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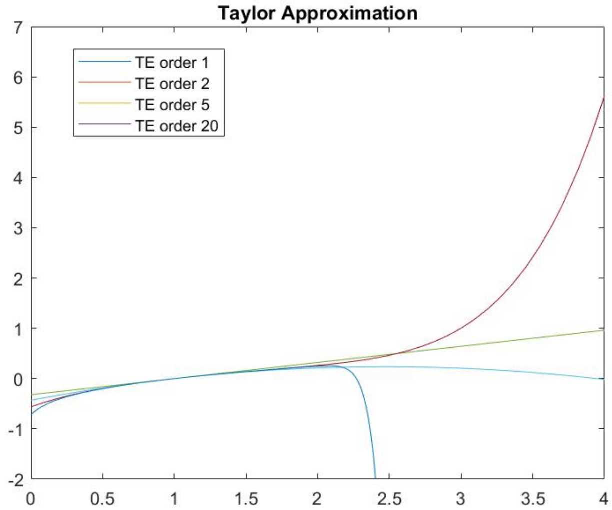
QUESTION 1. FUNCTION APPROXIMATION. UNIVARIATE.

Exercise 1.

For this exercise, we have created a MATLAB function called TE_approx because we did not know how the implemented Taylor function implemented in MATLAB. We saw that in both cases (applying the one created by us and the one in MATLAB report the same results which are in the following graph).



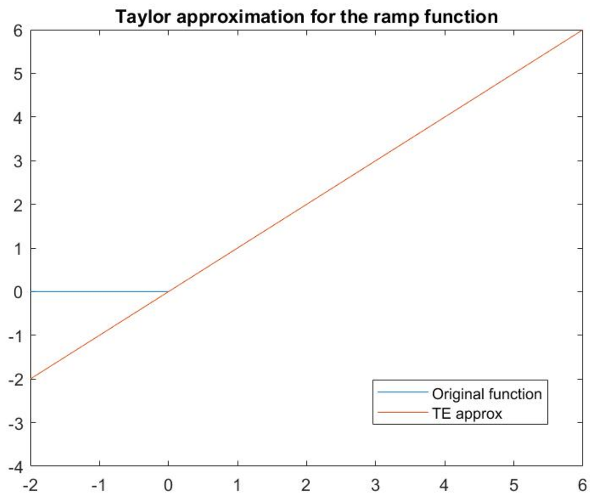
The main idea exhibited in this graph is that the best estimation using this method (Taylor approximation) is around 1 but as we move far from this number and increase the order, the estimation is worse off.

Exercise 2.

In this exercise, when we defined the function in MATLAB, an error arose, then we have split the function by applying the definition of the absolute value, that is:

$\frac{x+|x|}{2}$ divided in (a) $\frac{x+x}{2} = x$ and (b) $\frac{x-x}{2} = 0$

So given this separation, we have defined the ramp function by only taking into consideration the part where we have x (part a), which will be the same regardless the order. So, the graph for this approximation is as follows:



In this case, the approximation fits well in values close to 2. Moreover, as we can observe, there is a kink in x=0, so no matter the order, the approximation takes always the same value and the error jumps for values of x lower than 0.

