Full guide to the computation of stability derivatives in MSC Nastran

The process of computing aeroelastic stability (and control) derivatives in Nastran is straightforward.
A proper aircraft model must be defined. This model shall contain at least:
- the main wing
- a control surface (depending on the desired derivative)

A half span model can be used to model the aircraft thanks to symmetry options.

The main wing is modelled in a specific way: first, the geometry is defined and the elastic axis is chosen; then the main spar is modelled.
The movement of the nodes of the main spar can be determined through a bar/beam model or inputted through a GENEL entry from prevoius tests.
The fuselage of the aircraft can be considered rigid in a first test phase.

The tail can be modelled in a similar fashion to the wing or defined as fully rigid.
Infact, the deflection of the tail can be modelled through the control surface option.

Where needed, lumped masses can be inserted in the model.

When the structural model is complete, the aerodynamics are to be modelled.
The best way is to model the wing as an aerodynamic surface composed of aerodynamic panels.
The tail can be treated in the same way.

After the definition of aerodynamic panels, the splining process shall be carried out.

Finally, after defining a suitable reference system for the rotation of the tail, the latter can be assigned as control surface.

More reference systems can be introduced to compute the aerodynamics of the problem.

IMPORTANT: if a half span model is used with symmetry, when asked to input span and surface the ENTIRE SPAN and SURFACE are to be inserted (double the half span).
The same applies for the tail.
Beware that the tail can be assigned a reduced effectiveness.

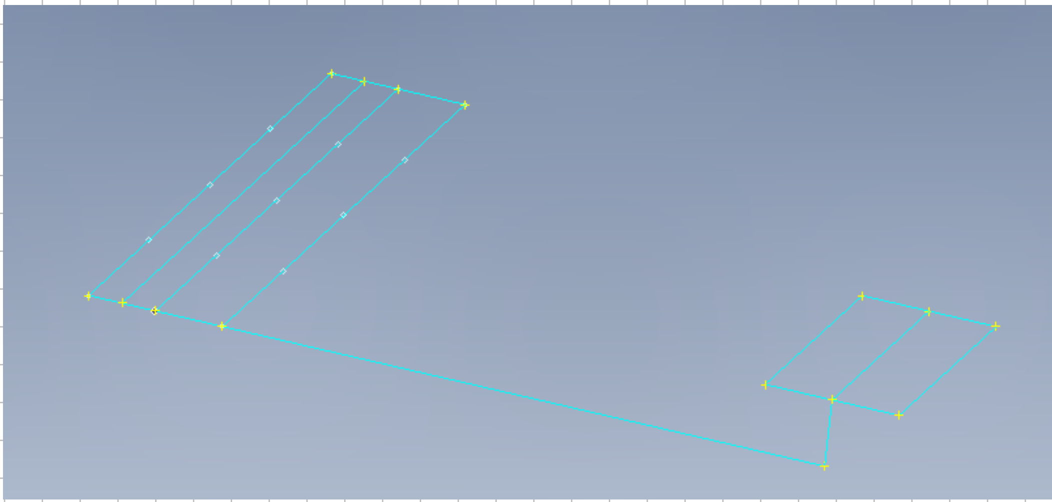
In order to perform a static aeroelastic analysis two constraints can be defined:

the first, positioned at the root of the wing, represents the blocked dofs, which are not studied in the problem at hand
the second set is called SUPORT and it's used by NASTRAN to determine the degrees of freedom of the problem, so it must be COMPLEMENTARY to the first one

Finally, trim conditions need to be applied. The trim conditions must be sufficient to fully determine the dofs. The use of a control surface is necessary.

