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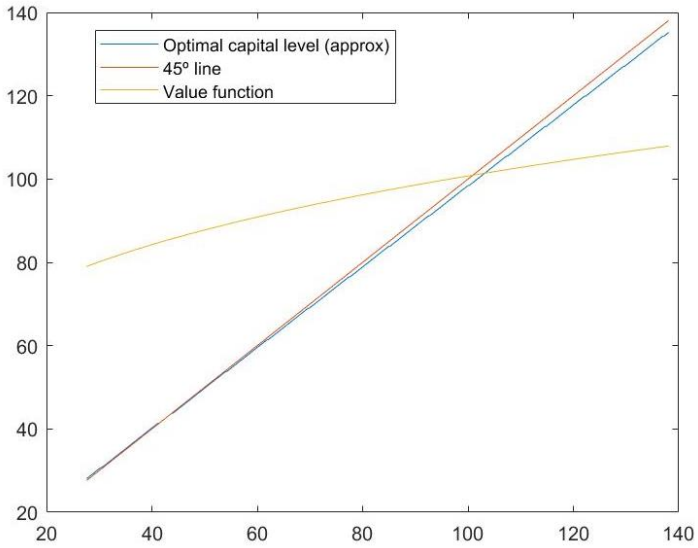
PROBLEM SET 4. VALUE FUNCTION ITERATION

QUESTION 1. CONTINUOUS CAPITAL CHOICE

PART A

In this part, we are considering labor as an exogenous variable, so we have to determine two state variables in this context, tomorrow capital and consumption today. In the figure below we can see the value function and the decision rule for capital associated to that value function.

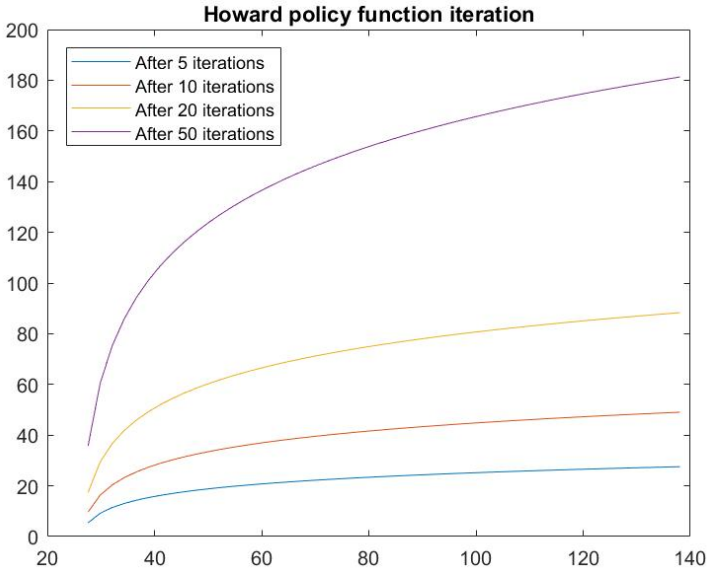


PART B, C, D, E, F AND G

The following table is a summary of times for each part:

Cases	Time (seconds)
A: Initial point (not taking into account anything)	15.7792
B: Monotonicity of the optimal decision rule	1.0336
C: Concavity of the value function	0.6391
D: Local search on the decision rule	0.9743
E: Concavity of the value function and monotonicity of the decision rule	0.7482
F: Howard's policy iterations	22.2779
G5: Policy iterations	0.1874
G10: Policy iterations	0.2840
G20: Policy iterations	0.5492
G50: Policy iterations	1.2554

Moreover, in the graph below we can observe that a larger amount of iterations entails more accuracy when we are computing the value function.



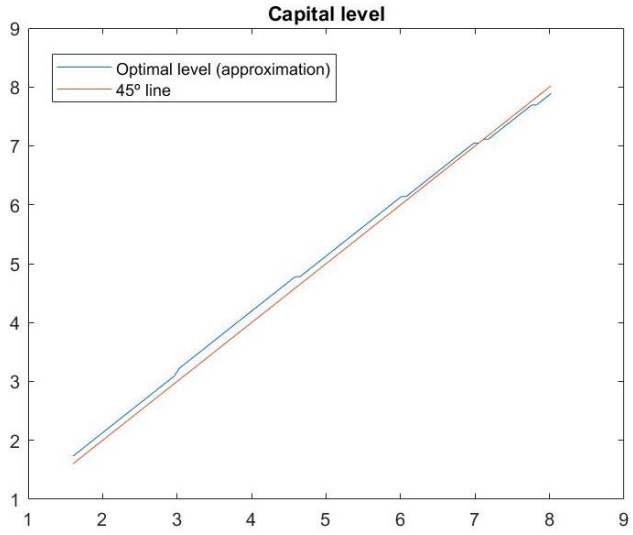
We explode ways of speeding up the VFI algorithm. The main conclusion we can achieve for this question is that as we take advantage of the characteristics of the functions (concavity, monotonicity) we are working with and applying the methods for local search and Howard's policy iteration step, the convergence speed towards the optimal level increases.

