Full derivation of the stability derivatives in Nastran

$$\{w_j\} = [A_{jj}]\{f_j/\bar{q}\}$$ 2-1

$$\{w_j\} = [D^1_{jk} + ikD^2_{jk}]\{u_k\} + \left\{w^g_j\right\}$$ 2-2

$$\{P_k\} = [S_{kj}]\{f_j\}$$ 2-3

w_j = downwash origin of lift

w^g_j = static aerodynamic downwash; it includes, primarily, the static incidence distribution that may arise from an initial angle of attack, camber, or twist

f_j = pressure on lifting element j

$\bar{q}$ = flight dynamic pressure

k = reduced frequency, $k = \omega b/V$ where ω is the angular frequency, b is a reference semichord, and V is the free-stream velocity

$A_{jj}(m, k)$ = aerodynamic influence coefficient matrix, a function of Mach number (m), and reduced frequency (k)

u_k, P_k = displacements and forces at aerodynamic grid points

D^1_{jk}, D^2_{jk} = real and imaginary parts of substantial differentiation matrix, respectively (dimensionless)

S_{kj} = integration matrix

2-1: definition of downwash

2-2: downwash from differentiation of deflections

2-3: integration of forces and moments

$$[Q_{kk}] = [S_{kj}][A_{jj}]^{-1}[D^1_{jk} + ikD^2_{jk}]$$ 2-4

All aerodynamic methods compute the S, D^1, and D^2 matrices at user-supplied Mach numbers and reduced frequencies. The Doublet-Lattice and ZONA51 theories compute the A matrix. Then, matrix decomposition and forward and backward substitution are used in the computation of the Q matrix. The remaining methods compute A^{-1} directly and use matrix multiplications to form Q.

Splining

Splines provide an interpolation capability that couples the disjoint structural and aerodynamic models in order to enable aeroelastic analysis. The aeroelastic splines are used for two distinct purposes: as a force interpolator to compute a **structurally equivalent** force distribution on the structure given a force distribution on the aerodynamic mesh and as a displacement interpolator to compute a set of aerodynamic displacements given a set of structural displacements. The force interpolation is represented mathematically as:

$$\left[F_s\right] = \left[G_{sa}\right]\left[F_a\right] \tag{2-22}$$

and the displacement interpolation as:

$$\left[U_a\right] = \left[G_{as}\right]\left[U_s\right] \tag{2-23}$$

Where $\boldsymbol{G}$ is the spline matrix, $\boldsymbol{F}$ and $\boldsymbol{U}$ refer to forces and displacements, respectively and the $\boldsymbol{s}$ and $\boldsymbol{a}$ subscripts refer to structure and aerodynamics, respectively.

The two splines given in the above relationships are used when making the force and displacement interpolations. However, virtual work principals can be applied to relate the two splines as being the transform of one another:

$$\left[G_{sa}\right] = \left[G_{as}\right]^T \tag{2-24}$$

As stated in (2-22) and (2-23), the two basic relationships that must be developed are the displacement transformation and the force transformation. In general, the structural displacements are the usual six global displacement degrees of freedom and the forces are the usual three forces and three moments. The aerodynamic degrees of freedom depend on the aerodynamic method, but must include displacements normal to a local surface and rotations about an axis lying in the osculatory plane since these are the degrees of freedom used in the internal aerodynamic methods. The corresponding aerodynamic forces are a normal force and a local pitching moment.

The force transformation must be computed such that the resultant structural loads are statically equivalent to the aerodynamic loads:

$$\sum_{i=1}^{3ns} [TBG]_i [F_s]_i = \sum_{j=1}^{3na} [TBA]_j [F_a]_j \tag{2-25}$$

where $[TBG]$ is the transformation from global to basic coordinates. For moments, the following condition must be satisfied:

$$\sum_{i=1}^{3ns} [r]_i \times [TBG]_i [F_s]_i = \sum_{j=1}^{3na} [r]_j \times [TBA]_j [F_a]_j \tag{2-26}$$

where the $[r]_i$, $[r]_j$ are, respectively, the vectors between the (arbitrary) moment center and the structural and aerodynamic mesh points in the basic coordinate system. These two requirements are imposed on the individual spline matrices on a component-by-component basis, thus ensuring that the relationship will hold for the assembled spline transformation.

Each of the spline methods yields a relationship:

$$\{U(x,y,z)\} = [R]\{a\} + [\bar{A}]\{P\} = [C][P] \tag{2-27}$$

where $[R]\{a\}$ are the weighted coefficients of the interpolant (usually determined by boundary conditions on the function, e.g., equilibrium) and $[\bar{A}]\{P\}$ are the coefficient matrix and the applied load respectively. The coefficient matrix is a function only of geometry and the form of the interpolant. The evaluation uses the structural geometry alone in (2-27) to evaluate the coefficients:

$$\{P_s\} = [C_{ss}]^{-1}\{U_s(x,y,z)\} \tag{2-28}$$

and then uses (2-27) again, with both geometries to evaluate the displacement function at the aerodynamic points given the solution of (2-28) (which are loads at the structural grids) for point displacements at the structural grids.

$$\{U_a(x,y,z)\} = [R_a]\{a\} + [\bar{A}_{as}]\{P_s\} \tag{2-29}$$

In other words, to create the spline transformation matrix, (2-27) is evaluated for point loads at the structural points to form basis vectors at the aerodynamic points that are the columns of the displacement transformation of (2-22).

It should be mentioned here that MSC Nastran supports the option of having a separate spline matrix for force and displacement transformation. The algorithms used are identical in the two cases but the set of points used in the splining could be different. The rationale for allowing separate force and displacement splines that a grid that is appropriate for predicting aerodynamic displacement may not be good for applying forces. For example, a grid point on a wing surface that is not attached to substructure may be needed to get a smooth pattern on the aerodynamic mesh but may load the structure inappropriately if it is included in the force transformation. The notations to describe these two splines are:

$[G^D_{kg}]$ - Transformation matrix to transform displacements from the structural grid to the aerodynamic grid.

$\left[G^P_{kg}\right]^T$ - Transformation matrix to transform forces from the aerodynamic grid to the structural grid.

Static Aeroelasticity

Static (or quasi-steady) aeroelastic problems deal with the interaction of aerodynamic and structural forces on a flexible vehicle that results in a redistribution of the aerodynamic loading as a function of airspeed. The aerodynamic load redistribution and consequent internal structural load and stress redistributions are of concern to the structural analyst. The possibility of a static aeroelastic instability, (that is, divergence) is also of concern to the structural analyst. The aerodynamic load redistribution and consequent modifications to aerodynamic stability and control derivatives are of interest to the aerodynamicist and the control systems analyst. The static aeroelastic capability in MSC Nastran addresses these needs by the computation of aircraft trim conditions, with subsequent recovery of structural responses, aeroelastic stability derivatives, and static aeroelastic divergence dynamic pressures.

For static aeroelasticity, the downwash relation of (2-2) becomes: $\{w_j\} = [D^1_{jk} + ikD^2_{jk}]\{u_k\} + \left\{w^g_j\right\}$

$$\{w_j\} = [D_{jk}]\{u_k\} + [D_{jx}]\{u_x\} + \left\{w^g_j\right\} \tag{2-104}$$

where:

$\{w_j\}$	= downwash vector (for example, angles of attack)
$\{u_k\}$	= vector of aerodynamic displacements (deformations)
$\{u_x\}$	= vector of "extra aerodynamic points" used to describe, for example, aerodynamic control surface deflections and overall rigid body motions
$\left\{w^g_j\right\}$	= represents an initial static aerodynamic downwash. It includes, primarily, the static incidence distribution that may arise from an initial angle of attack, camber, or washout (twist).
$[D_{jk}]$	= substantial derivative matrix for the aerodynamic displacements. This is the D^1_{jk} term of (2-2) and the D^2_{jk} term is not used for this quasi-steady analysis
$[D_{jx}]$	= substantial derivative matrix for the extra aerodynamic points

The theoretical aerodynamic pressures are given by (2-20).

$$\{f_j\} = \bar{q}[A_{jj}]^{-1}\{w_j\} \tag{2-105}$$

and the aerodynamic forces, based on (2-20) and (2-21), can be written

$$\{P_k\} = \bar{q}[W_{kk}][S_{kj}][A_{jj}]^{-1}\{w_j\} + \bar{q}[S_{kj}]\left\{f^e_j/\bar{q}\right\} \tag{2-106}$$

where all the terms have been defined in Internal Aerodynamic Theories.

Experimental Aerodynamic Corrections

The theoretical aerodynamic pressures are found from (2-1).

$$\{f_j\} = \bar{q}[A_{jj}]^{-1}\{w_j\} \tag{2-20}$$

Two experimental correction may be introduced into (2-3), so that the corrected force distribution becomes

$$\{P_k\} = [W_{kk}][S_{kj}]\{f_j\} + \bar{q}[S_{kj}]\left\{f^e_j/\bar{q}\right\} \tag{2-21}$$

where:

$[W_{kk}]$	= a matrix of empirical correction factors to adjust each theoretical aerodynamic box lift and moment to agree with experimental data for incidence changes; Giesing, Kalman, and Rodden (1976) [Reference 22] suggest one way of obtaining these factors.
$\left\{f^e_j/\bar{q}\right\}$	= vector of experimental pressure coefficients at some reference incidence (for example, zero angle of attack) for each aerodynamic element.