OBS: the flow is directed in the positive direction of the x-axis, as all aero refs. must be

LONGITUDINAL STABILITY DERIVATIVES COMPUTATION



Half span 5 m

Chord 1 m

Tail span 2 m

Tail chord 1 m

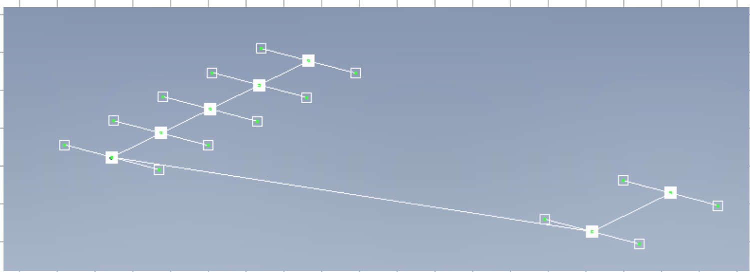
Distance of the tail EA from the wing EA 5 m

Material
E=7e10 Pa
nu=0.33

The elastic axis is at 50 % of the chord

Wing spar modelled as 4 bars, with circular section with r=0.3m

The tail is rigidly connected to the wing spar, ribs are modelled as rigid elements

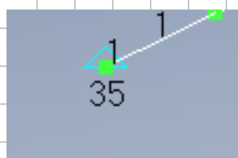


The tail is modelled as a single bar with the same properties as before

800 kg masses, representing the mass of the material, are positioned in the nodes

The constraints are defined: for the wing root all dofs are blocked except for vertical translation and pitch around the elastic axis.

constrained

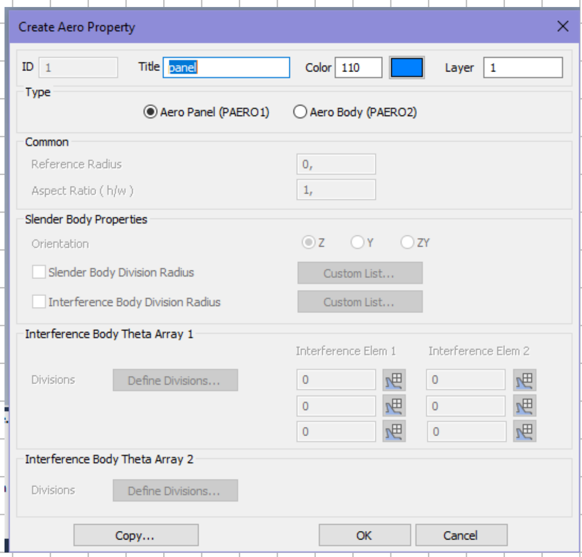


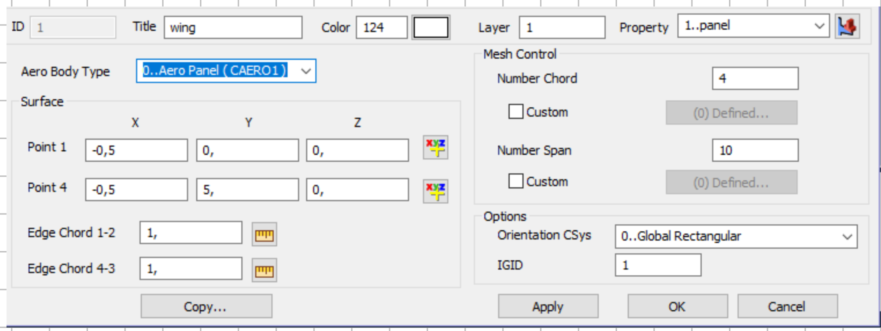
SUPORTed

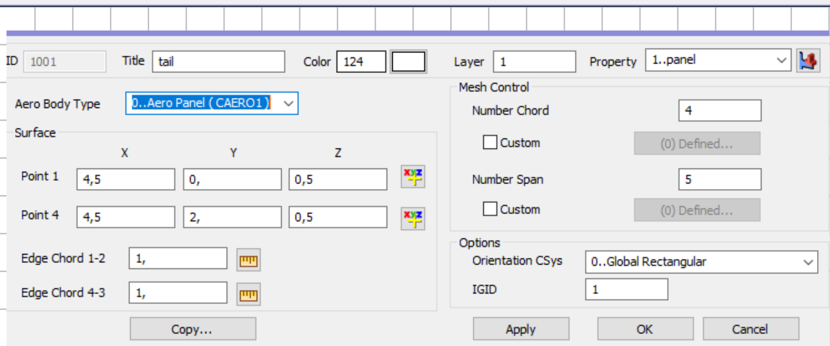
OBS: the wing spar model can be used to produce a GENEL entry, to be used as usual

The inclusion of the UD entry does not cause any variation of the derivatives related to the SUPORTed dofs.

