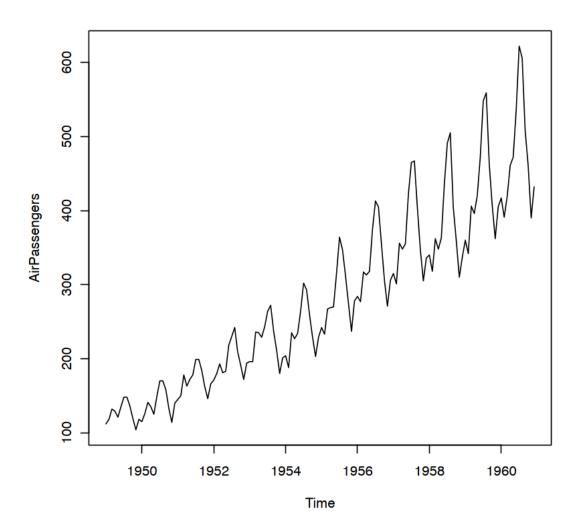
test1

November 26, 2024

```
[2]: library(astsa)
    library(datasets)

## dataset
    data("AirPassengers")
    summary(AirPassengers)
    plot(AirPassengers)
```

Min. 1st Qu. Median Mean 3rd Qu. Max. 104.0 180.0 265.5 280.3 360.5 622.0



```
[3]: data <- ts(AirPassengers)
sarima.for(data, xreg = data.frame(time = 1:length(data)), newxreg = data.

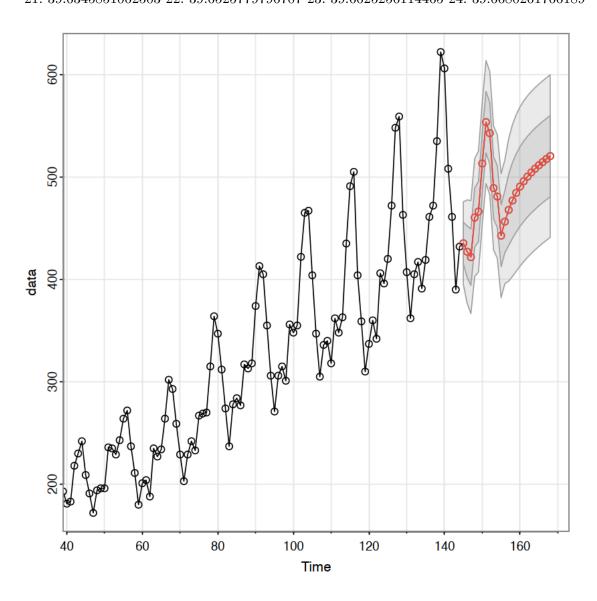
frame(time = 1:24 + length(data)), 24, 1, 0, 0, 0, 0, 1, 12)
```

\$pred A Time Series:

- $1. \quad 435.296404448951 \quad 2. \quad 427.132067810041 \quad 3. \quad 421.740058102871 \quad 4. \quad 460.354001689686$
- $5. \quad 466.215317459882 \quad 6. \quad 513.152786883938 \quad 7. \quad 553.530511194164 \quad 8. \quad 542.737439328013$
- $9. \ \ 489.167182676446 \ \ 10. \ \ 480.895509645266 \ \ 11. \ \ 442.72359302425 \ \ 12. \ \ 456.381396774694$
- $13. \ \ 467.826636698122 \ \ 14. \ \ 476.9821798826 \ \ 15. \ \ 484.434921417891 \ \ 16. \ \ 490.621323438804$
- 17. 495.865974101629 18. 500.410263123371 19. 504.433707206489 20. 508.069809330573
- $21.\ 511.417852990222\ 22.\ 514.551673333763\ 23.\ 517.526180083603\ 24.\ 520.382208501587$

\$se A Time Series:

 $1. \quad 20.2369465077224 \quad 2. \quad 25.2153420831165 \quad 3. \quad 27.5854874380948 \quad 4. \quad 28.8126935603087$



```
[4]: DIR <- "/Users/gianlucamastrantonio/Dropbox (Politecnico di Torino Staff)/

didattica/DataSpaces/dispense/Time Series/"

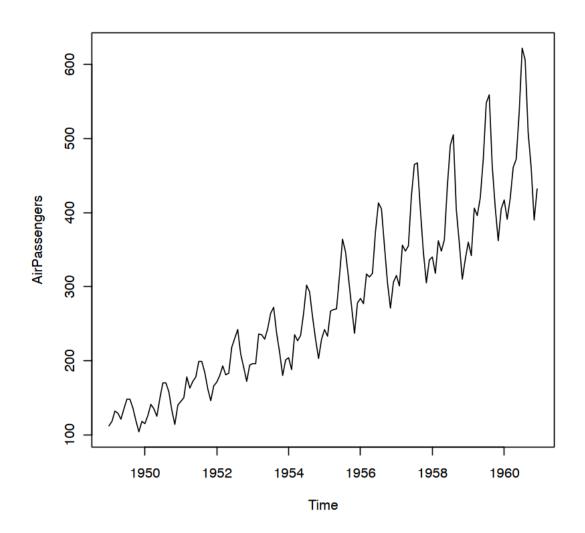
# remotes::install_github("nickpoison/astsa/astsa_build")
library(astsa)
library(astsa)
library(datasets)
```

```
## dataset
data("AirPassengers")
summary(AirPassengers)

# prendiamo lo stesso dataset precedente e confrontiao modelli ARIMA

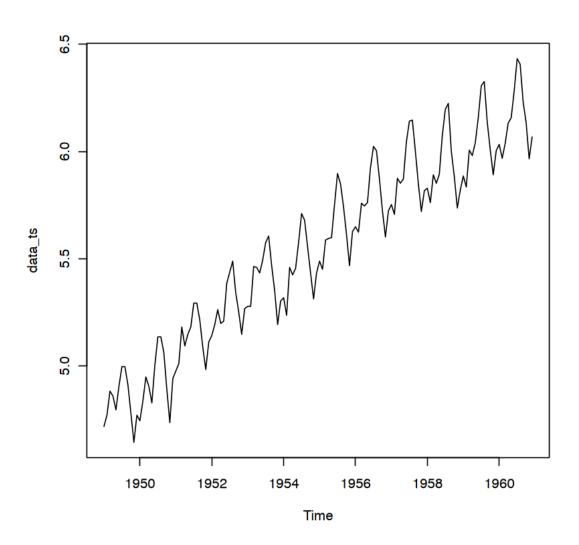
# vediamo dei plot
plot(AirPassengers)
```

```
Min. 1st Qu. Median Mean 3rd Qu. Max. 104.0 180.0 265.5 280.3 360.5 622.0
```



Applico la traasfornmazione logartmica

```
[5]: data_ts = log(AirPassengers)
    plot(data_ts)
```



```
[6]: data_ts <- ts(data_ts)
sarima(data_ts,
1,# AR
0,# I
0,# MA
0,# AR_s
0,# I_s
0,# MA_s
12)</pre>
```

```
initial value -0.829999
       2 value -2.248560
iter
iter
       3 value -2.250003
iter
       4 value -2.250493
iter
       5 value -2.250645
       6 value -2.251785
iter
iter
      7 value -2.252303
iter
      8 value -2.252413
      9 value -2.252541
iter
iter 10 value -2.252943
     11 value -2.253590
iter
     12 value -2.253823
iter
     13 value -2.253841
iter
     14 value -2.253859
iter
iter
     15 value -2.253919
     16 value -2.254000
iter
iter
     17 value -2.254051
     18 value -2.254055
iter
     19 value -2.254063
iter
iter
     20 value -2.254082
iter
     21 value -2.254113
     22 value -2.254129
iter
iter
    23 value -2.254130
     24 value -2.254132
iter
iter 25 value -2.254136
     26 value -2.254143
iter
     27 value -2.254147
iter
iter
     28 value -2.254147
     29 value -2.254148
iter
iter
     30 value -2.254149
     31 value -2.254151
iter
iter
     32 value -2.254152
iter
     32 value -2.254152
iter 32 value -2.254152
final value -2.254152
converged
initial value -2.222794
       2 value -2.224612
       3 value -2.228908
iter
       4 value -2.229816
iter
      5 value -2.230136
iter
       6 value -2.230206
iter
       7 value -2.230224
iter
       8 value -2.231597
iter
iter
       9 value -2.231670
iter
     10 value -2.231714
iter
      11 value -2.231729
     12 value -2.231741
iter
```