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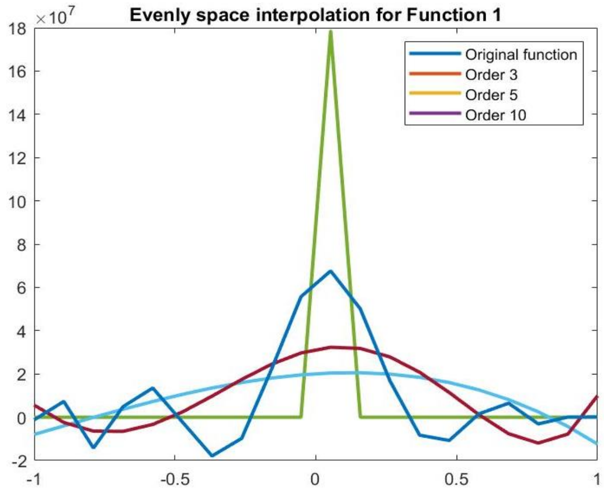
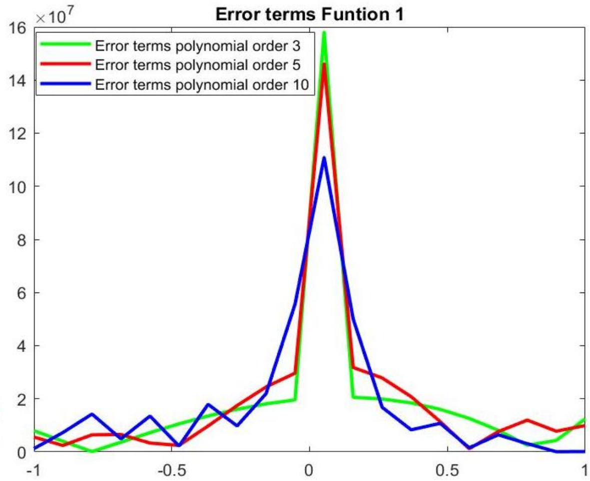
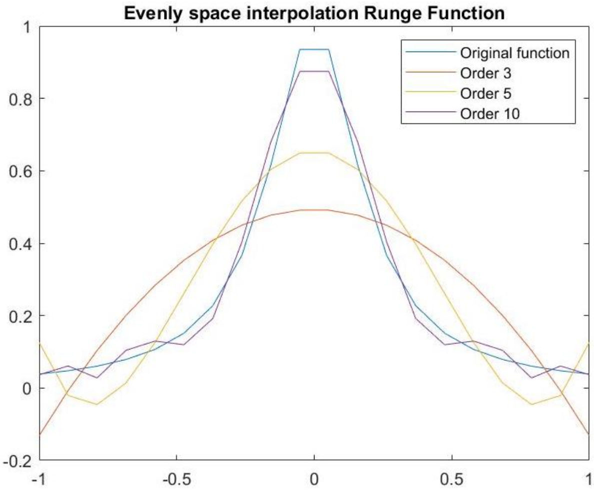
Exercise 3.

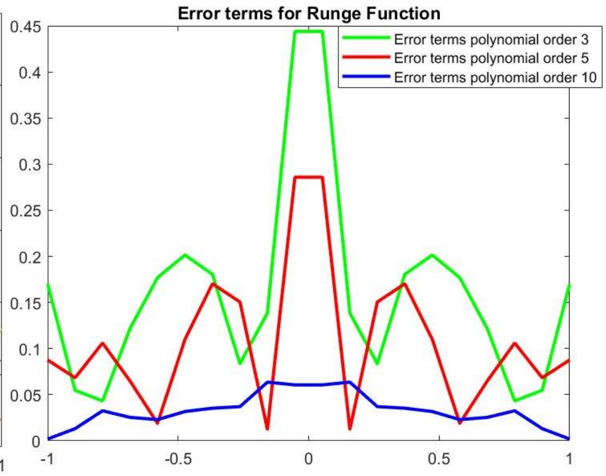
The first part referring to the evenly space interpolation nodes has been made in MATLAB whereas the second and the third have been done in Python because we have discovered some commands implemented in this software make easier the computations.

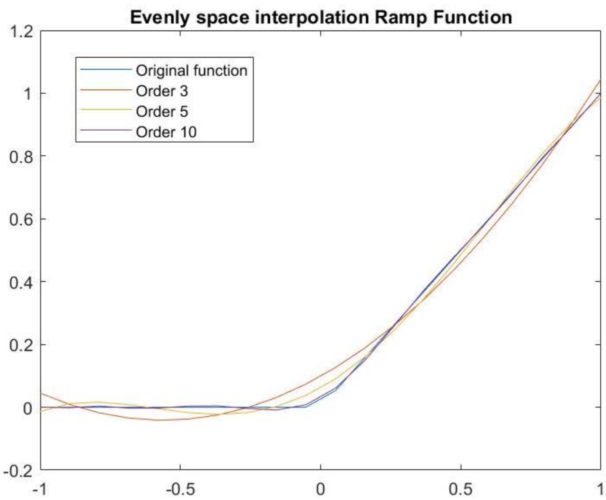
<u>Part 1. Evenly space interpolation nodes</u>.

