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Solow Model

IMPLEMENTING AND EXTENDING THE MODEL

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The Solow model

The Solow Model is used to explain economic growth in the long-run through the accumulation of capital, population growth, and technological progress.

According to the model, which assumes a Cobb Douglas form for the production function, every economy converges in the long run to a “steady state”, in which income per capita (Y/L) is related to the saving rate (s) and the population growth (n) in the following way:

$$\ln\left(\frac{Y(t)}{L(t)}\right) = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta)$$

where δ is the depreciation rate and g is the growth rate of technology, assumed to be constant and equal for all countries a.
According to the model, GDP per capita is higher in countries with a higher saving rate and lower in countries with a larger population growth.
In this report, we perform a cross-country analysis looking for empirical evidence in favour of the Solow Model, regressing GDP per capita on saving rate, population growth and trade as % of GDP.

Variable selection and preprocessing

To obtain measures of the variables mentioned in the Solow Model, we used the following datasets from the World Bank:

- **GDP per capita** (current $) - indicator code NY.GDP.PCAP.CD
- **Population Growth** (annual %) - indicator code SP.POP.GROW
- **Gross Saving Rate** (% of GDP) - indicator code NY.GNS.ICTR.ZS
- **Trade** (% of GDP) - indicator code NE.TRD.GNFS.ZS

We considered data from 2000 to 2023 included, and excluded countries lacking data about any of the variables mentioned above*. The analysis considered 168 countries.

Since we’re dealing with panel data, but our intention is to perform a cross-sectional analysis, we either considered only the year 2023, or averaged the data on the temporal dimension. A more detailed discussion behind these choices with a comparison of the different options is presented in the following slide.

We proceeded by converting the datasets from wide to long, excluding irrelevant columns and rows not corresponding to individual countries (e.g. “World”). The various datasets were then merged based on the “Country Name” column to form a unified dataset.

*We also excluded Ukraine, since it was the only country with pop_growth < -5%, causing the logarithm in the model to have a negative argument.

Model definition

We chose the following functional form for the regression:

$$\ln(\text{gdp}) = \alpha + \beta \cdot \text{lsaving} + \gamma \cdot \text{lpop} + \varepsilon$$

$$\text{lpop} = \ln\left(\text{pop\_growth} + \delta + g\right)$$

$$\text{lsaving} = \text{sign}(\text{saving}) \cdot \ln\left(|\text{saving}|\right)$$

Coherently with Mankiw et al. (1992), δ + g = 5% for all countries.
As regards the interpretation of the parameters, with δ + g fixed, c represents the elasticity between population growth and gdp per capita. For positive saving rates, b is the elasticity between saving rate and gdp per capita, while for negative values it is minus the elasticity computed with the absolute value.

Selecting the timeframes for our regression

We are working with panel data: multiple countries observed over multiple years. To test whether the Solow model explains cross-country differences, we considered three alternative ways to construct the regression dataset.

2023 only

All variables (both dependent and independent) refer to 2023 only.

We test whether the Solow model explains contemporary cross-sectional variation in GDP per capita using recent values for saving and population growth.

$R^2 = 0.16$ → The low value might suggest other short-run factors may be more relevant in the short term.

	Estimated Value	p-value
β (lsaving)	0.41	9.16e-05 ***
γ (lpop)	-1.73	5.11e-05 ***

→ Coefficients are significant and their signs are consistent with the Solow predictions

BP test: $p = 0.0017$ → Strong evidence of heteroskedasticity: variance of GDP per capita unexplained by the model differs across countries.

2000-2023; 2000-2023

All variables are averaged over 2000–2023.

Testing whether **long-run averages** of savings and population growth explain long-run average income levels.

$R^2 = 0.21$ → The model performs better with long-run averages, which give more information about the capital accumulation of countries.

	Estimated Value	p-value
β (lsaving)	0.36	0.00316 ***
γ (lpop)	-3.16	1.38e-08 ***

→ Coefficients are stronger and more significant, reflecting the cumulative effect of saving and population growth over time.

BP test: $p = 0.01$ → Heteroskedasticity remains, indicating that residual variation still differs across countries.