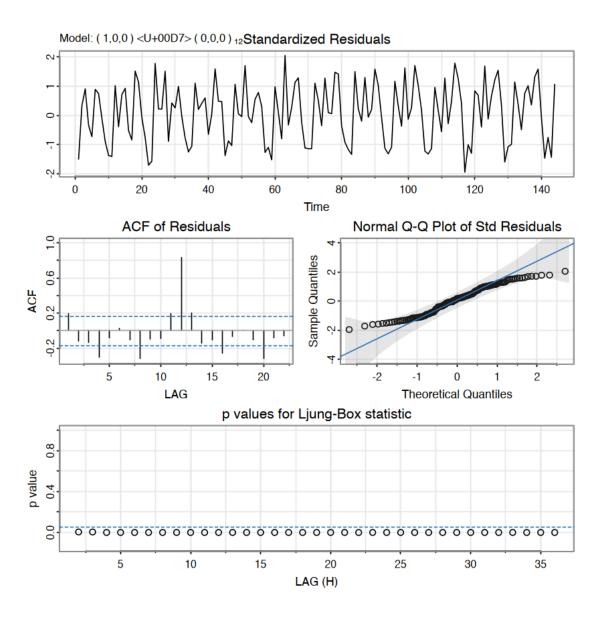
```
iter 13 value -2.231851
iter 14 value -2.231854
iter 15 value -2.231855
iter 16 value -2.231859
iter 17 value -2.231859
iter 18 value -2.231879
iter 19 value -2.231880
iter 20 value -2.231881
iter 21 value -2.231885
iter 22 value -2.231885
iter 23 value -2.231893
iter 23 value -2.231893
iter 23 value -2.231893
final value -2.231893
converged
```

Coefficients:

Estimate SE t.value p.value ar1 0.9781 0.0171 57.3534 0 xmean 5.4854 0.3181 17.2425 0

sigma^2 estimated as 0.01127042 on 142 degrees of freedom

AIC = -1.584242 AICc = -1.583651 BIC = -1.522371



```
[7]: mod <- sarima(
    data_ts,
    1, # AR
    0, # I
    0, # MA
    1, # AR_s
    0, # I_s
    0, # MA_s
    12
)
```

