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Question 1. Computing Transitions in a Representative Agent Economy

Part a

In order to choose z, we have normalized output equal to one (that, is y_t=1), and given this assumption our capital of the steady state is equal to 4. This let us to compute the depreciation rate:

$\frac{k_t}{y_t} = 4$ and $\frac{i_t}{y_t} = 0.25$, recall that yt =1, so combining both we get $\frac{i_t}{y_t} = \delta \cdot \frac{k_t}{y_t} \rightarrow \delta = \frac{1}{16}$

Using this solution and the given parameters in the exercise, we can find out z = 1.0768.

Part b

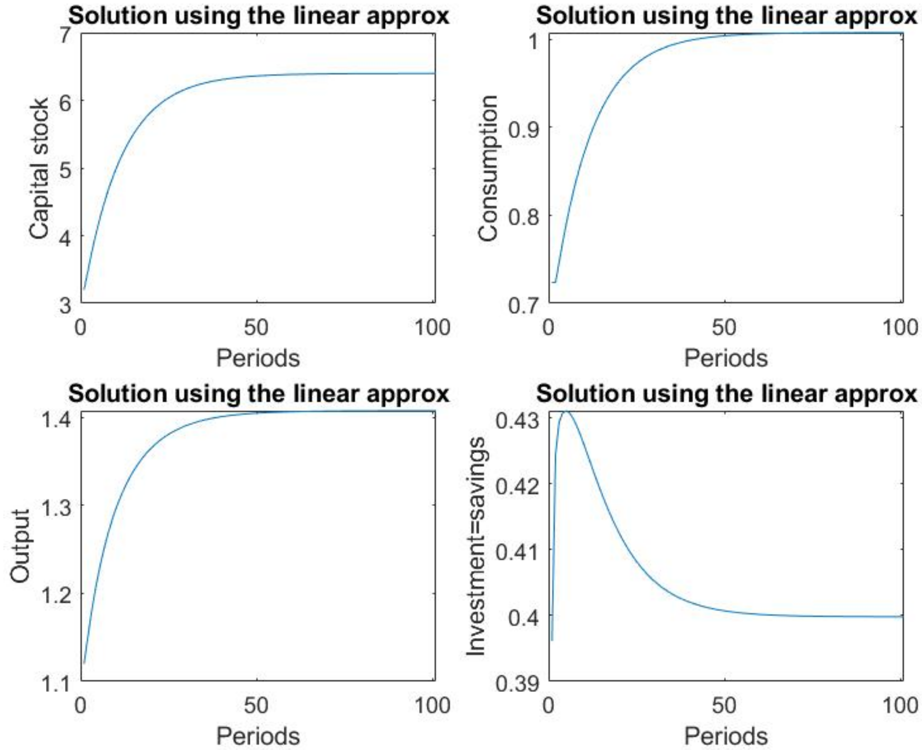
In this case, we have implemented a function called "steadystate.m" which collects the equilibrium conditions to reach the steady state point. Specifically, these conditions are:

1. The resource constraint.
2. The marginal utility of capital in t+1.
3. $C_t = C_{t+1} \rightarrow C_t - C_{t+1} = 0$
4. $K_t = K_{t+1} \rightarrow K_t - K_{t+1} = 0$

The level we got for the steady state capital was 6.3970 when the shock is the double with respect to the previous one.