Aerodynamics

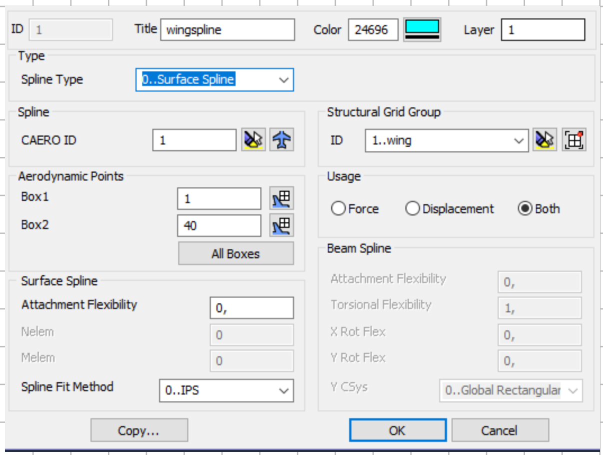




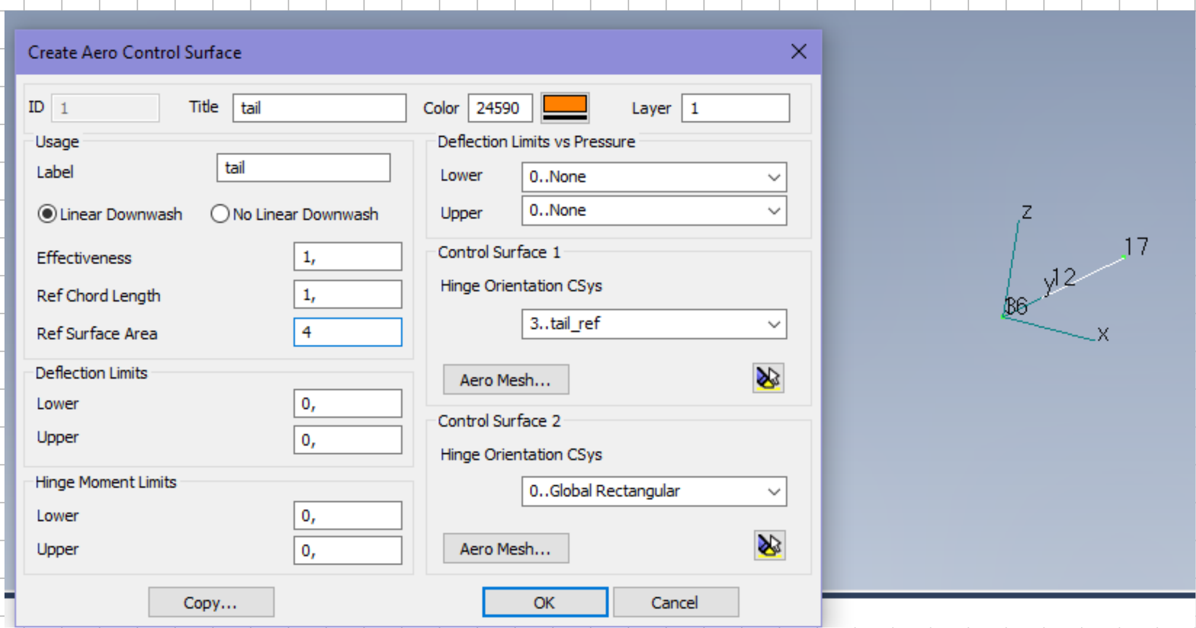


Creation of aerodynamic panel

Creation of aerodynamic surfaces

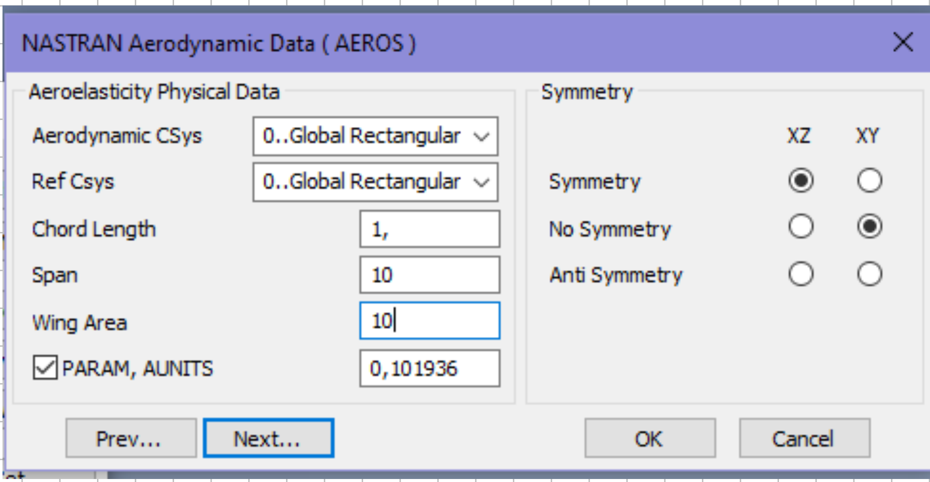


Splining procedure



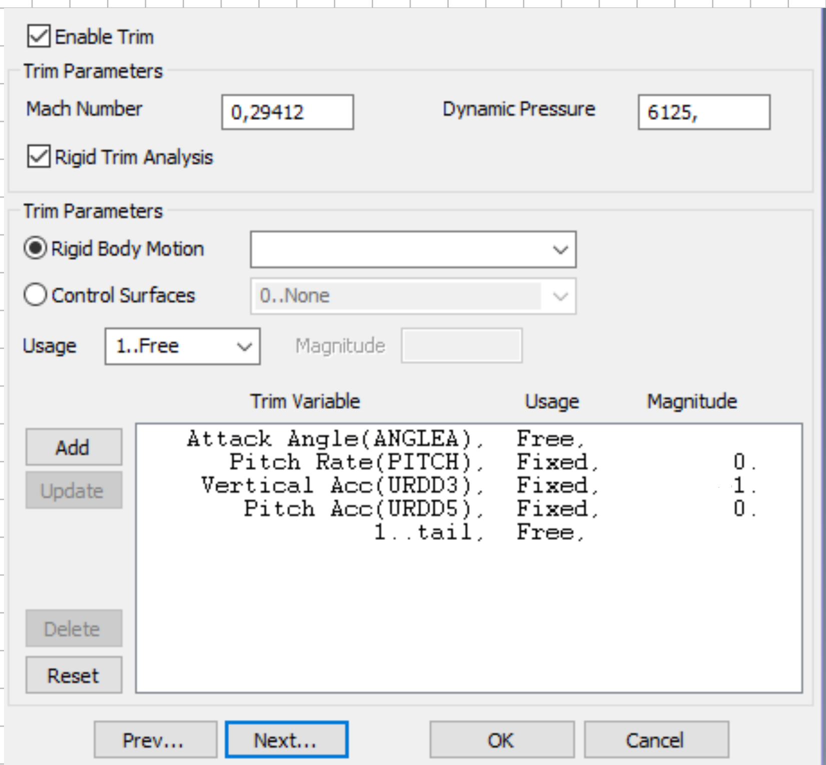
Creation of control surface, note that the ref area is 4, based on the full tail span

Definition of static aeroelastic analysis



Note the PARAM, AUNITS entry: if set to 1/9.81 allows for simpler input of the vertical acceleration as a load factor

