# Table Placeholders

# Methods

Table 1: Linear Model Simulation: data generating models, parameter values, and mis-specifications.

	Data Generating Model	Parameters	Data Fitting Model
Correct	$X_i \sim N(0, 1)$ $\mu_{i,j} = X_i \beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\beta = (4, -5)$ $\sigma_y = 1$	$X_i \sim N(0, 1)$ $\mu_{i,j} = X_i \beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$
Mis-specified	$X_{i} \sim N(0, 1)$ $\mu_{i,j} = X_{i}\beta$ $y'_{i,j} \sim N(\mu_{i,j}, exp(\sigma_{y}))$		$X_i \sim N(0, 1)$ $\mu_{i,j} = X_i \beta$ $y'_{i,j} \sim N(\mu_{i,j}, \sigma_y)$

Data Generating Model	Parameters Data Fitting Model	Data Generating Model	Parameters	Data Fitting Model
Linear Mixed Model Correct		Generalized Linear Mixed Model Correct		
$X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$	$\beta = (4, -8)$ $X_i \sim Unif(-0.5, 0.5)$ $\alpha = -3$ $u_j \sim N(0, \sigma_u)$	$u_j \sim N(0, \sigma_u)$ $\dots - c_{xm}(\beta + u_j)$	$\beta = log(2)$ $size - 1$	$u_j \sim N(0, \sigma_u)$ $\dots = \rho_{mn}(\beta + n_v)$
$\mu_{i,j} = X_i \beta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\sigma_y = 0.5$ $\mu_{i,j} = X_i \beta + u_j$ $\sigma_y = 0.5$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$y_{i,j} \sim NBinom(\mu_{i,j}, size)$	$\sigma_u = 1$	$y_{i,j} \sim NBinom(\mu_{i,j}, size)$
Mis-specified		Mis-specified		
1	Missing Random Effect $X_i \sim Unif(-0.5, 0.5)$			Missing Random Effect $\mu_{i,j} = exp(\beta)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$
$Correct \ X_i \sim Unif(-0.5, 0.5) \ \dots \sim N(0, \sigma)$	$\mu_{i,j} = X_i eta \ y_{i,j} \sim N(\mu_{i,j},\sigma_y)$	Correct $u_j \sim N(0, \sigma_u)$		Mis-specified Distribution
$u_j = X(0, \sigma_u)$ $\mu_{i,j} = X_i eta + u_j$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	Missing Covariate $u_j \sim N(0, \sigma_u)$	$\mu_{i,j} = exp(\beta + u_j)$ $y_{i,j} \sim NBinom(\mu_{i,j}, size)$		(Poisson) $u_j \sim N(0, \sigma_u)$
	$\mu_{i,j} = eta + u_j \ y_{i,j} \sim N(\mu_{i,j},\sigma_y)$			$\mu_{i,j} - exp(eta + a_j)$ $y_{i,j} \sim Poisson(\mu_{i,j})$
Lognormal Random Effect	Correct	Gamma		Correct
$X_i \sim Unif(-0.5, 0.5)$ $u_j \sim N(0, \sigma_u)$	$X_i \sim Unif(-0.5, 0.5)$ $\mu_{i,j} = X_i\beta + u_j$	Random Effect $u_j \sim Gamma(1,1)$ , — $com(\beta + n.)$		$u_j \sim N(0,\sigma_u) \ \mu_{i,j} = exp(eta + u_j)$
$egin{aligned} \mu'_{i,j} &= X_i eta + e^{u_j} \ y'_i &\stackrel{\cdot}{\sim} N(\mu'_i \ i, \sigma_n) \end{aligned}$	$y_{i,j}^{\prime} \sim N(\mu_{i,j},\sigma_y)$	$egin{aligned} \mu_{i,j} &= exp(eta + u_j) \ y_{i,j}^{'} &\sim NBinom(\mu_{i,j}, size) \end{aligned}$		$y_{i,j} \sim NBinom(\mu_{i,j}, size)$