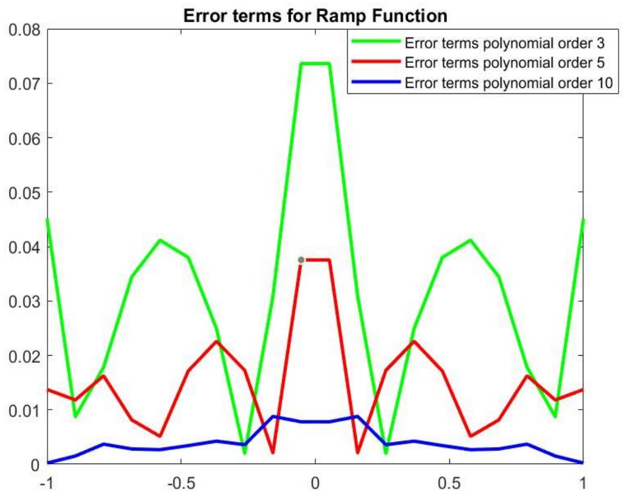












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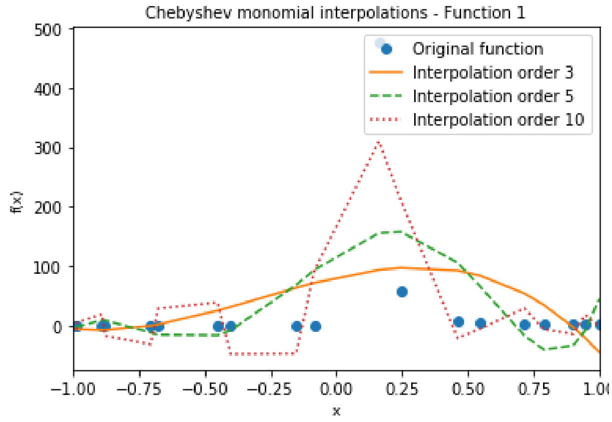
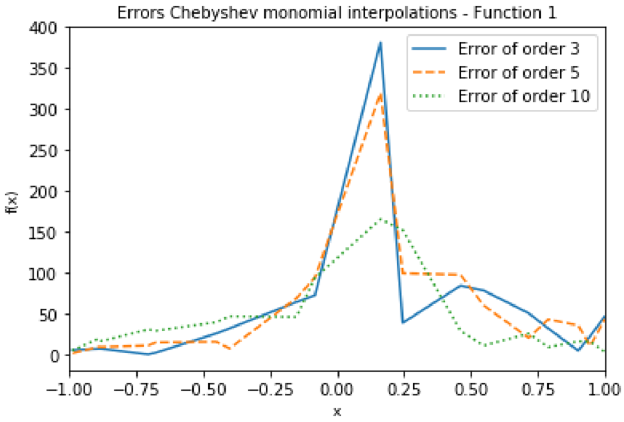
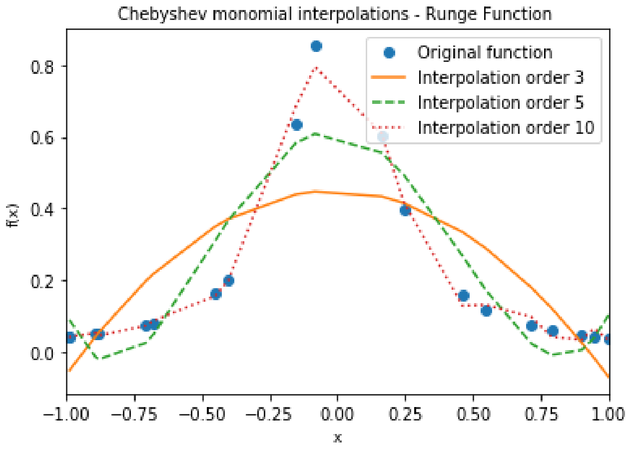
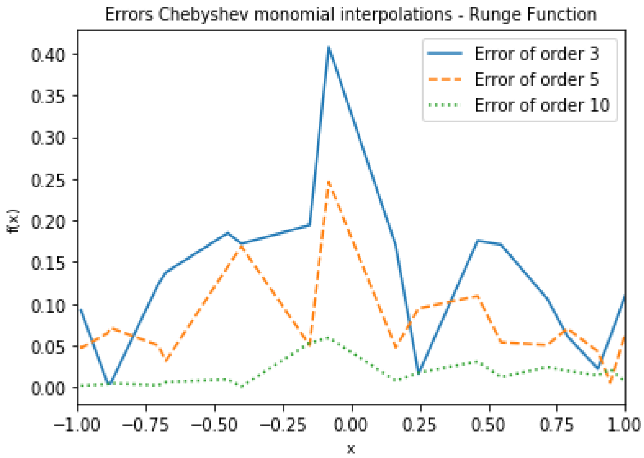
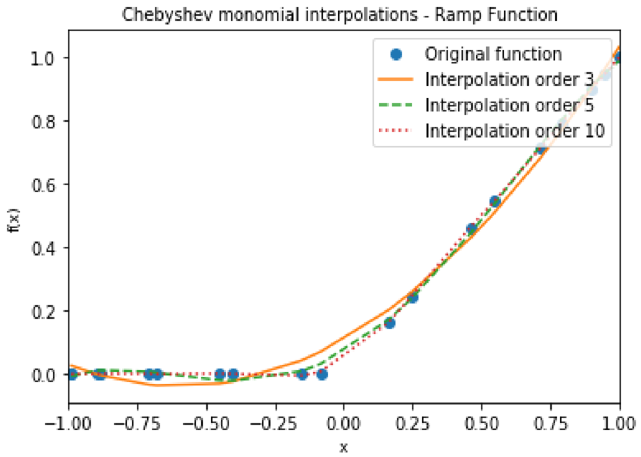
Part 2. Chebyshev interpolation nodes.