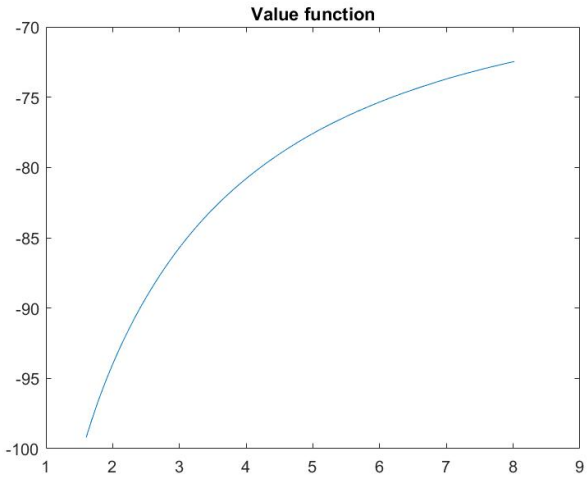
QUESTION 2. CONTINUOUS CAPITAL AND LABOR CHOICE

To solve this part of the problem set we have to add the labor as state variable a part from tomorrow capital and consumption.

PART A.

The following graph depicts the optimal choice of capital.



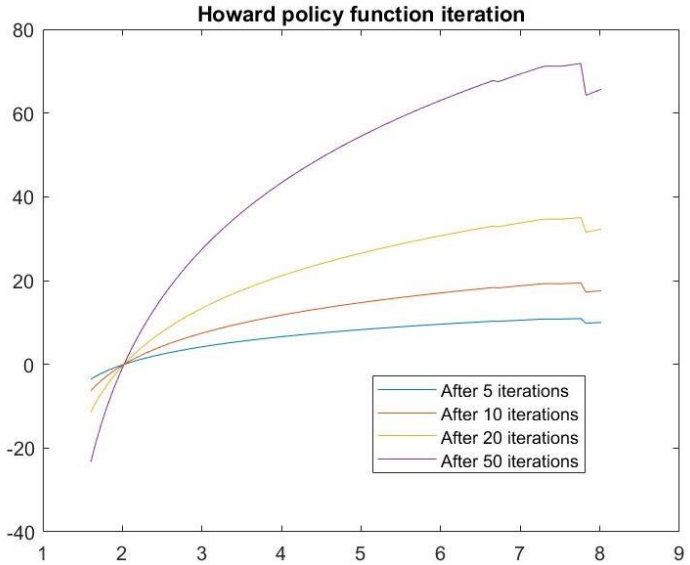


Notice that in this case, as we are including labor also, hence the number of iterations to get the optimal choice is larger than in part a of the previous exercise (953 vs 1525).

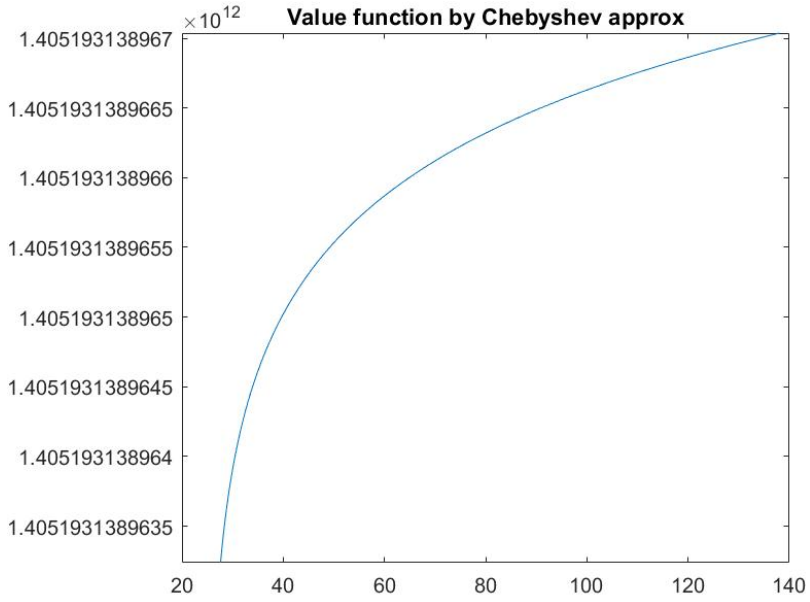
PART B, C, D, E, F AND G

Cases	Time (seconds)
A: Initial point (not taking into account anything)	26.7364
B: Monotonicity of the optimal decision rule	0.5823
C: Concavity of the value function	0.5247
D: Local search on the decision rule	0.6752
E: Concavity of the value function and monotonicity of the decision rule	0.5721
F: Howard's policy iterations	17.1800
G5: Policy iterations	0.2036
G10: Policy iterations	0.3300
G20: Policy iterations	0.6235
G50: Policy iterations	1.3586

Again, in the graph below (corresponding with the different value functions computed in part G) we can observe that a larger amount of iterations entails more accuracy when we are computing the value function.



QUESTION 3. CHEBYSHEV REGRESSION ALGORITHM APPROXIMATION



In this case, the speed was 58.1175 seconds for 100 iterations at a tolerance level of 0.0001. The main difference we can observe in this graph regarding the graph for the value function in question 1 is that in this case, the value achieved by the value function is much larger.