$a = 2$ $u[1] = 0$ $a_{u} = 1$ $a_{v} = 1$ $a_{v} = N(\mu_{i}, \sigma_{u})$ $\sigma_{y} = 1$ $d_{i} = N(u_{i}, \sigma_{y})$ $d_{i} = 1$ $d_{i} = N(u_{i}, \sigma_{y})$ $d_{i} = 1$ $d_{$	Data Generating Model Linear Mixed Model	Parameters	Data Fitting Model	Model Parameters Data Fitting Model   Data Generating Model   Parameters Data   Generalized Linear Mixed Model	Parameters	Data Fitting Model
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Correct			Correct		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	n := n : 1 + a	a = 2	$n : n : n \to \infty$	$\mu_i = u_{i-1} + a$	a = .01	$\mu_i = u_{i-1} + a$
$(x_{ij}) \qquad \sigma_{ij} = 1 \qquad y_{ij} \sim N(u_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2}) \qquad \sigma_{ij} = 0.05  y_{ij} \sim N(u_{ij}, \sigma_{ij}) \qquad \text{Missing}$ $(x_{ij}, \sigma_{ij}) \qquad \text{Missing Drift Term}$ $(x_{ij}, \sigma_{ij}) \qquad \text{Missing Drift Term}$ $(x_{ij}, \sigma_{ij}) \qquad y_{ij} \sim N(u_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2}) \qquad N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2}) \qquad N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2}) \qquad N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(0.5, 20) \qquad y_{ij} \sim N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(0.5, 20) \qquad y_{ij} \sim N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(0.5, 20) \qquad y_{ij} \sim N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(0.5, 20) \qquad y_{ij} \sim N(y_{ij}, \sigma_{ij}) \qquad y_{ij} \sim Gamma(0.5, 20) \qquad y_{ij} \sim Samma(0.5, 20) \qquad y_{ij$	$\sum_{n=\infty}^{\infty} N(n,\sigma)$	u[1] = 0	$p_i - w_{i-1} - w$	$u_i \sim N(\mu_i, \sigma_u)$	u[1] = 0	$u_i \sim N(\mu_i, \sigma_u)$
Missing  Missing  Random Effect $y_i \sim N(a(1:n), \sigma_y)$ $u_i \sim N(a(1:n), \sigma_y)$ Missing Drift Term $u_i \sim N(u_i, \sigma_y)$ $u_i \sim N(u_i, \sigma_y)$ Missing Drift Term $u_i \sim N(u_i, \sigma_y)$	$y_i \sim N(u_i, \sigma_y)$	$\sigma_u = 1$	$u_i \sim N(u_i, \sigma_u)$	$y_i \sim Gamma(\frac{1}{CVV}, e^{u_i}CV^2)$	$\sigma_u = 0.05$ $CV = 0.5$	$y_i \sim Gamma(\frac{1}{CV}^2, e^{u_i}CV^2)$
Missing Bandom Effect $y_i \sim N(a(1:n), \sigma_y)$ Rissing Drift Term $y_i \sim N(a(1:n), \sigma_y)$ Missing Drift Term $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim Correct$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i = u_{i-1} + a$ $y_i \sim Correct$ $y_i \sim N(\mu_i, \sigma_y)$ $y_i \sim Correct$ $y_i \sim Corr$		cy		2		2
Missing Random Effect $y_i \sim N(a(1:n), \sigma_y)$ Missing Drift Term $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim N(u_i, \sigma_y)$ $u_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Camma (0.5, 20)$ $y_i \sim N(\mu_i, \sigma_u)$ $y_i \sim Camma (0.5, 20)$	Mis-specified			Mis-specified		
Random Effect $y_i \sim N(a(1:n), \sigma_y)$ Missing Drift Term $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_y)$ $u_i = u_{i-1} + a$ Random Effect $u_i = u_{i-1} + a$ Random Effect $u_i = u_{i-1} + a$			Missing			Mising Dendem Effect
