Lab 3: Temporal models

April 14, 2016

Objectives

- Tools for detecting temporal autocorrelation
- Build a state-space model with autocorrelation in the state process

Temporal Models

- White noise process

$$W_t \sim Gau(0,\sigma^2)$$

- Random walk process

$$Y_t = Y_{t-1} + W_t$$
$$W_t \sim Gau(0,\sigma^2)$$

- Autoregressive Process

$$Y_t = Y_{t-1} + w_t$$
$$w_t \sim Gau(0,\sigma^2)$$

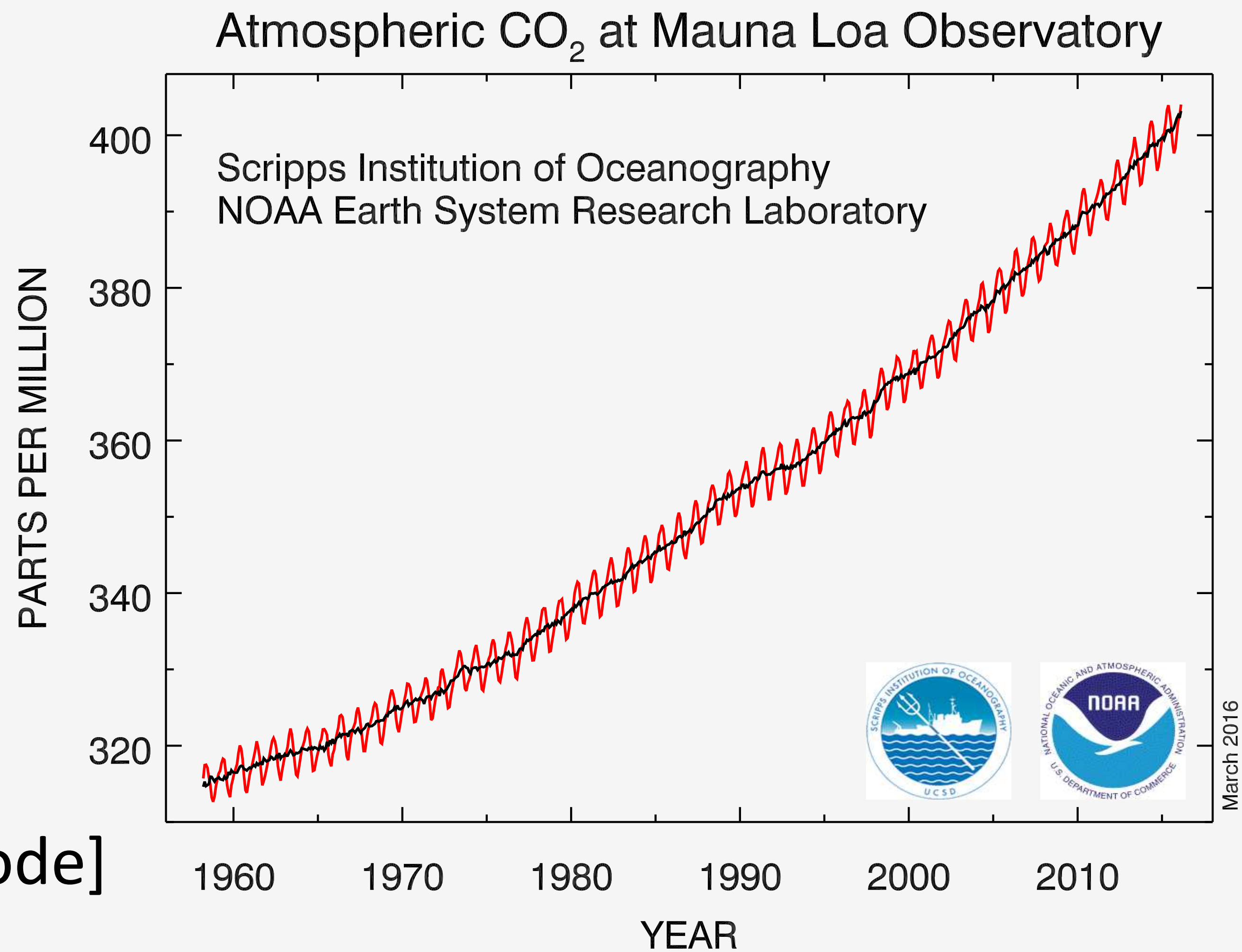
- Moving Average process

$$Y_t = W_t + \beta_1 W_{t-1} + \ldots \beta_q W_{t-q}$$
$$W_t \sim Normal(0,\sigma^2)$$

