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**Exercises on DAGs**

1. Consider a binary screening situation formulated in terms of the following random variables:

   - $D?$ = indicator whether a randomly sampled person is diseased
   - $T?$ = indicator whether a single test given to the person is positive
   - $T_1?$, $T_2?$ = similar indicators for two conditionally independent tests

   and compute the following probabilities.

   (a) Consider a single test. Assuming:
   - $\mathrm{P}(\mathrm{D?} = 1) = 0.01$ (prevalence),
   - $\mathrm{P}(\mathrm{T?} = 1|\mathrm{D?} = 1) = 0.90$ (sensitivity)
   - and $\mathrm{P}(\mathrm{T?} = 0|\mathrm{D?} = 0) = 0.95$ (specificity),

   compute
   $$\mathrm{P}(\mathrm{D?} = 1|\mathrm{T?} = 1) \quad \text{(positive predictive value)}$$

   (b) Consider repeating two conditionally independent tests on the same person. Assume sensitivity and specificity of $T_1?$ and $T_2?$ are the same as $T?$. Compute
   $$\mathrm{P}(\mathrm{D?} = 1|\mathrm{T}_1? = 1, \mathrm{T}_2? = 1).$$

2. Suppose you are working for a financial institution and you are asked to build a fraud detection system. You plan to use the following information. When the card holder is traveling abroad, fraudulent transaction are more likely since tourists are prime targets for thieves. More precisely, 2% of transactions are fraudulent when the card holder is traveling, whereas only 1% of the transactions are fraudulent when he is not traveling. On average, 5% of all transactions happen while card holder is traveling. If a transaction is fraudulent, then the likelihood of a purchase abroad increases, unless the card holder happens to be traveling. More precisely, when the card holder is not traveling, 10% of the fraudulent transactions are abroad purchases, whereas only 1% of the legitimate transactions are abroad purchases. On the other hand, when the card holder is traveling, then 90% of the transactions are abroad purchases regardless of the legitimacy of the transactions.

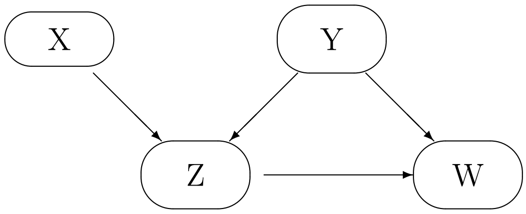
   (a) construct a DAG representation of $(T?, F?, A?)$, where
   - $T?$ indicates whether the card holder is traveling;
   - $F?$ indicates whether the transaction is fraudulent;
   - $A?$ indicates whether the purchase is abroad.

   and write its joint density function;

   (b) compute the conditional probability that $F? = 1$ if $A? = 1$;

   (c) compute the conditional probability that $F? = 1$ if $A? = 1$ and $T? = 1$.

3. Let $X$, $Y$, $Z$ and $W$ be binary variables with a joint distribution represented by the following DAG




and such that

- $P(X = 1) = 0.7$; $P(Y = 1) = 0.3$;
- $P(Z = 1|X = 0 \cap Y = 0) = 0.2$; $P(Z = 1|X = 0 \cap Y = 1) = 0.3$;
- $P(Z = 1|X = 1 \cap Y = 0) = 0.2$; $P(Z = 1|X = 1 \cap Y = 1) = 0.4$;
- $P(W = 1|Y = 0 \cap Z = 0) = 0.2$; $P(W = 1|Y = 0 \cap Z = 1) = 0.3$;
- $P(W = 1|Y = 1 \cap Z = 0) = 0.2$; $P(W = 1|Y = 1 \cap Z = 1) = 0.4$ .

(a) Compute the probability that at least one variable equals 1.

(b) Having observed $Y = 0$ and $Z = 0$, compute the probability that $X = 1$.

**Solutions**

- 1.(a) 0.154
- 1.(b) 0.766
- 2.(a) 0.03
- 2.(b) 0.02
- 3.(a) 0.8656
- 3.(b) 0.7