$y_{i} \sim N(a(1:n), \sigma_{y})$ $\text{Missing Drift Term} \qquad \text{Correct}$ $\mu_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{y})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim N(\mu_{i}, \sigma_{u})$ $u_{i} = u_{i-1} + a$ $u_{i} = u_{i-1} $			Random Effect			$\frac{1}{1}$
Missing Drift Term $ \begin{aligned} \mu_i &= u_{i-1} & \mu \\ \mu_i &= u_{i-1} + a \\ u_i &\sim N(\mu_i, \sigma_u) \\ y_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{aligned} \mu_i &= u_{i-1} + a \\ u_i &\sim N(\mu_i, \sigma_u) \\ y_i &\sim N(\mu_i, \sigma_u) \\ y_i' &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{aligned} & \text{Correct} \\ \mu_i &= u_{i-1} + a \\ u_i &\sim N(\mu_i, \sigma_y) \\ y_i' &\sim N(\mu_i, \sigma_y) \end{aligned} \qquad \begin{aligned} & \text{Gamma} \\ & \text{Gamma} \\ & u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{aligned} & u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{vmatrix} u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &\sim N(u_i, \sigma_y) \\ & u_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{vmatrix} u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{vmatrix} u_i &= u_{i-1} + Gamma(0.5, 20) \\ & u_i &= u_{i-1} + Gamma(0.$	Correct		$y_i \sim N(a(1:n), \sigma_y)$			$y_i \sim Gamma(\frac{1}{CV}, e^aCV^2)$
Missing Drift Term $\mu_{i} = u_{i-1} + a$ $\mu_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(\mu_{i}, \sigma_{u})$ $u_{i} = u_{i-1} + Gamma(0.5, 20)$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$	$\mu_i = u_{i-1} + a$					
$\begin{array}{ll} \mu_i = u_{i-1} & \mu_i = u_{i-1} + a \\ u_i \sim N(\mu_i, \sigma_u) & u_i \sim N(\mu_i, \sigma_u) \\ y_i \sim N(u_i, \sigma_y) & y_i \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2) \\ \\ \mu_i = u_{i-1} + a \\ u_i \sim N(\mu_i, \sigma_u) \\ y_i' \sim N(\mu_i, \sigma_u) & u_i = u_{i-1} + Gamma(0.5, 20) \\ \\ \mu_i = u_{i-1} + a \\ u_i \sim N(\mu_i, \sigma_u) & u_i = u_{i-1} + Gamma(0.5, 20) \\ y_i' \sim N(u_i, \sigma_y) & y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2) \\ \\ y_i' \sim N(u_i, \sigma_y) & y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2) \\ \end{array}$	$u_i \sim N(\mu_i, \sigma_u)$		Missing Drift Term	Correct		Missing Drift Term
$u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim Ramma(\frac{1}{CV}, e^{u_{i}}CV^{2})$ $N$ $Correct$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y'_{i} \sim N(u_{i}, \sigma_{y})$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $u_{i} = u_{i-1} + a$ $u_{i} = u_{i-1} + Gamma(0.5, 20)$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y'_{i} \sim Gamma(0.5, 20)$ $y'_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$ $y'_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$	$y_i \sim N(u_i, \sigma_y)$		$\mu_i = u_{i-1}$	$\mu_i = u_{i-1} + a$		$\mu_i = u_{i-1}$
$y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$ $N$ $Correct$ $u_{i} \sim N(u_{i}, \sigma_{u})$ $y'_{i} \sim N(u_{i}, \sigma_{u})$ $u_{i} = u_{i-1} + a$ $u_{i} = u_{i-1} + Gamma(0.5, 20)$ $u_{i} \sim N(u_{i}, \sigma_{u})$ $y'_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$ $y'_{i} \sim Ramma(0.5, 20)$ $y'_{i} \sim Ramma(\frac{1}{CV}, e^{u_{i}}CV^{2})$			$u_i \sim N(\mu_i, \sigma_u)$	$u_i \sim N(\mu_i, \sigma_u)$		$u_i \sim N(\mu_i, \sigma_u)$