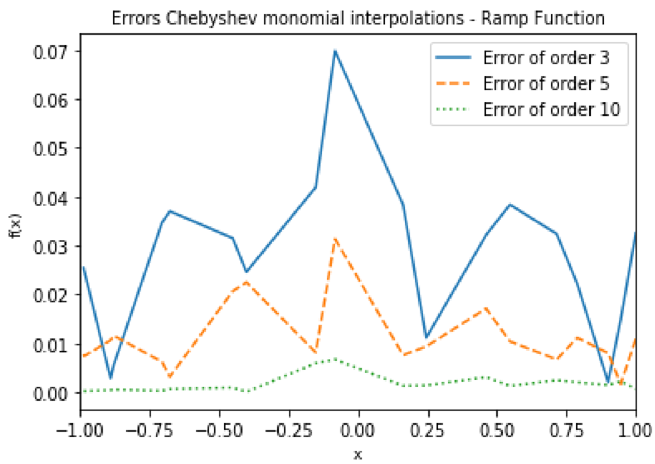




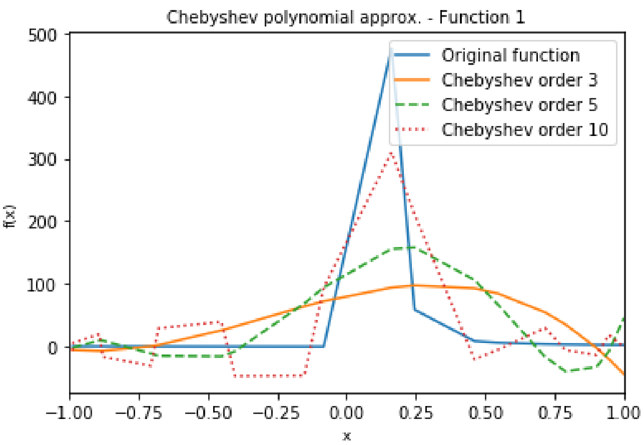


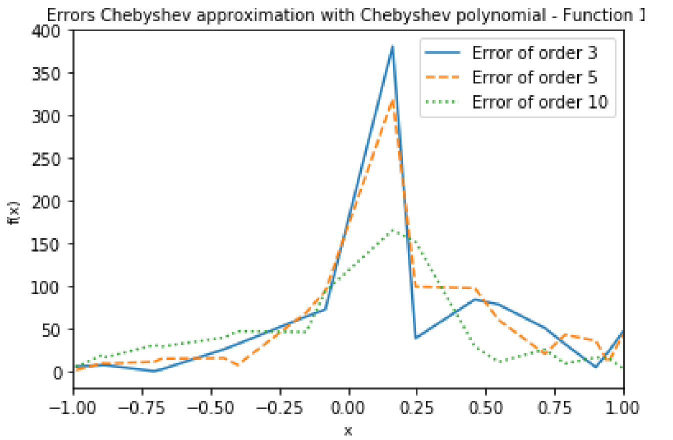


