Ratta Marco - Modelli Statistici
Politecnico di Torino - 24/10/24

Exercises on JAGS

1. Assume a population of $N = 100$ individuals. Each individual has a binary response variable $y_i \sim \text{Bernoulli}(p)$, with $i = 1, \dots N$. Assume a total of $S = \sum_{i=1}^{N} y_i = 34$ successes is observed and a Beta$(1, 1)$ prior distribution - equivalent to a Uniform$(0, 1)$ - is used for the parameter p.

   - Construct the appropriate model using JAGS and find the posterior distribution for p.
   - Repeat the previous analysis assuming $N = 200$ and $S = 68$. How does the posterior on p change? Why?
   - Repeat the analysis considering different Beta priors (pessimistic, optimistic). How does the posterior on p change?

2. In a randomized controlled trial (RCT) a medicine is given to a group of patients (treatment group), while a placebo is given to another group of patients (control group) in order to detect the mean effect of the medicine. Assume 100 patients are randomized in 1:1 proportion to the two groups and their binary responses y_i^c and y_i^t is available (1 if the patients recovers, 0 otherwise). Assume the following:

   - Responses are Bernoulli, so that $y_i^c \sim \text{Bernoulli}(p^c)$ and $y_i^t \sim \text{Bernoulli}(p^t)$.
   - No prior information is available on the parameters p^c and p^t.
   - The trial is declared successful if the posterior probability of the log odds-ratio being larger than 0 is above 95%, where $\log(OR) = \log\left(\frac{p^t(1-p^c)}{p^c(1-p^t)}\right)$.

   Answer the following questions:

   - What is the probability of having a successful trial (Power) if $p^c = 0.5$ and $p^t = 0.6$?
   - How does the Power change if 200 patients are recruited (keeping 1:1 proportion between arms)? Explains why.
   - How does the Power change if $p^c = 0.5$ and $p^t = 0.7$? Explains why.

   Hint: Do a *for* cycle with many iterations (1000 is enough) generating each time the binary responses y_i^c and y_i^t and approximate the Power as the fraction of successful trials.

3. The owner of a gym organizes a challenge, which consists in lifting a 200 kilos block of stone. Among the users of the gym, 100 decide to participate to the competition.
   The body-weight of each athlete (X) and his success in the challenge (Y) are available in the data-frame in the *lifting.RData* file.
   Suppose the following assumptions hold:

   - The body-weights are normally distributed with mean $\mu = 75$ Kg and standard deviation $\sigma = 10$ Kg. Call x_i the standardization of the body-weight of each athlete.

- The log-odds of being successful in the challenge vary linearly with the body weight, so that $\text{logit}(p_i) = a + bx_i$.
- No prior information is available on the parameters of the linear relationship a and b (use Normal prior distribution with large variance).

Answer the following questions:

- Compute the posterior probability that the body-weight co-variate is positively correlated with the success of the challenge
- What is the mean probability of success in the challenge for an athlete whose body-weight is 75 Kg?
- How much should an athlete weigh to have a 80% mean probability of success in the challenge?