CQF Exam
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Question 1

The answers to Question 1 are reported in the following table:

VaR stats	**21 days rolling window**	**42 days rolling window**
Number of observations	1219	1198
Number of breaches	25	19
VaR breaches	2.05%	1.59%
Number of consecutive breaches	14	9
Conditional breach probability	56%	47.37%

The results are represented in the following plot:

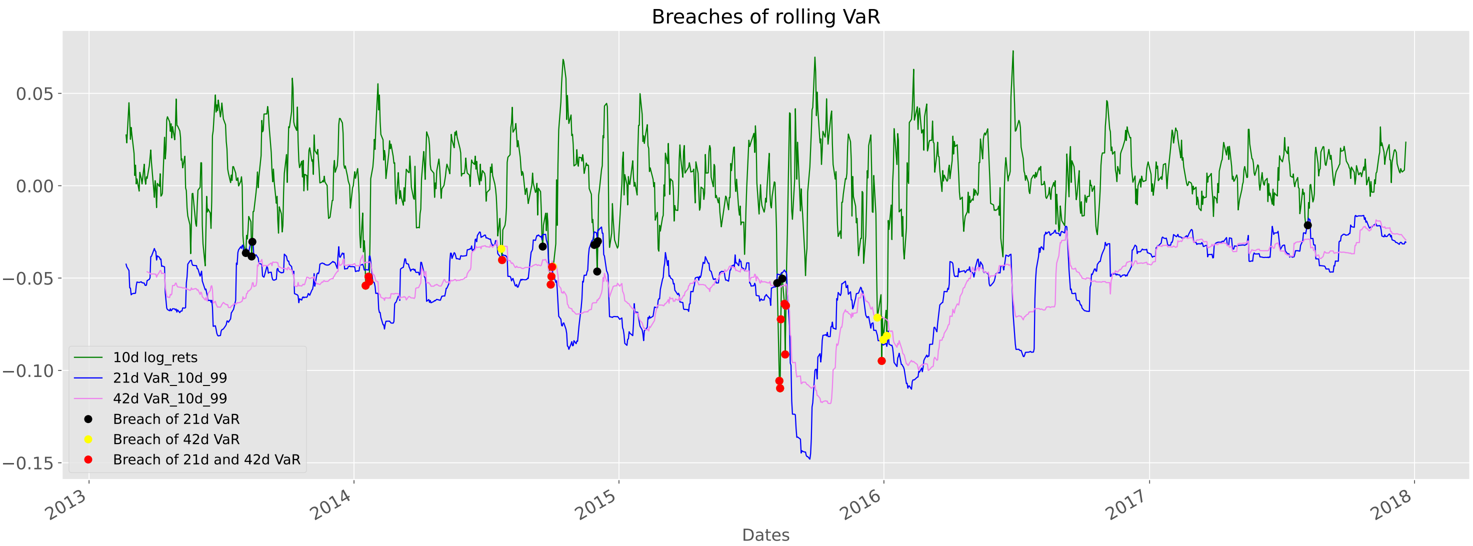

Figure 1: The figure shows breaches in the 21 days and 42 days rolling VaR at 99% confidence. The black, yellow and red dots mark (respectively) the values of the 10 days future log returns which cross the boundaries delimited by the 21 days, 42 days and both VaRs

Detailed explanation on how these results have been obtained can be found in the relevant Python script.

Question 2

a)

The relevant article titles regarding stable funding are:

- Article 413: Stable Funding
- Article 427: Items providing stable funding
- Article 428: Items requiring stable funding
- Article 510: Net stable funding requirements

The Net Stable Funding Ratio (NSFR) is defined as a liquidity standard requiring banks to hold enough stable funding to cover the duration of their long-term assets. For both funding and assets, long-term is mainly defined as more than one year, with lower requirements applying to anything between six months and a year to avoid a cliff-edge effect. Banks must maintain a ratio of 100% to satisfy the requirement. Mathematically, it can be defined as the ration of the Available amount of Stable Funding (ASF) and the Required amount of Stable Funding (RSF):

$$\text{NSFR} = \frac{\text{ASF}}{\text{RSF}} \geq 100\%.$$

b)

The relevant articles on requirements for Internal risk models for ”correlation trading” are:

- Article 338: Own funds requirement for the correlation trading portfolio
- Article 377: Requirements for an internal model for correlation trading.

These are the most relevant articles. However, complete understanding of these articles requires understanding of the institutions that are allowed to use an internal model for specific risk which need to meet the requirements specified in Article 367(1) and (3), Article 368, Article 369(1) and points (a), (b), (c), (e) and (f) of Article 370. The main risks that need to be captured by the model are:

1. the cumulative risk arising from multiple defaults, including different ordering of defaults, in tranched products;
2. credit spread risk, including the gamma and cross-gamma effects;
3. volatility of implied correlations, including the cross effect between spreads and correlations;
4. basis risk, including both of the following:
   - the basis between the spread of an index and those of its constituent single names;
   - the basis between the implied correlation of an index and that of bespoke portfolios
5. recovery rate volatility, as it relates to the propensity for recovery rates to affect tranche prices;
6. to the extent the comprehensive risk measure incorporates benefits from dynamic hedging, the risk of hedge slippage and the potential costs of rebalancing such hedges;
7. any other material price risks of positions in the correlation trading portfolio.

ATTENTION: The articles presented above were obtained from the 2013 CCRR as provided by CQF. It should be noted that there exists a new version (CRR2) which was completed in 2019 and will replace the old CRR. The new version provides additional information and should be consulted for up to date regulations on the subject.

Question 3

Given the assets volatility σ, the portfolio weights w and the correlation matrix ρ, the values for the derivative of the Value at Risk and the Expected Shortfall with respect to the weights are reported in the following table. The covariance matrix employed for the computation of the results has been calculated using the matrix multiplication

$$\Sigma = \Omega\rho\Omega = \begin{pmatrix} 0.09 & 0.048 & 0.0225 \\ 0.048 & 0.04 & 0.009 \\ 0.0225 & 0.009 & 0.0225 \end{pmatrix} \tag{1}$$

where Ω is the diagonal matrix with the volatility of the assets along the diagonal and zeros on the off diagonal entries.
The derivatives are reported in the following table:

Asset	$\frac{\partial \mathbf{VaR}}{\partial \mathbf{w_i}}$	$\frac{\partial \mathbf{ES}}{\partial \mathbf{w_i}}$
1	-0.68	-0.78
2	-0.39	-0.44
3	-0.22	-0.25

Detailed explanation on how these results have been obtained can be found in the relevant Python script.

Question 4

The Expected Shortfall (ES), also known as the conditional value at risk (CVaR), is a coherent risk measure defined to be more sensitive to the shape of the distribution with respect to the Value at Risk (VaR). Informally, it is defined as the expected loss during the time horizon T conditional on the loss being greater than the c-th percentile of the loss distribution. Formally, this is expressed as

$$ES_c(X) = \mathbb{E}[X|X \leq VaT_c(X)] = \frac{1}{1-c}\int_0^{1-c} VaR_u(X)du$$

where $c \in \mathbb{R}$ and X can generally belong to any L^p space but, for the sake of simplicity, we will consider $X \in (\Omega, \mathcal{F}, \mathbb{P}) = (\mathbb{R}, \mathcal{B}, \mathbb{P})$ to belong to the probability space over the reals. X is then a probabilistic variable representing either losses or returns (here we have chosen it to represent losses) such that the underlying probability distribution of X is the Normal Distribution.
This implies

$$VaR_c(X) = \mu + \Phi^{-1}(1-c)\sigma$$

where $\Phi^{-1}(1-c)$ is the inverse f the cumulative distribution function for the Normal Distribution and $\mu, \sigma \in \mathbb{R}$. Given that X represents the losses, we integrate over the right tail of the distribution:

$$\begin{aligned}
ES_c(X) &= \frac{1}{1-c}\int_c^1 VaR_u(X)du \\
&= \frac{1}{1-c}\int_c^1 (\mu + \Phi^{-1}(1-u)\sigma)du \\
&= \frac{\mu}{1-c}\int_c^1 du + \frac{\sigma}{1-c}\int_c^1 \Phi^{-1}(1-u)du \\
&= \frac{\mu}{1-c}\big[1\big]_c^1 + \frac{\sigma}{1-c}\int_c^1 \Phi^{-1}(1-u)du \\
&= \frac{\mu}{1-c}(1-c) + \frac{\sigma}{1-c}\int_c^1 \Phi^{-1}(1-u)du \\
&= \mu + \frac{\sigma}{1-c}\int_c^1 \Phi^{-1}(1-u)du
\end{aligned}$$

We now make the substitution $u = \Phi(v)$, where $\Phi(v)$ is the standard Normal cumulative distribution function.

$$\Phi_V(v) = \int_{-\infty}^{v} \phi_V(t)dt \tag{2}$$

Hence,

$$\begin{aligned}
&du = d\Phi_V(v) = \phi_V(v)dv \\
&\Phi_V(v) \to c \quad \Rightarrow \quad v \to \Phi_V^{-1}(c) \\
&\Phi_V(v) \to 1 \quad \Rightarrow \quad v \to \infty
\end{aligned}$$

where $\phi_V(v)$ is the Normal probability density function and we recall the property of the CDF which goes to 1 when its integration limits span over the entire probability space.

Therefore, suppressing subscripts to declutter the notation and exploiting the fact that $\Phi^{-1}(1-c) = -\Phi^{-1}(c)$:

$$
\begin{aligned}
ES_c(X) &= \mu + \frac{\sigma}{1-c}\int_c^1 \Phi^{-1}(1-u)du \\
&= \mu + \frac{\sigma}{1-c}\int_{\Phi^{-1}(c)}^{\infty} \Phi^{-1}(1-\Phi(v))\phi(v)dv \\
&= \mu - \frac{\sigma}{1-c}\int_{\Phi^{-1}(c)}^{\infty} \Phi^{-1}(\Phi(v))\phi(v)dv \\
&= \mu - \frac{\sigma}{1-c}\int_{\Phi^{-1}(c)}^{\infty} v\phi(v)dv \\
&= \mu - \frac{\sigma}{1-c}\int_{\Phi^{-1}(c)}^{\infty} ve^{-\frac{v^2}{2}}dv \\
&= \mu + \frac{\sigma}{1-c}\left[e^{-\frac{v^2}{2}}\right]_{\Phi^{-1}(c)}^{\infty} \\
&= \mu + \frac{\sigma}{1-c}\left[\phi(v)\right]_{\Phi^{-1}(c)}^{\infty} \\
&= \mu + \frac{\sigma}{1-c}\big(\phi(\infty) - \phi(\Phi^{-1}(c))\big)
\end{aligned}
$$

given that $\phi(\infty) = 0$, we are left with

$$
\boxed{ES_c(X) = \mu - \frac{\sigma}{1-c}\phi(\Phi^{-1}(c))}
$$