, Rotated	0.111	1	0.445	0.545	1.000
GOF.ad	Unconditional ecdf, Not Rotated	0.290	1	0.381	0.295	1.000
GOF.ad	Conditional ecdf, Rotated	0.009	1	0.309	0.000	0.000
GOF.ad	Conditional ecdf, Not Rotated	0.005	1	0.203	0.000	0.000
GOF.ks	Pearson	0.135	1	0.000	1.000	1.000
GOF.ks	one-step Generic	0.000	1	0.016	0.979	1.000
GOF.ks	one-step Gaussian	0.000	1	0.016	0.979	1.000
GOF.ks	full Gaussian	0.000	1	0.017	0.984	1.000
GOF.ks	cdf	0.000	1	0.016	1.000	1.000
GOF.ks	MCMC	0.060	1	0.472	0.037	0.053
GOF.ks	Unconditional ecdf, Rotated	0.000	1	0.283	0.882	1.000
GOF.ks	Unconditional ecdf, Not Rotated	0.998	1	0.773	1.000	1.000
GOF.ks	Conditional ecdf, Rotated	0.081	1	0.000	1.000	1.000
GOF.ks	Conditional ecdf, Not Rotated	0.128	1	0.000	1.000	1.000

Spatial

Table 9: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

		Type I Error	Power		
test	method	Correct	A: Missing RE	B: Lognorm error	C: Misp RE
GOF.ad	Pearson	0.052	0.909	1.000	1.000
GOF.ad	one-step Generic	0.034	0.909	1.000	0.939
GOF.ad	one-step Gaussian	0.034	0.909	1.000	0.957
GOF.ad	full Gaussian	0.034	0.909	1.000	0.957
GOF.ad	cdf	0.034	0.909	1.000	0.955
GOF.ad	MCMC	0.057	0.909	1.000	0.358
GOF.ad	Unconditional ecdf, Rotated	0.050	0.968	1.000	0.973
GOF.ad	Unconditional ecdf, Not Rotated	0.490	0.925	1.000	0.998
GOF.ad	Conditional ecdf, Rotated	0.104	0.975	1.000	1.000
GOF.ad	Conditional ecdf, Not Rotated	0.059	0.932	1.000	1.000
GOF.ks	Pearson	0.050	0.753	1.000	1.000
GOF.ks	one-step Generic	0.048	0.753	1.000	0.714
GOF.ks	one-step Gaussian	0.048	0.753	1.000	0.714
GOF.ks	full Gaussian	0.048	0.753	1.000	0.714
GOF.ks	cdf	0.048	0.753	1.000	0.719
GOF.ks	MCMC	0.061	0.753	1.000	0.166
GOF.ks	Unconditional ecdf, Rotated	0.041	0.794	1.000	0.764
GOF.ks	Unconditional ecdf, Not Rotated	0.442	0.739	1.000	0.998
GOF.ks	Conditional ecdf, Rotated	0.066	0.816	1.000	1.000
GOF.ks	Conditional ecdf, Not Rotated	0.045	0.748	1.000	1.000
SAC	Pearson	0.061	0.932	0.066	0.608
SAC	one-step Generic	0.063	0.932	0.005	0.213
SAC	one-step Gaussian	0.063	0.932	0.002	0.272
SAC	full Gaussian	0.063	0.932	0.002	0.272
SAC	cdf	0.063	0.932	NA	0.218
SAC	MCMC	0.073	0.932	0.000	0.098
SAC	Unconditional ecdf, Rotated	0.052	0.912	0.007	0.063
SAC	Unconditional ecdf, Not Rotated	0.934	0.912	0.175	0.880
SAC	Conditional ecdf, Rotated	0.066	0.909	0.070	0.163
SAC	Conditional ecdf, Not Rotated	0.059	0.912	0.068	0.204

Table 10: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

		Type I Error	Power		
test	method	Correct	A: Missing RE	B: Lognorm error	C: Misp RE
GOF.ad	Pearson	0.005	0.030	0.053	0.009
GOF.ad	one-step Generic	0.048	0.025	NA	NA
GOF.ad	one-step Gaussian	0.030	0.032	0.085	0.052
GOF.ad	full Gaussian	0.025	0.021	0.124	0.047
GOF.ad	cdf	0.045	0.025	0.108	0.104
GOF.ad	MCMC	0.043	0.018	0.083	0.047
GOF.ad	Unconditional ecdf, Rotated	0.149	0.115	0.702	0.325
GOF.ad	Unconditional ecdf, Not Rotated	0.060	0.078	0.743	0.295
GOF.ad	Conditional ecdf, Rotated	0.015	0.131	0.681	0.092
GOF.ad	Conditional ecdf, Not Rotated	0.020	0.064	0.729	0.099
GOF.ks	Pearson	0.113	0.000	1.000	0.587
GOF.ks	one-step Generic	0.008	0.000	NA	NA
GOF.ks	one-step Gaussian	0.008	0.000	1.000	0.250
GOF.ks	full Gaussian	0.008	0.000	1.000	0.304
GOF.ks	cdf	0.023	0.000	1.000	0.283
GOF.ks	MCMC	0.035	0.000	1.000	0.153
GOF.ks	Unconditional ecdf, Rotated	0.000	0.000	0.993	0.163
GOF.ks	Unconditional ecdf, Not Rotated	0.045	0.000	1.000	0.427
GOF.ks	Conditional ecdf, Rotated	0.103	0.000	0.995	0.531
GOF.ks	Conditional ecdf, Not Rotated	0.108	0.000	1.000	0.592
SAC	Pearson	0.025	0.931	0.124	0.160
SAC	one-step Generic	0.053	0.931	NA	NA
SAC	one-step Gaussian	0.053	0.931	0.119	0.231
SAC	full Gaussian	0.053	0.931	0.102	0.211
SAC	cdf	0.053	0.931	0.115	0.210
SAC	MCMC	0.050	0.931	0.124	0.186
SAC	Unconditional ecdf, Rotated	0.050	0.915	0.147	0.193
SAC	Unconditional ecdf, Not Rotated	0.945	0.927	0.140	0.932
SAC	Conditional ecdf, Rotated	0.025	0.904	0.138	0.158
SAC	Conditional ecdf, Not Rotated	0.023	0.931	0.140	0.160

Table 11: Overview of issues and recommendations for common classes of models. Correlation and distributions refer to predicted data from a fitted model, against which observed points are compared. A linear rotation refers to a multiplication of the simulated and observed data by a Cholesky decomposition of the estimated covariance matrix of the observed data, z'=Lz, as available in DHARMa.

Model class	Case studies	Issues and causes	Recommendation
Linear model	Linear model	No issues	Pearson residuals
Generalized linear model (GLM)	Skewed Gamma	Non-normality resulting from response variable. Quantile residuals are needed if not approximately normal.	Quantile residual
Linear mixed model (LMM), Multivariate model	Random walk, Spatial LMM, Multinomial	Linear correlations caused by non-independence in observations.	Use a method that linearly decorrelates in order to transform to a unit iid normal. OSA Full Gaussian, OSA one-step Gaussian, or simulation residuals with rotation.
Generalized linear mixed model (GLMM)	Spatial Poisson, Repeated measures Tweedie	Non-normality and non-linear correlations caused by response variable and non-independence in observations.	Needs non-linear decorrelation and quantiles. Needs non-linear decorrelation and quantiles. Best approach depends on case study and sample size.