$\{u_x\}$	= vector of "extra aerodynamic points" used to describe, for example, aerodynamic control surface deflections and overall rigid body motions

The vector of aerodynamic extra points specifies the values of aerodynamic trim variables. MSC Nastran has a number of predefined variables, including incidence angles (α and β), roll, pitch, and yaw rates (p, q, and r) and two translational ($\ddot{u}_2$ and $\ddot{u}_3$) and three rotational ($\dot{p}$, $\dot{q}$ and $\dot{r}$) accelerations. The DJX matrix, then, provides the vector of downwash velocities for unit values of these aerodynamic extra points. Note that accelerations do not result in any downwash for the quasi-steady assumption so that the corresponding columns of the DJX matrix are null.

$$\sum_{i=1}^{3ns} [TBG]_i [F_s]_i = \sum_{j=1}^{3na} [TBA]_j [F_a]_j$$

Static Aeroelastic Equations of Motion

$$[F_s] = [G_{sa}][F_a]$$

The aerodynamic forces are transferred to the structure using the spline matrix in (2-22) and (2-25) reduced to the a-set to form an aerodynamic influence coefficient matrix, Q_{aa}, which provides the forces at the structural grid points due to structural deformations

$$[Q_{aa}] = [G_{ka}]^T[W_{kk}][S_{kj}][A_{jj}]^{-1}[D_{jk}][G_{ka}] \tag{2-107}$$

and a second matrix, Q_{ax}, which provides forces at the structural grid points due to unit deflections of the aerodynamic extra points:

$$[Q_{ax}] = [G_{ka}]^T[W_{kk}][S_{kj}][A_{jj}]^{-1}[D_{jx}] \tag{2-108}$$

The complete equations of motion in the a-set degrees of freedom require

K_{aa}	Structural stiffness matrix
M_{aa}	Structural mass matrix
P_a	Vector of applied loads (for example, mechanical, thermal, and gravity loads plus aerodynamic terms due to user input pressures and/or downwash velocities)

The a-set equations are then:

u_a: displacement of dofs

aerodynamic stiffness

trim variables

$$[K_{aa} - \bar{q}Q_{aa}]\{u_a\} + [M_{aa}]\{\ddot{U}_a\} = \bar{q}[Q_{ax}]\{u_x\} + \{P_a\} \text{ loads} \tag{2-109}$$

structural stiffness

forces due to variation in trim values

This is the basic set of equations used for static aeroelastic analysis. In the general case, rigid body motions are included in the equations to represent the free-flying characteristic of an air vehicle. This is addressed in MSC Nastran by a requirement that the user identify reference degrees of freedom equal in number to the number of rigid body motions using the SUPORT Bulk Data entry. (2-109) is then partitioned into r-set (supported) and l-set (left over) degrees of freedom yielding

$$\begin{bmatrix} K^a_{ll} & K^a_{lr} \\ K^a_{rl} & K^a_{rr} \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} + \begin{bmatrix} M_{ll} & M_{lr} \\ M_{rl} & M_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{u}_l \\ \ddot{u}_r \end{Bmatrix} = -\begin{bmatrix} K^a_{lx} \\ K^a_{rx} \end{bmatrix} \{u_x\} + \begin{Bmatrix} P_l \\ P_r \end{Bmatrix} \tag{2-110}$$

r-set: these were the rigid body motions

l-set: all other dofs

where the notation

$$[K^a_{aa}] = [K_{aa} - \bar{q}Q_{aa}]$$

$$[K^a_{ax}] = -\bar{q}[Q_{ax}]$$

has been introduced.

At this point the MSC Nastran implementation of aeroelastic analysis introduces a mathematical technique that is based on the MSC Nastran inertia relief analysis without aeroelastic effects. The technique entails multiplying the first row of (2-110) by D^T and adding the result to the second row, where

$$D = -[K_{ll}]^{-1}[K_{lr}] \tag{2-111}$$

is known as the rigid body mode matrix and can be shown to be only a function of the geometry of the model. The resulting set of equations is then

$$\begin{bmatrix} K_{ll}^a & K_{lr}^a \\ (D^T K_{ll}^a + K_{rl}^a) & (D^T K_{lr}^a + K_{rr}^a) \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} + \begin{bmatrix} M_{ll} & M_{lr} \\ (D^T M_{ll} + M_{rl}) & (D^T M_{lr} + M_{rr}) \end{bmatrix} \begin{Bmatrix} \ddot{u}_l \\ \ddot{u}_r \end{Bmatrix} = -\begin{bmatrix} K_{lx}^a \\ D^T K_{lx}^a + K_{rx}^a \end{bmatrix} \{u_x\} + \begin{Bmatrix} P_l \\ D^T P_l + P_r \end{Bmatrix} \tag{2-112}$$

If there were no aerodynamic terms, the $D^T K_{ll} + K_{rl}$ and $D^T K_{lr} + K_{rr}$ would sum to zero so that the second row of equations could be solved for $\{\ddot{u}_r\}$. With the aerodynamic coupling, this simplification is not possible.

It is seen that (2-112) contains $nl + nr$ equations with $2(nl + nr)$ undetermined quantities, where nl, nr, and nx are the number of degrees of freedom in the l and r sets and the number of aerodynamic extra points, respectively.

The undetermined accelerations can be directly specified using two relations. The first relation comes from the assumption of quasi-steady equilibrium and specifies that

$$\{\ddot{u}_l\} = [D]\{\ddot{u}_r\} \tag{2-113}$$

where $[D]$ is the rigid body mode matrix of (2-111), and (2-112) simplifies to

$$\begin{bmatrix} K_{ll}^a & K_{lr}^a \\ (D^T K_{ll}^a + K_{rl}^a) & (D^T K_{lr}^a + K_{rr}^a) \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} + \begin{bmatrix} M_{ll} D & M_{lr} \\ & m_r \end{bmatrix} \{\ddot{u}_r\} = -\begin{bmatrix} K_{lx}^a \\ D^T K_{lx}^a + K_{rx}^a \end{bmatrix} \{u_x\} + \begin{Bmatrix} P_l \\ D^T P_l + P_r \end{Bmatrix} \tag{2-114}$$

$$\begin{bmatrix} K_{ll}^a & K_{lr}^a \\ (D^T K_{ll}^a + K_{rl}^a) & (D^T K_{lr}^a + K_{rr}^a) \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} + \begin{bmatrix} M_{ll} & M_{lr} \\ (D^T M_{ll} + M_{rl}) & (D^T M_{lr} + M_{rr}) \end{bmatrix} \begin{Bmatrix} \ddot{u}_l \\ \ddot{u}_r \end{Bmatrix} = -\begin{bmatrix} K_{lx}^a \\ D^T K_{lx}^a + K_{rx}^a \end{bmatrix} \{u_x\} + \begin{Bmatrix} P_l \\ D^T P_l + P_r \end{Bmatrix} \tag{2-112}$$

where $[mr] = [M_{rr} + M_{rl}D + D^T M_{lr} + D^T M_{ll} D]$ is the “total” mass matrix relative to the u_r points.

The second relation recognizes that the $\{\ddot{u}_r\}$ structural accelerations are related to the aerodynamic extra points $\{u_x\}$ via

$$\{\ddot{u}_r\} = [TR]^T [TRX]\{u_x\} \tag{2-115}$$

where TRX is a Boolean matrix that selects accelerations from the aerodynamic extra points and $[TR]^T$ is a matrix that transforms accelerations from the aerodynamic reference point to the “supported” degrees of freedom. This second matrix is a function of only the geometry of the model.

The further solution of the static aeroelastic equations is dependent on the type of analysis required. The remainder of the section is divided into four subsections that treat (1) restrained analysis for trim and stability derivative analysis, (2) unrestrained stability derivative analysis, (3) rigid stability derivatives, and (4) divergence analysis.

Restrained Analysis

Significant simplification is made by assuming that the $\{u_r\}$ terms can be set to zero with the remaining displacements then computed relative to this assumption. Therefore, setting $u_r = 0$ in (2-114), and solving for u_l from the first row gives:

$$\{u_l\} = [K^a_{ll}]^{-1}[-[M_{ll}D + M_{lr}]\{\ddot{u}_r\} - [K^a_{lx}]\{u_x\} + \{P_l\}] \tag{2-116}$$

This is then substituted into the second row of (2-114) and the relationship for $\{\ddot{u}_r\}$ in terms of $\{u_x\}$ of (2-115) is used to give nr equations with only the u_x quantities undetermined:

$$[ZZX]\{u_x\} = \{PZ\} \qquad \{\ddot{u}_r\} = [TR]^T[TRX]\{u_x\} \tag{2-117}$$

$$\begin{bmatrix} K^a_{ll} & \cancel{K^a_{lr}} \\ (D^TK^a_{ll} + K^a_{rl}) & \cancel{(D^TK^a_{lr} + K^a_{rr})} \end{bmatrix}\begin{Bmatrix} u_l \\ \cancel{u_r} \end{Bmatrix} + \begin{bmatrix} M_{ll}D \; M_{lr} \\ m_r \end{bmatrix}\{\ddot{u}_r\}$$

$$= -\begin{bmatrix} K^a_{lx} \\ D^TK^a_{lx} + K^a_{rx} \end{bmatrix}\{u_x\} + \begin{Bmatrix} P_l \\ D^TP_l + P_r \end{Bmatrix}$$

where:

$$[ZZX] = [m_r][TR]^T[TRX] + -[D^TK^a_{ll} + K^a_{rl}][K^a_{ll}]^{-1}[M_{ll}D + M_{lr}][TR]^T[TRX] \tag{2-118}$$
$$-[D^TK^a_{lx} + K^a_{rx}][D^TK^a_{ll} + K^a_{rl}][K^a_{ll}]^{-1}[K^a_{lx}]$$

$$[PZ] = [D]^T\{P_l\} + \{P_r\} - [D^TK^a_{ll} + K^a_{rl}][K^a_{ll}]^{-1}\{P_l\} \tag{2-119}$$