Fitting models to data

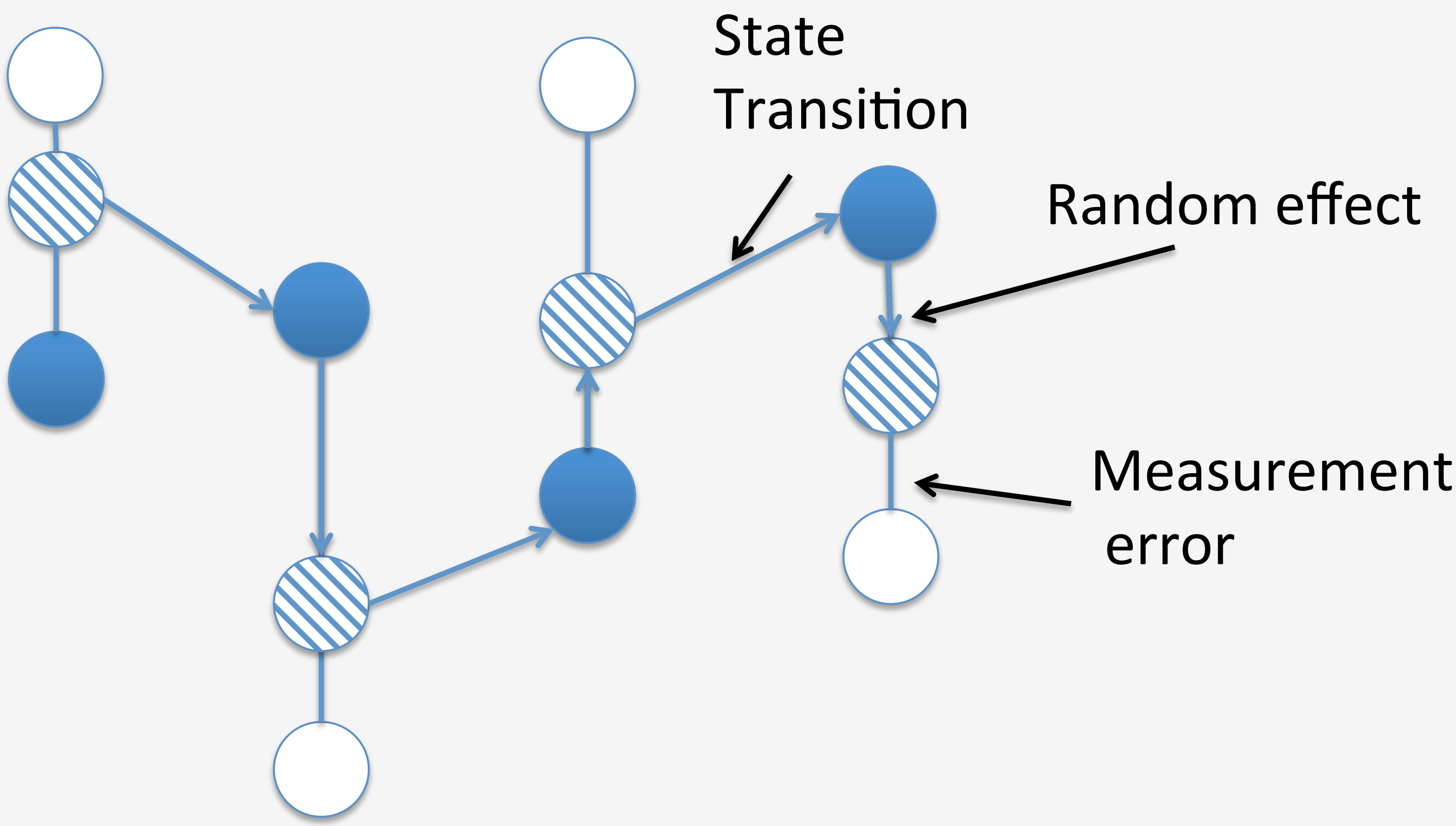
- Transform to obtain a stationary time series
  - Remove trends and seasonal components
  - Differentiate (if needed)
  - Test for stationarity
- Determine order of AR and MA processes
  - Compute partial auto-correlation function to determine order of model for AR,
  - partial auto-correlation of lag k is the autocorrelation between z_t and z_{t+k} with the linear dependence of z_t on z_{t+1} through z_{t+k-1} removed
  - Compute auto-correlation function to determine order of MA model
  - Estimate auto-regression and variance using Yule-Walker (AR) or maximum likelihood (AR, MA, or ARMA)
- Compute residuals and test for white noise