initial value -0.912435 iter 2 value -3.106051

```
3 value -3.118288
iter
       4 value -3.118682
iter
       5 value -3.118841
iter
iter
       6 value -3.118922
iter
       7 value -3.119519
       8 value -3.121222
iter
iter
       9 value -3.122354
iter
      10 value -3.122715
      11 value -3.123006
iter
      12 value -3.123424
iter
      13 value -3.123719
iter
      14 value -3.127010
iter
      15 value -3.142954
iter
      16 value -3.152645
iter
iter
      17 value -3.163801
      18 value -3.173404
iter
iter
      19 value -3.180734
      20 value -3.181914
iter
      21 value -3.182794
iter
iter
      22 value -3.183073
iter
      23 value -3.183084
      24 value -3.183338
iter
iter
      25 value -3.186227
      26 value -3.188607
iter
iter
      27 value -3.191853
      28 value -3.192674
iter
      29 value -3.193119
iter
iter
      30 value -3.193122
      31 value -3.193131
iter
iter
      32 value -3.193137
      33 value -3.193138
iter
      34 value -3.193154
iter
iter
      35 value -3.193178
      36 value -3.193223
iter
      37 value -3.193225
iter
iter
      38 value -3.193248
      39 value -3.193253
iter
iter
      40 value -3.193254
      41 value -3.193267
iter
     42 value -3.193278
iter
     43 value -3.193290
iter
      44 value -3.193291
iter
      45 value -3.193292
iter
      46 value -3.193293
iter
iter
      46 value -3.193293
      46 value -3.193293
final value -3.193293
converged
```

```
initial value -2.333650
iter
       2 value -2.982095
       3 value -2.988231
iter
       4 value -3.017026
iter
iter
       5 value -3.018893
       6 value -3.020718
iter
iter
       7 value -3.021906
iter
       8 value -3.024465
iter
       9 value -3.024555
iter
     10 value -3.024714
      11 value -3.024751
iter
iter
     12 value -3.024864
     13 value -3.024942
iter
iter
     14 value -3.024985
iter
     15 value -3.025017
     16 value -3.025037
iter
iter
     17 value -3.025210
     18 value -3.025257
iter
     19 value -3.025364
iter
     20 value -3.025422
iter
iter
     21 value -3.025507
iter
     22 value -3.025607
iter
     23 value -3.025686
iter
     24 value -3.025814
     25 value -3.025843
iter
     26 value -3.025957
iter
     27 value -3.026058
iter
iter
     28 value -3.026131
iter
     29 value -3.026181
iter
     30 value -3.026214
     31 value -3.026529
iter
iter
     32 value -3.026567
iter
     33 value -3.026660
     34 value -3.026705
iter
iter
     35 value -3.026822
iter
     36 value -3.027039
     37 value -3.027199
iter
     38 value -3.027353
     39 value -3.027372
iter
iter
     40 value -3.027471
    41 value -3.027632
iter
iter
     42 value -3.027950
     43 value -3.028316
iter
iter
     44 value -3.028456
iter
     45 value -3.029088
iter
     46 value -3.029223
     47 value -3.029538
iter
    48 value -3.029624
iter
```

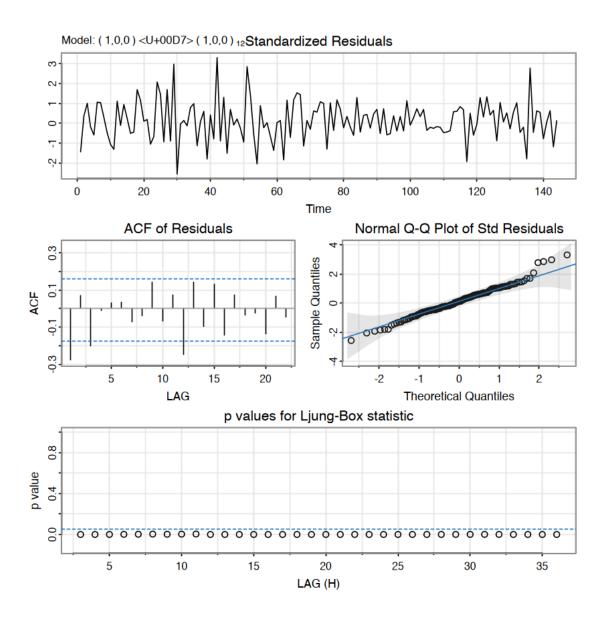
49 value -3.029652 iter iter 50 value -3.029763 iter 51 value -3.029803 52 value -3.030163 iter iter 53 value -3.030200 iter 54 value -3.030426 55 value -3.030635 iter 56 value -3.031067 iter 57 value -3.031960 iter 58 value -3.032545 59 value -3.033705 iter iter 60 value -3.033902 61 value -3.034408 iter iter 62 value -3.034738 iter 63 value -3.034781 64 value -3.034848 iter iter 65 value -3.034883 66 value -3.035200 iter 67 value -3.035299 iter 68 value -3.035548 iter iter 69 value -3.035819 iter 70 value -3.036228 iter 71 value -3.036968 iter 72 value -3.037472 iter 73 value -3.038425 74 value -3.038632 iter 75 value -3.038890 iter iter 76 value -3.039009 iter 77 value -3.039086 iter 78 value -3.039114 79 value -3.039137 iter iter 80 value -3.039286 iter 81 value -3.039349 82 value -3.039634 iter iter 83 value -3.040038 iter 84 value -3.040541 85 value -3.040854 iter 86 value -3.041001 87 value -3.041532 iter iter 88 value -3.041584 89 value -3.041587 iter iter 90 value -3.041588 91 value -3.041589 iter iter 92 value -3.041591 iter 93 value -3.041591 iter 94 value -3.041593 iter 95 value -3.041594 96 value -3.041598 iter

```
iter 97 value -3.041602
iter 98 value -3.041608
iter 99 value -3.041613
iter 100 value -3.041614
final value -3.041614
stopped after 100 iterations
Warning message in arima(xdata, order = c(p, d, q), seasonal = list(order = c(P, d, q))
"possibile errore di convergenza: optim ha restituito codice = 1"
Coefficients:
```