The solution of (2-117) for u_x requires that the equation be augmented by user input relations that specify all but nr terms in the u_x vector. These user specifications can be done by either specifying u_x values directly or by specifying a linkage that makes a term (or terms) in the u_x vector dependent on an independently varying term. The augmented trim equation then has the form:

$$\begin{bmatrix} ZZX \\ IP \\ AEL \end{bmatrix}\{u_x\} = \begin{Bmatrix} PZ \\ Y \\ O \end{Bmatrix} \tag{2-120}$$

where IP is a pseudo-identity matrix with as many rows as there are user-specified constraints on the values of u_x terms. The IP matrix has ones in the row and columns corresponding to the constrained variables and zero elsewhere. The Y vector contains the values of the user-specified constraints. The AEL matrix contains any user-specified relationships between (or among) aerodynamic extra points. (2-120) can then be solved for the remaining terms in the u_x vector.

Once the u_x vector has been evaluated, $\{\ddot{u}_l\}$ and $\{u_l\}$ can be recovered using (2-115), and (2-116), respectively. Standard MSC Nastran data recovery techniques are used to compute user-requested values of displacements, stresses, etc.

The user may also request pressures and forces on the aerodynamic boxes or elements. These terms are calculated using (2-105) for the pressures and (2-106) for the forces.

Stability derivatives are also calculated using the ZZX and PZ matrices with some modifications. For stability derivatives associated with aerodynamic extra points, dimensional stability derivatives are obtained from

$$[KRZX] = [ZZX] - [m_r][TR]^T[TRX] \tag{2-121}$$

Stability Derivatives. The second term on the right-hand side represents removing the vehicle accelerations from the ZZX matrix. Nondimensional stability derivatives are then calculated using:

$$\begin{Bmatrix} C_x \\ C_y \\ C_z \\ C_{mx} \\ C_{my} \\ C_{mz} \end{Bmatrix} = \frac{1}{\bar{q}S}[NDIM][TR][KRZX] \tag{2-122}$$

where S is the reference area of the vehicle, TR transforms forces from the support location to the aerodynamic reference point and

$$[NDIM] = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0/b_{ref} & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0/c_{ref} & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0/b_{ref} \end{bmatrix} \tag{2-123}$$

where c_{ref} and b_{ref} are the reference chord and span, respectively.

The intercept coefficients are computed using only partitions of the PZ vector that are dependent on $\bar{q}$, that is, terms associated with user input downwashes and user input pressure coefficients. This means that applied mechanical, thermal, and gravity loads are neglected in the following calculation:

$$\begin{Bmatrix} C_x \\ C_y \\ C_z \\ C_{mx} \\ C_{my} \\ C_{mz} \end{Bmatrix}_0 = \frac{1}{\bar{q}S}[NDIM][TR][PZ] \tag{2-124}$$

Unrestrained Stability Derivatives

The stability derivatives of the previous section were computed under the assumption of $\{u_r\} = 0$. This calculates these quantities in a structural axis system that is dependent on the arbitrary selection of the support point location. The stability derivatives of an unrestrained vehicle must be invariant with the selection of the support point location. This invariance is obtained by introducing a mean axis system. The characteristic of the mean axis is that deformations of the structure about it occur such that there is neither movement of the center of gravity nor rotation of the principal axes of inertia. Stated another way, the displacements in the mean axis system are orthogonal to the rigid body modes of the vehicle. In terms of the rigid body mode matrix $[D]$ and the mass matrix of the system $[M_{aa}]$, the mean axis constraint is defined by

$$\begin{bmatrix} D \\ I \end{bmatrix}^T \begin{bmatrix} M_{ll} & M_{lr} \\ M_{rl} & M_{rr} \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} = 0 \qquad (2\text{-}125)$$

or

$$[D^T M_{ll} + M_{rl}]\{u_l\} + [D^T M_{lr} + M_{rr}]\{u_r\} = 0 \qquad (2\text{-}126)$$

(2-114) and (2-126) can be combined to give an overall system of equations:

$$\begin{bmatrix} K^a_{ll} & K^a_{lr} \\ D^T M_{ll} + M_{rl} & D^T M_{lr} + M_{rr} \\ D^T K^a_{ll} + K^a_{rl} & D^T K^a_{lr} + K^a_{rr} \end{bmatrix} \begin{Bmatrix} u_l \\ u_r \end{Bmatrix} + \begin{bmatrix} M_{ll} D + M_{lr} \\ 0 \\ m_r \end{bmatrix} \{\ddot{u}_r\} \qquad (2\text{-}127)$$

$$= -\begin{bmatrix} K^a_{lx} \\ 0 \\ D^T K^a_{lx} + K^a_{rx} \end{bmatrix} \{u_x\} + \begin{Bmatrix} P_l \\ 0 \\ D^T P_l + P_r \end{Bmatrix}$$

neglectable

The first row of this equation can be solved for u_l in terms of u_r and $\ddot{u}_r$ and u_x

$$\{u_l\} = -[K^a_{ll}]^{-1}[[K^a_{ll}]\{u_r\} + [M_{ll}D + M_{lr}]\{\ddot{u}_r\} + [K^a_{lx}]\{u_x\} - INTL] \qquad (2\text{-}128)$$

$$\equiv -[AMLR]\{\ddot{u}_r\} - [ARLR]\{u_r\} - [ALX]\{u_x\} + \{UINTL\}$$

where the ARLR, AMLR, ALX, and UINTL terms can be inferred. The INTL vector is the aerodynamic related portion of the P_l vector.

The expression for u_l of (2-128) can be placed in the second row of (2-127) to give

$$[M2RR]\{u_r\} + [M3RR]\{\ddot{u}_r\} + [K3LX]\{u_x\} = -[TMP1] \qquad (2\text{-}129)$$

where:

$$[M2RR] = [D^T M_{lr} + M_{rr}] - [D^T M_{ll} + M_{rl}][ARLR]$$

$$[M3RR] = -[D^T M_{ll} + M_{rl}][AMLR]$$

$$[K3LX] = -[D^T M_{ll} + M_{rl}][ALX]$$

$$[TMP1] = [D^T M_{ll} + M_{rl}]\{UINTL\}$$

(2-129) is solved for u_r in terms of $\ddot{u}_r$ and u_x to give:

$$\{u_r\} = -[M4RR]\{\ddot{u}_r\} - [K4LX]\{u_x\} - \{TMP2\} \tag{2-130}$$

where:

$$[M4RR] = [M2RR]^{-1}[M3RR]$$
$$[K4LX] = [M2RR]^{-1}[K3LX]$$
$$\{TMP2\} = [M2RR]^{-1}\{TMP1\}$$

If the expression for u_l from (2-128) is placed in the third row of (2-127), then:

$$[K2RR]\{u_r\} + [MSRR - KAZL \cdot AMLR]\{\ddot{u}_r\} + [KARZX]\{u_x\} = \{IPZ\} \tag{2-131}$$

where:

$$[K2RR] = -[D^T K^a_{ll} + K^a_{rl}][ARLR] + [D^T K^a_{lr} + K^a_{rr}]$$
$$MSRR = m_r$$
$$KAZL = D^T K^a_{ll} + K^a_{rl}$$
$$KARZX = KAZL - KAXL \cdot ALX$$
$$\{IPZ\} = \{INTZ\} - [D^T K^a_{ll} + K^a_{rl}]\{UINTL\}$$

where: $\{INTZ\}$ is the aerodynamic portion of $\left\{D^T P_l + P_r\right\}$

Next, the u_r expression of (2-130) is placed in (2-131) to give:

$$[MIRR]\{\ddot{u}_r\} + [KR1ZX]\{u_x\} = \{IPZF\} \tag{2-132}$$

where:

$$[M5RR] = -[K2RR][M4RR] + [MSRR]$$
$$[MIRR] = -[KAZL][AMLR] + [M5RR]$$
$$[KR1ZX] = -[K2RR][K4LX] + [KARZX]$$
$$\{IPZF\} = [K2RR]\{TMP2\} + \{IPZ\}$$

The stability derivatives require an equation that pre multiplies the $\ddot{u}_r$ term by the rigid body mass matrix. This is achieved by pre multiplying (2-132) by $[MSRR][MIRR]^{-1}$

$$[MSRR]\{\ddot{u}_r\} = [Z1ZX]\{u_x\} + \{IPZF2\} \tag{2-133}$$

where:

$$\{IPZF1\} = [MIRR]^{-1}\{IPZF\}$$
$$\{IPZF2\} = [MSRR]\{IPZF1\}$$
$$[KR2ZX] = -[MIRR]^{-1}[KR1ZX]$$
$$[Z1ZX] = [MSRR][K2RZX]$$