Note the full span/surface and the symmetry indication

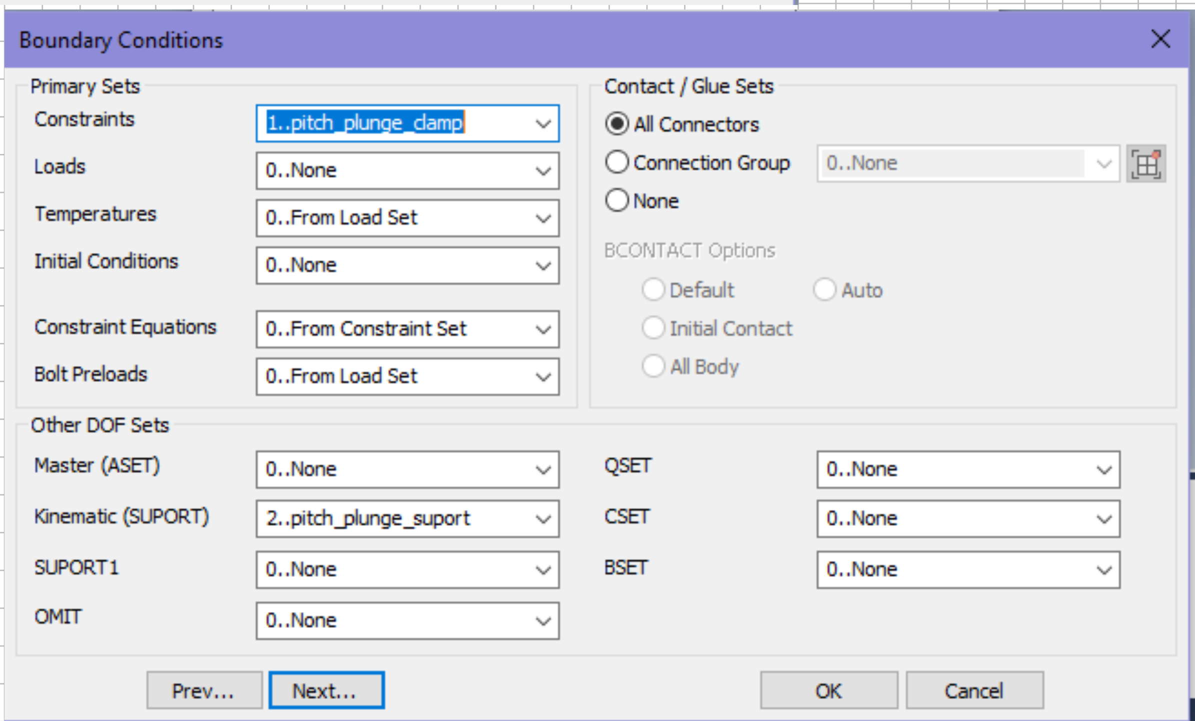


The test is performed at 100 m/s

This is a static trim test, where q=0, theta acc = 0, vert acc=1

The angle of attack and the tail deflection are left free

The vertical acc is set to 1, as z is upwards and AUNITS has been employed



This last tab requires the indication of the constraints and SUPORT

Discussion of the results

TRIM VARIABLE	COEFFICIENT	RIGID UNSPLINED	RIGID SPLINED	ELASTIC RESTRAINED	ELASTIC UNRESTRAINED	INERTIAL RESTRAINED	INERTIAL UNRESTRAINED
REF. COEFF.	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
TAIL	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.527181E-02
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	8.176011E-01	8.176011E-01	8.176011E-01	8.176011E-01	0.000000E+00	8.176011E-01
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-3.722689E+00	-3.722689E+00	-3.722689E+00	-3.722689E+00	0.000000E+00	-3.722689E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
ANGLEA	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	3.341201E-02
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	3.189289E+00	3.189289E+00	3.189289E+00	3.189289E+00	0.000000E+00	3.189289E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-2.136143E+00	-2.136143E+00	-2.136143E+00	-2.136143E+00	0.000000E+00	-2.136143E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
PITCH	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.438663E-01
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	9.823486E+00	9.823486E+00	9.823486E+00	9.823486E+00	0.000000E+00	9.823486E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	-3.893929E+01	-3.893929E+01	-3.893929E+01	-3.893929E+01	0.000000E+00	-3.893929E+01
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
URDD3	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	8.969213E-01	0.000000E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.281316E+00	0.000000E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
URDD5	CX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	6.406581E-02	0.000000E+00
	CY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-1.281316E+00	0.000000E+00
	CMX	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	CMY	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	6.470647E+00	0.000000E+00
	CMZ	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

