

## *The Battle of the Sexes*

### Project specifications

**Definitions.** A *population* is a set of individuals; individuals are grouped into *types*. We define *state* of a type  $T$  the ratio between the number of individuals of type  $T$  over the total number of individuals. The *global* state associates each type with its state. We assume a notion of *closeness* is given: two population are *close* if the difference between the values of their state is small enough to be negligible.

Populations evolve according to (possibly non-deterministic) *rules* which determine how the individuals of each type thrive (grow in their number) or languish (decrease) depending on the global state. An *evolution trace* is an infinite sequence  $\langle p_0, p_1, \dots \rangle$  of populations, where  $p_0$  is called *initial* and each  $p_{n+1}$  is obtained from  $p_n$  by applying the evolution rules. We say that a trace reaches *stability* if it includes a population  $p_n$  such that for each  $m > n$  the state of  $p_m$  is close to that of  $p_n$ .

**Four types of people.** This project is inspired by Chapter 9 of *The Selfish Gene*, a 1976 book by Richard Dawkins. The chapter describes “the battle of the sexes”, where a model is provided of a population, featuring two male types, the *faithful* ( $F$ ) and the *philanderers* ( $P$ ), and two female types, the *coy* ( $C$ ) and the *fast* ( $S$ ), characterised as follows (the names are from the book):

- *faithfull* men: they are willing to engage in a long courtship and participate in rearing their children;
- *philanderers*: reckless men, they don’t waste time in courting women: if not immediately accepted, they move away and try somewhere else; moreover, if accepted, they mate and then leave anyway, ignoring the destiny of their children;
- *coy* women: they accept a partner only after a long courtship;
- *fast* women: if they feel so, they don’t mind copulating with whoever they like, even if just met.

**The FPCS payoffs table.** Let’s give names to the evolutionary payoffs involved in the battle of the sexes:

$a$  : the evolutionary benefit for having a baby

$b$  : the cost of parenting a child

$c$  : the cost of courtship

Next we represent the payoffs resulting from a woman of type  $X$  engaging with a man of type  $Y$  by means of a pair  $(x, y)$ , where  $x$  is the payoff of  $X$ , and  $y$  that of  $Y$ . According to the characters described above, the battle of the sexes can be formalised as in the following matrix, which may be used in defining the evolution rules.

	F	P
C	$(a - b/2 - c, a - b/2 - c)$	$(0, 0)$
S	$(a - b/2, a - b/2)$	$(a - b, a)$

**Dawkins' case study.** Dawkins described the battle of the sexes by adopting the following values:  $a = 15$ ,  $b = 20$  e  $c = 3$ . Here is the corresponding FPCS table:

	F	P
C	$(2, 2)$	$(0, 0)$
S	$(5, 5)$	$(-5, 15)$

When using these values, the system stabilises in a state where  $5/6$  of the women are coy and  $5/8$  of the men are faithful. It is easy to check that this is indeed a stable solution: given these ratios, the average gain of a coy woman, that is  $2 \cdot 5/8$ , equals that of a fast, which is  $5 \cdot 5/8 - 5 \cdot 3/8$ . Therefore none of the two would have any interest in changing her strategy. And similarly for men.

### Assignment

Implement in Java a simulator for the battle of the sexes.

The simulator must be parametric in the values of  $a$ ,  $b$  and  $c$ . The rules of evolution should be suitably defined according with the FPCS table. Given an initial population, the simulator will iteratively apply the rules of evolution till, if ever, an evolutionary stable state is reached, in which case the percentages of the four types are returned as result. Ideally, independently of the initial state, the simulator should compute the values discussed above. Or should it?

Together with the Java program, a short paper (approx 10 pages) is required, describing the adopted model (e.g. the rules of evolution, the mechanism for modelling how people match, etc.), the structure of the software, and discussing some interesting case studies, which may be obtained by choosing different values for the parameters  $a$ ,  $b$  and  $c$ , or modifying the initial population.