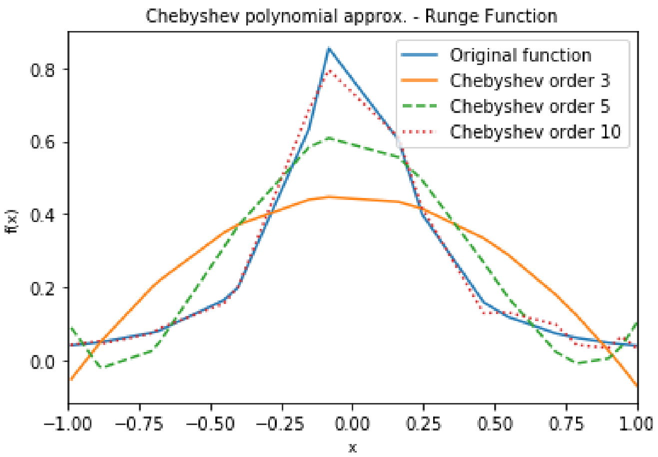


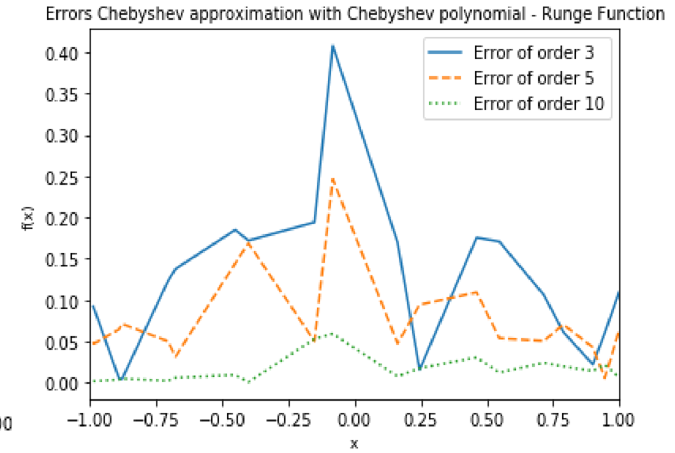


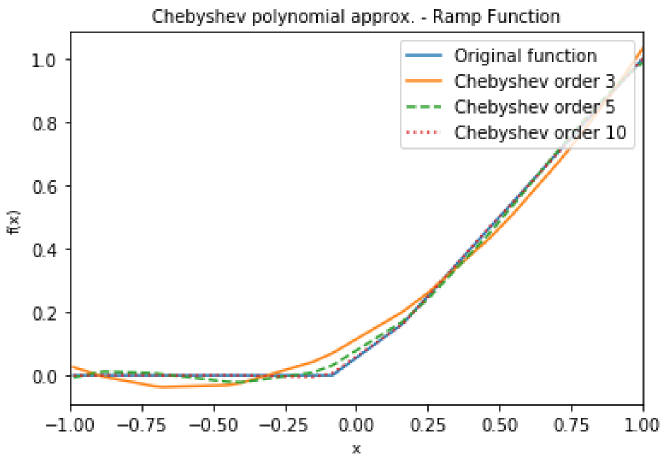
Part 3. Chebyshev polynomial interpolation

