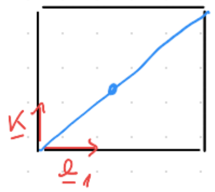
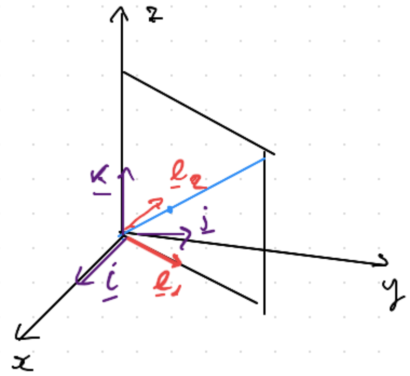
1) Per l'osservatore mobile:

$P_r(t) = \lambda s(t) (\underline{e}_1 + \underline{K})$

$\lambda = \frac{\sqrt{2}}{2}$





$$\begin{cases} \underline{e}_1 = \cos\omega t\, \underline{i} + \sin\omega t\, \underline{j} \\ \underline{e}_2 = -\sin\omega t\, \underline{i} + \cos\omega t\, \underline{j} \\ \underline{e}_3 = \underline{K} \end{cases} \qquad \begin{cases} \underline{i} = \cos\omega t\, \underline{e}_1 - \sin\omega t\, \underline{e}_2 \\ \underline{j} = \sin\omega t\, \underline{e}_1 + \cos\omega t\, \underline{e}_2 \\ \underline{K} = \underline{e}_3 \end{cases}$$

a) Per l'osservatore fisso (riferimento assoluto)

$$\underline{v}_a(t) = \frac{d}{dt} P_r(t) = \lambda \dot{s}(t) (\cos\omega t\, \underline{i} + \sin\omega t\, \underline{j} + \underline{K}) + \lambda \omega s(t) (-\sin\omega t\, \underline{i} + \cos\omega t\, \underline{j})$$

$$\underline{a}_a(t) = \lambda \ddot{s}(t) (\cos\omega t\, \underline{i} + \sin\omega t\, \underline{j} + \underline{K}) + 2\lambda\omega \dot{s}(t) (-\sin\omega t\, \underline{i} + \cos\omega t\, \underline{j}) + \lambda \omega^2 s(t) (-\cos\omega t\, \underline{i} - \sin\omega t\, \underline{j})$$

Notare che:

$$\underline{v}_a(t) = \lambda \dot{s}(t) (\underline{e}_1 + \underline{K}) + \lambda\omega s(t)\, \underline{e}_2$$

$$\underline{a}_a(t) = \lambda \ddot{s}(t) (\underline{e}_1 + \underline{K}) + 2\lambda\omega \dot{s}(t)\, \underline{e}_2 - \lambda\omega^2 s(t)\, \underline{e}_1$$

b) $\underline{v}_a = \underline{v}_r + \underline{v}_\tau$

$\underline{v}_r = \lambda \dot{s}(t) (\underline{e}_1 + \underline{K})$

$\underline{v}_\tau = \underset{\overset{\shortparallel}{0}}{\underline{v}_O} + \underline{\omega} \wedge (P-O)$ in questo caso: $\underline{v}_\tau = \underline{\omega} \wedge (P-O)$

$\underline{\omega} = \omega \underline{K}$

$\underline{v}_\tau = \omega \underline{K} \wedge \lambda s(t) (\underline{e}_1 + \underline{K}) = \lambda\omega s(t)\, \underline{e}_2$

c) $\underline{a}_a = \underline{a}_r + \underline{a}_\tau + \underline{a}_c$

$\underline{a}_r = \lambda \ddot{s}\,(\underline{e}_1 + \underline{k})$
(relativa)

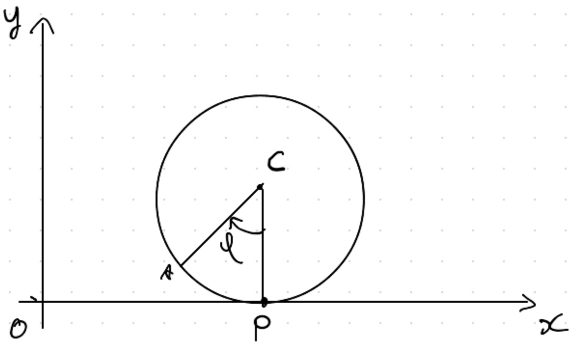
$\underline{a}_\tau = \underset{\overset{\shortparallel}{\underline{0}}}{\underline{a}_0} + \underset{\overset{\shortparallel}{\underline{0}}}{\dot{\underline{\omega}}} \wedge (P-O) + \underline{\omega} \wedge \overbrace{(\underline{\omega} \wedge (P-O))}^{\underline{v}_\tau}$