2023; 2000-2023*

*This is the timeframe also implemented by Mankiw et. al (1992)

GDP per capita is from 2023, while explanatory variables are averaged over 2000–2023.

We test whether **historical fundamentals help predict a country's final position** in the income distribution (aligning with the steady-state view of the Solow model).

$R^2 = 0.27$ → Highest explanatory power: past, averaged fundamentals predict current income levels most effectively. This is in line with the steady state logic of the Solow Model. In this case, the steady state is approximated empirically by the current year.

	Estimated Value	p-value
β (lsaving)	0.42	0.000319 ***
γ (lpop)	-3.5982	2.78e-11 ***

→ Strong and significant coefficients. Supports the Solow view that persistent structural factors shape income levels in the long run. As in the previous case, a plausible reason could be the cumulative effect which is now stronger as we're regressing on the current GDP.

BP test: $p = 0.003$ → Stronger evidence of heteroskedasticity. It may be due to due to diverging long-run fundamentals across countries.

Selecting Country Groups Using the Chow Test

To identify structural differences, we apply the **Chow test** to the following categories:

CHOW TEST FOR GROUP SPLITTING

Test	F-statistic	p-value
High Resource Rents vs Others	3.492	0.01697 *
OECD vs Non-OECD	24.48	3.233e-13 ***

The Chow test compares restricted and unrestricted models across groups to detect parameter instability.

We conclude that that is a **significant discrepancy** between high resource rents countries and regular countries and between OECD and non OECD countries. As a result, we estimate separate regressions for the following groups:

1. All countries
2. Countries excluding major natural resource exporters *
3. OECD countries only

*We define "countries with high resource rents" as those for which resource_rents >= 10. That is, the countries where revenues from natural resources such as oil, gas, and minerals account for at least 10% of GDP. This threshold is frequently used in empirical studies to identify resource-dependent economies, which often show distinct macroeconomic dynamics due to volatility, rentier effects, and non-steady-state behaviour, which may reduce the stability of our regression framework.

OBSERVATIONS

In the following analysis we consider the timeframe '2023; 2000-2023' as is the one that is most consistent with the solow model theory, as previously discussed.

Feature	All Countries	Excl. Resource Exporters	OECD Countries
R^2	0.26	0.26	0.34
β (lsaving)	0.41 *** ($p = 0.0005$)	0.35 ** ($p = 0.0043$)	1.26 ** ($p = 0.0024$)
γ (lpop)	-3.52 *** ($p < 1e-10$)	-3.57 ***($p \approx 3e-8$)	+1.55 * ($p = 0.038$)
BP test (p-value)	0.0022	0.22	0.016

Excluding High Resources Countries

- **The BP test is not significant:** this means removing high-resource exporters eliminates much of the **heteroskedasticity**. This may be due to the fact that these countries:
  - may have more volatile export revenues (and consequentially noisy GDP) as they are driven by global markets
  - resource exporters countries may not converge toward a steady-state level of the economy, which is an important assumption of the solow model (which is the reason that justificates the choice of this timeframe).
- The sign and the magnitude of the estimated parameters is quite similar, and also the R^2.

Only OECD Countries

- **Savings effect is much stronger:**
  - this may be due to a higher efficiency of capital accumulation of OECD economies.
- **Population growth** has a **positive association** with GDP, clearly deviating from Solow. It's important to note that also the significance of the parameter is dropped (due to a higher p-value).
  - in advanced economies population growth might be associated with labor force expansion and strengthening, urban growth or (skilled) immigration which offset the negative effect of a capital diluition. This may be even stronger in aging economies, where population growth may rejuvenate the workforce.

An additional explanatory variable: trade

We initially considered **two potential extensions** to the Solow model: education, measured by the **percentage of the population attaining secondary education**, and **trade** as a percentage of GDP.

While education was statistically significant, it **did not provide clear or interpretable results**, and made the other variables non significant, leading us to shift our focus to trade.

Trade openness offers a **strong theoretical foundation** for influencing GDP per capita, by facilitating technology diffusion, improving resource allocation, expanding market access, and fostering competition, trade enhances productivity and economic efficiency. Greater **trade exposure** allows countries to access a wider variety of goods and services, integrate into global value chains, and attract foreign investment, further boosting economic activity.

Empirical results confirmed trade's positive and significant impact on GDP per capita while preserving the significance of population growth.

Unlike education, trade aligns well with macroeconomic theory and provides a clearer, more interpretable mechanism for economic growth.

We consider the log of trade as a percentage of GDP:

$$\text{ltrade} = \ln(\text{trade (\% of GDP)})$$

Feature	All Countries	Excl. Resource Exporters	OECD Countries
R^2	0.34	0.31	0.30
β (lsaving)	0.37*** (p = 0.00072)	0.32*** (p = 0.0065)	1.11* (p = 0.010)
γ (lpop)	-3.11*** (p = 1.73e-09)	-3.11*** (p = 7.64e-07)	1.66* (p = 0.027)
π (ltrade)	0.84*** (p = 1.10e-05)	0.71*** (p = 0.00045)	0.20 (p = 0.29)
BP test (p-value)	0.1392	0.6225	0.0154

We observe that accounting for trade behaviour we have more **similarity between the parameters** found for all countries and those found excluding big resource exporters: the **sign** and **magnitude** are consistent and **also the R^2**.

Excluding High Resources Countries

- The BP test has a higher p-value for the same reasons pointed out before: removing high-resource exporters eliminates much of the **heteroskedasticity** as the exports are less volatile and resource exporters may not converge to a steady state.
- Coefficients are consistent with the ones found regressing over the whole sample.

Only OECD Countries

- The BP test a much lower p-value, suggesting heteroskedasticity if we regress only on OECD countries.
- The significance of the coefficients is lower as in the previous regression. This may be due again to the higher efficiency of developed countries, which makes their GDP per capital less tied to the fundamentals. Moreover, the value of **γ** increases while that of **β** decreases, suggesting trade exhibits some multicollinearity with the other variables in this group.

Observations and conclusion

1. Differentiated Effects Across Country Groups

The impact of trade on economic growth **vary significantly across different country groups.**

Advanced economies that are already well-integrated into global markets, experience **moderate but steady gains from trade**, mainly through productivity improvements and innovation diffusion.

On the other hand, **emerging economies** get to benefit substantially more, as trade enhances capital accumulation, industrial diversification, and access to foreign investment.

Lastly, in lower income economies, trade effects are less consistent, likely due to weaker institutional frameworks, infrastructure bottlenecks, or reliance on commodity exports with volatile price fluctuations.

2. Trade as a Growth Driver

The empirical results reinforce the role of trade openness as a key determinant of GDP per capita growth. Beyond the direct economic benefits, such as improved resource allocation, technology diffusion, and economies of scale, trade also fosters institutional development.

Countries with higher trade to GDP ratios may in fact be associated with better governance and regulatory practices that enhance their competitiveness in global markets.

This aligns with previous literature emphasizing that trade is not only a source of efficiency gains but also a mechanism for long-term structural transformation.

3. Revisiting the Solow Model Extensions

Integrating trade into the Solow framework provides a more complete picture of growth dynamics, particularly for countries where capital accumulation alone does not explain observed GDP variations.

The interplay between trade, savings, and population growth highlights the need for a **broader theoretical framework** that accounts for globalization's role in shaping long-run economic outcomes.

4. Results in line with the theory

The Solow Model describes the behavior of economies in the long run ("steady states"): our choice of the timeframes reflects this property, and the model is able to explain a greater variation in the target variable, when this is set to the last year only.
Coherently with Mankiw et al. (1992), we run a restricted version of the model, in which the constraint $\beta = -\gamma$ is enforced. Then we perform the t-test for this coefficient to be equal to 0.5, as this implies a value for the alpha parameter in the Solow Model equal to 0.33, confirmed empirically. The null hypothesis that the coefficient is equal to 0.5 cannot be rejected (p-value = 0.75), providing evidence in favor of alpha=0.33, so in favor of the theoretical Solow Model.