Estimate SE t.value p.value 0.9488 0.0220 43.1396 ar1 0.9094 0.0271 33.5774 0 sar1 xmean 5.5325 0.4599 12.0306 0

sigma^2 estimated as 0.001921555 on 141 degrees of freedom

AIC = -3.189796 AICc = -3.188606 BIC = -3.107301



```
[8]: mod1 <- sarima(
    data_ts,
    1, # AR
    0, # I
    0, # MA
    0, # AR_s
    1, # I_s
    0, # MA_s
    12
)</pre>
```

initial value -2.795206 iter 2 value -3.163852

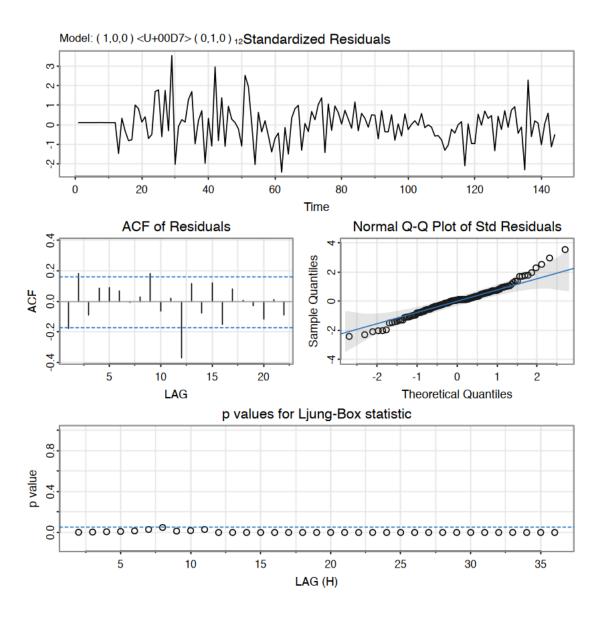
```
iter
      3 value -3.163901
iter 4 value -3.163932
iter 5 value -3.163935
iter 6 value -3.163944
iter 7 value -3.163944
iter 8 value -3.163944
iter 8 value -3.163944
      8 value -3.163944
iter
final value -3.163944
converged
initial value -3.155776
iter
    2 value -3.155917
     3 value -3.156145
iter
iter 4 value -3.156193
iter 5 value -3.156229
iter 6 value -3.156234
iter 7 value -3.156234
iter 8 value -3.156234
iter 8 value -3.156234
iter 8 value -3.156234
final value -3.156234
converged
```

Coefficients:

