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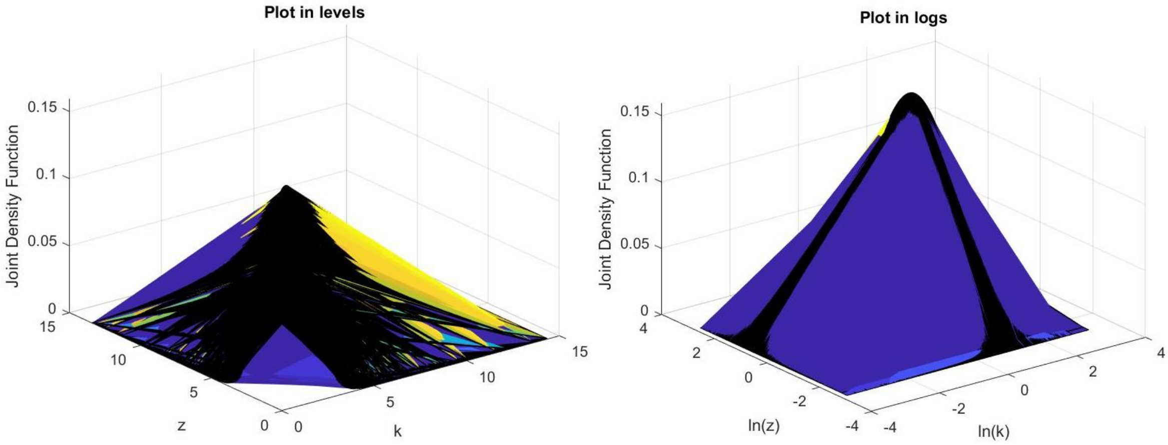
PROBLEM SET 5. MISALLOCATION

QUESTION 1. FACTOR INPUT MISALLOCATION

PART 1

In this part, we are considering the following distributions for the variables:



The covariance of both variables is zero, and since the joint distribution is multivariate normal, then we can state that both variables are independent. Hence, the joint is the product of the marginal distributions, meaning that we can generate both variables assuming they are normally distributed. Moreover, given the expectation of both variables without logarithms is 1, by taking them we concluded that E(ln(z)) = E(ln(k)) = ln(1) = 0. Therefore, ln(z)~N(0,1) and ln(k)~N(0,1).

PART 2

To compute the output, we have used the formula and the definitions which appear in the slides, which is equivalent to that one:

$$y_i = s_i k_i^{\gamma} \quad \text{where} \quad s_i = z_i^{1-\gamma}$$

PART 3

Then the problem we are solving with the nomenclature in the slides is as follows:

Then, the planners problem is the maximization of aggregate output,

$$\max_{\{k_1,..,k_I\}} \sum_{i=1}^{I} s_i k_i^{\gamma}$$

subject to

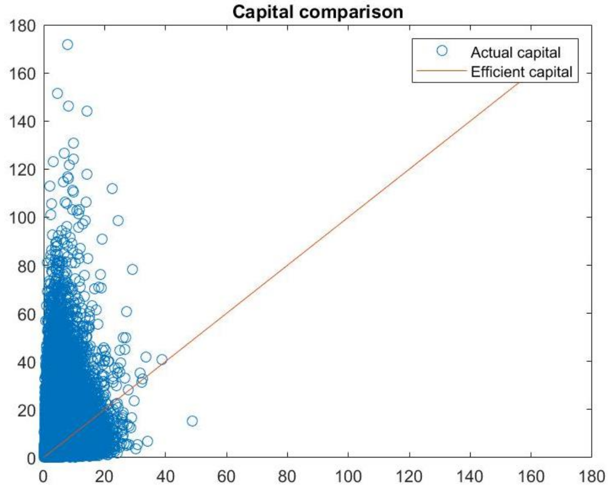
$$\sum_{i=1}^{I} k_i = K$$

To compute the efficient capital, we have used the first order conditions which appear in the slides:

$$k_1 = \frac{z_1}{\sum_{i=1}^{I} z_i} \sum_{i=1}^{I} k_i = \frac{z_1}{Z} K$$

PART 4

In the following graph we can see the comparison between the actual and the efficient capital:



In the previous graph we can see that the actual capital is concentrated among the less productive part of the population.