The derivatives are expressed exactly in the global reference system

TRANSFORMATION FROM BASIC TO REFERENCE COORDINATES:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{REF} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}_{BAS} + \begin{Bmatrix} 0.0000E+00 \\ 0.0000E+00 \\ 0.0000E+00 \end{Bmatrix}$$

No transf. is applied

N O N - D I M E N S I O N A L   H I N G E   M O M E N T   D E R I V A T I V E   C O E F F I C I E N T S

CONFIGURATION = AEROSG2D   XY-SYMMETRY = ASYMMETRIC   XZ-SYMMETRY = SYMMETRIC

MACH = 2.9412E-01   Q = 6.1250E+03

CONTROL SURFACE = TAIL   REFERENCE CHORD LENGTH = 1.000000E+00   REFERENCE AREA = 2.000000E+00

TRIM VARIABLE	RIGID	ELASTIC RESTRAINED	ELASTIC UNRESTRAINED	INERTIAL RESTRAINED	INERTIAL UNRESTRAINED
AT REFERENCE	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
TAIL	1.047701E+00	1.047701E+00	1.047701E+00	0.000000E+00	0.000000E+00
ANGLEA	7.826452E-01	7.826452E-01	7.826452E-01	0.000000E+00	0.000000E+00
PITCH	1.015387E+01	1.015387E+01	1.015387E+01	0.000000E+00	0.000000E+00
URDD3	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
URDD5	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

Another relevant output is the collection of hinge moment derivatives

```
                    AEROELASTIC TRIM VARIABLES

 ID     LABEL                 TYPE        TRIM STATUS      VALUE OF UX

        INTERCEPT          RIGID BODY          FIXED      1.000000E+00
  1     TAIL           CONTROL SURFACE          FREE      2.143481E-01  RADIANS
  1     ANGLEA             RIGID BODY           FREE      2.262793E-01  RADIANS
  2     PITCH              RIGID BODY          FIXED      0.000000E+00  NONDIMEN. RATE
  3     URDD3              RIGID BODY          FIXED      1.000000E+00  LOAD FACTOR
  4     URDD5              RIGID BODY          FIXED      0.000000E+00  RAD/S/S PER G


                 CONTROL SURFACE POSITION AND HINGE MOMENT RESULTS

         ACTIVE LIMITS ARE FLAGGED WITH AN (A),  VIOLATED LIMITS ARE FLAGGED WITH A (V).

                            POSITION                                   HINGE MOMENT
CONTROL SURFACE      LOWER LIMIT      VALUE        UPPER LIMIT       LOWER LIMIT      VALUE         UPPER LIMIT
     TAIL           -1.570796E+00   2.143481E-01   1.570796E+00          N/A        4.920446E+03        N/A
```

The full collection of trim data is provided

```
             A E R O S T A T I C   D A T A   R E C O V E R Y   O U T P U T   T A B L E S
     CONFIGURATION = AEROSG2D      XY-SYMMETRY = ASYMMETRIC      XZ-SYMMETRY = SYMMETRIC
                        MACH = 2.941200E-01                 Q = 6.125000E+03
     CHORD = 1.0000E+00           SPAN = 1.0000E+01           AREA = 1.0000E+01


                      TRANSFORMATION FROM REFERENCE TO WIND AXES:
            ANGLE OF ATTACK    =  2.262793E-01 RADIANS (     12.964851 DEGREES)
            ANGLE OF SIDESLIP =  0.000000E+00 RADIANS (      0.000000 DEGREES)

                 { X }         [  0.974508  0.000000  0.224353 ]  { X }
                 { Y }         [  0.000000  1.000000  0.000000 ]  { Y }
                 { Z }WIND =   [ -0.224353  0.000000  0.974508 ]  { Z }REF
```