Correct $\mu_i = u_{i-1} + a$ $u_i \sim N(\mu_i, \sigma_u)$ $y'_i \sim N(u_i, \sigma_y)$ $u_i = u_{i-1} + a$ $u_i = u_{i-1} +$			$y_i \sim N(u_i, \sigma_y)$	$\frac{1}{2} \frac{1}{n!} \frac{2}{2n!} \frac{1}{2n!}$		$\frac{1}{m} \approx Gamma(\frac{1}{2} \frac{2}{a^{n_i}CV^2})$
Correct $\mu_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y'_{i} \sim N(u_{i}, \sigma_{y})$ $\psi_{i} = u_{i-1} + a$ $u_{i} = u_{i-1} + Gamma(0.5, 20)$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y'_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$				$y_i \sim Gamma(\frac{CV}{CV}, e^{-iCV^2})$		$g_i \sim Gamma \left( \frac{CV}{CV} \right)$
Correct $\mu_i$ $\mu_i = u_{i-1} + a$ $\mu_i = u_{i-1} + a$ $\mu_i = v_{i-1} + a$ $\mu_i = u_{i-1} + a$ $\mu_i = u_{i$	Heteroscedasticity					
$\mu_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $h_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$	$\mu_i = u_{i-1} + a$		Correct			Mis-specified Distribution
$u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $Correct$ $\mu_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim Ramma(0.5, 20)$ $u_{i} = u_{i-1} + Gamma(0.5, 20)$ $y_{i} \sim Gamma(\frac{1}{CV}, e^{u_{i}}CV^{2})$	$u_i \sim N(\mu_i, \sigma_u)$		$\mu_i = u_{i-1} + a$			INOLITICAL
$y_{i} \sim N(u_{i}, \sigma_{y})$ $Correct$ $\mu_{i} = u_{i-1} + a$ $u_{i} \sim N(\mu_{i}, \sigma_{u})$ $y_{i} \sim N(u_{i}, \sigma_{y})$ $y_{i} \sim Gamma(0.5, 20)$ $y_{i} \sim Gamma(0.5, 20)$ $y_{i} \sim Gamma(0.5, 20)$	$\sqrt{1.3}$		$u_i \sim N(\mu_i, \sigma_u)$			$\mu_i = u_{i-1} + a$
Correct $\mu_i = u_{i-1} + a \\ u_i \sim N(\mu_i, \sigma_u) \\ y)$ $y_i' \sim N(u_i, \sigma_y)$ $y_i' \sim Gamma(0.5, 20) \\ y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$	$\sigma_y = \sqrt{(1:rac{2}{2})}$		$y_i^{'} \sim N(u_i, \sigma_u)$			$u_i \sim N(\mu_i, \sigma_u)$
Correct Random Effect Random Effect $\mu_i = u_{i-1} + a$ $u_i = u_{i-1} + Gamma(0.5, 20)$ $y_i' \sim N(u_i, \sigma_y)$ $y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$	$y_i^{'} \sim N(u_i,\sigma_y)$					$y_i \sim N(u_i, \sigma_y)$
Correct Random Effect Random Effect $\mu_i = u_{i-1} + a$ $u_i = u_{i-1} + Gamma(0.5, 20)$ $i, i, i, i, j$ $i, i, i, i, j$ $i, i, i, j, j$ $i, i, i, j, j$ $i, i, j, j, j$ $i, i, j, j, j$ $i, i, j, j,$					ı	
Random Effect $ \begin{aligned} \mu_i &= u_{i-1} + a \\ u_i &\sim N(\mu_i, \sigma_u) \\ y) \\ y'_i &\sim N(u_i, \sigma_y) \end{aligned} \qquad \begin{aligned} u_i &= u_{i-1} + Gamma(0.5, 20) \\ u_i &= u_{i-1} + Gamma(0.5, 20) \\ y'_i &\sim Gamma(\frac{1}{CV}^2) \end{aligned}$	Lognormal		Correct	Gamma		Correct
$u_i \sim N(\mu_i, \sigma_u)$ $u_i = u_{i-1} + Gamma(0.5, 20)$ $y_i' \sim N(u_i, \sigma_y)$ $y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$	Random Effect		$\mu_i = u_{i-1} + a$	Random Effect		$\mu_i = u_{i-1} + a$
$y_i' \sim N(u_i, \sigma_y)$ $y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$	$\mu_i = u_{i-1} + u \ u_i \sim N(u_i,\sigma_u)$		$u_i \sim N(\mu_i, \sigma_u)$	$u_i = u_{i-1} + Gamma(0.5, 20)$		$u_i \sim N(\mu_i, \sigma_u)$
	$y_i^{'} \sim N(e^{u_i},\sigma_u)$		$y_i^{'} \sim N(u_i, \sigma_y)$	$y_i' \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$		$y_i \sim Gamma(\frac{1}{CV}, e^{u_i}CV^2)$

