

Table Placeholders

Methods

Table 1: Overview of issues and recommendations for common classes of models. Correlation and distributions refer to predicted data from a fitted model, against which observed points are compared. A linear rotation refers to a multiplication of the simulated and observed data by a Cholesky decomposition of the estimated covariance matrix of the observed data, $z'=Lz$, as available in DHARMA.

Model class	Case studies	Issues and causes	Recommendation
Linear model	Linear model	No issues	Pearson residuals
Generalized linear model (GLM)	Skewed Gamma, Non-normality caused by response variable.	Quantile residuals are needed if not approximately normal.	Conditional simulation residuals or OSA CDF
Linear mixed model (LMM)	Random walk, spatial LMM	Random effects cause linear correlation in predicted data.	Use a method that linearly decorrelates in order to scale to a unit iid normal. OSA Full Gaussian or simulation residuals with rotation.
Generalized linear mixed model (GLMM)	Spatial Poisson, Repeated measures gamma	Non-normality and non-linear correlations caused by random effect structure and non-normal response variable.	Needs non-linear decorrelation and quantiles. OSA is only viable option.

Table 2: Linear Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$X_i \sim N(0, 1)$ $\mu_{i,j} = X_i\beta$ $y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$	$\beta = (4, -5)$ $\sigma_y = 1$	Data simulated with lognormal overdispersion: $\mu_{i,j} = X_i\beta + \exp(\epsilon)$ $\epsilon \sim N(0, 1)$ Data fit to model without drift term

Table 3: Mixed Model Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$X_i \sim N(0, 1)$	$\beta = (4, -8)$ $\sigma_u = 2$ $\sigma_y = 0.5$	Data simulated with covariate term Data fit to model without covariate term
$u_j \sim N(0, \sigma_u)$		
$\mu_{i,j} = X_i\beta + u_j$		
$y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$		
$X_i \sim Unif(-0.5, 0.5)$	$\beta = (4, -8)$ $\sigma_u = 2$ $\sigma_y = 0.5$	Data simulated with covariate term Data fit to model without covariate term
$u_j \sim N(0, \sigma_u)$		
$\mu_{i,j} = X_i\beta + u_j$		
$y_{i,j} \sim N(\mu_{i,j}, \sigma_y)$		
$u_j \sim N(0, \sigma_u)$	$\beta = 1.5$ $\sigma_u = 1.4$ $\phi = 1.4$ $p = 1.2$	Data simulated with random effect term Data fit to model without random effect term
$\mu_{i,j} = \exp(\beta_0 + u_j)$		
$y_{i,j} \sim Tweedie(\mu_{i,j}, \phi, p)$		

Table 4: Randomwalk Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
$\mu_i = u_{i-1} + a$	$a = 0.75$	
$u_i \sim N(\mu_i, \tau)$	$\tau = 1$	Data simulated with drift term, a
$y_i \sim N(u_i, \sigma)$	$\sigma = 1$	Data fit to model without drift term

Table 5: Spatial Simulation: data generating models, parameter values, and mis-specifications.

Data Generating Model	Parameters	Mis-specified Model
	spatial range = 50	
$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$	$\kappa = \sqrt{8}/50$	Data simulated with $\exp(\omega_i)$ Data fit to model without covariate term
$\eta_i = \beta_0 + \omega_i$	$\sigma_\omega^2 = 1$	
$y \sim N(\eta, \sigma_y)$	$\beta_0 = 1$	
	$\sigma_y = 1$	
	spatial range = 50	
$\omega \sim GMRF(Q[\kappa, \sigma_\omega^2])$	$\kappa = \sqrt{8}/50$	Data simulated with random effect term Data fit to model without random effect term
$\eta_i = \beta_0 + \omega_i$	$\sigma_\omega^2 = 2$	
$y \sim Pois(\exp(\eta))$	$\beta_0 = 0.5$	

Results

Linear Model

Table 6: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals. Results are partitioned out by residual type (top to bottom).

method	Overdispersion	
	Type I Error	Power
Pearson	0.048	1
one-step Generic	0.048	1
one-step Gaussian	0.048	1
full Gaussian	0.048	1
cdf	0.048	1
Unconditional ecdf, Not Rotated	0.041	1
Conditional ecdf, Not Rotated	0.044	1

Table 7: Linear Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals. Results are partitioned out by residual type (top to bottom).

method	Overdispersion	
	Type I Error	Power
Pearson	0	0.962
one-step Generic	0	0.963
one-step Gaussian	0	0.963
full Gaussian	0	0.963
cdf	0	0.963
Unconditional ecdf, Not Rotated	0	0.962
Conditional ecdf, Not Rotated	0	0.961