Estimate SE t.value p.value ar1 0.7255 0.0601 12.0676 0 constant 0.0097 0.0011 8.8101 0

sigma^2 estimated as 0.001803314 on 130 degrees of freedom

AIC = -3.429137 AICc = -3.428432 BIC = -3.363619



```
[9]: mod2 <- sarima(
    data_ts,
    1, # AR
    1, # I
    0, # MA
    0, # AR_s
    0, # I_s
    1, # MA_s
    12
)</pre>
```

initial value -2.239650 iter 2 value -2.556407

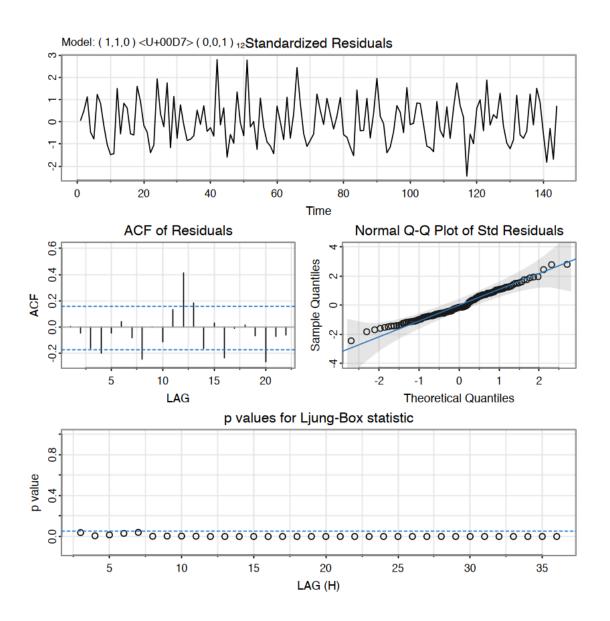
```
iter
      3 value -2.594623
iter 4 value -2.603738
iter 5 value -2.604535
iter 6 value -2.604618
iter 7 value -2.604619
iter 7 value -2.604619
iter 7 value -2.604619
final value -2.604619
converged
initial value -2.604052
iter
     2 value -2.604254
iter
      3 value -2.605169
iter 4 value -2.605170
iter 5 value -2.605170
iter 5 value -2.605170
      5 value -2.605170
iter
final value -2.605170
converged
<><><><><>
```

Coefficients:

Estimate SE t.value p.value ar1 0.1053 0.0834 1.2629 0.2087 sma1 0.7401 0.0607 12.1952 0.0000 constant 0.0078 0.0112 0.6984 0.4861

sigma^2 estimated as 0.005107866 on 140 degrees of freedom

AIC = -2.316519 AICc = -2.315312 BIC = -2.233642



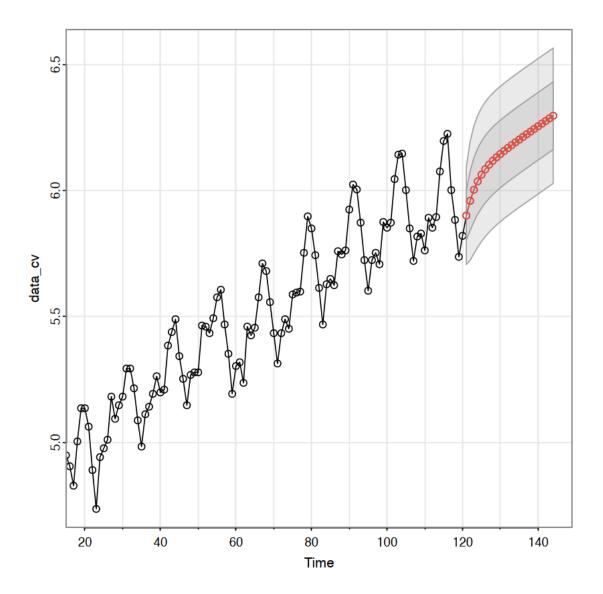
```
[10]: mod1$fit$aic
    mod2$fit$aic

-452.646059544319
    -331.262213554599

[11]: n = length(data_ts)
    n
    data_cv <- ts(data_ts[1:120])
    #data_ts

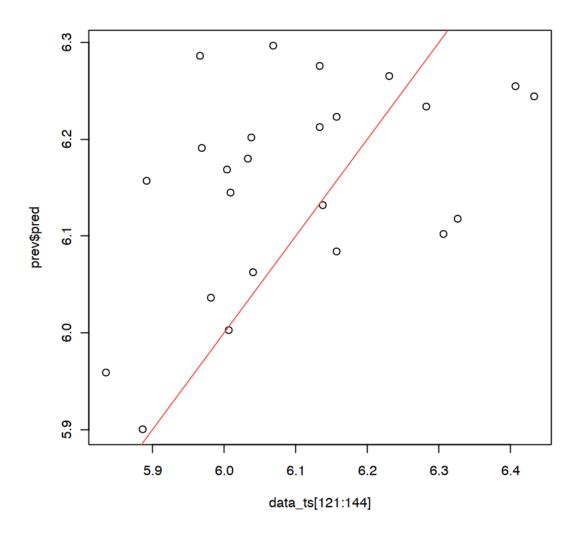
    prev <- mod2 <- sarima.for(
        data_cv,</pre>
```

```
xreg = data.frame(time = 1:120),
newxreg = data.frame(time = 121:144),
24,
1, # AR
0, # I
0, # MA
0, # AR_s
0, # I_s
0, # MA_s
12
```



```
[12]: plot(data_ts[121:144],prev$pred)
abline(a=0, b = 1, col=2)
mean((data_ts[121:144] - prev$pred)^2)
```