In a manner similar to the restrained case, the unrestrained stability derivatives can be obtained using

$$\left.\begin{array}{l}\begin{bmatrix} C_x \\ C_y \\ C_z \\ C_{mx} \\ C_{my} \\ C_{mz} \end{bmatrix} = \frac{1}{\bar{q}S}[NDIM][TR][Z1ZX] \\ \begin{Bmatrix} C_x \\ C_y \\ C_z \\ C_{mx} \\ C_{my} \\ C_{mz} \end{Bmatrix}_0 = \frac{1}{\bar{q}S}[NDIM][TR]\{IPZF2\} \end{array}\right\} \tag{2-134}$$

Rigid Stability Derivatives and Mean Axis Rotations

This subsection briefly provides a theoretical description of several data blocks that are used to provide output to the user. Two sets of rigid stability derivatives are printed out. These are stability derivatives that are computed when elastic deformations are neglected. The first set of rigid derivatives are denoted as "unsplined" and are the values obtained directly from the aerodynamic calculations before they have been transferred to the structure. The dimensional matrix that contains these derivatives is

$$[RSTAB] = \bar{q}[SRKT]^T[Q_{kx}]$$

where:

$$[Q_{kx}] = [W_{kk}][S_{kj}][A_{jj}]^{-1}[D_{jx}]$$

[see (2-108)], and $SRKT$ is a matrix that sums forces acting on each of the aerodynamic boxes or elements to the supported degrees of freedom. This matrix is only a function of the geometry of the aerodynamic model and the locations of the support degrees of freedom.

The intercept stability dimensional derivatives are computed using

$$\{RINT\} = \bar{q}[SRKT]^T\left([W_{kk}][S_{kj}][A_{jj}]^{-1}\{w_j\} + [S_{kj}]\left\{f_j^e/\bar{q}\right\}\right) \tag{2-135}$$

Similar expressions are available to the splined rigid stability derivatives and intercepts

$$[KSAZX] = [D^T K_{lx}^a + K_{rx}^a] \tag{2-136}$$

$$\{INTZ\} = [G_{ka}]^T\{RINT\} \tag{2-137}$$

Nondimensionalization of these matrices is performed in a fashion similar to that given in (2-122) for stability derivatives and (2-124) for intercept values.

The availability of these rigid terms is useful in several ways. A comparison between the splined and unsplined derivatives provides an assessment of the quality of the splining. If the numbers differ significantly, this may indicate that not all aerodynamic elements have been transferred to the structure. This may be the user's intent or it may indicate a user error. If the two sets of rigid numbers bear little resemblance to one another, a serious splining error has been made. The guidelines of Aeroelastic Modeling should be consulted to determine if there is a modeling error.

Similarly, experience should allow the user to assess the reasonableness of the flexible results when compared with the rigid numbers. Large differences indicate large structural deformations and may point up conditions such as local weaknesses in the structure, an aerodynamic model displaced from the structural model, or errors in the input of the flight condition.

The rotations of the mean axes relative to the structural axes through the support points are required when restrained aeroelastic coefficients (stability derivatives and intercept coefficients) are used in the equations of motion [see Rodden and Love (1985) [Reference 51]]. The deflections of the mean axes relative to the origin of the structural axes are derived as follows.

The grid point deflections relative to the mean axes $\{\bar{u}_l\}$ are related to the deflections of the SUPORT points $\{\bar{u}_r\}$ through the rigid body mode matrix $[D]$ (see Figure 2-5).

$$\{\bar{u}_l\} = \{u_l\} + [D]\{\bar{u}_r\} \tag{2-138}$$

The requirement for the mean axes is that deformation occurs about them such that the center of gravity does not move and the axes do not rotate. In terms of the rigid body mode matrix and the mass matrix, this condition is expressed by

$$\begin{bmatrix} D \\ I \end{bmatrix}^T \begin{bmatrix} M_{ll} & M_{lr} \\ M_{rl} & M_{rr} \end{bmatrix} \begin{Bmatrix} \bar{u}_l \\ \bar{u}_r \end{Bmatrix} = 0 \tag{2-139}$$

(2-138) and (2-139) lead to

$$\{\bar{u}_r\} = -[m_r]^{-1}[D^T M_{ll} + M_{rl}]\{u_l\} \tag{2-140}$$

Introducing (2-116) yields

$$\{\bar{u}_r\} = -[m_r]^{-1}[D^T M_{ll} + M_{rl}][K^a_{ll}]^{-1} \\ ([M_{ll}D + M_{lr}]\{\ddot{u}_r\} + [K^a_{lx}]\{u_x\} - \{P_l\}) \tag{2-141}$$

and with (2-115) we finally obtain

$$\{\bar{u}_r\} = -[m_r]^{-1}[D^T M_{ll} + M_{rl}][K^a_{ll}]^{-1}[([M_{ll}D \\ + M_{lr}][TR]^T[TRX] + [K^a_{lx}])\{u_x\} - \{P_l\}] \tag{2-142}$$

The coefficient of $\{u_x\}$ in (2-142) is defined as the matrix $[HP]$ corresponding to the aerodynamic extra points

$$[HP] = [m_r]^{-1}[D^T M_{ll} + M_{rl}][K^a_{ll}]^{-1}([M_{ll}D + M_{lr}][TR]^T[TRX] + [K^a_{lx}])$$

The second term in (2-142) is defined as the matrix $[HP0]$ corresponding to the user inputs of $\{w^g_j\}$ and $\left\{f^e_j/\bar{q}\right\}$.

$$[HP0] = -[m_r]^{-1}[D^T M_{ll} + M_{rl}][K^a_{ll}]^{-1}\{P_l\}$$

The columns of deflections in $[HP]$ are used to find the mean axis rotations for each aerodynamic extra point, α_{m_i} in the longitudinal case, β_{m_i} in the lateral case, and γ_{m_i} in the directional case, for the user inputs; $[HP0]$ is used in like manner to find the longitudinal mean axis rotation α_{m_0}.

The general problem of obtaining the mean axis rotations with multiple SUPORT points is a problem in solid analytical geometry that is beyond the scope of this guide, but is also not regarded as a practical situation. If there is only a clamped SUPORT at one point (see Figure 2-5) the deflections in $[HP]$ and $[HP0]$ are not needed; only the rotations are of interest, and these are illustrated in the examples of Static Aeroelastic Analysis Problems. In the more general longitudinal case of two SUPORTed grid points (see Figure 2-6) the longitudinal mean axis rotations α_{m_i} and α_{m_0} are found by dividing the difference between the upstream and downstream deflections by the distance between the two grid points. Note that "upstream" and "downstream" must be determined by PARAM,USETPRT,11 in the Bulk Data to account for any resequencing of grid points in the MSC Nastran solution.

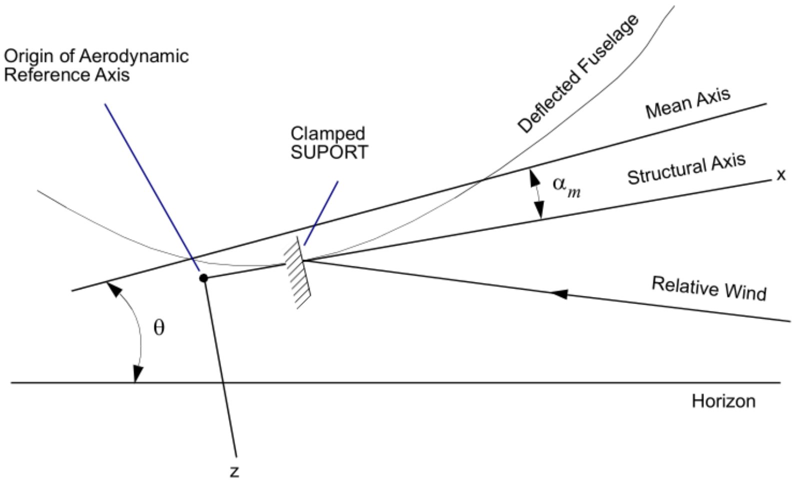

Figure 2-5 Geometry of Deformed Flight Vehicle with Clamped SUPORT

our case

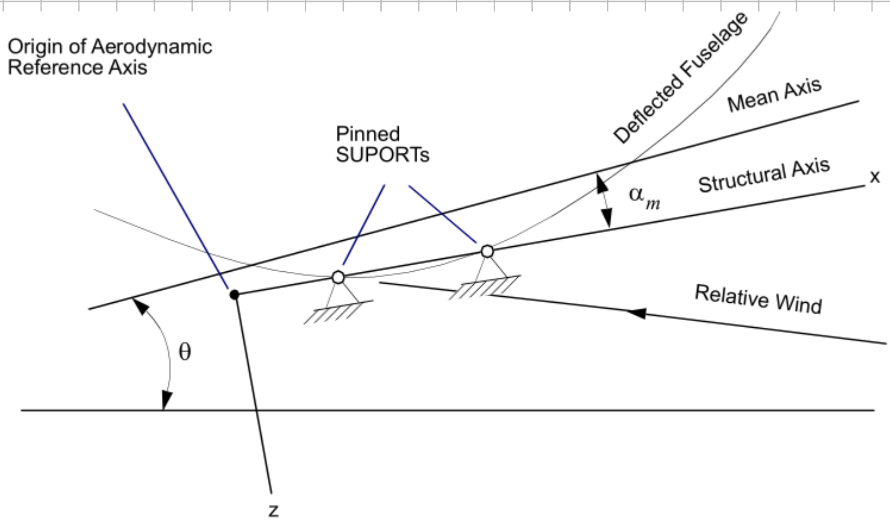

Figure 2-6 Geometry of Deformed Flight Vehicle with Multiple SUPORTs

Static Aeroelastic Analysis

Static aeroelastic analysis is intended to obtain both structural and aerodynamic data. The structural data of interest include loads, deflections, and stresses. The aerodynamic data include stability and control derivatives, trim conditions, and pressures and forces. The analysis presupposes a structural model (both stiffness and inertial data), an aerodynamic model, and the interconnection between the two.