Analyze a time series of $C0_2$ concentrations from Mauna Loa Observatory from 1958 to 2008



[See R code]

State-space models



Temporal models

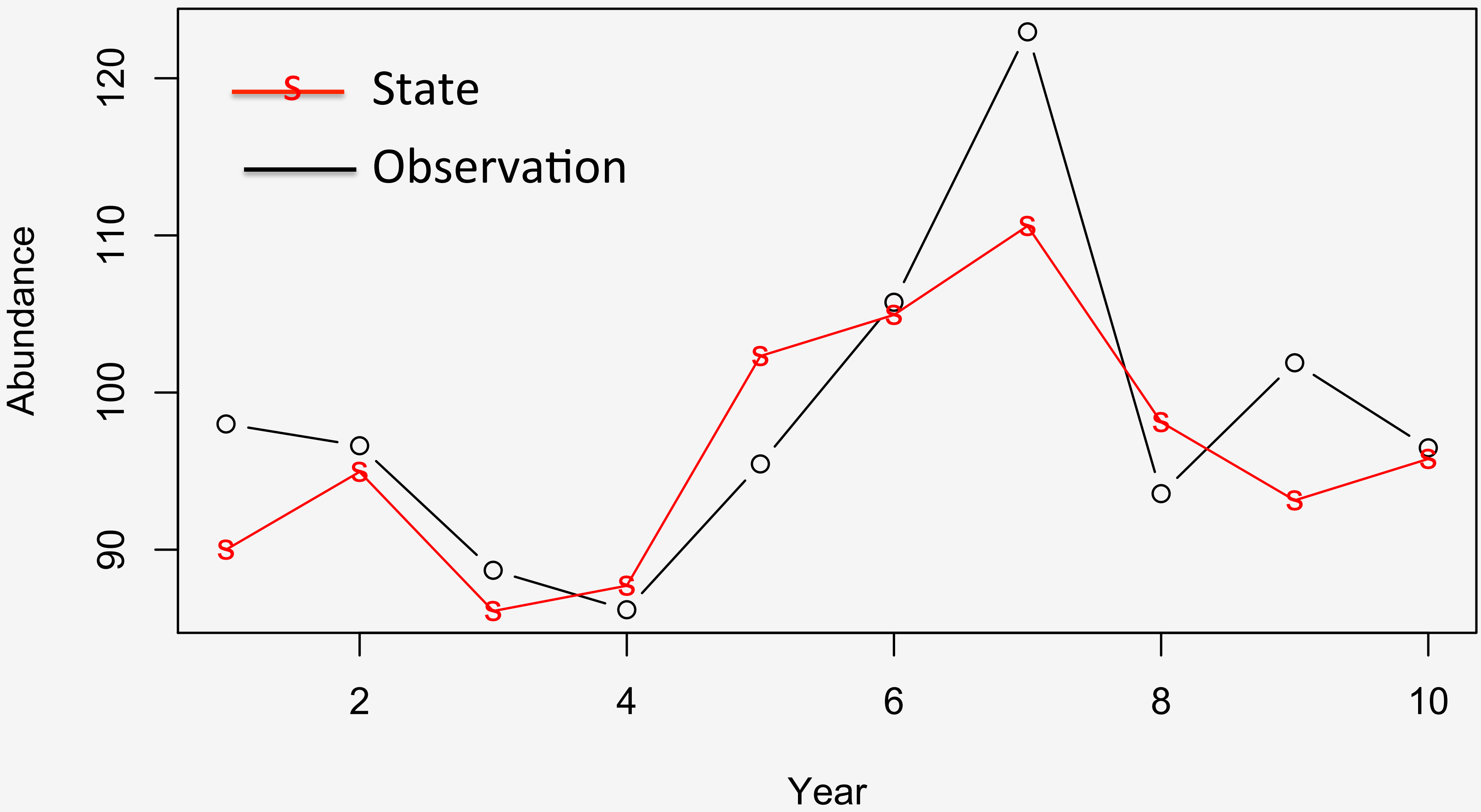
State-space model example - Dynamic Linear Model

$$S_t | S_{t-1} \sim N(\lambda S_{t-1}, \sigma_S^2)$$

$$y_t | S_t \sim N(S_t, \sigma_y^2)$$

- The state S_t is a stochastic function of the state the previous time step and the population growth rate λ and the level of process noise σ_S
- The observation process is a function of the measurement error σ_y

Temporal models



State-space models

- Dynamic Linear Model
- Gompertz state space model

- [See R code and TMB code]