Table 4:	: Spatial Model	Simulation: data generating	Table 4: Spatial Model Simulation: data generating models, parameter values, and mis-specifications	mis-specification	1S.
Data Generating Model	Parameters	Data Fitting Model	Data Generating Model	Parameters	Data Fitting Model
Linear Mixed Model			Generalized Linear Mixed Model	del	
Correct	$\beta_0 = 4$		Correct	1	
$\omega \sim GMRF(Q[\kappa,\sigma_{\omega}^2])$	$\sigma_y^2 = 1$	$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$	$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$	$eta = 0.5$ $\phi = 50$	$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$
$\eta_i = \beta_0 + \omega_i$	$\phi = 50$	$\eta_i = \beta_0 + \omega_i$	$\eta_i = exp(\beta_0 + \omega_i)$		$\eta_i = exp(\beta_0 + \omega_i)$
$y \sim N(\eta, \sigma_y)$	$\kappa = \sqrt{8}/\phi$	$y \sim N(\eta, \sigma_y)$	$y \sim Pois(\eta, \sigma_y)$	$\sigma_{\omega}^2 = 0.25$	$y \sim Pois(\eta, \sigma_y)$
	$\sigma_\omega^- = 1$				
Mis-specified			Mis-specified		
		Missing			
		Random Effect			
		$\eta_i = \beta_0$			
Correct		$y \sim N(\eta, \sigma_y)$	Correct		Missing
$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$			$\omega \sim GMRF(Q[\kappa,\sigma_{\omega}^2])$		Random Effect
$\eta_i = eta_0 + \omega_i$		Lognormal Error	$\eta_i = exp(\beta_0 + \omega_i)$		$\eta_i = exp(\beta_0)$
$y \sim N(\eta, \sigma_y)$		$\omega \sim GMRF(Q[\kappa,\sigma_{\omega}^2])$	$y \sim Pois(\eta)$		$y \sim Pois(\eta)$
		$\eta_i = \beta_0 + \omega_i$			
		$\sigma_y = exp(N(0,1))$			
		$y \sim W(\eta, \sigma_y)$			
Lognormal		Correct	Zero-Inflated		Correct
Kandom Effect $\omega \sim GMRF(O[\kappa,\sigma^2])$		$\omega \sim GMRF(Q[\kappa,\sigma_{\omega}^2])$	Formoup $\omega \sim GMRF(O[\kappa, \sigma^2])$		$\omega \sim GMRF(Q[\kappa,\sigma_{\omega}^2])$
$\eta_i = eta_0 + exp(\omega_i)$			$\eta_i = exp(\beta_0 + \omega_i)$		$\eta_i = exp(\beta_0 + \omega_i)$
$y^{'} \sim N(\eta, \sigma_y)$		$y \sim N(\eta, \sigma_y)$	$y^{'} \sim B(1,0.7) * Pois(\eta)$		$y \sim Pois(\eta)$
				·	
			Lognormal		Correct
			Random Effect		$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$
			$\omega \sim GMRF(Q[\kappa, \sigma_{\omega}^2])$		$\eta_i = exp(\beta_0 + \omega_i)$
			$\eta_i = exp(\beta_0 + exp(\omega_i))$		$y^{'} \sim Pois(\eta)$
			$y^{'} \sim Pois(\eta)$		