The requirements for static aeroelastic analysis beyond those for the structural and aerodynamic models are nominal. The Executive Control requires the statement SOL 144 to call for the Static Aeroelastic Response DMAP sequence, SOL 144. Optional case control entries are AESYMXZ and AESYMXY, which specify symmetry flight conditions (these can be alternatively entered using the AEROS entry if a single symmetry is being used in the analysis). The AECONFIG command designates a parameter that can be utilized by the database with AECONFIG=AEROSG2D the default. The AERCONFIG command is a required parameter when executing the flexible part of a rigid/flexible aeroelastic analysis.

AEROS is present in femap

The stability derivatives are obtained as part of the solution process and are always printed. All other derived quantities of interest must be requested in the Case Control: The subcase commands APRES $= n$ and AEROF $= n$ request the aerodynamic pressures and forces for a set of aerodynamic grid points defined by n. The flight conditions are specified for each SUBCASE on the required TRIM commands that invokes AELINK and TRIM (or TRIM2) entries.

The reference geometry for the dimensionless stability derivatives are specified on the AEROS entry. Three bulk data entries; viz., AESTAT, AESURF and AEPARM, can be used to define aerodynamic extra points used in the trim analysis. The AESTAT entry is used to invoke specific aerodynamic conditions with the LABEL field defining each, such as angle of attack (ANGLEA) or vertical acceleration (URDD3). The presence of this entry triggers the generation of a downwash vector that provides the aerodynamic force for the extra point. Trimming surfaces are specified on a AESURF entry and are used to define the aerodynamic boxes that make up the surface. Note that up to two sets of aerodynamic elements and hinge lines can be defined for a single AESURF, enabling; for example, specifying right and left elevators as a single control surface. There are a number of other optional parameters that can be input using the AESURF bulk data entry to support control surface effectiveness, reference dimensions and deflection and hinge moment limits. If the aerodynamics for the control surface are associated with external aerodynamics, NOLDW is specified and the aerodynamic are input using some form of direct input, such as the AEFORCE entry. The AESURFS entry is a companion to the AESURF entry and can be used to specify structural grids whose mass can be used to derive the moments of inertia about the control surface hinge line. The AEPARM entry provides an additional means of specifying an aerodynamic extra point. In this case, you must supply not only the LABEL of the extra point but also any loading data associated with it. This entails the specification of bulk data entries including UXVEC, AEFORCE, AEDW, AEPRESS and DMIJ,DMIJI and DMIK. The UXVEC entry defines a state that can be any combination of the available extra points. This leads to a number of complexities, so the capability must be used with care. For a given trim analysis, the number of trim states is equal to the number of AESTAT,AESURF and AEPARM entries, but the total number of states is equal to the number of AESTAT,AESURF entries NOT mentioned on a UXVEC entry plus the number of relevant UXVEC entries. An important exception is when the UXVEC contains a LABELi corresponding to an AESTAT with UXi = 1.0 and either LABELj = INTERCPT with UXj = 1.0 or no INTERCPT. In this case the state is added to the existing one from the AESTAT. This can be used; e.g., to input aerodynamics on a fuselage that are not contained in the aerodynamic model. If the UXVEC contains a LABELi corresponding to an AESTAT and UXi is not equal to 1.0, it is an error.

The UXVEC entry defines the associated state while associated AEDW,AEPRESS,AEFORCE entries point to the user defined data for the state. Note that these three entries require a Mach number specification that corresponds with trim Mach number specified on the TRIM entry, a UXID that identifies the UXVEC state and a character label that points to the actual data. The AEDW and AEPRESS entries provide a way of specifying downwash and pressure data on the aerodynamic model. The AEDW points to DMIJ/DMIJI entries that specify downwash values at the j-set points of the model as specified in Figure 3-8 while the AEPRESS entry points to a DMIK entry with pressure values specified in the k-set as specified in Figure 3-7. The AEFORCE entry can point to the structural or the aerodynamic mesh as specified by the MESH describer (AERO or STRUCT). For AERO forces, an DMIK entry is invoked while the STRUCT option leads to a set of loads that can be any of the types of loads supported in a statics analysis. Care must be taken while interpreting the full vehicle integrated loads on aerodynamic monitor points when using UXVEC to specify AEFORCE with STRUCT loads. Since the forces affect the structural mesh directly, the breakdown of integrated loads into rigid, elastic and inertial portions for structural monitor points are from direct integration. However, since no aerodynamic mesh is specified using AEGRID, the integrated loads output for aerodynamic monitor points are not meaningful. There is a special case when there are no AESTAT entries and all the aerodynamics are input using AEPARM entries. In this case, it is necessary to establish the INTERCPT aerodynamics explicitly so that at least one UXVEC must have label INTERCPT=1.0, either explicitly or by default.

The AELINK bulk data entry enables the specification of any linear relationship among the aerodynamic extra points (AESTAT, AESURF or AEPARM entries). For example, the inboard and outboard ailerons could be linked together by a schedule specified by flight control engineers. The two trim entries differ in that TRIM entry requires the specification of all fixed variables, including the ones that are set to 0.0. Any variable that is not specified is considered to be free. Note that a valid trim specification requires

$$nr + nt + nael = nx \tag{3-2}$$

where,

nr	=	number of SUPORTed degrees of freedom	Ex. vertical displacement and pitch
nt	=	number of constrained variables on the trim entry	Ex. pitch rate, pitch acc. and vert. acc
nael	=	number of AELINK entries for this trim condition	Zero
nx	=	number of aerodynamic extra points (i.e. number of AESTAT, AESURF, AEPARM entries)	5: AoA, tail rotation, pitch rate, pitch acc. and vert. acc.

By contrast, the TRIM2 entry requires the specification of only non-zero fixed variables and the designation of the aerodynamic extra points that are free. Any aerodynamic extra point that is not otherwise specified in this case is assumed to be fixed at 0.0. This entry is useful; e.g., when there are many control surfaces that are undeflected in the trim state.

Both the TRIM and the TRIM2 entries support the AEQR parameter which governs how much aeroelastic feedback is included in the analysis. The default value of AEQR=1.0 provides for the full aeroelastic effect while values less than one multiply the dynamic pressure of the feedback to result in a smaller correction.

The TRIMF case control command enables the output of trimmed forces to the MSC Nastran punch file (or other user specified location) in the form of FORCE and MOMENT bulk data entries. Considerable flexibility is provided in this output with a default printout of the total elastic forces and options for the additional print of rigid and flexible inertial, applied, aerodynamic and total loads.

Stability derivatives are typically computed about the zero state of all the aerodynamic analysis. If the analysis is linear, this is sufficient. For nonlinear analyses, the AEUXREF case control command can be used to specify the computation of stability derivatives about the trim state or about a state specified by a UXVEC entry invoked by the AEUXREF case control command.

CSV Output of Static Aeroelastic Results

Two CSV (comma separated values) files are available to provide a summary of static aeroelastic results that would otherwise have to be gleaned from disparate parts of the .f06 file. This is particularly valuable when hundreds of subcases are being analyzed in a single run. The intent is that these files can be viewed and manipulated in a spreadsheet application.

LDSUM

PARAM LDSUM (see Chapter 6 in the *MSC Nastran Quick Reference Guide*) can be placed in case control or the bulk data packet to activate the output of critical results for each subcase. These results include subcase id, Mach and dynamic pressure value, trim configuration, mass and CG information and monitor point results that are controlled by the value of LDSUM as explained in the QRG. The unit of the CSV file is specified by PARAM XYUNIT and an assign statement such as:

```
assign userfile='aecsv1.csv' status=unknown form=formatted unit=52
```

defines the file where the results are stored. The unit 52 corresponds to PARAM XYUNIT 52. The first row in the spreadsheet provides titles for the columns that contain the results. Each subsequent row contains the requested results for a single subcase. PARAM XYUNIT is also used in SOL 200 (see *MSC Nastran Design Sensitivity and Optimization User's Guide*) to provide design optimization results. If the SOL 200 run includes static aeroelastic subcases and PARAM LDSUM is used, the resulting spreadsheet will have a row for each static aero subcase for each design iteration followed by design optimization results.

SDCSV

PARAM SDCSV (see Chapter 6 in the *MSC Nastran Quick Reference Guide*) can be placed in case control or the bulk data packet to activate the output of stability derivatives for each subcase. As shown in the QRG, the user can select up to 6 forms for each stability derivative. For each row of the output, the stability derivative data is preceded by subcase id, Mach and dynamic pressure value, trim configuration, and mass and CG information The unit of the CSV file is specified by PARAM SDUNIT and an assign statement such as:

```
assign userfile='aecsv2.csv' status=unknown form=formatted unit=51
```

defines the file where the results are stored. The unit 51 corresponds to PARAM SDUNIT 51. The first row in the spreadsheet provides titles for the columns that contain the results. Each subsequent row contains the requested results for a single subcase.