This section determines the rotation matrix from body axes to wind axes, aligned with the asymptotic wind

```
STRUCTURAL  MONITOR POINT TOTAL VEHICLE COEFFICIENTS:
        AXIS         RIGID AIR  +  RESTRAINED INCR.  -  INERTIAL  +   RIGID-APPLIED + RESTRAINED INCR.  =  BALANCE
        ----         ---------     ----------------     --------      -------------   ----------------     -------
BODY  CX          0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
WIND  CD          2.012272E-01     0.000000E+00     2.012272E-01     0.000000E+00     0.000000E+00     2.775558E-17

BODY  CY          0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
WIND  CY-WIND     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00

BODY  CZ          8.969213E-01     0.000000E+00     8.969213E-01     0.000000E+00     0.000000E+00     1.110223E-16
WIND  CL          8.740569E-01     0.000000E+00     8.740569E-01     0.000000E+00     0.000000E+00     1.110223E-16

BODY  CMX         1.649093E-01     0.000000E+00     1.857908E-01     0.000000E+00     0.000000E+00    -2.088157E-02
WIND  CM-ROLL     1.607054E-01     0.000000E+00     1.810546E-01     0.000000E+00     0.000000E+00    -2.034925E-02

BODY  CMY        -1.281316E+00     0.000000E+00    -1.281316E+00     0.000000E+00     0.000000E+00     0.000000E+00
WIND  CM-PITCH   -1.281316E+00     0.000000E+00    -1.281316E+00     0.000000E+00     0.000000E+00     0.000000E+00

BODY  CMZ         0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
WIND  CM-YAW     -3.699793E-02     0.000000E+00    -4.168278E-02     0.000000E+00     0.000000E+00     4.684847E-03


AERODYNAMIC MONITOR POINT TOTAL VEHICLE COEFFICIENTS:
        AXIS         RIGID AIR  +  RESTRAINED INCR.  -  INERTIAL  +   RIGID-APPLIED + RESTRAINED INCR.  =  BALANCE
        ----         ---------     ----------------     --------      -------------   ----------------     -------
BODY  CX          0.000000E+00     0.000000E+00          N/A              N/A         0.000000E+00     0.000000E+00
WIND  CD          2.012272E-01     0.000000E+00          N/A              N/A         0.000000E+00     2.012272E-01

BODY  CY          0.000000E+00     0.000000E+00          N/A              N/A         0.000000E+00     0.000000E+00
WIND  CY-WIND     0.000000E+00     0.000000E+00          N/A              N/A         0.000000E+00     0.000000E+00

BODY  CZ          8.969213E-01     0.000000E+00          N/A              N/A         0.000000E+00     8.969213E-01
WIND  CL          8.740569E-01     0.000000E+00          N/A              N/A         0.000000E+00     8.740569E-01

BODY  CMX         1.649093E-01     0.000000E+00          N/A              N/A         0.000000E+00     1.649093E-01
WIND  CM-ROLL     1.607054E-01     0.000000E+00          N/A              N/A         0.000000E+00     1.607054E-01

BODY  CMY        -1.281316E+00     0.000000E+00          N/A              N/A         0.000000E+00    -1.281316E+00
WIND  CM-PITCH   -1.281316E+00     0.000000E+00          N/A              N/A         0.000000E+00    -1.281316E+00

BODY  CMZ         0.000000E+00     0.000000E+00          N/A              N/A         0.000000E+00     0.000000E+00
WIND  CM-YAW     -3.699793E-02     0.000000E+00          N/A              N/A         0.000000E+00    -3.699793E-02
```