Part c

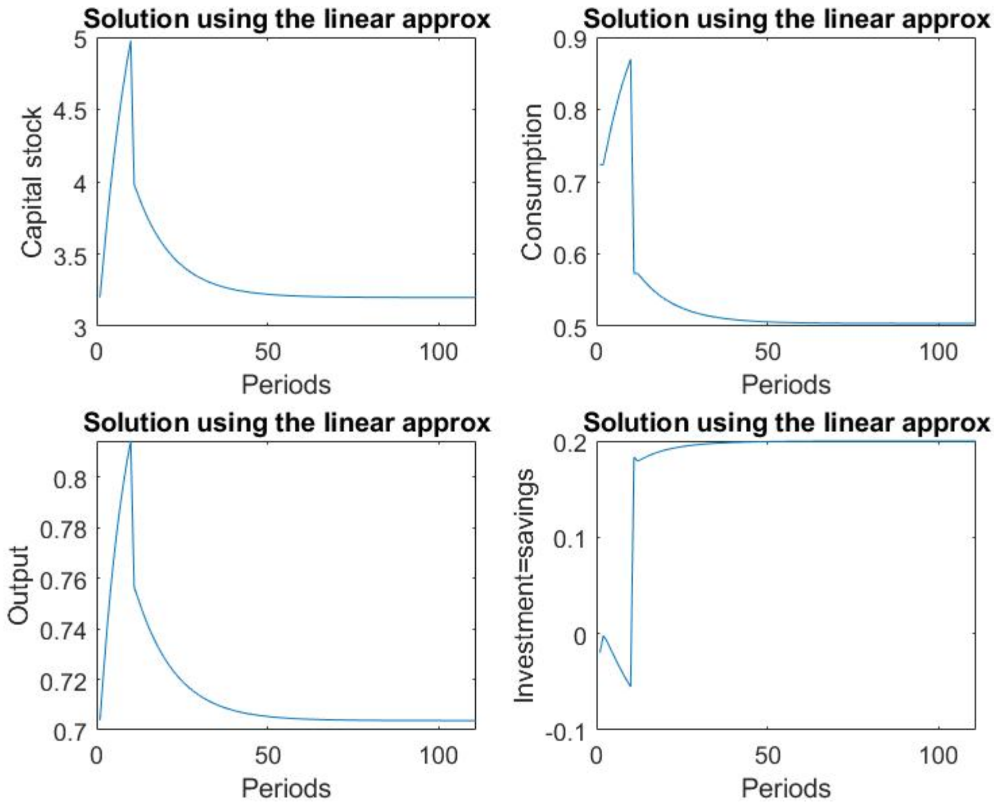
For this exercise, we have used the method of decomposition values in eigen values and eigen vectors. This technique linearly approximates around the steady state of the model in order to get a stability condition which later on it will replace the Euler equation. To apply this method in MATLAB, we are going to use the function called "jacobian.m" developed by J. Ruiz. With this routine we have got the following time-path for consumption, savings, output and capital:



As we can observe in the graphs below, the larger shock in the productivity gives as a result a higher convergence level for the steady state. In general, the new steady state for all variables is higher than the first steady state but notice that in the case of savings, the shock affects firstly positively and then it decreases along the time path. The opposite happens for the rest.

Part d

Now, we have analyzed unexpected shocks where the productivity shocks doubles for 10 periods and then the productivity returns to the initial value (with the original z). The following graphs exhibit the transitions in this set-up:



Again, the shock affects in the opposite direction to saving than the others variables. The logic is similar to before, but now, the new steady state is lower than the initial one (we increase for ten periods the productivity and after that, it comes back to the initial value), that is why we observe a negative convergence to the new steady state.

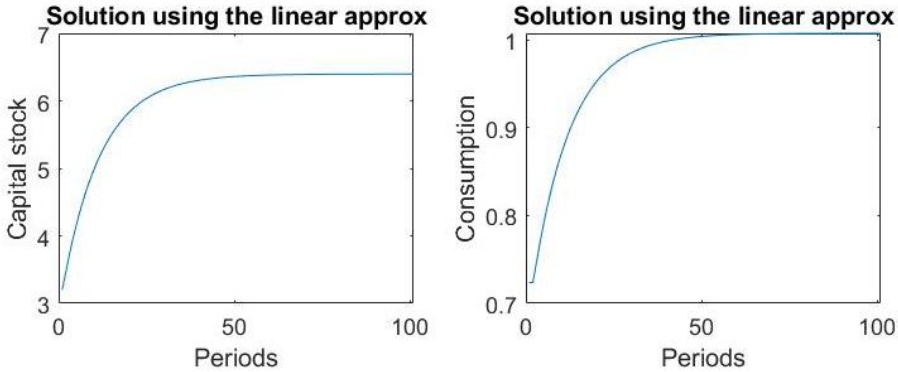
Part e

For this exercise, firstly we include a tax on consumption at a tax rate $\tau_C = 0.057$[1] and for capital $T_K = 0.04287$[2].