0.0237953299801296



```
[13]: data_diff <- diff(data_cv)
#plot(data_diff)

prev <- mod2 <- sarima.for(
    data_diff,</pre>
```

```
24,

10, # AR

0, # I

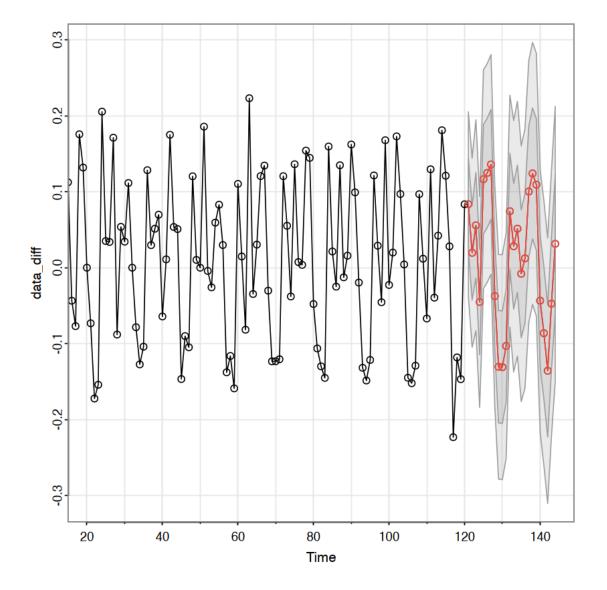
0, # MA

0, # AR_s

0, # I_s

0, # MA_s

12
```



1 GP

Simuliamo un GP con

$$C()=\sigma^2\exp(-\phi h)$$

o

$$C()=\sigma^2\exp(-\frac{h}{\phi})$$

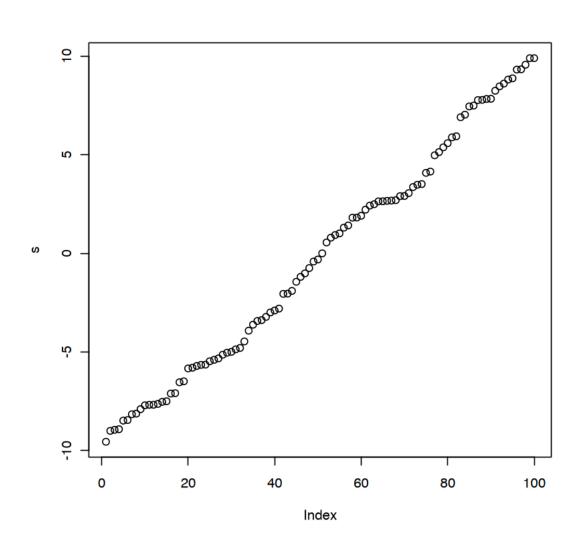
e media 0.