## Results

#### Linear Model

Table 5: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by residual type (top to bottom).

	Overdisper	sion
method	Type I Error	Power
Pearson	0.045	1
one-step Generic	0.045	1
one-step Gaussian	0.045	1
full Gaussian	0.045	1
$\operatorname{cdf}$	0.045	1
Unconditional ecdf, Not Rotated	0.034	1
Conditional ecdf, Not Rotated	0.034	1

Table 6: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by residual type (top to bottom).

	Overdisper	sion
method	Type I Error	Power
Pearson	0	0.959
one-step Generic	0	0.959
one-step Gaussian	0	0.959
full Gaussian	0	0.959
$\operatorname{cdf}$	0	0.959
Unconditional ecdf, Not Rotated	0	0.961
Conditional ecdf, Not Rotated	0	0.961

#### Mixed Model

Table 7: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Missing	g X	C: Misp 1	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.045	1	0.045	1.000	0.045	0.995
$_{ m LMM}$	one-step Generic	0.041	1	0.041	1.000	0.041	0.045
$_{ m LMM}$	one-step Gaussian	0.041	1	0.041	1.000	0.041	0.045
$_{ m LMM}$	full Gaussian	0.041	1	0.041	1.000	0.041	0.045
LMM	$\operatorname{cdf}$	0.043	1	0.043	1.000	0.043	0.048
LMM	MCMC	0.032	1	0.032	1.000	0.032	0.054
$_{ m LMM}$	Unconditional ecdf, Rotated	0.052	1	0.052	1.000	0.052	0.057
$_{ m LMM}$	Unconditional ecdf, Not Rotated	0.905	1	0.905	0.821	0.905	1.000
LMM	Conditional ecdf, Rotated	0.041	1	0.041	1.000	0.041	0.998
LMM	Conditional ecdf, Not Rotated	0.029	1	0.029	1.000	0.029	0.995

Table 8: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Missing	g X	C: Misp 1	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.000	0.449	0.000	0.159	0.000	0.000
LMM	one-step Generic	0.032	0.449	0.032	0.324	0.032	0.016
LMM	one-step Gaussian	0.032	0.449	0.032	0.324	0.032	0.016
LMM	full Gaussian	0.032	0.449	0.032	0.324	0.032	0.016
LMM	$\operatorname{cdf}$	0.032	0.449	0.032	0.324	0.032	0.016
LMM	MCMC	0.039	0.449	0.039	0.186	0.039	0.034
$_{\rm LMM}$	Unconditional ecdf, Rotated	0.034	0.372	0.034	0.315	0.034	0.016
$_{\rm LMM}$	Unconditional ecdf, Not Rotated	0.456	0.460	0.456	0.002	0.456	0.163
LMM	Conditional ecdf, Rotated	0.002	0.381	0.002	0.093	0.002	0.002
LMM	Conditional ecdf, Not Rotated	0.000	0.442	0.000	0.159	0.000	0.000

#### Randomwalk

Table 9: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Heteros	ced.	D: Misp l	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.059	1	0.059	1.000	0.059	1
LMM	one-step Generic	0.036	1	0.036	1.000	0.036	1
LMM	one-step Gaussian	0.036	1	0.036	1.000	0.036	1
LMM	full Gaussian	0.036	1	0.036	1.000	0.036	1
LMM	$\operatorname{cdf}$	0.034	1	0.034	1.000	0.034	1
LMM	MCMC	0.039	1	0.039	1.000	0.039	1
LMM	Unconditional ecdf, Rotated	0.045	1	0.045	1.000	0.045	1
$_{\rm LMM}$	Unconditional ecdf, Not Rotated	0.989	1	0.989	0.764	0.989	1
LMM	Conditional ecdf, Rotated	0.061	1	0.061	1.000	0.061	1
LMM	Conditional ecdf, Not Rotated	0.057	1	0.057	1.000	0.057	1