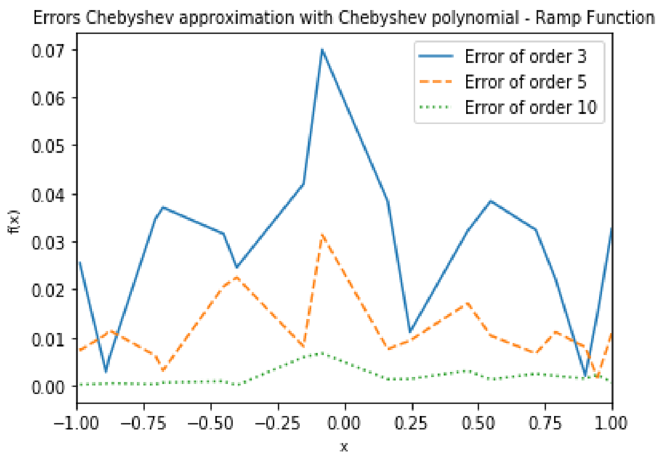












Comparing all the previous graphs with different techniques to approximate functions, we can observe some remarkable facts. Generally speaking, the approximation errors are smaller with the approximations made by Chebyshev (interpolation nodes and polynomials) than with the evenly space method. Firstly, in the evenly space interpolation, the errors are higher than in the second and third case at the tails. In the second case, the errors are lower because roots are closer among them in the tails and further in the centered points which is basically what Chebyshev technique does (populate more the extremes than the center). Finally, the last approximation by is smooth because now the nodes are interpolated by using Chebyshev polynomials. This approximation is better than the other ones due to the orthogonality of the basis but notice that it will not work properly when there are kinks and singularities as we can see with ramp and runge functions.

Question 2. Function Approximation. Multivariate.

1. Show that σ is the ES:

We are working with the following function called CES function, where σ is the elasticity of substitution between capital and labor:

$$f(k, h) = \left[(1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

In order to show the previous statement, we need to calculate the marginal productivity for both inputs (h, k):

$$MPH = \left[\frac{\sigma}{\sigma-1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}\right]\frac{\alpha(\sigma-1)}{\sigma}h^{\frac{-1}{\sigma}}$$

$$MPK = \left[\frac{\sigma}{\sigma-1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}\right]\frac{(1-\alpha)(\sigma-1)}{\sigma}k^{\frac{-1}{\sigma}}$$

In the next step we have to divide the MPK wrt MPH

$$\frac{MPK}{MPH} = \frac{(1-\alpha)h^{\frac{1}{\sigma}}}{\sigma k^{\frac{1}{\sigma}}} = \frac{1-\alpha}{\sigma} \cdot \left(\frac{h}{k}\right)^{\frac{1}{\sigma}}$$

Let's take logarithms: $\log(\frac{MPK}{MPH}) = \log(\frac{1-\alpha}{\sigma}) + \frac{1}{\sigma}\log(\frac{h}{k})$