Asssumiamo di aver opsservato iol processo nei punti

$$s_1, \dots s_n$$

con

$$s_i \sim R$$



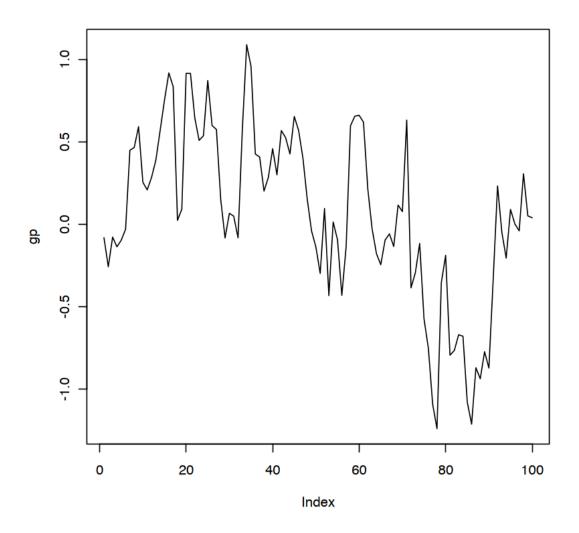
```
[18]: mean_function = rep(0, n)
    Sigma = matrix(NA, nrow=n , ncol= n)
    sigma2 = 1
    phi = 3/10

    for(i in 1:n)
    {
        for(j in 1:n)
        {
            Sigma[i, j] = sigma2 * exp(-phi * abs(s[i] - s[j]))
        }
    }
}
```

```
Sigma_chol = t(chol(Sigma))

gp <- mean_function + Sigma_chol%*% matrix(rnorm(n,0,1), ncol=1)

plot(gp, type = "l")</pre>
```



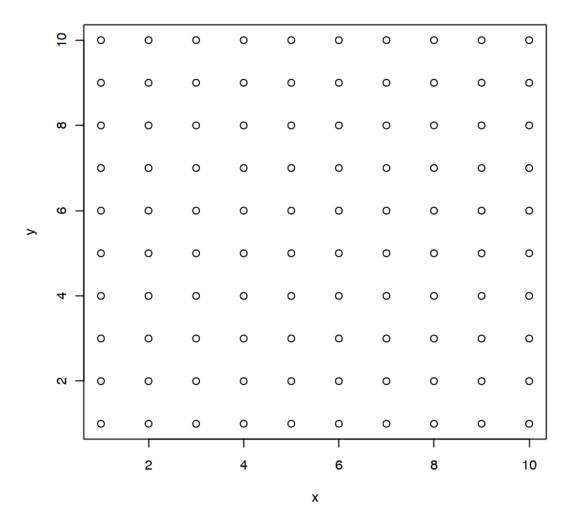
```
[]:
```

Simulazione di un processo gaussiano definito su \mathcal{R}^2 con funzione di covarianza esponenziale

```
[19]: n_grid = 10
x_grid = seq(1, n_grid, by = 1)
y_grid = seq(1, n_grid, by = 1)

data_grid = data.frame(x = rep(x_grid, times = n_grid), y = rep(y_grid, each = n_grid))

plot(data_grid)
```



```
[21]: n = n_grid^2
mu = rep(0, n)
```

```
sigma2 = 0.2
phi = 3/5
Sigma = matrix(NA, ncol=n, nrow=n)
dist_mat = as.matrix(dist(data_grid))
for(i in 1:n)
{
    for(j in 1:n)
    {
        Sigma[i,j] = sigma2 * exp(- phi * sqrt((data_grid[i, 1] - data_grid[j,u=1])^2 + (data_grid[i, 2] - data_grid[j, 2])^2))
    }
}
gp2 <- mu + t(chol(Sigma))%*%matrix(rnorm(n,0,1), ncol=1)</pre>
```

[24]: image(matrix(gp2, ncol=n_grid))

