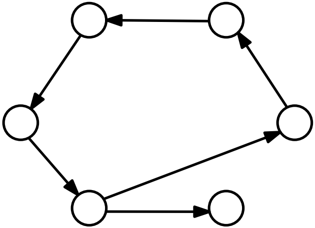
Problems in graph theory

Giacomo Como, DISMA, Politecnico di Torino

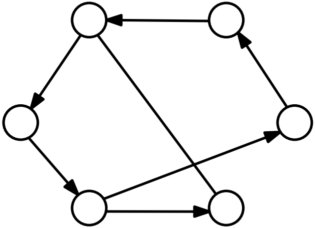
Fabio Fagnani, DISMA, Politecnico di Torino

Connectivity, condensation graph

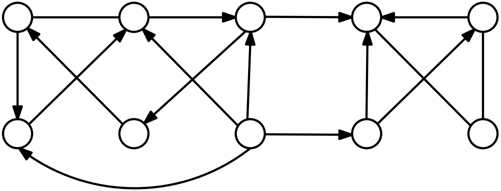
Study the connectivity of the following graphs (connected components, period, condensation graph).



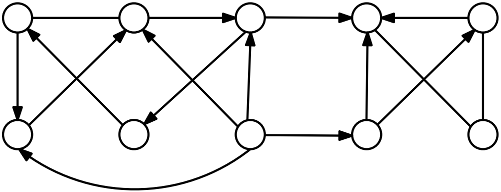
Connectivity, condensation graph



Connectivity, condensation graph



Connectivity, condensation graph



what is the minimal number of directed edges that needs to be added to make the graph strongly connected?

Bipartite graphs, chromatic number

1. Suppose $\mathcal{G}^1 = (\mathcal{V}^1, \mathcal{E}^1)$ and $\mathcal{G}^2 = (\mathcal{V}^2, \mathcal{E}^2)$ are two simple bipartite graphs. Prove that

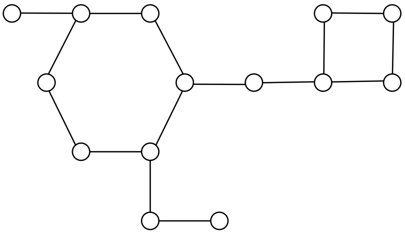
$$\mathcal{G} = (\mathcal{V}^1 \cup \mathcal{V}^2, \mathcal{E}^1 \cup \mathcal{E}^2 \cup \{(i,j),(j,i)\})$$

where $i \in \mathcal{V}^1$ and $j \in \mathcal{V}^2$ is a bipartite graph.

2. Prove that trees are bipartite.

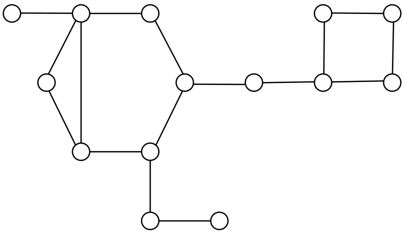
Bipartite graphs, chromatic number

Are the following graphs bipartite?



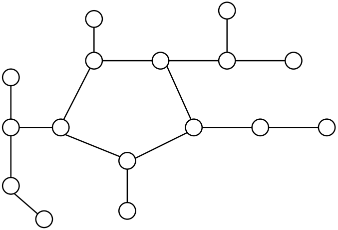
Bipartite graphs, chromatic number

Are the following graphs bipartite?



Bipartite graphs, chromatic number

Compute the chromatic number of the following graph

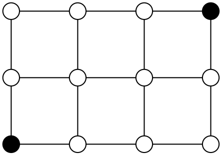


Geodesic distance, diameter

Compute the diameter of the graphs considered before.

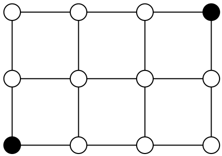
Geodesic distance, diameter

Compute the number of geodetic paths between the two nodes below, in the grid graph:



Geodesic distance, diameter

Compute the number of geodetic paths between the two nodes below, in the grid graph:



What about in the grid $L_h \times L_k$ for generic h and k?

Regular graphs

An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$ is called *regular* if all nodes have the same degree: $w_i = w_j$ for every $i, j \in \mathcal{V}$.

1. Is it possible to find simple regular graphs with 6 nodes having common degree, respectively, 2, 3, 4, and 5?
2. Is it possible to find a simple regular graph with 7 nodes having common degree 3?