Finally, we have to derivate the last expression $\log(\frac{MPK}{MPH})$ wrt $\log(\frac{k}{h})$ to obtain:

$$\sigma = \varepsilon_{kh}$$

2. Compute labor share for an economy with that CES

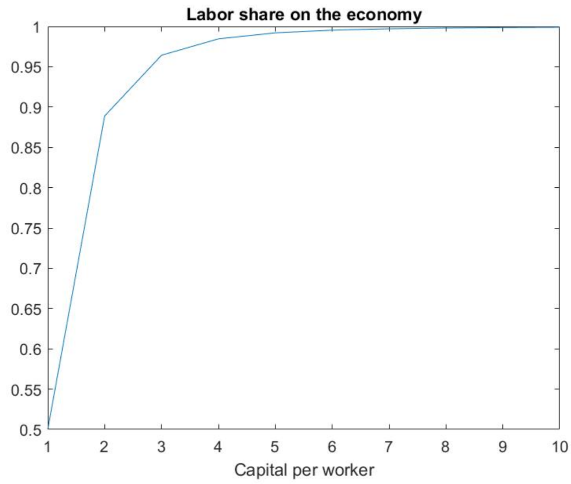
$$LS = \frac{MPH * h}{f(k,\ h)} = \frac{w * h}{Y}$$

Analytically,

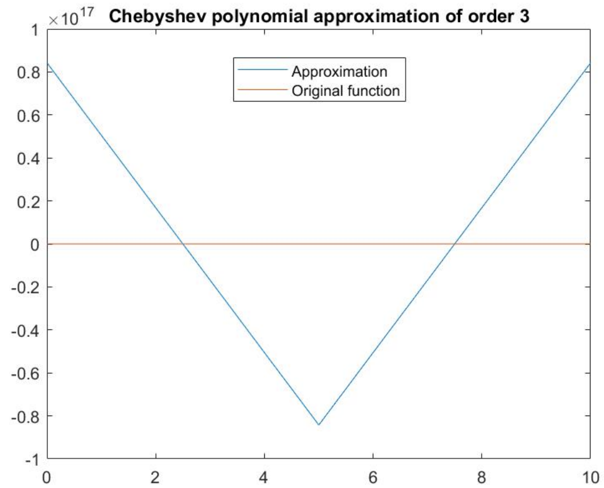
$$LS=\frac{\left[\frac{\sigma}{\sigma-1}((1-\alpha)k^{\frac{\sigma-1}{\sigma}}+\alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}\right]\frac{\alpha(\sigma-1)}{\sigma}h^{\frac{-1}{\sigma}}}{\left[(1-\alpha)k^{\frac{\sigma-1}{\sigma}}+\alpha h^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}$$

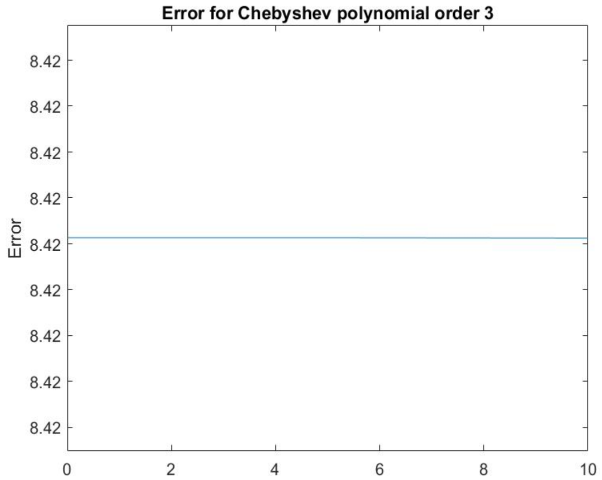
$$LS=\alpha h^{\frac{\sigma-1}{\sigma}}\left[(1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}\right]^{-1} \rightarrow LS = \left[\frac{1-\alpha}{\alpha} \cdot \left(\frac{k}{h}\right)^{\frac{\sigma-1}{\sigma}} + 1\right]^{-1}$$

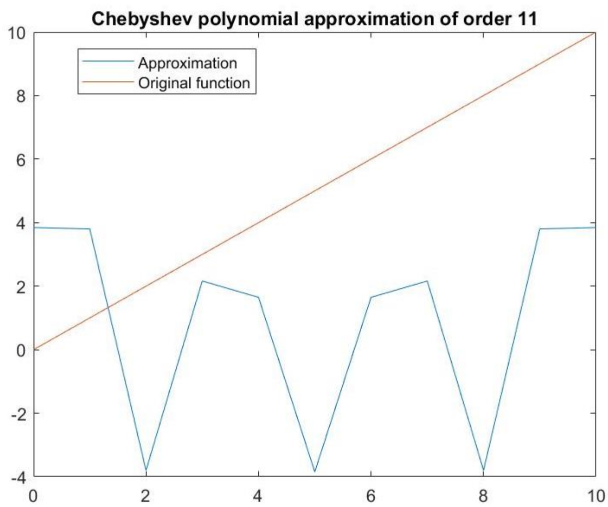
By plugging this formula in MATLAB, the labor share has the following shape