Divergence Analysis

The Static Aeroelastic Solution Sequence can also perform a divergence analysis. The analysis is invoked by a `DIVERG` command in Case Control which, in turn, invokes a `DIVERG` Bulk Data entry. The eigenanalysis of (2-145) is carried out using a complex eigensolver.

- A `CMETHOD` Case Control command invokes an EIGC Bulk Data entry that specifies the attributes for the eigenanalysis. The `CMETHOD` request invokes a complex Lanczos eigenanalysis that asks for five roots to be extracted.
- The `DIVERG` Bulk Data entry allows the user to extract a desired number of divergence pressures (typically one, since the second and higher pressures are not of practical interest) for the Mach numbers given on the entry.

  The `DIVERG` Bulk Data entry indicates that the analysis is to be performed using incompressible aerodynamics ($m = 0.0$) and that five divergence roots are requested. Results from executing this input file are given at the end of Static Aeroelasticity in lieu of an example in Static Aeroelastic Analysis Problems.

Sample Case Control commands and Bulk Data entries for a divergence analysis are given in Listing 3-1.

Listing 3-1 Divergence Analysis Sample

```
TITLE = EXAMPLE HA144B:  BAH JET TRANSPORT WING DYNAMIC ANALYSIS
SUBTI = DIVERGENCE ANALYSIS
  ECHO = BOTH
  SPC = 13 $
  MPC = 1 $ CONTROL SURFACE RELATIVE MOTION
  SET 2 = 7 THRU 12
  SET 3 = 11
  DISP = 2
  SPCF = 3
  AEROF = ALL
  APRES = ALL
  DIVERG = 100
  CMETHOD = 100
BEGIN BULK
.
.
.
DIVERG     100      5         0.0
EIGC       100      CLAN      MAX                           5
$
ENDDATA
```

Monitor Points

"Monitor Points" is a concept originally developed for static aeroelasticity but has since been applied in the solution sequences 101,103,105,108,109,111,112,144,146 and 200. They are basically postprocessing operators that allow for the monitoring of key results in an analysis that is beyond what is available in standard data recovery. This section provides an overview of the available monitor point types and examples are each are shown in the results of Chapters 7 and 9.

The MSC Nastran user interface for Monitor Points starts with the MONITOR (Case) Case Control command, which provides control over the printing of the monitor points results. Toggles can be used to control the print of the individual monitor types. This command must be placed above the subcase level or in the first subcase and applies to all subcases.

- MONPNT1 - The MONPNT1 entry provides integrated loads at a user defined point in a user defined coordinate system that are output in a user defined output coordinate system. The user also identifies the nodes (on either the structural or the aerodynamic model) whose loads are to be integrated. This enables the output of the applied loading for the specified set of nodes and can be used; e.g., for the batch calculation of VMT (shear moment and torque) data.
- MONPNT2 - The MONPNT2 entry provides element results from the TABLEs Stress, Strain, or Force in a tabulated fashion. This can be used to pinpoint a particular response for output, as opposed to finding a particular item in a large OFP listing. For the results to be accurate, the term selected must be a linear function of the displacements.

  The user must identify the element TYPE and NDDL item. The type and nddl item is obtained from the nddl description for each table. See QRG Remark 5 of this entry. There are separate types for composite and element corner results.
- MONPNT3 - The MONPNT3 entry provides a summation of grid point forces at user specified integration points and in a user defined coordinate system. The summation of the internal loads is useful in calculating resultant forces at a cut in the structure. This can be used to provide the net load acting at a fuselage or wing station by making a "cut" in the structure and then identifying all the grids and elements on one side of the cut.

  The entry NAME is identified with a piece of structure by listing the elements and nodes associated with it. The grid point force data associated with these entities is then integrated to the location specified on the entry. The XFLAG can be used to exclude certain grid point force types from consideration.

- MONDSP1 - The MONDSP1 entry allows for the sampling of a displacement vector to create a blended displacement response at a user specified point and coordinate system. The displacement monitor point is essentially an RBE3 element (limited to a single dependent point) and the dependent grid is now an arbitrary point. The averaged displacement can be seen as providing a qualitative assessment of the elastic deflection of a vehicle. An example is to monitor the nominal pitch and plunge at a station along the wing.

  This entry is similar to the MONPNT1 entry but now it is displacements that are being averaged. As with the MONPNT1, the COMP field, points to a AECOMP entry which specifies a SET1 entry or entries (or AELIST for aerodynamic boxes) that specify the grids to be monitored.
- MONSUM - The MONSUM entry defines a new monitor result that is the weighted sum of existing monitor results. The existing monitor points do not need to be of the same type but they must be of similar type.

  This entry can be used for both updating and summing of monitor point results. It allows for the modification of existing component results from MONDSP1, MONPNT1 and MONPNT3. A scalar multiple, for example, can provide a change in sign or a change in units. Enabling the weighted summation or two or more MONDSP1, MONPNT1 or MONPNT3's, that are of the same type, can provide the ability to present running results along a wing or fuselage. Note that a differentiation is made between aerodynamic (AEMONDSP1 and AEMONPNT1) and structural (SMONDSP1 and SMONPNT1) monitor points. AEMONDSP1 and SMONDSP1 are of the same type and can be summed. Similarly, AEMONPNT1 and SMONPNT1 are of the same type. There are numerous comments in the QRG for the MONSUM (and all monitor point entries) and you should study these carefully to understand and appreciate their features.

- MONSUM1 - The MONSUM1 Bulk data entry specifies the location of the summed quantity. This enables the summation of monitor points from disparate points to a single location.

  The first continuation defines a CP,X,Y,Z, CD combination that specifies the point where the summation is said to occur and performs the summation using the equation provided below. By contrast, the MONSUM does not specify the location and therefore cannot be used in subsequent MONSUMT processing.

  The MONSUM1 does not support the combination of MONDSP1's since no physically meaningful interpretation or application can be envisioned for combining MONDSP1's in this manner.

  The underlying equation that is executed for the MONSUM1 is:

$$MONSUM1_j = \sum_i^n COEF_{ij} MR_i$$

  Where MR_i is the result from the individual component.

  Relative to the MONSUM, the MONSUM1 allows the user to specify where the summed output is requested. MSC Nastran uses this information plus the locations of the referenced monitor points that are being summed to perform a coordinate transformation as part of the summation.

- MONSUMT - The MONSUMT Bulk data entry provides the ability to transfer moments and thereby allows the specification of a monitor point location that is apart from the finite element model. This facilitates communication between the loads group and the stress group in the development/simulation process.

  With the MONSUMT entry moment transfers do occur. The first continuation defines a CP,X,Y,Z, CD combination that specifies the point where the loads are to be monitored and performs the summation/transfer using the equation provided below. By contrast, the MONSUM1 location is user specified and does not involve moment transfer.

  The MONSUMT like MONSUM1 does not support the combination of MONDSP1's since no physically meaningful interpretation or application can be envisioned for combining MONDSP1's in this manner.

  The MONSUMT can reference the results of another MONSUMT or MONSUM1 as long as there is not a circular reference.

  The underlying equation that is executed for the MONSUMT is:

$$MONSUMT_j = \sum_i^n T_{ji} MR_i$$

  Where T_{ij} is a partial rigid body vector for the location of the monitor points being summed and MR_i is the result from the individual component.

- MONCNCM - The MONCNCM entry is a special purpose monitor point that is restricted to SOL 144 and CAERO1 entries. It provides stripwise aerodynamic lift and pitching moment results. Similar results could be extracted using the MONPNT1 entry, but the MONCNCM entry provides a simple user interface that could help in visualizing the aerodynamics and aid in generating corrections factors. The code internally determines if flat plate panels are abutting in the streamwise direction and, if they are, considers this a single strip from the leading to the trailing edge of the multiple CAER01 entries. The output is in the aerodynamic panel coordinate system so that the sign of the forces and moments are a function of the numbering of the corners of the panel. The results are given for the aeroelastic trim state.

CORD2R 100 is for the rigid body motions of the aerodynamic reference point; specifically the pitch and moment axis is at the canard midchord at GRID 90. This coordinate system is the standard NACA body axis system with the x-axis forward and the z-axis downward. The stability derivatives are output using this coordinate system.