$\underline{a}_\tau = \omega \underline{k} \wedge \lambda \omega s(t)\, \underline{e}_2 = -\lambda \omega^2 s(t)\, \underline{e}_1$
(Trascinamento)

$\underline{a}_c = 2\, \underline{\omega} \wedge \underline{v}_r = 2 \omega \underline{k} \wedge \lambda \dot{s}(t) (\underline{e}_1 + \underline{k}) = 2 \omega \lambda \dot{s}(t)\, \underline{e}_2$
(Coriolis)

2) $C - O = x\,\underline{i} + R\,\underline{j}$ $\qquad \underline{\omega} = \omega \underline{k}$

$P - O = x\,\underline{i}$



$\underline{v}_c = \dot{x}\,\underline{i}$

$\underline{v}_p = \dot{x}\,\underline{i}$ punto geometrico

$\underline{v}'_p = \underline{0}$ velocità del punto materiale del disco che è istantaneamente a contatto con il piano (o con la guida orizzontale)

$\underline{v}_c - \underset{\overset{\shortparallel}{\underline{0}}}{\underline{v}'_p} = \omega \underline{k} \wedge (C - P)$ (legge di distr. delle vel.)

$\underline{v}_c = \omega \underline{k} \wedge R \underline{j} = -\omega R\, \underline{i}$

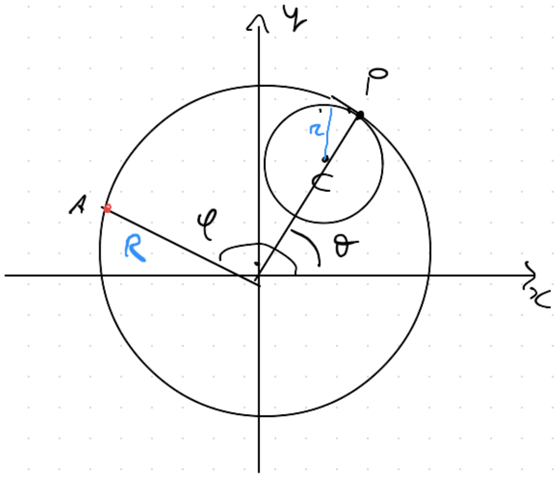
$\dot{x}\,\underline{i} = -\omega R\, \underline{i} \quad \Rightarrow \quad \omega = -\dfrac{\dot{x}}{R} \qquad x(t) = R \int_0^t \dot{\varphi}(t')\,dt' =$

$\dot{\varphi} = \dfrac{\dot{x}}{R} \qquad = R(\varphi(t) - \varphi_0)$

$\underline{\omega} = -\dot{\varphi}\, \underline{k}$

3)

a) $\underline{\omega}_G = \dot{\varphi}\,\underline{k}$



b) P punto geometrico:

$$P = (R\cos\theta,\ R\sin\theta,\ 0)$$

$$C = ((R-r)\cos\theta,\ (R-r)\sin\theta,\ 0)$$

$$\underline{v}_C = (R-r)\dot{\theta}\,(-\sin\theta,\ \cos\theta,\ 0)$$

Voglio applicare la legge di distribuzione delle velocità

$$\underline{v}'_P - \underline{v}_C = \underline{\omega}_D \wedge (P-C) \qquad \underline{v}'_P \neq R\dot{\varphi}\,(-\sin\varphi,\ \cos\varphi,\ 0)$$

$$\underline{v}'_P = R\dot{\varphi}\,(-\sin\theta,\ \cos\theta,\ 0)$$

$$\underline{v}'_P = \underline{\omega}_G \wedge (P-O) = \dot{\varphi}\,\underline{k} \wedge (R\cos\theta\,\underline{i} + R\sin\theta\,\underline{j}) = -R\dot{\varphi}\sin\theta\,\underline{i} + \dot{\varphi}R\cos\theta\,\underline{j}$$