PART 5

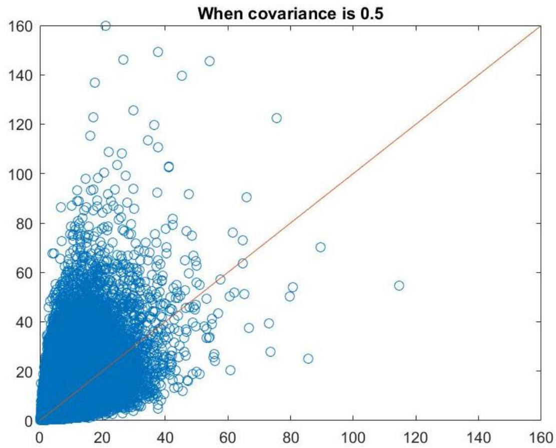
Notice that the population is randomly created so the value of this population misallocation rate can vary when the code is run every time. In this case, the rate is 27.1310 which means that the country would gain a 27.13% of output if the capital was reallocated efficiently.

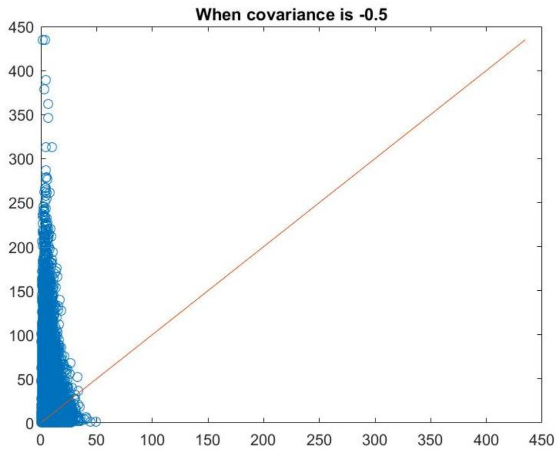
PART 6

In this case, the assumption of independence of the variables made before cannot be applied anymore. So now, in order to generate the population, we are going to use the following formula:

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{p/2} |\mathbf{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})' \mathbf{\Sigma}^{-1} (\mathbf{x}-\boldsymbol{\mu})}$$

Which in MATLAB is collected in the command “mvnrnd” so if you provide the mean for both variables and the variance covariance matrix, it returns the two variables. In this case, the output gains are 12.7522% for a covariance equals to 0.5 and 43.3222% for the covariance equals to -0.5. The graphs comparing the two optimal allocations of capital against the data are as follows:

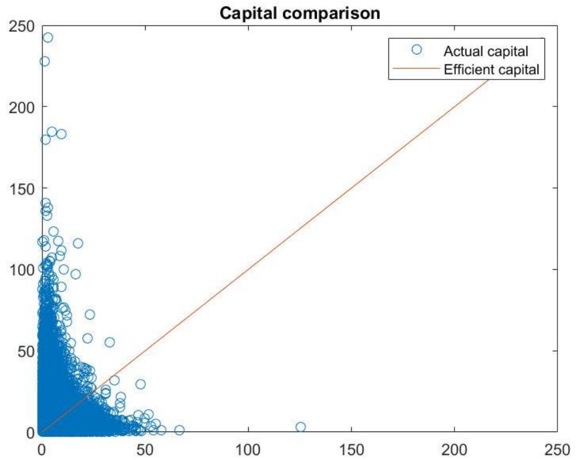


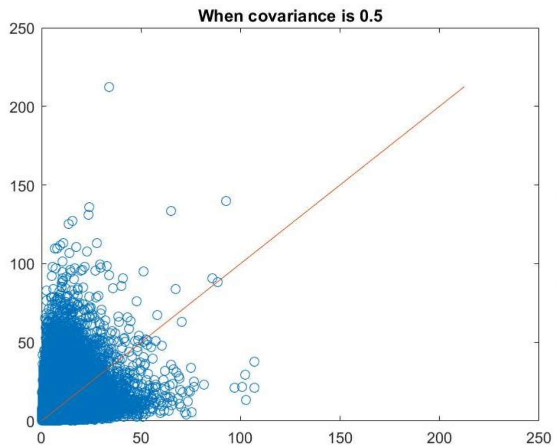


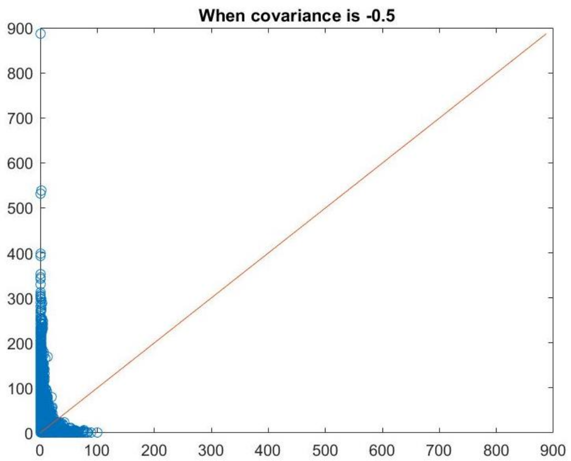
QUESTION 2. HIGHER SPAN OF CONTROL

To solve this exercise, we only have changed γ=0.8. The results that we have obtained are similar to the previous case. The only remarkable difference is that now the output misallocation rate when both variables are independent is not in between as it happened before. So, the misallocation rates are 17.3463% for the case of 0 covariance, 8.3328% for the case of covariance equals to 0.5 and 27.0992% for the case of the covariance equals to -0.5.

We can observe that in this exercise and in exercise 1, the case in which both variables are positively related is the most efficient one, that is, lowest rate of output gain. The graphs for this exercise where we compare the efficient allocation of capital against the data are as follows:







QUESTION 3. FROM COMPLETE DISTRIBUTIONS TO RANDOM SAMPLES

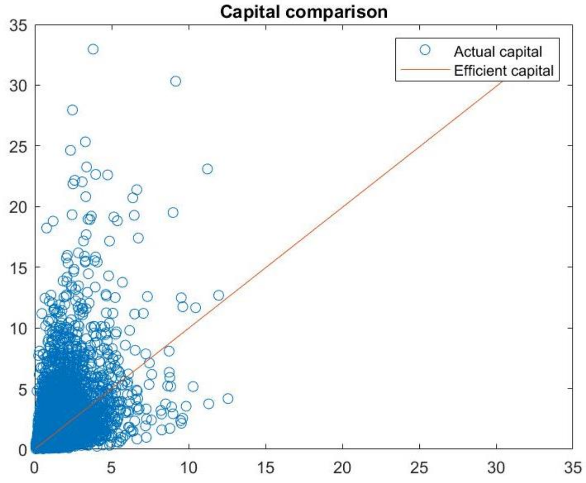
To generate the data for this exercise, we use the first case where covariance is 0.

PART 1

The variance for the ln(k) for the random sample of 10,000 observations is 1.0283 which is quite close to 1 (it makes sense since we are using a N(0,1) to generate the data). The same happens for the variance of ln(z) which is 1.0117. The covariance between the two variables is -0.0154 which is close to 0.

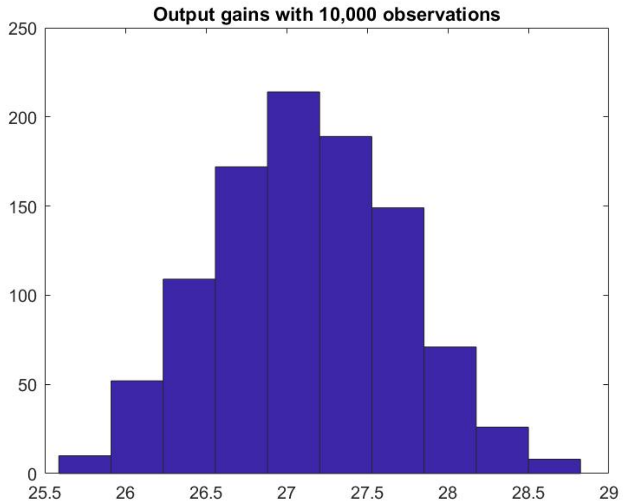
PART 2

The output gain in this case is 27.5586%. The graph comparing the actual capital and the efficient one is as follows:



PART 3

The histogram for this third part is as follows:



The median of these rates is 27.0958% which is quite close to the value achieved by the population.

PART 4 & 5

Given that we have used a for loop to create the samples with different sizes, we have done this exercise at once for samples with size equals to 100,1000,10000 and 100000. The probabilities that a random sample delivers the misallocation gains within an interval of 10% with respect to the gains from the data are the following:

Sample size	10% probability
100	0.1870
1000	0.5210
10000	0.9830
100000	0.9952

As we can observe, the probability increases as the sample size does (as it can be seen in the graph below). Therefore, we can conclude that the literature is accurate enough when we are working with sufficiently large samples.