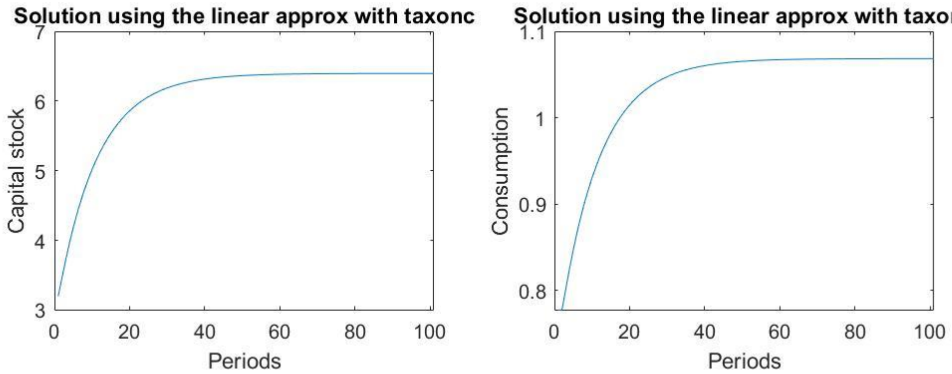
[1] Data taken from Mendoza et al. (1994): Effective tax rate in macroeconomics. Cross-country estimates of tax rates on factor incomes and consumption. Journal of Monetary economics, 287-313.

[2] Ibidem.

Transitions without taxes

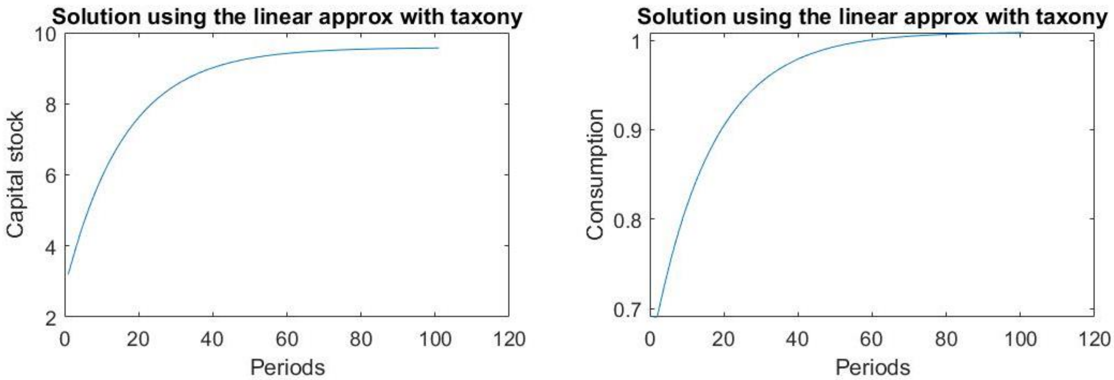


Transitions with consumption taxes



The effect of the introduction of a consumption tax is almost worthless to steady state level. It might affect the transition but not the level reached. The function we have used to compute it is called “steadystate_taxonc.m”.

Transitions with capital taxes



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Question 2. Multicountry model with free mobility of capital (not labor) and progressive labor income tax

Part 2.1.

Firstly, we have done a code for a representative agent economy (because there is no any friction in the economy so the representative agent is the average of all agents in the economy). Therefore, we have assumed that there is a mass of agents (that is, an amount for of high and low-skill workers) for this economy which has been determined by a random normally distributed vector with 100 agents (the code appears below):

```
random = randn(100,1);
%     rand=[];
%     for i=1:length(random)
%     if random(i)<0
%         rand(i,1)=0.5;
%     else
%         rand(i,1)=5.5;
%     end
%     end
% num_h=size(find(rand==0.5),1);
% %num_h=95; pro_h=0.95
% %num_l=5;  pro_h=0.05
```

Moreover, we have run a model with a no-representative agent economy where we have distinguished between low and high-skill workers, solving the economy for each case (the corresponding code is “PS3_Q2_noRA.m”).

Given the parameters of the model, the competitive equilibrium can be defined as the allocations of capital demand, capital supply, labor demand and labor supply and prices (wages and interest rate) such that:

-Given prices, the representative agent maximizes the utility subject to his budget constraint.