$$\underline{v}'_P - \underline{v}_C = \omega_D\,\underline{k} \wedge (P-C)$$

$$P-C = r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}$$

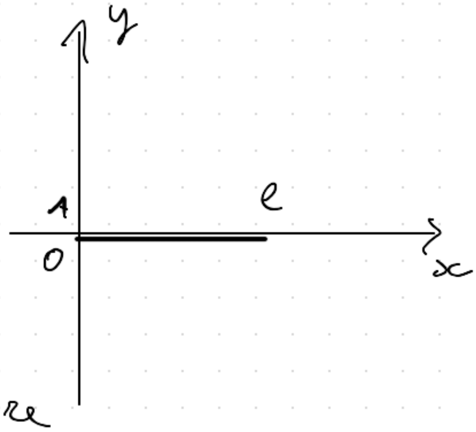
$$\overbrace{(-R\dot{\varphi} + (R-r)\dot{\theta})\sin\theta\,\underline{i} + (R\dot{\varphi} - (R-r)\dot{\theta})\cos\theta\,\underline{j}}^{\underline{v}'_P - \underline{v}_C} =$$

$$= \omega_D\,\underline{k} \wedge (r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}) = r\omega_D\cos\theta\,\underline{j} - r\omega_D\sin\theta\,\underline{i}$$

$$R\dot{\varphi} - (R-r)\dot{\theta} = r\,\omega_D$$

$$\omega_D = \frac{R\dot{\varphi} - (R-r)\dot{\theta}}{r} \quad ; \quad \underline{\omega}_D = \frac{R\dot{\varphi} - (R-r)\dot{\theta}}{r}\,\underline{k}$$

a)



$\underline{I} = \sum_i m_i r_i^2$

$I_A = \int_0^\ell x^2 \, dm$

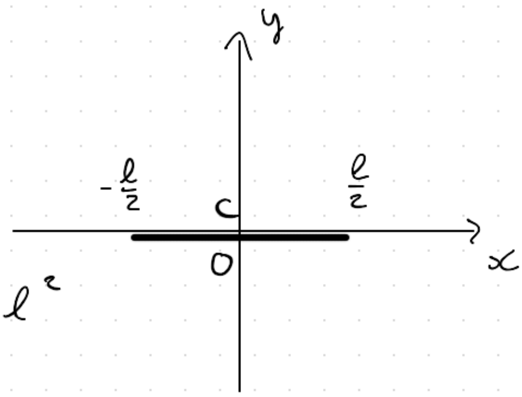
$dm = \lambda \, dx$ $\quad \lambda$ = densità lineare

$I_A = \int_0^\ell x^2 \lambda \, dx = \lambda \int_0^\ell x^2 dx = \frac{\lambda}{3} \ell^3$

$\lambda = \frac{m}{\ell}$

$I_A = \frac{1}{3} \frac{m}{\ell} \ell^3 = \frac{1}{3} m \ell^2$

b)



$I_C = \lambda \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} x^2 dx = \frac{1}{3} \lambda \left(\frac{\ell^3}{8} + \frac{\ell^3}{8} \right) =$

$= \frac{1}{12} \lambda \ell^3 = \frac{1}{12} \frac{m}{\ell} \ell^3 = \frac{1}{12} m \ell^2$

c) Huygens - Steiner

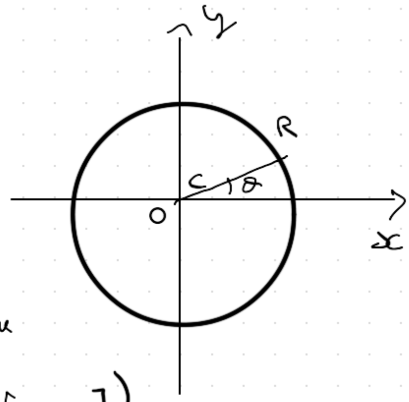
$I_A = I_G + m d^2$ $\quad$ Qui: $G = C$ (simmetria)

$I_C + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{12} m \ell^2 + \frac{1}{4} m \ell^2 = \frac{4}{12} m \ell^2 = \frac{1}{3} m \ell^2$

$I_A = \frac{1}{3} m \ell^2$

5)

$$I_c = \int_{\mathcal{C}} r^2 \, dm = \int_{\mathcal{C}} r^2 \lambda \, dl$$



Uso θ come variabile di integrazione

Il punto sulla corona ha coordinate $(R\cos\theta, R\sin\theta)$ $(\theta \in [0, 2\pi])$

$dl = R\,d\theta$

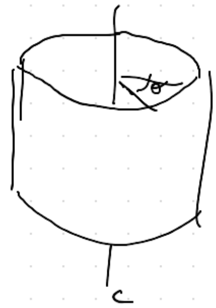
$$I_c = \lambda \int_0^{2\pi} R^2 R\,d\theta = 2\pi \lambda R^3$$

$$\lambda = \frac{m}{2\pi R}$$

$$I_c = 2\pi \frac{m}{2\pi R} R^3 = m R^2$$

6)

a) $$I_c = \iint_S R^2 \, dm = \iint_S R^2 \sigma \, dA$$



In questo caso $dm = \sigma \, dA$

σ densità superficiale.

Un punto sul bordo ha coordinate: $(R\cos\theta, R\sin\theta, z)$

$\theta \in [0, 2\pi]$; $z \in [0, h]$

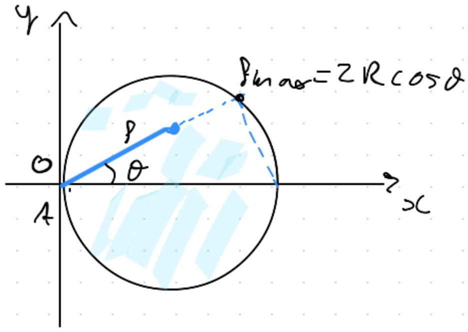
$dA = R\,d\theta\,dz$

$$I_c = \sigma \int_0^{2\pi} d\theta \int_0^{h} dz \, R^2 R = \sigma R^3 2\pi h$$

$$\sigma = \frac{m}{2\pi R h} \; ; \qquad I_c = \frac{m}{2\pi R h} R^3 2\pi h = m R^2$$

b) Per Huygens – Steiner : asse // al precedente e che giace sulla superficie

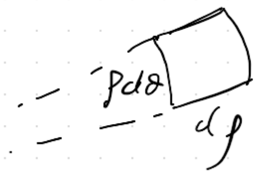
$$I_A = I_c + mR^2 = mR^2 + mR^2 = 2mR^2$$



$$\underline{I_A} = \iint_S r^2 \sigma \, dA$$

In questo caso il punto generico ha coordinate (polari) $(\rho\cos\theta, \rho\sin\theta)$

$$I_A = \sigma \iint_S \rho^2 \rho \, d\rho \, d\theta$$



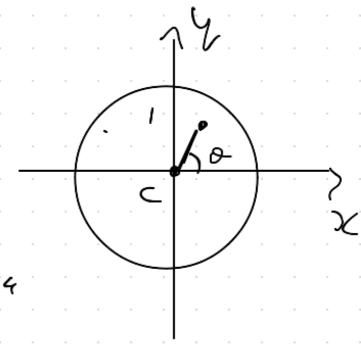
$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; $\rho \in [0, 2R\cos\theta]$

$$I_A = \sigma \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{2R\cos\theta} \rho^3 d\rho$$

7

$I_A = \frac{3}{2}\pi\sigma R^4$; $\sigma = \frac{m}{\pi R^2}$

$$I_A = \frac{3}{2}\pi \frac{m}{\pi R^2} R^4 = \frac{3}{2} m R^2$$



b)

$\theta \in [0, 2\pi]$; $\rho \in [0, R]$

$$\underline{I_c} = \sigma \int_0^{2\pi} d\theta \int_0^R d\rho \, \rho^3 = \frac{1}{4}\sigma \, 2\pi \, R^4 = \frac{1}{2}\pi\sigma R^4$$

$$I_c = \frac{1}{2}\pi \frac{m}{\pi R^2} R^4 = \frac{1}{2} m R^2$$

c) H-S. $I_c + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 = I_A$