24-11-19

November 19, 2024

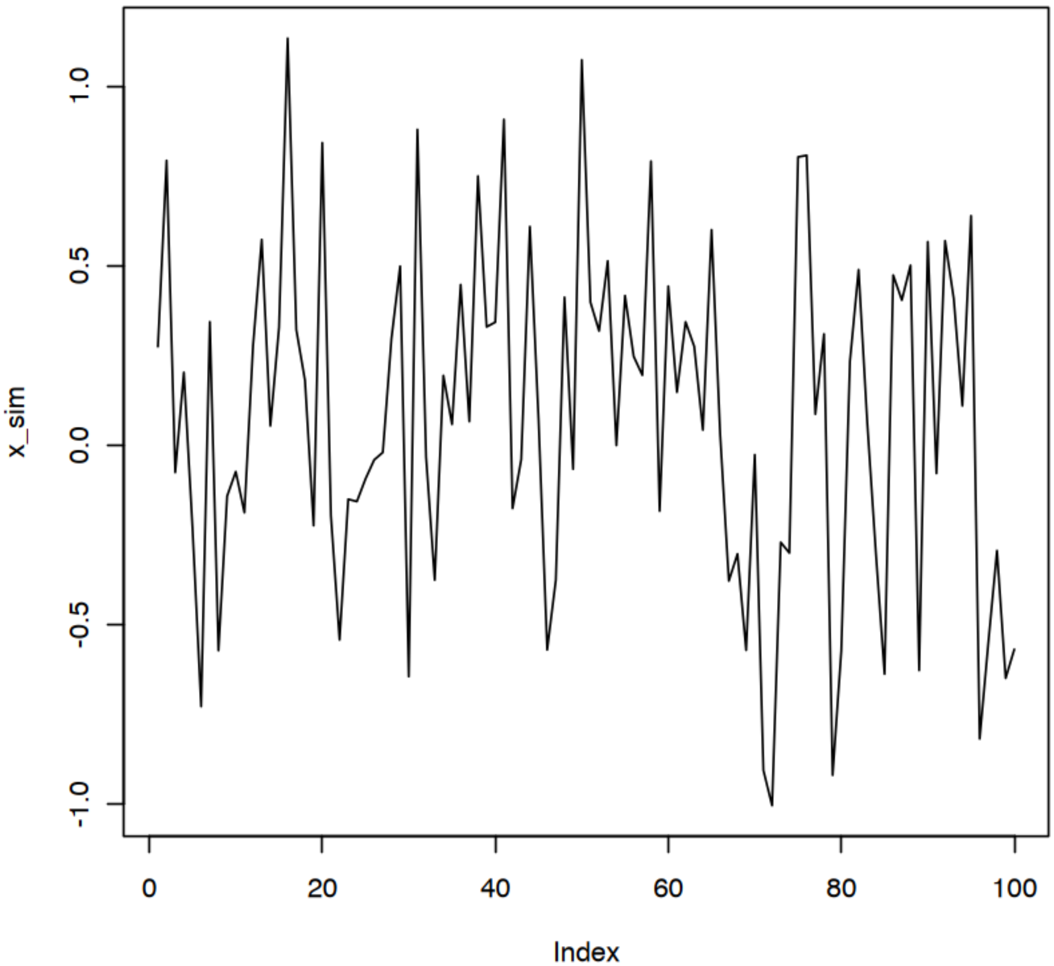
1 Serie temporali

$$x_t = \alpha x_{t-1} + w_t$$
$$w_t \sim N(0, \sigma^2)$$

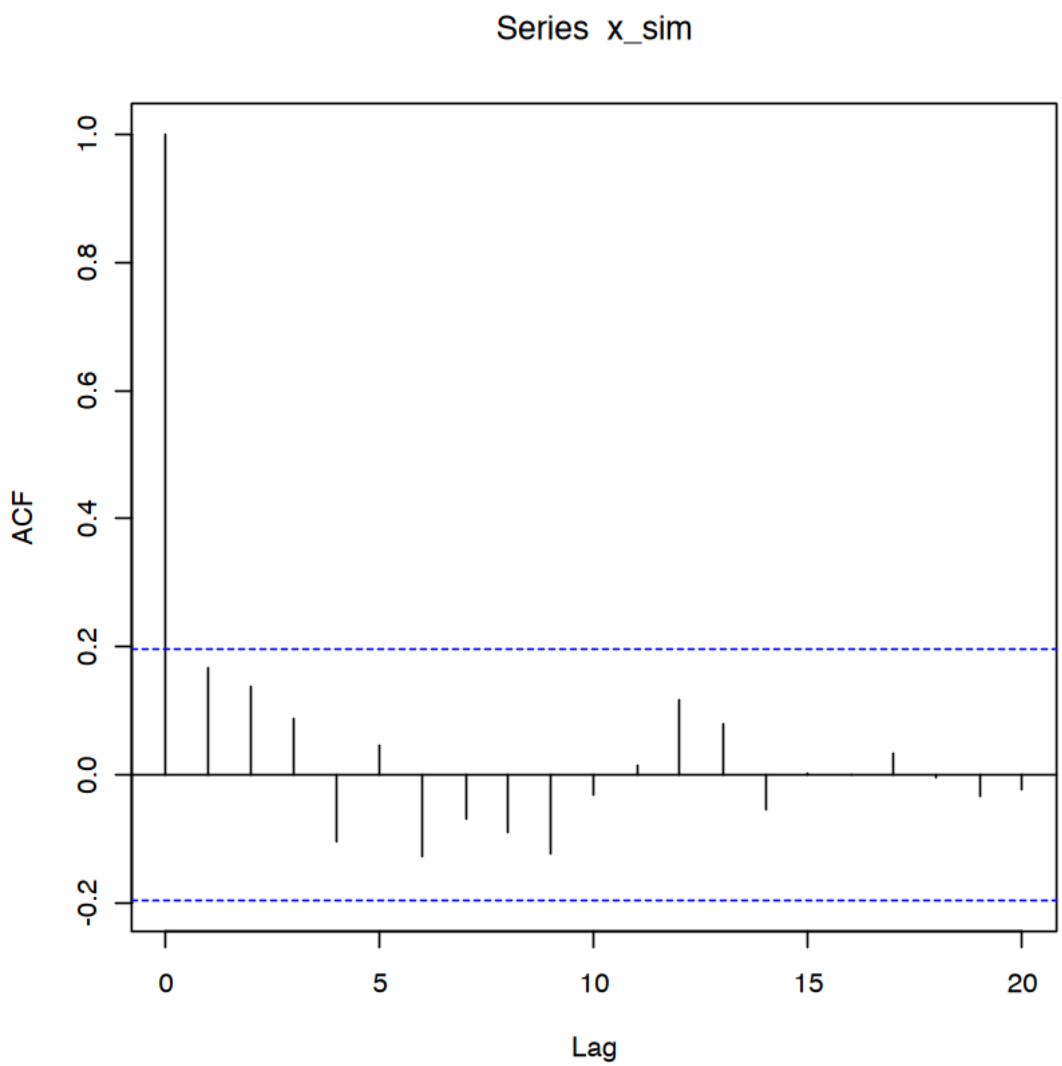
$$Cor(X_t, X_{t+k}) = \alpha^k$$

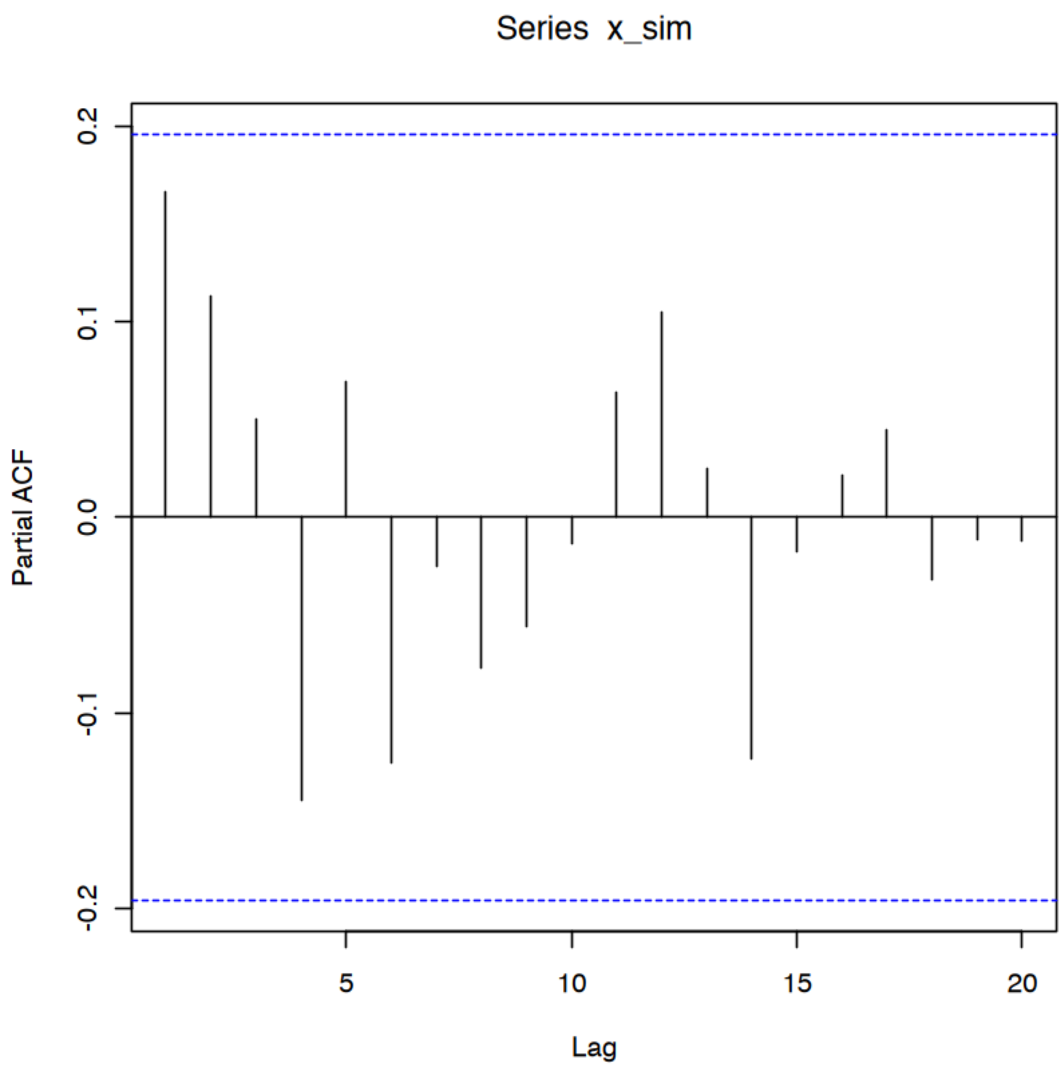
[1]:
```
n = 100
x_sim <- rep(NA, n)
sigma2 <- 0.2
alpha = 0.1


x_sim[1] <- rnorm(1, 0, (sigma2 / (1 - alpha^2))^0.5)
for(i in 2:n)
{
  x_sim[i] = rnorm(1, alpha * x_sim[i - 1], sigma2^0.5) ## f(x_t | x_{t-1},\
 ↪dots x_{1}) = f(x_t | x_{t-1})
}
```

[8]:
```
plot(x_sim, type="l")
acf(x_sim)
pacf(x_sim)
```



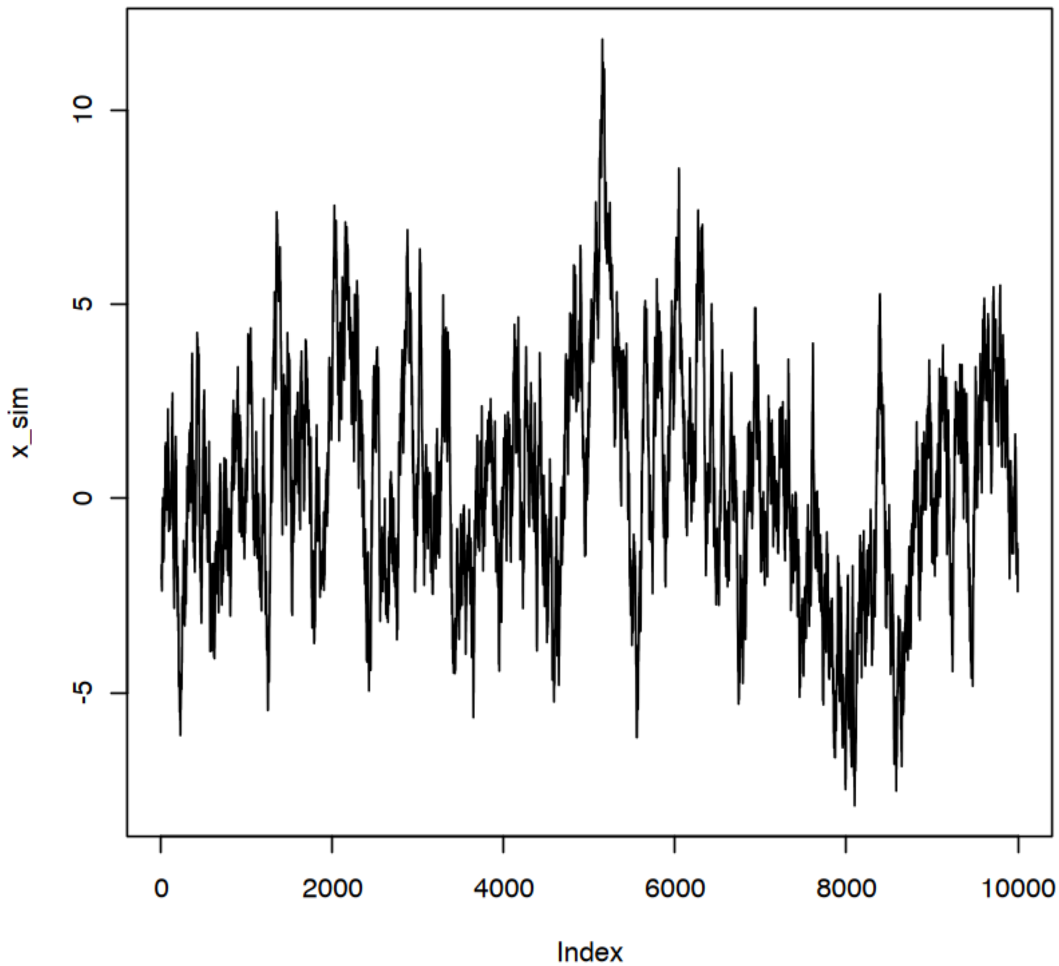
x_sim
1.0
0.5
0.0
-0.5
-1.0
0
20
40
60
80
100
Index

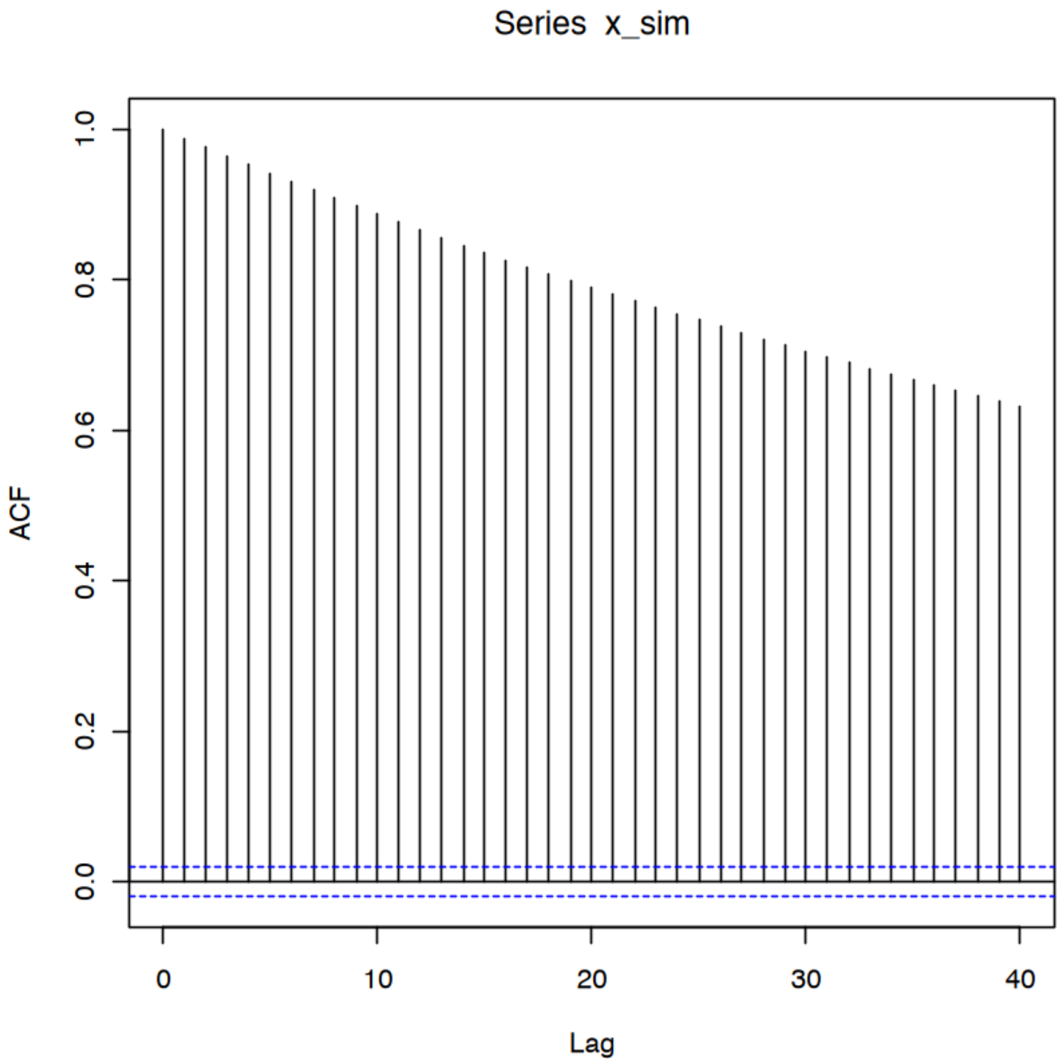

Series x_sim
ACF
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
5
10
15
20
Lag

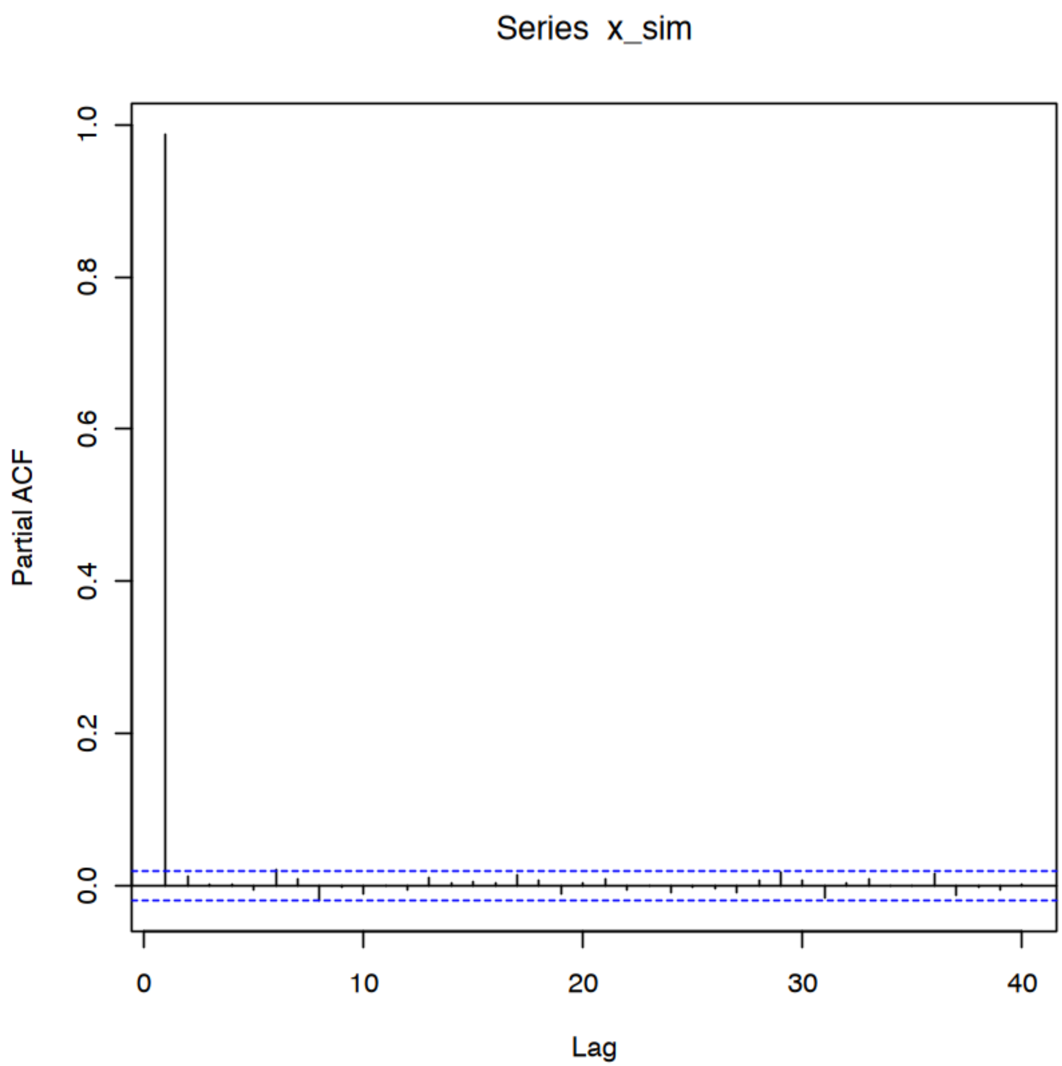


[13]:
```
n <- 10000
x_sim <- rep(NA, n)
sigma2 <- 0.2
alpha <- 0.99


x_sim[1] <- rnorm(1, 0, (sigma2 / (1 - alpha^2))^0.5)
for (i in 2:n)
{
  x_sim[i] <- rnorm(1, alpha * x_sim[i - 1], sigma2^0.5) ## f(x_t | x_{t-1},\
 ↪dots x_{1}) = f(x_t | x_{t-1})
}
```

[ ]:
```
plot(x_sim, type = "l")
acf(x_sim)
pacf(x_sim)
```

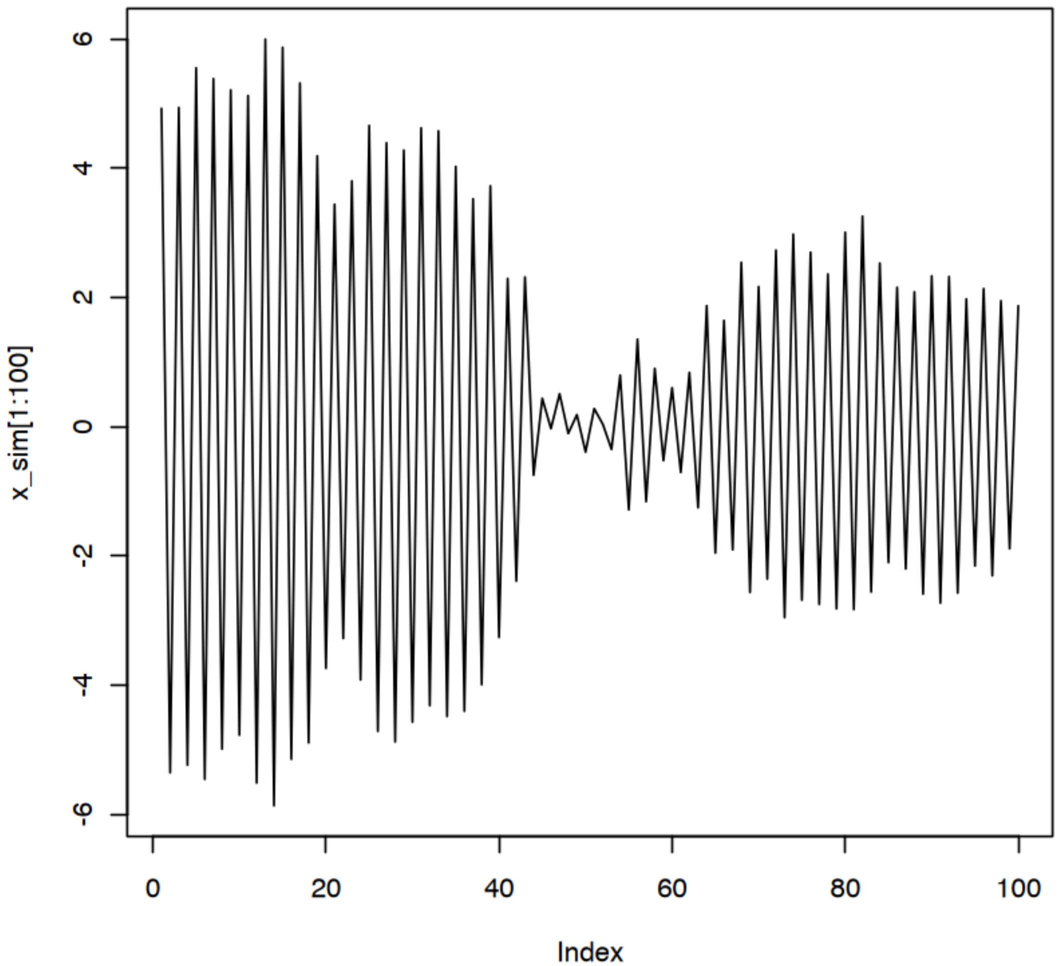


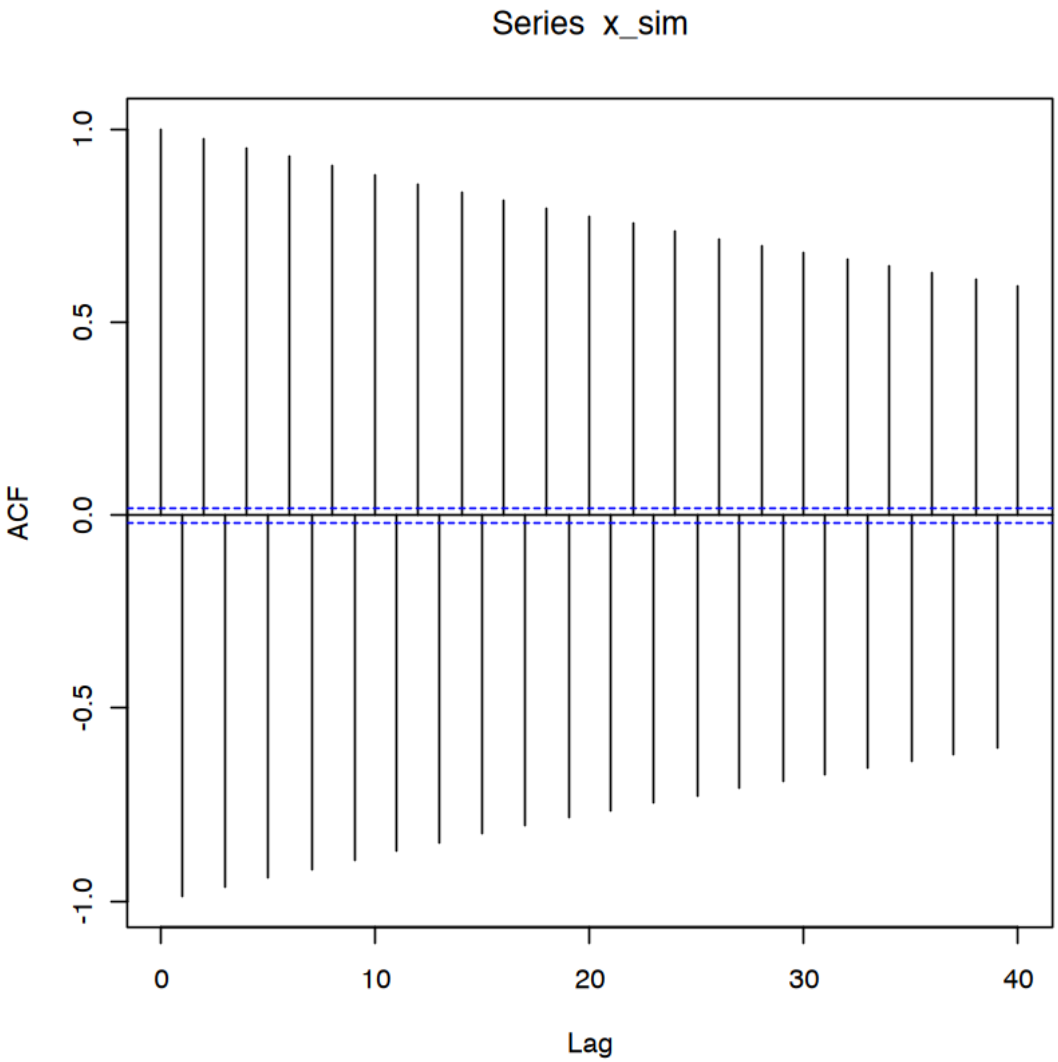

Series x_sim
ACF
Lag
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40

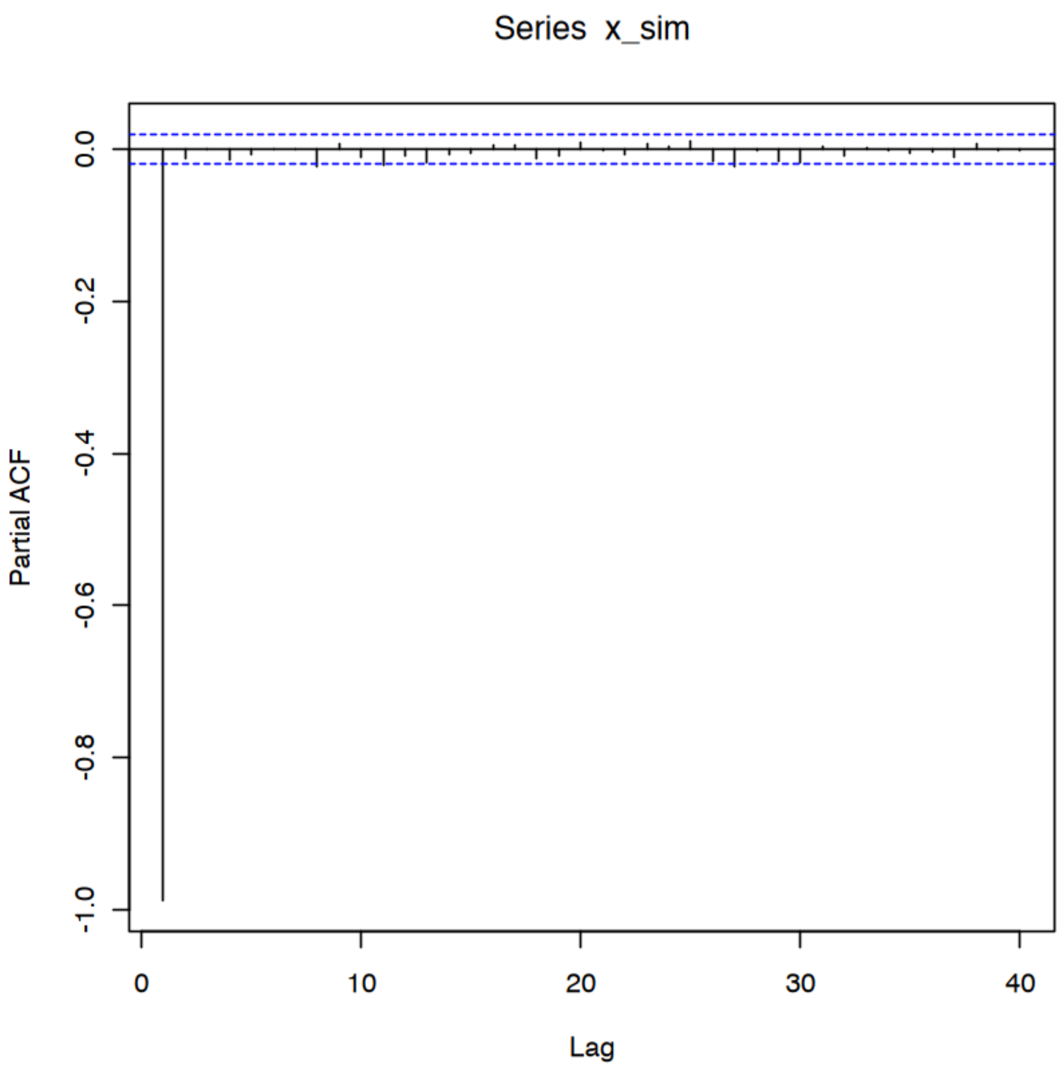



```
[ ]: n <- 10000
     x_sim <- rep(NA, n)
     sigma2 <- 0.2
     alpha <- -0.99


     x_sim[1] <- rnorm(1, 0, (sigma2 / (1 - alpha^2))^0.5)
     for (i in 2:n)
     {
       x_sim[i] <- rnorm(1, alpha * x_sim[i - 1], sigma2^0.5) ## f(x_t | x_{t-1},\
      ↪dots x_{1}) = f(x_t | x_{t-1})
     }
```

[17]:
```
plot(x_sim[1:100], type = "l")
acf(x_sim)
pacf(x_sim)
```

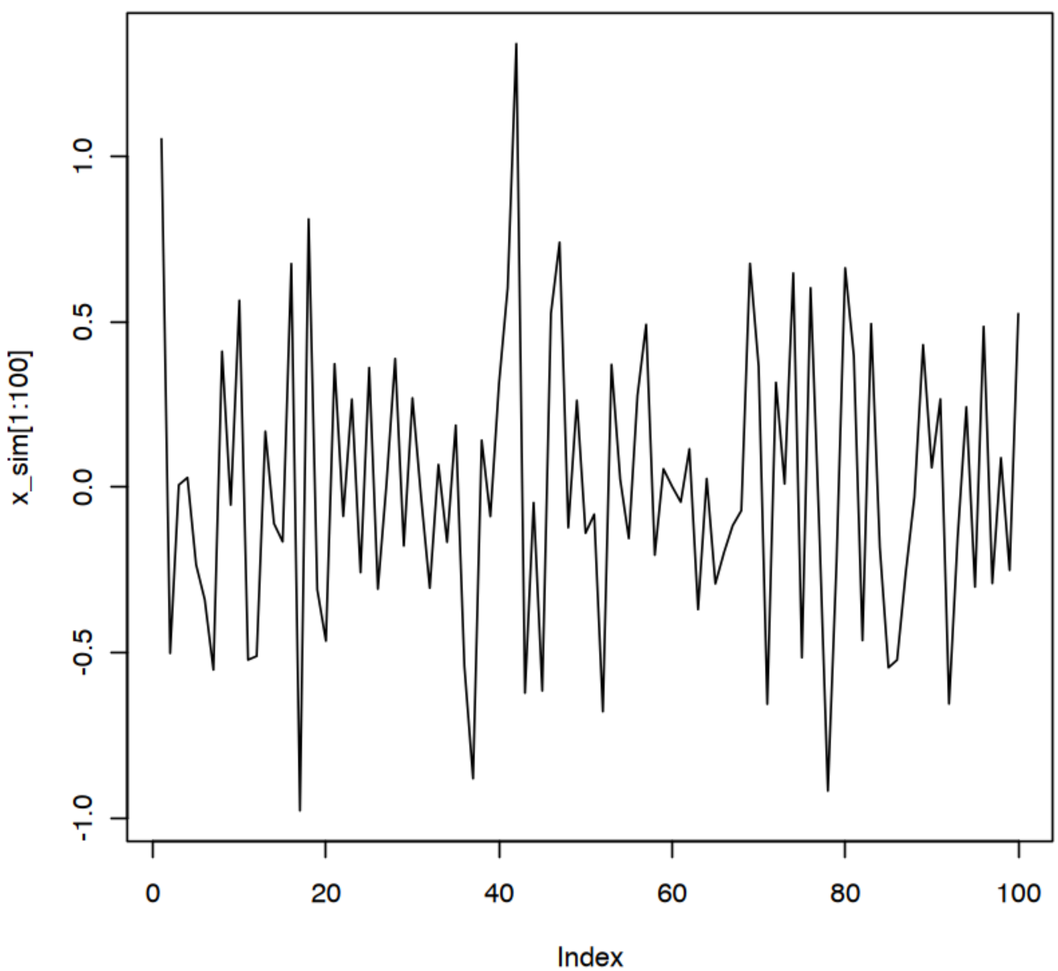


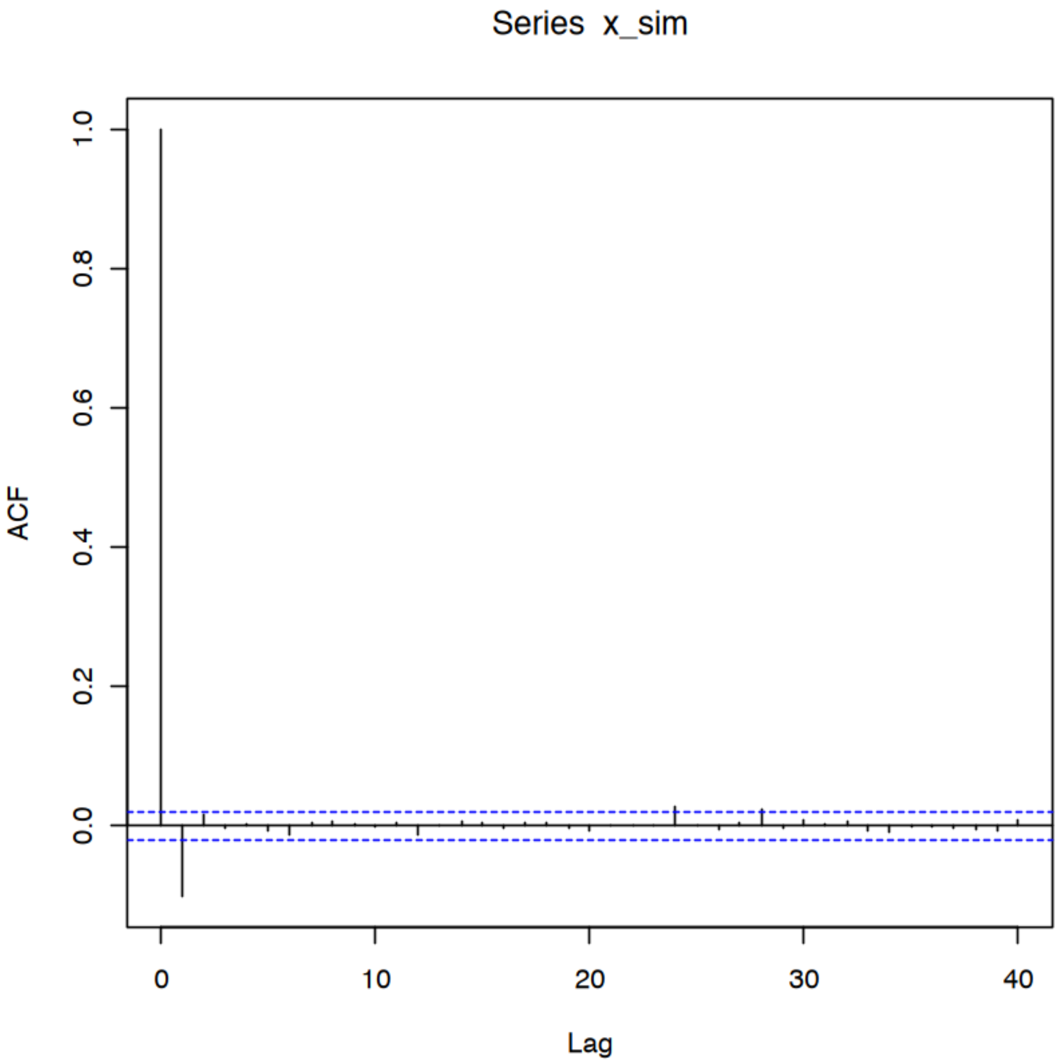

Series x_sim
ACF
Lag
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40

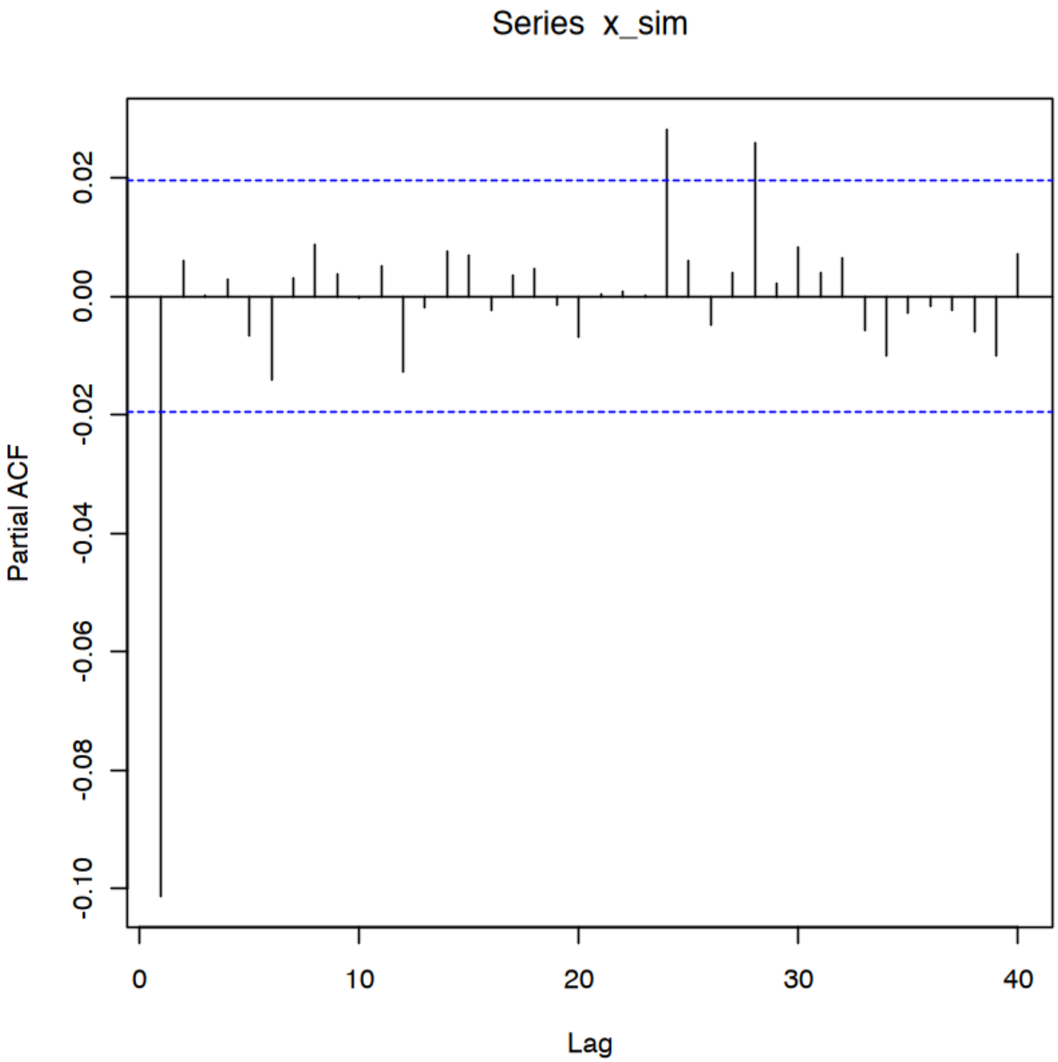


[18]:
```
n <- 10000
x_sim <- rep(NA, n)
sigma2 <- 0.2
alpha <- -0.1


x_sim[1] <- rnorm(1, 0, (sigma2 / (1 - alpha^2))^0.5)
for (i in 2:n)
{
  x_sim[i] <- rnorm(1, alpha * x_sim[i - 1], sigma2^0.5) ## f(x_t | x_{t-1},\
 ↪dots x_{1}) = f(x_t | x_{t-1})
}
plot(x_sim[1:100], type = "l")
```

```
acf(x_sim)
pacf(x_sim)
```




Series x_sim
ACF
Lag
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40



2 MA(1)

$$x_t = w_t + \beta w_{t-1}$$

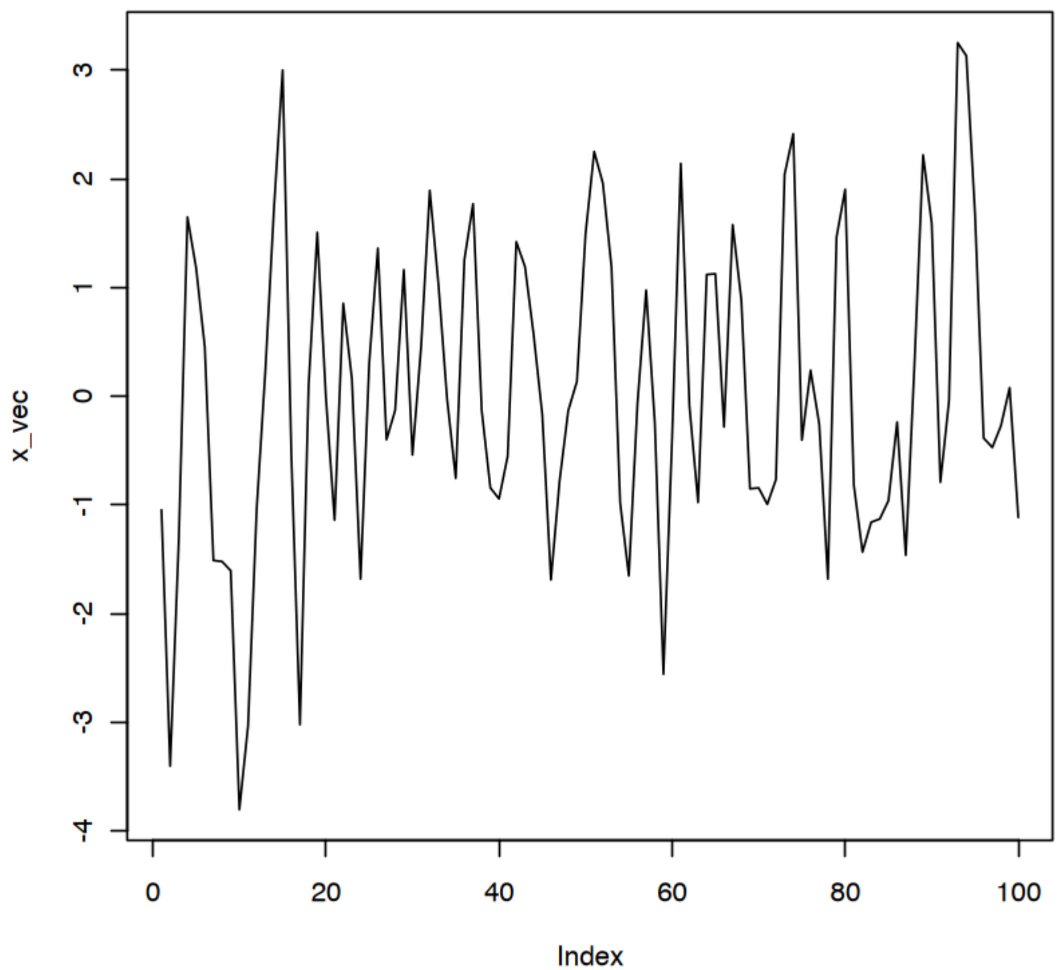
Se voglio simulare devo simulare da

$$f(\mathbf{x}) = f(x_1)f(x_2|x_1)f(x_3|x_2,x_1)f(x_4|x_3,x_2,x_1)\ldots f(x_t|x_{t-1},\ldots,x_1)$$

[35]:
```
sigma2 = 1.2
beta = 0.5
n = 100
x_vec = rep(NA, n)
w <- rnorm(n, 0, sigma2^0.5)
```

```r
w_0 <- rnorm(1, 0, sigma2^0.5)
x_vec[1] = w[1] + beta * w_0
for(i in 2:n)
{
  x_vec[i] = w[i] + beta * w[i-1]
}
```

[32]:
```r
plot(x_vec, type="l")
```



$$\mathbf{X} \sim N(\mathbf{0}, \Sigma)$$
$$[\Sigma]_{j,j} = \sigma^2(1 + \beta^2)$$
$$[\Sigma]_{j,j+1} = [\Sigma]_{j+1,j} = \beta\sigma^2$$

$$[\Sigma]_{j,j+c} = [\Sigma]_{j+c,j} = 0$$
$$c > 2$$

[ ]:
```
Sigma = matrix(0, nrow=n, ncol=n)
for(i in 1:n)
{
  Sigma[i,i] = sigma2*(1+beta^2)
}
for(j in 1:(n-1))
{
  Sigma[j, j+1] = sigma2 *beta
  Sigma[ j + 1,j] = sigma2 * beta
}
Sigma[1:10,1:10]
```

A matrix: 10 x 10 of type dbl

1.5	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	1.5	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.6	1.5	0.6	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.6	1.5	0.6	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.6	1.5	0.6	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.6	1.5	0.6	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.6	1.5	0.6	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.6	1.5	0.6	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	1.5	0.6
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	1.5

$$\Sigma = \mathbf{C}\mathbf{C}^T$$
$$\mathbf{X} = \mathbf{C}\mathbf{Z}$$

con

$$\mathbf{Z} \sim N(0, \mathbf{I})$$

[40]:
```
C = t(chol(Sigma))
(C %*% t(C))[1:5,1:5]
Sigma[1:5, 1:5]

Z = matrix(rnorm(n, 0, 1), ncol=1)
X = C %*% Z
```

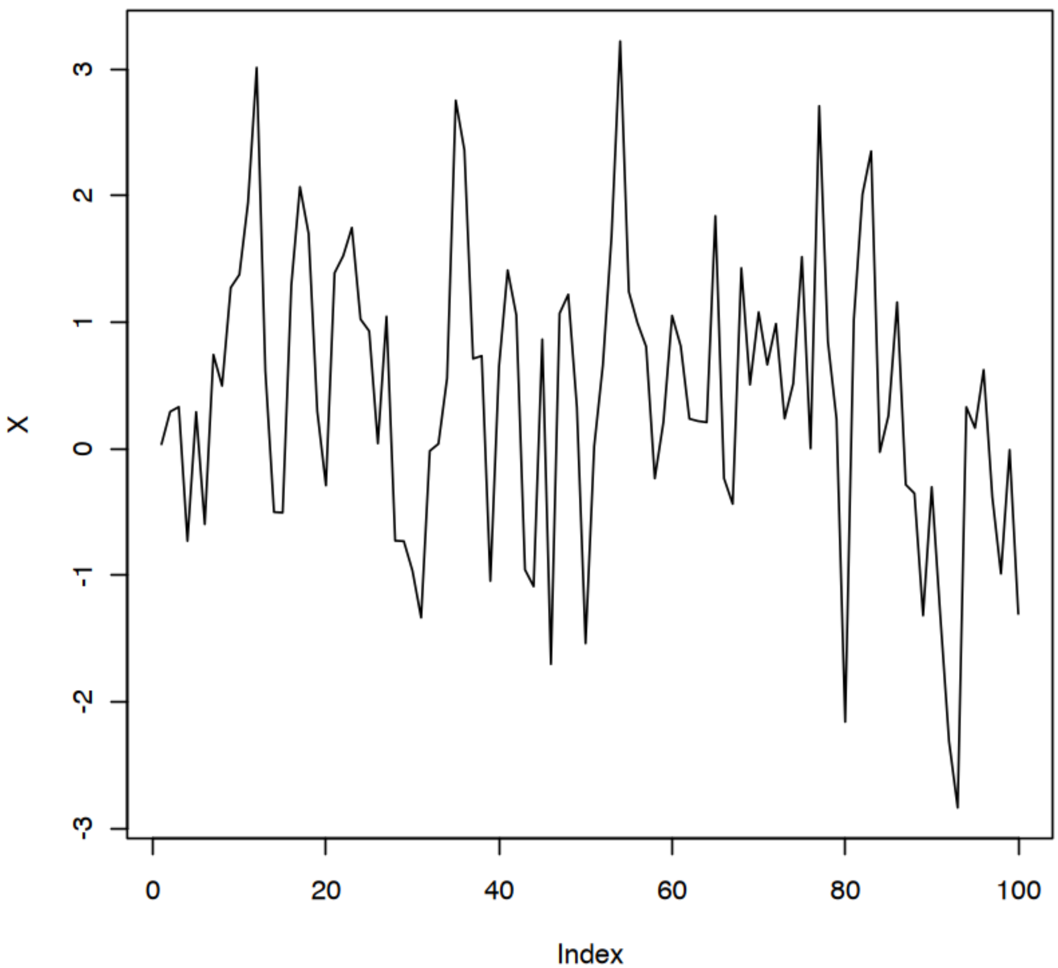
A matrix: 5 x 5 of type dbl

1.5	0.6	0.0	0.0	0.0
0.6	1.5	0.6	0.0	0.0
0.0	0.6	1.5	0.6	0.0
0.0	0.0	0.6	1.5	0.6
0.0	0.0	0.0	0.6	1.5

A matrix: 5 x 5 of type dbl

1.5	0.6	0.0	0.0	0.0
0.6	1.5	0.6	0.0	0.0
0.0	0.6	1.5	0.6	0.0
0.0	0.0	0.6	1.5	0.6
0.0	0.0	0.0	0.6	1.5

[41]:
```
plot(X, type="l")
```



[43]:
```
solve(Sigma)[1:5, 1:5]
```

A matrix: 5 x 5 of type dbl

0.83333333	-0.4166667	0.2083333	-0.1041667	0.05208333
-0.41666667	1.0416667	-0.5208333	0.2604167	-0.13020833
0.20833333	-0.5208333	1.0937500	-0.5468750	0.27343750
-0.10416667	0.2604167	-0.5468750	1.1067708	-0.55338542
0.05208333	-0.1302083	0.2734375	-0.5533854	1.11002604

$$x_t = \alpha x_{t-1} + w_t$$

$$Cov(X_t, X_{t-k}) = \frac{\sigma^2 \alpha^k}{1 - \alpha^2}$$

[46]:

```
n = 10
alpha = 0.5
sigma=1
Sigma_ar = matrix(0, ncol=n , nrow=n)


for(irow in 1:n)
{
  for(icol in 1:n)
  {
    Sigma_ar[irow, icol] = (sigma2 / (1 - alpha^2))* alpha^( abs(irow-icol) )
  }
}
Sigma_ar
```

A matrix: 10 x 10 of type dbl

1.600000	0.80000	0.4000	0.200	0.10	0.05	0.025	0.0125	0.00625	0.0031
0.800000	1.60000	0.8000	0.400	0.20	0.10	0.050	0.0250	0.01250	0.0062
0.400000	0.80000	1.6000	0.800	0.40	0.20	0.100	0.0500	0.02500	0.0125
0.200000	0.40000	0.8000	1.600	0.80	0.40	0.200	0.1000	0.05000	0.0250
0.100000	0.20000	0.4000	0.800	1.60	0.80	0.400	0.2000	0.10000	0.0500
0.050000	0.10000	0.2000	0.400	0.80	1.60	0.800	0.4000	0.20000	0.1000
0.025000	0.05000	0.1000	0.200	0.40	0.80	1.600	0.8000	0.40000	0.2000
0.012500	0.02500	0.0500	0.100	0.20	0.40	0.800	1.6000	0.80000	0.4000
0.006250	0.01250	0.0250	0.050	0.10	0.20	0.400	0.8000	1.60000	0.8000
0.003125	0.00625	0.0125	0.025	0.05	0.10	0.200	0.4000	0.80000	1.6000

[48]:

```
round(solve(Sigma_ar),12)
```

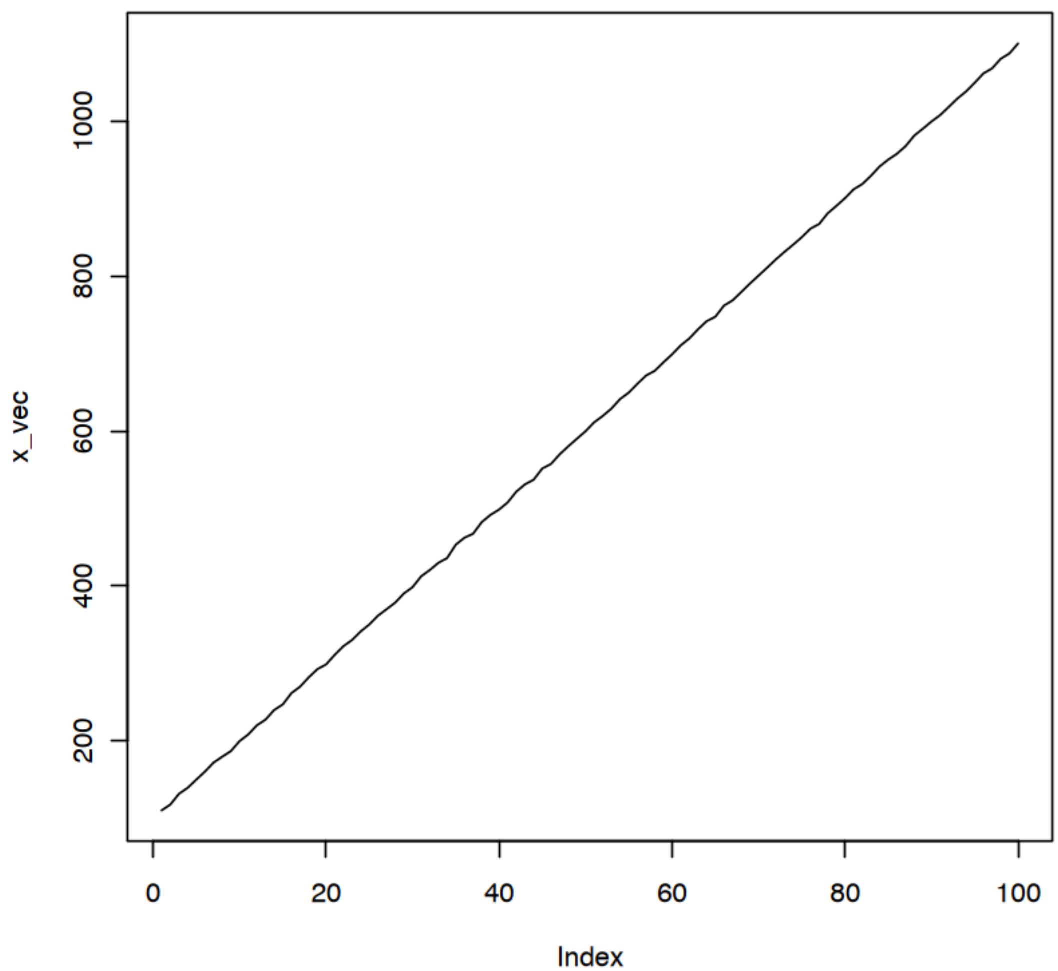
A matrix: 10 x 10 of type dbl

0.8333333	-0.4166667	0.0000000	0.0000000	0.0000000	0.0000000	0.000
-0.4166667	1.0416667	-0.4166667	0.0000000	0.0000000	0.0000000	0.000
0.0000000	-0.4166667	1.0416667	-0.4166667	0.0000000	0.0000000	0.000
0.0000000	0.0000000	-0.4166667	1.0416667	-0.4166667	0.0000000	0.000
0.0000000	0.0000000	0.0000000	-0.4166667	1.0416667	-0.4166667	0.000
0.0000000	0.0000000	0.0000000	0.0000000	-0.4166667	1.0416667	-0.41
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.4166667	1.041
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	-0.41
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.000

$$x_t = a + bt + w_t$$

[58]:
```
n = 100
a = 100
b = 10
sigma2 = 2

x_vec = rnorm(n, a+b*(1:n), sigma2^0.5)
```

[59]:
```
plot(x_vec , type="l")
```

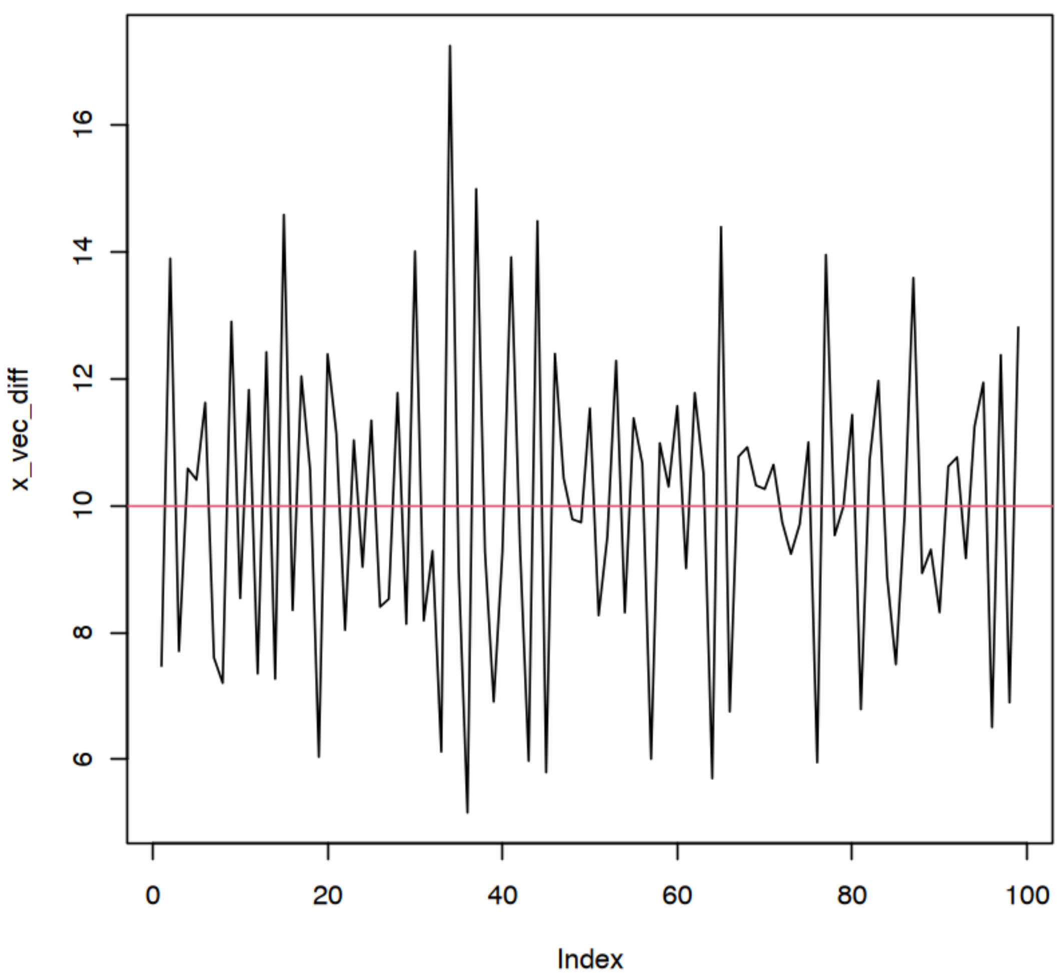


[60]:

```
x_vec_diff = rep(NA, n)

for(i in 2:n)
{
  x_vec_diff[i] = x_vec[i] - x_vec[i-1]
}
x_vec_diff = x_vec_diff[-1]
```

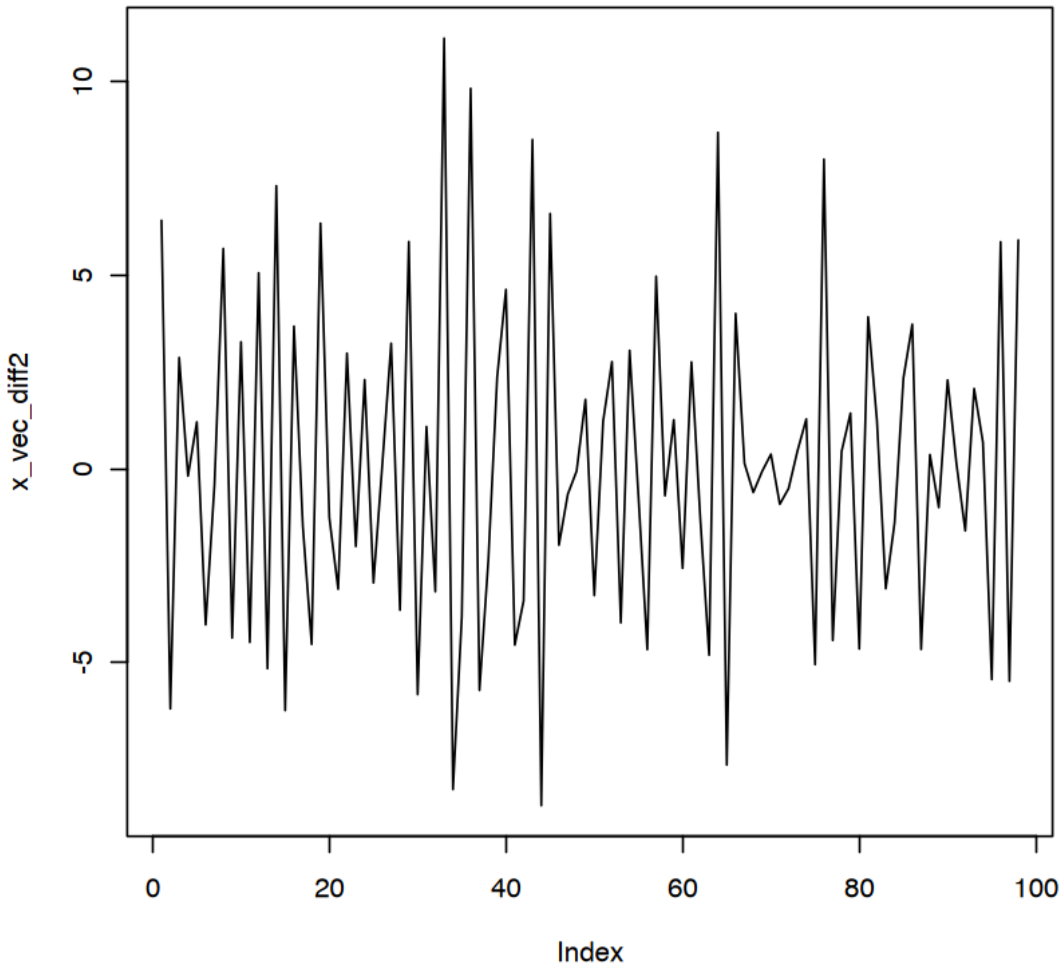
[62]:

```
plot(x_vec_diff, type="l")
abline(h = b , col=2)
```



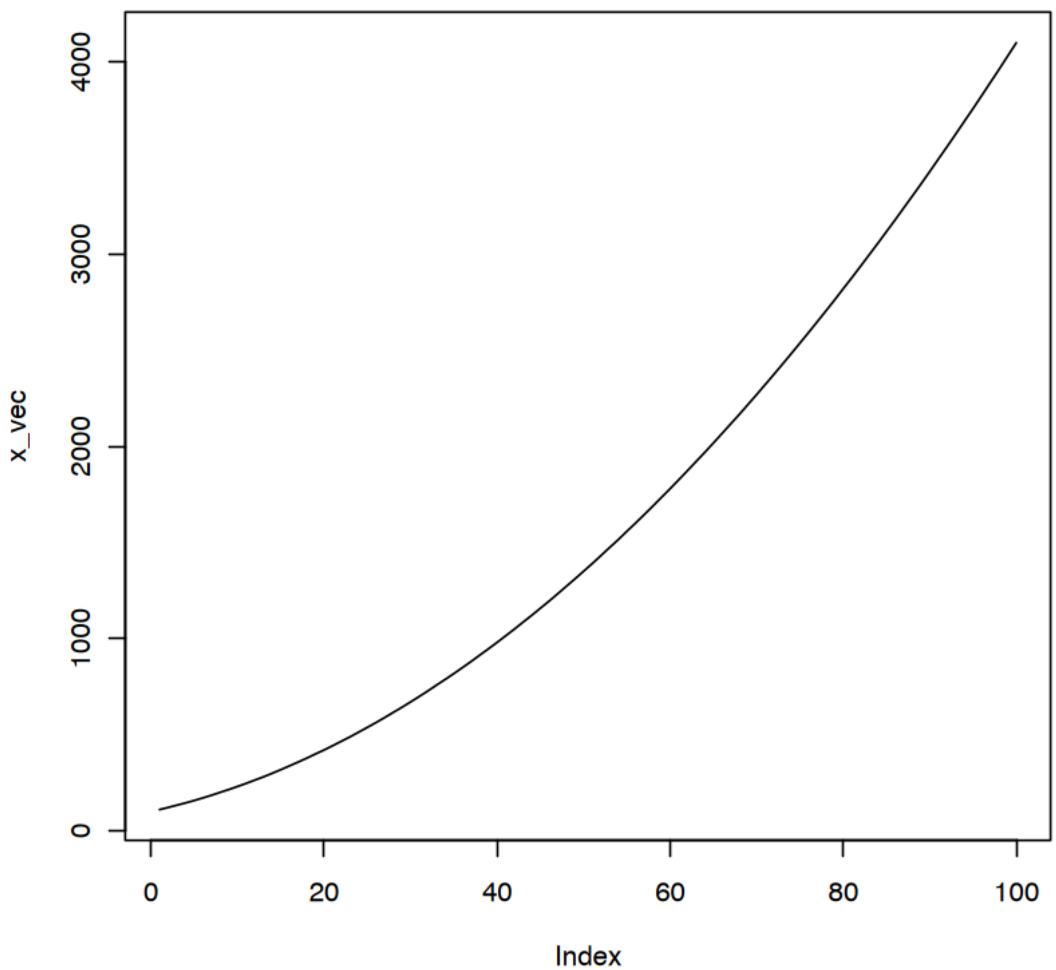
[64]:

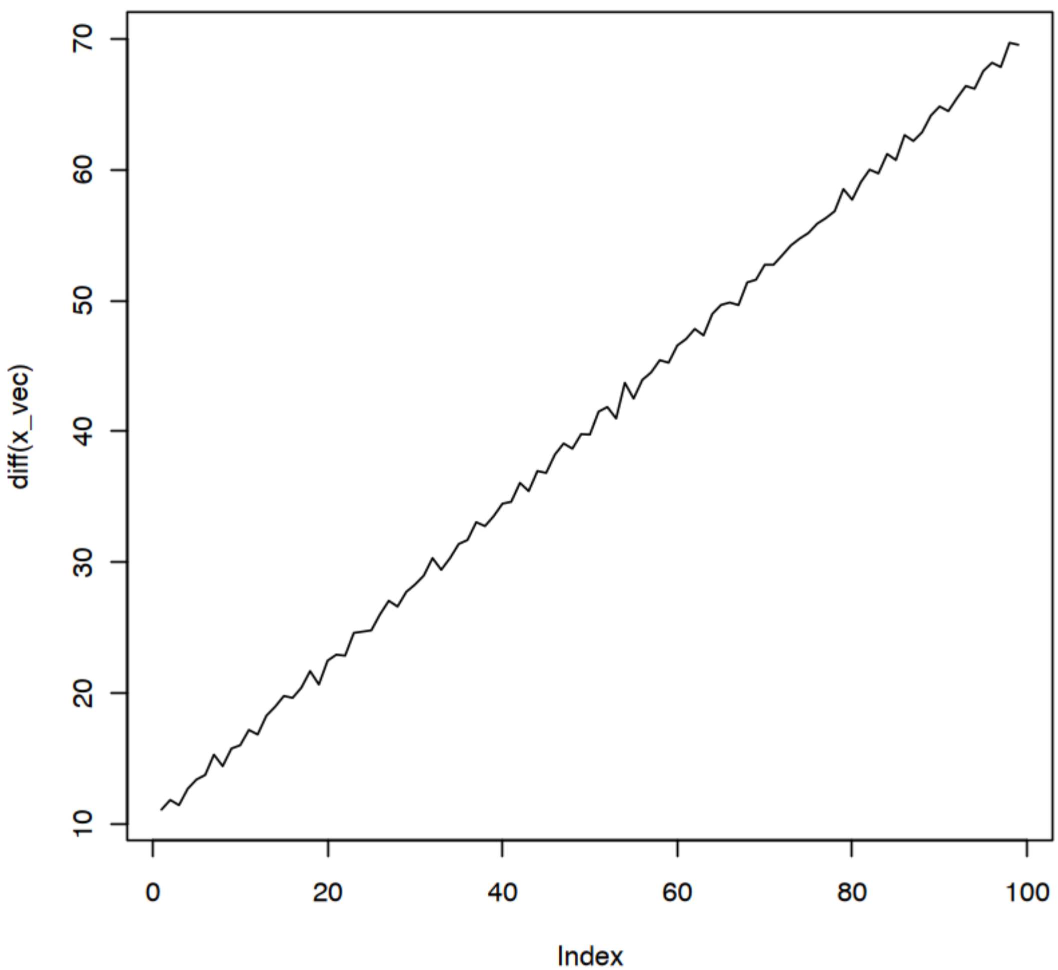
```
x_vec_diff2 = diff(x_vec_diff)
plot(x_vec_diff2, type = "l")
```



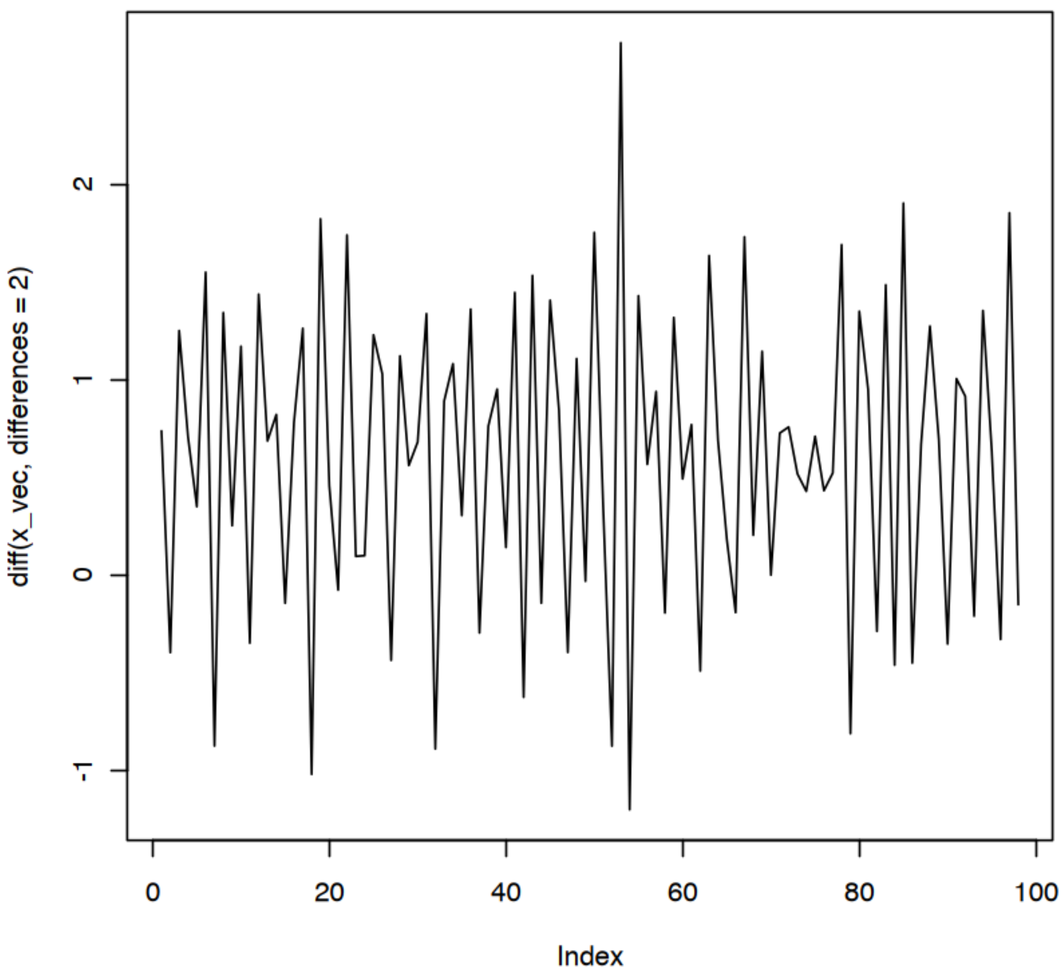
[73]:
```
n <- 100
a <- 100
b <- 10
c <- 0.3
sigma2 <- 0.1

x_vec <- rnorm(n, a + b * (1:n) +c* (1:n)^2, sigma2^0.5)
```

[74]:
```
plot(x_vec, type="l")
plot(diff(x_vec), type="l")
plot(diff(x_vec, differences = 2), type = "l")
```


x_vec
4000
3000
2000
1000
0
0
20
40
60
80
100
Index



diff(x_vec)
10
20
30
40
50
60
70
0
20
40
60
80
100
Index


diff(x_vec, differences = 2)
Index
0
20
40
60
80
100
-1
0
1
2