-Given prices, the firm of the economy maximizes its profits.

-Labor and capital markets clear (goods market does as well by Walras’s law).

The following table is a summary of all the values obtained in the equilibrium for each country in the case of representative agent economy:

Equilibirum variables	Country A	Country B
Labor supply	0.2992	0.3058
Wage	1.4394	0.8746
Interest rate	0.1077	0.2273

Notice that, based on the rationality of the agents, we have assumed that they are going to use all the available capital, so we take it as given.

Here it is the table with the value obtained in the case of no representative agent economy:

Equilibirum variables	Country A	Country B
Labor supply high	0.1510	0.2878
Labor supply low	0.3546	0.3155
Wage	0.5969	0.6226
Interest rate	0.4031	0.3784

Part 2.2.

Same idea as before but in this case where there is a union economy, market clearing is given by the union capital markets (that is the main difference with the previous set-up) and country-specific for labor markets. The following table exhibits the equilibrium values for each variable:

Equilibirum variables	Country A	Country B
Consumption high	0.8883	3.0513
Consumption low	6.8167	4.09
Labor supply high	0.3814	0.4470
Labor supply low	0.4844	0.4602
Capital supply high	2.4687	0.3762
Capital supply low	0.8252	0.3299
Wage	4.4055	4.4843
Interest rate	0.0047	0.001

As it was expected, marginal productivity of both inputs (labor and capital) are almost equal in the equilibrium, otherwise there would be an excess of demand/supply due to the higher remuneration in one of the countries.

Part 2.3.

In order to choose the optimal progressive taxation of labor income for this economy, we have to solve the problem of the social planner.

Given the parameters of the model, the social planner has to maximize the welfare function, that in this case we have chosen a utilitarian one (just the sum of both utilities). Notice that we have introduced a public good g[3] (public expenditure for instance) in this function in an additively separable way subject to the feasibility constraint, the government budget constraint which has to be balanced and the limit of the capital. The solution of this maximization problem gives us the amount of capital and labor for each country that is optimal and the optimal progressive taxation.

Notice that in the budget constraint cannot appear prices, so we have replaced them for the corresponding marginal utilities of capital and labor.

Hence, the problem we are maximizing in this exercise is as follows:

[3] The value of g according to the NIPA is g=0.3846 (on average the public consumption over GDP in USA is 16%).

$$\underset{\{c_l,\tau,k_l\epsilon[0,\bar{k}_l],h_l\epsilon[0,1]\}_{l=\{A,B\}}}{\text{Max}} \left(\frac{c_A{}^{1-\sigma}}{1-\sigma} - \kappa\frac{h_A{}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right) + \left(\frac{c_B{}^{1-\sigma}}{1-\sigma} - \kappa\frac{h_B{}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right) + g$$

Subject to

$$c_A = prob_H \cdot \lambda\left(MPH_A(1-\tau)h_A\eta_A^H\right)^{1-\phi_A} + prob_L \cdot \lambda\left(MPH_A(1-\tau)h_A\eta_A^L\right)^{1-\phi_A} + MPK_A k_A$$

$$c_B = prob_H \cdot \lambda\left(MPH_B(1-\tau)h_B\eta_B^H\right)^{1-\phi_B} + prob_L \cdot \lambda\left(MPH_B(1-\tau)h_B\eta_B^L\right)^{1-\phi_B} + MPK_B k_B$$

$$c_A + c_B = 2 \cdot \left[ZK_d{}^{1-\theta}H_d{}^{\theta} - MPH \cdot H_d - MPK \cdot K_d\right]$$

$$k_A + k_B \leq 4$$

$$g \leq prob_H \cdot \lambda\left(MPH_A(\tau)h_A\eta_A^H\right)^{1-\phi_A} + prob_L \cdot \lambda\left(MPH_A(\tau)h_A\eta_A^L\right)^{1-\phi_A} + prob_H \cdot \lambda\left(MPH_B(\tau)h_B\eta_B^H\right)^{1-\phi_B} + prob_L \cdot \lambda\left(MPH_B(\tau)h_B\eta_B^L\right)^{1-\phi_B}$$

The algorithm proposed is written in the code and the name of the function where the equilibrium conditions are defined is “steadystate_social.m”