Mixed Model

Table 8: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	GLMM - Drop RE		LMM - Missing X Normal		LMM - Missing X Uniform	
	Type I Error	Power	Type I Error	Power	Type I Error	Power
MCMC	0.042	0.038	0.050	1	0.050	1
Unconditional ecdf, Rotated	0.809	0.998	0.996	1	0.994	1
Unconditional ecdf, Not Rotated	0.991	0.032	0.999	1	0.999	1
Conditional ecdf, Rotated	0.995	0.998	0.996	1	0.996	1
Conditional ecdf, Not Rotated	0.035	0.030	0.048	1	0.050	1
Pearson	NA	NA	0.045	1	0.047	1
one-step Generic	NA	NA	0.045	1	0.044	1
one-step Gaussian	NA	NA	0.045	1	0.044	1
full Gaussian	NA	NA	0.045	1	0.044	1
cdf	NA	NA	0.045	1	0.044	1

Table 9: Mixed Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	GLMM - Drop RE		LMM - Missing X Normal		LMM - Missing X Uniform	
	Type I Error	Power	Type I Error	Power	Type I Error	Power
MCMC	0.050	0.667	0.036	0.259	0.042	0.861
Unconditional ecdf, Rotated	0.935	0.999	0.998	0.999	0.998	1.000
Unconditional ecdf, Not Rotated	0.710	0.654	0.914	0.073	0.914	0.070
Conditional ecdf, Rotated	0.999	1.000	0.999	0.999	0.997	1.000
Conditional ecdf, Not Rotated	0.000	0.664	0.000	0.207	0.000	0.782
Pearson	NA	NA	0.000	0.216	0.000	0.785
one-step Generic	NA	NA	0.035	0.281	0.030	0.902
one-step Gaussian	NA	NA	0.035	0.281	0.030	0.902
full Gaussian	NA	NA	0.035	0.281	0.030	0.902
cdf	NA	NA	0.035	0.281	0.030	0.902

Randomwalk

Table 10: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	mu0	
	Type I Error	Power
Pearson	0.039	1.000
one-step Generic	0.038	1.000
one-step Gaussian	0.038	1.000
full Gaussian	0.038	1.000
cdf	0.041	1.000
MCMC	0.050	0.055
Unconditional ecdf, Rotated	0.058	1.000
Unconditional ecdf, Not Rotated	0.987	1.000
Conditional ecdf, Rotated	0.053	1.000
Conditional ecdf, Not Rotated	0.046	1.000

Table 11: Randomwalk Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	mu0	
	Type I Error	Power
Pearson	0.128	1.000
one-step Generic	0.000	1.000
one-step Gaussian	0.000	1.000
full Gaussian	0.000	1.000
cdf	0.000	1.000
MCMC	0.045	0.042
Unconditional ecdf, Rotated	0.000	1.000
Unconditional ecdf, Not Rotated	0.996	1.000
Conditional ecdf, Rotated	0.080	1.000
Conditional ecdf, Not Rotated	0.113	1.000

Spatial

Table 12: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for theoretical residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	GLMM - Drop RE		LMM - Lognormal RE	
	Type I Error	Power	Type I Error	Power
Pearson	0.452	1.000	0.046	1.000
one-step Generic	0.033	0.979	0.042	0.286
cdf	0.047	0.979	0.042	0.292
MCMC	0.039	0.979	0.042	0.088
Unconditional ecdf, Rotated	0.381	0.987	0.044	0.372
Unconditional ecdf, Not Rotated	0.662	0.973	0.689	1.000
Conditional ecdf, Rotated	0.081	0.989	0.049	1.000
Conditional ecdf, Not Rotated	0.037	0.976	0.039	1.000
one-step Gaussian	NA	NA	0.042	0.286
full Gaussian	NA	NA	0.042	0.286

Table 13: Spatial Model. Type I error rates and Power evaluated for each analytical and simulation method for estimated residuals. Results are partitioned out by model mis-specification (from left to right) and residual type (top to bottom).

method	GLMM - Drop RE		LMM - Lognormal RE	
	Type I Error	Power	Type I Error	Power
Pearson	0.859	1.000	0.183	0.655
one-step Generic	0.007	0.965	0.013	0.366
cdf	0.006	0.964	0.015	0.368
MCMC	0.039	0.969	0.043	0.297
Unconditional ecdf, Rotated	0.193	0.970	0.027	0.325
Unconditional ecdf, Not Rotated	0.270	0.966	0.272	0.636
Conditional ecdf, Rotated	0.345	0.969	0.109	0.575
Conditional ecdf, Not Rotated	0.415	0.960	0.186	0.660
one-step Gaussian	NA	NA	0.013	0.366
full Gaussian	NA	NA	0.013	0.366