Table 10: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Heteros	sced.	D: Misp l	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.135	1	0.135	0.000	0.135	1.000
LMM	one-step Generic	0.000	1	0.000	0.016	0.000	0.979
$_{\rm LMM}$	one-step Gaussian	0.000	1	0.000	0.016	0.000	0.979
LMM	full Gaussian	0.000	1	0.000	0.017	0.000	0.984
LMM	$\operatorname{cdf}$	0.000	1	0.000	0.016	0.000	1.000
LMM	MCMC	0.060	1	0.060	0.472	0.060	0.037
LMM	Unconditional ecdf, Rotated	0.000	1	0.000	0.283	0.000	0.882
$_{\rm LMM}$	Unconditional ecdf, Not Rotated	0.998	1	0.998	0.773	0.998	1.000
LMM	Conditional ecdf, Rotated	0.081	1	0.081	0.000	0.081	1.000
LMM	Conditional ecdf, Not Rotated	0.128	1	0.128	0.000	0.128	1.000

### Spatial

Table 11: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Lognorm	error	C: Misp ]	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.050	0.755	0.050	1	0.050	1.000
$_{\rm LMM}$	one-step Generic	0.011	0.755	0.011	1	0.011	0.388
$_{ m LMM}$	one-step Gaussian	0.011	0.755	0.011	1	0.011	0.388
$_{\rm LMM}$	full Gaussian	0.011	0.755	0.011	1	0.011	0.388
LMM	$\operatorname{cdf}$	0.011	0.755	0.011	1	0.011	0.390
LMM	MCMC	0.036	0.755	0.036	1	0.036	0.095
$_{\rm LMM}$	Unconditional ecdf, Rotated	0.014	0.789	0.014	1	0.014	0.438
$_{\rm LMM}$	Unconditional ecdf, Not Rotated	0.560	0.751	0.560	1	0.560	0.995
LMM	Conditional ecdf, Rotated	0.066	0.800	0.066	1	0.066	1.000
LMM	Conditional ecdf, Not Rotated	0.045	0.762	0.045	1	0.045	1.000

Table 12: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

	A: Missing RE		B: Lognorm	error	C: Misp 1	RE	
method	Type I Error	Power	Type I Error	Power	Type I Error	Power	NA
LMM	Pearson	0.082	0	0.082	1.000	0.082	0.360
$_{ m LMM}$	one-step Generic	0.005	0	NA	NA	NA	NA
$_{ m LMM}$	one-step Gaussian	0.005	0	0.005	1.000	0.005	0.167
$_{ m LMM}$	full Gaussian	0.005	0	0.005	1.000	0.005	0.213
LMM	$\operatorname{cdf}$	0.027	0	0.027	1.000	0.027	0.200
LMM	MCMC	0.039	0	0.039	1.000	0.039	0.155
$_{ m LMM}$	Unconditional ecdf, Rotated	0.007	0	0.007	0.991	0.007	0.116
$_{ m LMM}$	Unconditional ecdf, Not Rotated	0.139	0	0.139	1.000	0.139	0.360
$_{ m LMM}$	Conditional ecdf, Rotated	0.078	0	0.078	0.998	0.078	0.292
LMM	Conditional ecdf, Not Rotated	0.084	0	0.084	1.000	0.084	0.364

Table 13: Overview of issues and recommendations for common classes of models. Correlation and distributions refer to predicted data from a fitted model, against which observed points are compared. A linear rotation refers to a multiplication of the simulated and observed data by a Cholesky decomposition of the estimated covariance matrix of the observed data, z'=Lz, as available in DHARMa.

Model class	Case studies	Issues and causes	Recommendation
Linear model	Linear model	No issues	Pearson residuals
Generalized linear model (GLM)	Skewed Gamma	Non-normality resulting from response variable. Quantile residuals are needed if not approximately normal.	Quantile residual
Linear mixed model (LMM), Multivariate model	Random walk, Spatial LMM, Multinomial	Linear correlations caused by non-independence in observations.	Use a method that linearly decorrelates in order to transform to a unit iid normal. OSA Full Gaussian, OSA one-step Gaussian, or simulation residuals with rotation.
Generalized linear mixed model (GLMM)	Spatial Poisson, Repeated measures Tweedie	Non-normality and non-linear correlations caused by response variable and non-independence in observations.	Needs non-linear decorrelation and quantiles. Needs non-linear decorrelation and quantiles. Best approach depends on case study and sample size.