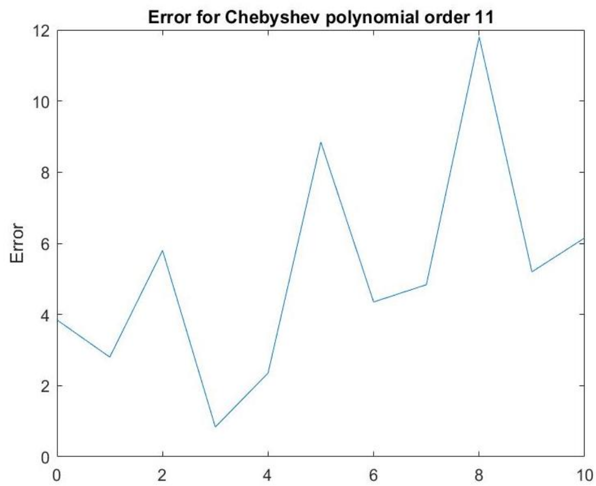


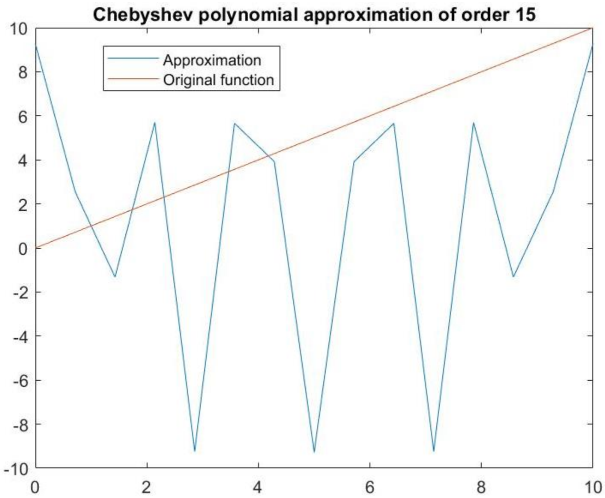
Now, for this part of the exercise, we compute the Chebyshev polynomial approximation using the Chebyshev regression algorithm adapted to the case of two variables. In the following plots, we have represented the approximation for order 3, 11 and 15 (in a 2-dimensional graph because we do not how to do it in MATLAB):

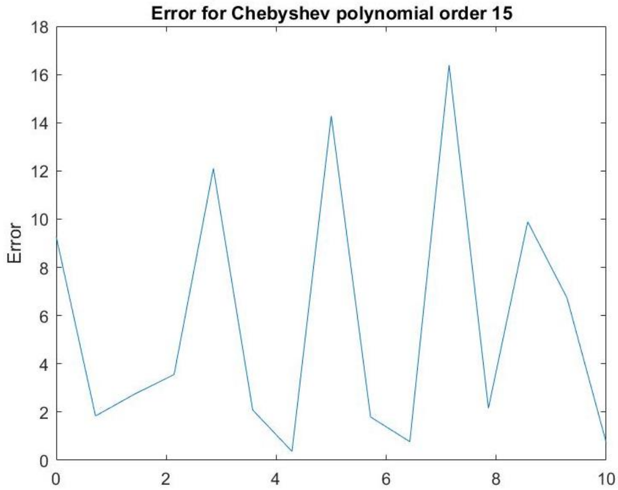












Finally, we plot the isoquants associated with the percentiles 5, 10, 25, 50, 75, 90 and 95 of output: