

Table Placeholders

Methods

Table 1: Linear Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$X_i \sim N(0, 1)$	$\beta = (4, -5)$ $\sigma_y = 1$	Data simulated with lognormal overdispersion: $\mu_{i,j} = X_i\beta + \exp(\epsilon)$ $\epsilon \sim N(0, 1)$ Data fit to model without drift term
$\mu_{i,j} = X_i\beta$		
$y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$		

Table 2: Mixed Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$X_i \sim N(0, 1)$	$\beta = (4, -8)$ $\sigma_u = 2$ $\sigma_y = 0.5$	Data simulated with covariate term Data fit to model without covariate term
$u_j \sim N(0, \sigma_u)$		
$\mu_{i,j} = X_i\beta + u_j$		
$y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\beta = (4, -8)$ $\sigma_u = 2$ $\sigma_y = 0.5$	Data simulated with covariate term Data fit to model without covariate term
$X_i \sim Unif(-0.5, 0.5)$		
$u_j \sim N(0, \sigma_u)$		
$\mu_{i,j} = X_i\beta + u_j$	$\beta = 1.5$ $\sigma_u = 1.4$ $\phi = 1.4$ $p = 1.2$	Data simulated with random effect term Data fit to model without random effect term
$y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$		
$u_j \sim N(0, \sigma_u)$		
$\mu_{i,j} = \exp(\beta_0 + u_j)$	$\phi = 1.4$ $p = 1.2$	Data simulated with random effect term Data fit to model without random effect term
$y_{i,j} \sim Tweedie(\mu_{i,j}, \phi, p)$		

Table 3: Randomwalk Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$\mu_i = u_{i-1} + a$	$a = 0.75$	Data simulated with drift term, a Data fit to model without drift term
$u_i \sim N(\mu_i, \tau)$	$\tau = 1$	
$y_i \sim N(u_i, \sigma)$	$\sigma = 1$	

Table 4: Spatial Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
spatial range = 50		
$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$	$\kappa = \sqrt{8}/50$	
$\eta_i = \beta_0 + \omega_i$	$\sigma_\omega^2 = 1$	Data simulated with $\exp(\omega_i)$
$y \sim N(\eta, \sigma_y)$	$\beta_0 = 1$	Data fit to model without covariate term
	$\sigma_y = 1$	
spatial range = 50		
$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$	$\kappa = \sqrt{8}/50$	
$\eta_i = \beta_0 + \omega_i$	$\sigma_\omega^2 = 2$	Data simulated with random effect term
$y \sim Pois(\exp(\eta))$	$\beta_0 = 0.5$	Data fit to model without random effect term

Results

Linear Model

Table 5: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by residual type (top to bottom).

method	Overdispersion	
	Type I Error	Power
Pearson	0.048	1
one-step Generic	0.048	1
one-step Gaussian	0.048	1
full Gaussian	0.048	1
cdf	0.048	1
Unconditional ecdf, Not Rotated	0.041	1
Conditional ecdf, Not Rotated	0.044	1

Table 6: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by residual type (top to bottom).

method	Overdispersion	
	Type I Error	Power
Pearson	0	0.962
one-step Generic	0	0.963
one-step Gaussian	0	0.963
full Gaussian	0	0.963
cdf	0	0.963
Unconditional ecdf, Not Rotated	0	0.962
Conditional ecdf, Not Rotated	0	0.961

Mixed Model

Table 7: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	A: LMM - Normal X		B: LMM - Uniform X		C: Tweedie GLMM	
	Type I Error	Power	Type I Error	Power	Type I Error	Power
Pearson	0.045	1	0.045	1.000	NA	NA
one-step Generic	0.043	1	0.043	1.000	NA	NA
one-step Gaussian	0.043	1	0.043	1.000	NA	NA
full Gaussian	0.043	1	0.043	1.000	NA	NA
cdf	0.043	1	0.043	1.000	NA	NA
MCMC	0.053	1	0.051	1.000	0.042	1
Unconditional ecdf, Rotated	0.994	1	0.994	1.000	0.809	1
Unconditional ecdf, Not Rotated	0.999	1	0.999	0.999	0.991	1
Conditional ecdf, Rotated	0.996	1	0.996	1.000	0.995	1
Conditional ecdf, Not Rotated	0.047	1	0.047	1.000	0.035	1

Table 8: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	A: LMM - Normal X		B: LMM - Uniform X		C: Tweedie GLMM	
	Type I Error	Power	Type I Error	Power	Type I Error	Power
Pearson	0.000	0.210	0.000	0.780	NA	NA
one-step Generic	0.034	0.278	0.029	0.901	NA	NA
one-step Gaussian	0.034	0.278	0.029	0.901	NA	NA
full Gaussian	0.034	0.278	0.029	0.901	NA	NA
cdf	0.034	0.278	0.029	0.901	NA	NA
MCMC	0.035	0.257	0.044	0.858	0.050	0.667
Unconditional ecdf, Rotated	0.998	0.999	0.998	1.000	0.935	0.999
Unconditional ecdf, Not Rotated	0.916	0.074	0.916	0.067	0.710	0.654
Conditional ecdf, Rotated	0.999	0.998	0.997	1.000	0.999	1.000
Conditional ecdf, Not Rotated	0.000	0.203	0.000	0.780	0.000	0.664

Randomwalk

Table 9: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

method	mu0	
	Type I Error	Power
Pearson	0.039	1.000
one-step Generic	0.038	1.000
one-step Gaussian	0.038	1.000
full Gaussian	0.038	1.000
cdf	0.041	1.000
MCMC	0.050	0.055
Unconditional ecdf, Rotated	0.058	1.000
Unconditional ecdf, Not Rotated	0.987	1.000
Conditional ecdf, Rotated	0.053	1.000
Conditional ecdf, Not Rotated	0.046	1.000

Table 10: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model misspecification (from left to right) and residual type (top to bottom).

method	mu0	
	Type I Error	Power
Pearson	0.128	1.000
one-step Generic	0.000	1.000
one-step Gaussian	0.000	1.000
full Gaussian	0.000	1.000
cdf	0.000	1.000
MCMC	0.045	0.042
Unconditional ecdf, Rotated	0.000	1.000
Unconditional ecdf, Not Rotated	0.996	1.000
Conditional ecdf, Rotated	0.080	1.000
Conditional ecdf, Not Rotated	0.113	1.000

Spatial

Table 11: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	A: LMM		B: Poisson GLMM	
	Type I Error	Power	Type I Error	Power
Pearson	0.046	1.000	0.452	1.000
one-step Generic	0.042	0.286	0.033	0.979
one-step Gaussian	0.042	0.286	NA	NA
full Gaussian	0.042	0.286	NA	NA
cdf	0.042	0.292	0.047	0.979
MCMC	0.042	0.088	0.039	0.979
Unconditional ecdf, Rotated	0.044	0.372	0.381	0.987
Unconditional ecdf, Not Rotated	0.689	1.000	0.662	0.973
Conditional ecdf, Rotated	0.049	1.000	0.081	0.989
Conditional ecdf, Not Rotated	0.039	1.000	0.037	0.976

Table 12: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals using the KS normality test. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	A: LMM		B: Poisson GLMM	
	Type I Error	Power	Type I Error	Power
Pearson	0.183	0.655	0.859	1.000
one-step Generic	0.013	0.366	0.007	0.965
one-step Gaussian	0.013	0.366	NA	NA
full Gaussian	0.013	0.366	NA	NA
cdf	0.015	0.368	0.006	0.964
MCMC	0.043	0.297	0.039	0.969
Unconditional ecdf, Rotated	0.027	0.325	0.193	0.970
Unconditional ecdf, Not Rotated	0.272	0.636	0.270	0.966
Conditional ecdf, Rotated	0.109	0.575	0.345	0.969
Conditional ecdf, Not Rotated	0.186	0.660	0.415	0.960

Table 13: Overview of issues and recommendations for common classes of models. Correlation and distributions refer to predicted data from a fitted model, against which observed points are compared. A linear rotation refers to a multiplication of the simulated and observed data by a Cholesky decomposition of the estimated covariance matrix of the observed data, $z'=Lz$, as available in DHARMA.

Model class	Case studies	Issues and causes	Recommendation
Linear model	Linear model	No issues	Pearson residuals
Generalized linear model (GLM)	Skewed Gamma	Non-normality caused by response variable. Quantile residuals are needed if not approximately normal.	Quantile residual
Linear mixed model (LMM), Multivariate model	Random walk, Spatial LMM, Multinomial	Non-linear correlations caused by non-independence in observations.	Use a method that linearly decorrelates in order to scale to a unit iid normal. OSA Full Gaussian or simulation residuals with rotation.
Generalized linear mixed model (GLMM)	Spatial Poisson, Repeated measures Gamma	Non-normality and non-linear correlations caused by response variable and non-independence in observations.	Needs non-linear decorrelation and quantiles. OSA is only viable option.