Static Aeroelastic Input

The foregoing input is typical for any aeroelastic analysis. The entries in the Bulk Data specifically for static aeroelastic analysis begin with the AESTAT entries, which specify the trim parameters. The parameters are angle of attack, $\alpha = \text{ANGLEA}$; pitch rate, $q\bar{c}/2V = \text{PITCH}$; normal load factor $\ddot{z}/g = URDD3$; and pitch acceleration, $\ddot{\theta}/g = URDD5$. Next, the trim surface is defined by an AESURF entry as the elevator (canard) ELEV using coordinate system CORD2R 1 for its hinge line and defining the aerodynamic boxes using AELIST 1000, which specifies aerodynamic box numbers 1000 through 1007. The reference geometry is specified on the AEROS entry. This entry specifies the aerodynamic coordinate system CORD2R 1, the aerodynamic reference coordinate system for rigid body motions CORD2R 100, a reference chord of $\text{REFC} = \bar{c} = 10.0\text{ ft}$, a reference span of $\text{REFB} = b = 40.0\text{ ft}$ (the full span), a reference area of $\text{REFS} = S = 200.0\text{ sq ft}$ (half-model), and symmetric aerodynamic loading (SYMXZ = 1). The first two TRIM entries specify the flight condition at Mach number, $m = 0.9$ and level flight with no pitch rate, $q\bar{c}/2V = \text{PITCH} = 0.0$, a one-g load factor, $\ddot{z}/g = \text{URDD3} = -1.0$, and no pitching acceleration, $\dot{q}/g = \ddot{\theta}/g = \text{URDD5} = 0.0$. The first entry, TRIM 1, specifies the low speed condition with dynamic pressure $\bar{q} = Q = 40\text{ psf}$, and the second entry, TRIM 2, specifies $\bar{q} = Q = 1200\text{ psf}$, both at $m = 0.9$ at sea level. The third entry, TRIM 3, specifies supersonic level flight at $m = 1.3$ at 20,000 ft with $Q = 1151\text{ psf}$.

.f06 Output

Portions of the output results from the .f06 filethat are unique to aeroelastic analysis are shown in Listing 7-2 and summarized here.

Stability Derivatives

The typical definition of the stability derivatives in the restrained longitudinal case may be illustrated by the lift coefficient [see, for example, Rodden and Love (1985) [Reference 51], and note that $C_z = -C_L$]: as the output ref sys has z downwards

$$C_z = C_{z_o} + C_{z_\alpha}\alpha + C_{z_{\delta_e}}\delta_e + C_{z_q}\frac{q\bar{c}}{2V} + \cancel{C_{z_{\dot{\alpha}}}\frac{\dot{\alpha}\bar{c}}{2V}} + C_{z_{\ddot{z}}}\frac{\ddot{z}}{g} + C_{z_{\ddot{\theta}}}\frac{\ddot{\theta}\bar{c}}{2g} \tag{7-1}$$

The $\dot{\alpha}$-derivatives are not obtained from the quasi-steady considerations here and will not be discussed further. The rotations of the mean axis in the restrained longitudinal case are defined in terms of rotational derivatives defined by Rodden and Love (1985) [Reference 51].

$$\alpha_m = \alpha_{m_o} + \alpha_{m_\alpha}\alpha + \alpha_{m_\delta}\delta_e + \alpha_{m_q}\frac{q\bar{c}}{2V} + \cancel{\alpha_{m_{\dot{\alpha}}}\frac{\dot{\alpha}\bar{c}}{2V}} + \alpha_{m_{\ddot{z}}}\frac{\ddot{z}}{g} + \alpha_{m_{\ddot{\theta}}}\frac{\ddot{\theta}\bar{c}}{2g} \tag{7-2}$$

Again, the $\dot{\alpha}$-term will not be discussed further. In the unrestrained case, all inertial derivatives vanish because their effects appear in the remaining derivatives, and the mean axis rotations do not have to be considered in the equations of motion.

Six sets of stability derivatives are generated for the system for each flight condition:

- Rigid unsplined
- Rigid splined
- Elastic restrained at the SUPORTed degrees of freedom
- Elastic unrestrained
- Inertia Restrained
- Inertia Unrestrained

Before the stability derivatives are tabulated, the transformation from the basic to the reference coordinates is shown. This transformation provides a check on the input of the aerodynamic reference coordinate system for the stability derivatives. The stability derivatives for the rigid and elastic vehicle are shown next. The rigid derivatives are those that are obtained while neglecting elastic deformation of the vehicle. These derivatives are presented in two ways: unsplined and splined coefficients, which provide checks on the splining. The unsplined coefficients are based on all of the boxes in the aerodynamic model and are independent of the spline. Usually, the two sets of coefficients are nearly identical unless there is an error in the spline input, such as not including all of the boxes. However, there may be situations where some boxes intentionally may not be connected to the spline, as in the case when no motion of certain boxes is desired.

The stability derivatives are summarized in Table 7-1. For the first dynamic pressure, $\bar{q} = Q = 40$ psf, the rigid and elastic coefficients are all quite close except that the inertial derivatives have finite values for the low dynamic pressure. In the rigid case, the inertial derivatives vanish, but in the limit of zero dynamic pressure, the flexible inertial derivatives remain finite. By virtue of the definitions of the inertial derivatives in (7-1), the printed output values corresponding to unit URDD5 $= \ddot{\theta}/g$ must be divided by $\bar{c}/2 = 5.0$ ft. The inertial derivatives are absorbed into the basic stability derivatives in the unrestrained case. The aerodynamic center for each loading may be found by dividing its moment coefficient by its lift coefficient and multiplying by the reference chord, for example, for the angle of attack α loading in unrestrained flight $x_{a.c.} = -C_{m_\alpha}\bar{c}/C_{z_\alpha} = 2.835$ ft aft of GRID 90 at low $\bar{q}$.

Table 7-1 Subsonic Derivatives for Example FSW Airplane

Derivative	Value for Rigid Airplane	Restrained Value at $\bar{q}$ = 40 psf	Unrestrained Value at $\bar{q}$ = 40 psf	Restrained Value at $\bar{q}$ = 1200 psf	Unrestrained Value at $\bar{q}$ = 1200 psf
C_{z_o}	-0.008421	-0.008464	-0.008509	-0.010332	-0.012653
C_{m_o}	-0.006008	-0.006031	-0.006064	-0.007074	-0.008678
C_{z_α}	-5.071	-5.103	-5.127	-6.463	-7.772
C_{m_α}	-2.871	-2.889	-2.907	-3.667	-4.577
$C_{z_{\delta_e}}$	-0.2461	-0.2538	-0.2520	-0.5430	-0.5219
$C_{m_{\delta_e}}$	0.5715	0.5667	0.5678	0.3860	0.3956
C_{z_q}	-12.074	-12.087	-12.158	-12.856	-16.100
C_{m_q}	-9.954	-9.956	-10.007	-10.274	-12.499
$C_{z_{\ddot{z}}}$	0.0	0.003154	-	0.003634	-
$C_{m_{\ddot{z}}}$	0.0	0.002369	-	0.002624	-
$C_{z_{\ddot{\theta}}}$	0.0	0.01181	-	0.01449	-
$C_{m_{\ddot{\theta}}}$	0.0	0.007900	-	0.009404	-

Hinge Moment Derivatives

Hinge moment coefficient output shows the moment produced at the control surface hinge line due to each of the aerodynamic extra points

Mean Axis Deformation

The mean axis translations and rotations for the SUPORT degrees of freedom follow next in the output and are shown as INTERMEDIATE MATRIX...HP. The rotational derivatives are presented in the second row (that is, printed in the second column shown). These derivatives are only associated with the restrained case, since they must be included in the equations of motion that utilize restrained stability derivatives, as discussed by Rodden and Love (1985) [Reference 51]. Equations of motion relative to the SUPORT using unrestrained stability derivatives are already expressed in terms of mean axis rotations.

The mean axis rotational derivatives are also summarized in Table 7-1. The derivative α_{m_α} is obtained by adding 1.0 to the tabulated value, and the derivative $\alpha_{m_{\ddot{\theta}}}$ is obtained by dividing by $\bar{c}/2 = 5.0$ ft.

α_{m_o}	-	-0.000004062	-	-0.0001419	-
α_{m_α}	-	0.9980	-	0.9251	-
$\alpha_{m_{\delta_e}}$	-	0.0003336	-	0.006420	-
α_{m_q}	-	-0.006942	-	-0.2136	-
$\alpha_{m_{\ddot{z}}}$	-	-0.0001624	-	-0.0001108	-
$\alpha_{m_{\ddot{\theta}}}$	-	-0.001641	-	-0.001457	-

The level flight trim solution follows the mean axis rotations. For the low speed condition, the angle of attack of the structural axis is necessarily high, ANGLEA $= \alpha = 0.169191$ rad $=$ 9.69 deg, and the corresponding canard incidence is ELEV $= \delta_e = 0.492457$ rad $=$ 28.22 deg.

Following the trim results, the listing shows default or user specified limits on control surfaces and compares them with the actual trim values.

The Structural and Aerodynamic Total Vehicle Coefficients which follow provide insight on the various loadings on the vehicle and how they balance one another. A description of this output is provided in Hinge Moment and Total Vehicle Coefficients.

The aerodynamic pressure and load data follow the trim solution. The pressure coefficients on each box are also high at the high angle of attack and low dynamic pressure. The pressures and box normal forces balance the weight of the airplane. The aerodynamic moments are taken about the midchord of each box: at subsonic speeds the box force acts at the box quarter-chord and causes a moment about the box reference midchord; at supersonic speeds the box force acts at the box midchord and results in a zero moment on each box.

```
                    TRANSFORMATION FROM BASIC TO REFERENCE COORDINATES:

                  { X }          [ -1.0000    0.0000    0.0000  ]  { X }          { 1.5000E+01 }
                  { Y }    =     [  0.0000    1.0000    0.0000  ]  { Y }     +    { 0.0000E+00 }
                  { Z }REF       [  0.0000    0.0000   -1.0000  ]  { Z }BAS       { 0.0000E+00 }

        CONTROLLER STATE: INTERCEPT ONLY, ALL CONTROLLERS ARE ZERO
```

TRIM VARIABLE	COEFFICIENT	RIGID		ELASTIC		INERTIAL	
		UNSPLINED	SPLINED	RESTRAINED	UNRESTRAINED	RESTRAINED	UNRESTRAINED
REF. COEFF.	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	-8.420786E-03	-8.420786E-03	-8.463757E-03	-8.508834E-03	0.000000E+00	-8.508834E-03
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-6.008128E-03	-6.008128E-03	-6.031396E-03	-6.063684E-03	0.000000E+00	-6.063684E-03
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
ANGLEA	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	-5.070975E+00	-5.070975E+00	-5.103213E+00	-5.127250E+00	0.000000E+00	-5.127250E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-2.870931E+00	-2.870931E+00	-2.889137E+00	-2.906502E+00	0.000000E+00	-2.906502E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
PITCH	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	-1.207429E+01	-1.207429E+01	-1.208659E+01	-1.215825E+01	0.000000E+00	-1.215825E+01
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-9.953998E+00	-9.953998E+00	-9.956322E+00	-1.000715E+01	0.000000E+00	-1.000715E+01
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
URDD3	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	0.000000E+00	0.000000E+00	3.154313E-03	0.000000E+00	1.000000E+00	0.000000E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	0.000000E+00	0.000000E+00	2.368632E-03	0.000000E+00	2.181625E-01	0.000000E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
URDD5	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	0.000000E+00	0.000000E+00	5.904780E-02	0.000000E+00	2.181625E+00	0.000000E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	0.000000E+00	0.000000E+00	3.950050E-02	0.000000E+00	1.163713E+01	0.000000E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
ELEV	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	-2.461394E-01	-2.461394E-01	-2.537563E-01	-2.519653E-01	0.000000E+00	-2.519653E-01
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	5.715301E-01	5.715301E-01	5.666561E-01	5.677910E-01	0.000000E+00	5.677910E-01
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

N O N - D I M E N S I O N A L   H I N G E   M O M E N T   D E R I V A T I V E   C O E F F I C I E N T S

CONFIGURATION = AEROSG2D   XY-SYMMETRY = ASYMMETRIC   XZ-SYMMETRY = SYMMETRIC
MACH = 9.0000E-01   Q = 4.0000E+01

CONTROL SURFACE = ELEV   REFERENCE CHORD LENGTH = 1.000000E+00   REFERENCE AREA = 1.000000E+00

TRIM VARIABLE	RIGID	ELASTIC		INERTIAL	
		RESTRAINED	UNRESTRAINED	RESTRAINED	UNRESTRAINED
AT REFERENCE	-1.541222E-01	-1.553415E-01	-1.559161E-01	0.000000E+00	0.000000E+00
ANGLEA	3.103143E+01	3.020692E+01	2.989496E+01	0.000000E+00	0.000000E+00
PITCH	-6.124346E+02	-6.132635E+02	-6.141365E+02	0.000000E+00	0.000000E+00
URDD3	0.000000E+00	4.527646E-02	0.000000E+00	0.000000E+00	0.000000E+00
URDD5	0.000000E+00	5.787790E-01	0.000000E+00	0.000000E+00	0.000000E+00
ELEV	1.193384E+02	1.192125E+02	1.192298E+02	0.000000E+00	0.000000E+00

AEROELASTIC TRIM VARIABLES

ID	LABEL	TYPE	TRIM STATUS	VALUE OF UX	
	INTERCEPT	RIGID BODY	FIXED	1.000000E+00	
501	ANGLEA	RIGID BODY	FREE	1.691910E-01	RADIANS
502	PITCH	RIGID BODY	FIXED	0.000000E+00	NONDIMEN. RATE
503	URDD3	RIGID BODY	FIXED	-1.000000E+00	LOAD FACTOR
504	URDD5	RIGID BODY	FIXED	0.000000E+00	RAD/S/S PER G
505	ELEV	CONTROL SURFACE	FREE	4.924567E-01	RADIANS

TRANSFORMATION FROM REFERENCE TO WIND AXES:

ANGLE OF ATTACK = 1.691910E-01 RADIANS (9.693934 DEGREES)

ANGLE OF SIDESLIP = 0.000000E+00 RADIANS (0.000000 DEGREES)

```
{ X }          [ -0.985721  0.000000 -0.168385 ]  { X }
{ Y }          [  0.000000  1.000000  0.000000 ]  { Y }
{ Z }WIND =    [  0.168385  0.000000 -0.985721 ]  { Z }REF
```

Output

The input data files are shown in Listing 7-7 followed by the sorted Bulk Data entries in Listing 7-8 and the output in Listing 7-9. The highlights of the computed results are discussed below.

The lateral stability derivatives are

$$C_l = C_{l_{\delta_a}}\delta_a + C_{l_p}\frac{pb}{2V} + C_{l_{\dot{p}}}\frac{\dot{p}b}{2g} \tag{7-3}$$

while the rotation of the lateral mean axis is given by

$$\gamma_m = \gamma_{m_{\delta_a}}\delta_a + \gamma_{m_p}\frac{pb}{2V} + \gamma_{m_{\dot{p}}}\frac{\dot{p}b}{2g} \tag{7-4}$$

In the unrestrained case, the inertial derivative vanishes, its effect is included in the other two derivatives, and the rotations are not needed in the rolling equation of motion.

At the Mach number $m = 0.0$, the restrained derivatives are found to be

$$C_{l_p} = -0.518943/\text{rad}$$

$$C_{l_{\dot{p}}} = -0.00021282/\text{rad}$$

$$C_{l_{\delta_a}} = 0.105493/\text{rad}$$

The output value for URDD4 is $C_{l_{\dot{p}}}\,b/2$; therefore, a division by $b/2 = 500$ in. leads to the value above.

The rotational derivatives are found from the intermediate matrix HP to be

$$\gamma_{m_p} = \frac{\partial\alpha_m}{\partial\left(\frac{pb}{2V}\right)} = 0.305455$$

$$\gamma_{m_{\dot{p}}} = \frac{\partial\alpha_p}{\partial\left(\frac{\dot{p}b}{2g}\right)} = 0.00419677$$

$$\gamma_{m_{\delta_a}} = \frac{\partial\alpha_p}{\partial\delta_a} = -0.0730480$$

where $\gamma_{m_{\dot{p}}}$ is obtained from the output by dividing by b/2. In the unrestrained case, the derivatives are

$$C_{l_p} = -0.505960/\text{rad}$$

and

$$C_{l_{\delta_a}} = 0.102853/\text{rad}$$

The trim solution gives a rolling helix angle of

$$pb/2V = -C_{l_{\delta_a}}\delta_a/C_{l_p} = 0.203284$$

for the aileron command of $\delta_a = 1.0\ \text{rad}$. A dynamic response solution gives $pb/2V\delta_a = 0.197$ from its graphical solution (see Example HA146B (p. 667)).

The remaining output of interest gives the pressure coefficients, pressures, and aerodynamic box forces in the steady roll, and the deformations near the wing tip (GRIDs 7, 8, 9, and 10) and at the aileron trailing edge (GRID 12). The rotation of the aileron actuator spring is given by GRID12, R2 and is -0.05056 rad; this is the aeroelastic effect on the aileron, that is, the net commanded aileron rotation is only $1.0 - 0.05056 = 0.9435\ \text{rad}$.

Structural element loads and stresses are not available in this example since the GENEL stiffness model does not contain any details of the structure.

The complete solution for aileron effectiveness as a function of dynamic pressure is shown in Figure 7-9. The aileron reversal dynamic pressure is found to be $\bar{q} = 11.06\ \text{psi}$ by interpolation and corresponds to $V = 789\ \text{mph}$ at sea level based on the incompressible aerodynamics assumed. Note that the curve in Figure 7-9 is not a straight line, although in this example its curvature is very small. Note also that the curve has not included the reduction in commanded aileron rotation due to the actuator flexibility.

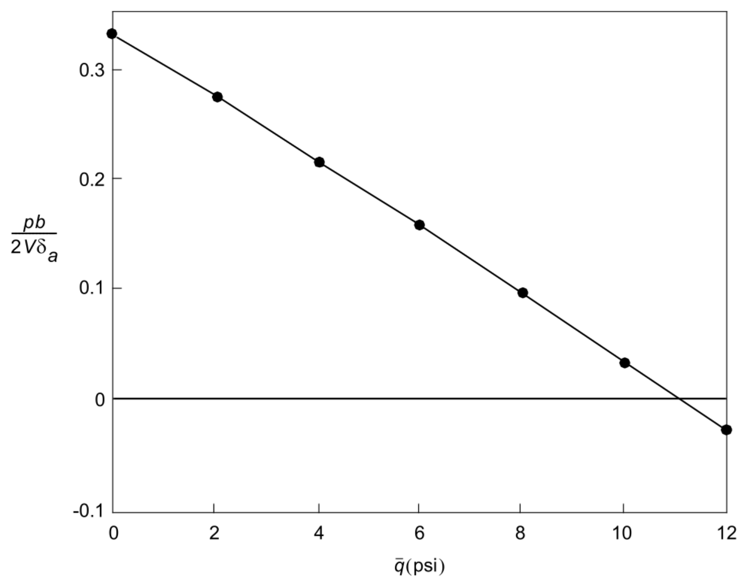

Figure 7-9 Aileron Effectiveness of BAH Wing