???

```
                 A E R O S T A T I C   D A T A   R E C O V E R Y   O U T P U T   T A B L E S
        CONFIGURATION = AEROSG2D      XY-SYMMETRY = ASYMMETRIC      XZ-SYMMETRY = SYMMETRIC
                           MACH = 2.941200E-01                  Q = 6.125000E+03
        CHORD = 1.0000E+00            SPAN = 1.0000E+01            AREA = 1.0000E+01


                            AERODYNAMIC PRESSURES ON THE AERODYNAMIC ELEMENTS

                                              AERODYNAMIC PRES.       AERODYNAMIC
                      GRID    LABEL             COEFFICIENTS           PRESSURES
                        1      LS              2.923780E+00          1.790815E+04
                        2      LS              1.248628E+00          7.647844E+03
                        3      LS              7.470595E-01          4.575739E+03
                        4      LS              4.142232E-01          2.537117E+03
                        5      LS              2.915091E+00          1.785493E+04
                        6      LS              1.244403E+00          7.621971E+03
                        7      LS              7.442420E-01          4.558482E+03
                        8      LS              4.125138E-01          2.526647E+03
                        9      LS              2.896927E+00          1.774368E+04
                       10      LS              1.235543E+00          7.567698E+03
                       11      LS              7.383217E-01          4.522221E+03
                       12      LS              4.089218E-01          2.504646E+03
                       13      LS              2.867513E+00          1.756352E+04
                       14      LS              1.221107E+00          7.479278E+03
```

The pressure is computed in all aerodynamic elements

```
                     A E R O S T A T I C   D A T A   R E C O V E R Y   O U T P U T   T A B L E S
                 CONFIGURATION = AEROSG2D     XY-SYMMETRY = ASYMMETRIC     XZ-SYMMETRY = SYMMETRIC
                                 MACH = 2.941200E-01                    Q = 6.125000E+03
                 CHORD = 1.0000E+00           SPAN = 1.0000E+01           AREA = 1.0000E+01


                                      AERODYNAMIC FORCES ON THE AERODYNAMIC ELEMENTS

GROUP   GRID ID  LABEL        T1              T2              T3              R1              R2              R3
    1         1   LS    0.000000E+00    0.000000E+00    2.238519E+03    0.000000E+00    1.399074E+02    0.000000E+00
    1         2   LS    0.000000E+00    0.000000E+00    9.559805E+02    0.000000E+00    5.974878E+01    0.000000E+00
    1         3   LS    0.000000E+00    0.000000E+00    5.719674E+02    0.000000E+00    3.574796E+01    0.000000E+00
    1         4   LS    0.000000E+00    0.000000E+00    3.171397E+02    0.000000E+00    1.982123E+01    0.000000E+00
    1         5   LS    0.000000E+00    0.000000E+00    2.231867E+03    0.000000E+00    1.394917E+02    0.000000E+00
```

The forces are computed for all aero elements

```
                    S T R U C T U R A L   M O N I T O R   P O I N T   I N T E G R A T E D   L O A D S
                  CONFIGURATION = AEROSG2D     XY-SYMMETRY = ASYMMETRIC     XZ-SYMMETRY = SYMMETRIC
                                  MACH = 2.941200E-01                  Q = 6.125000E+03

      CONTROLLER STATE:
      TAIL      =   2.1435E-01        ANGLEA    =   2.2628E-01        URDD3     =   1.0000E+00

      MONITOR POINT NAME = AEROSG2D          COMPONENT =                     CLASS = COEFFICIENT
      LABEL = Full Vehicle Integrated Loads
       CP =         0          X =  0.00000E+00          Y =  0.00000E+00          Z =  0.00000E+00          CD =        0

        AXIS      RIGID AIR       ELASTIC REST.       INERTIAL      RIGID APPLIED    REST. APPLIED
        ----    -------------    -------------    -------------    -------------    -------------
         CX     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
         CY     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
         CZ     5.493643E+04     5.493643E+04     5.493643E+04     0.000000E+00     0.000000E+00
         CMX    1.010069E+05     1.010069E+05     1.137969E+05     0.000000E+00     0.000000E+00
         CMY   -7.848062E+04    -7.848062E+04    -7.848062E+04     0.000000E+00     0.000000E+00
         CMZ    0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00     0.000000E+00
1    STATIC_AERO_CLASSIC                          **STUDENT EDITION* FEBRUARY  14, 2024  MSC Nastran  3/30/23   PAGE    21

0

                    A E R O D Y N A M I C   M O N I T O R   P O I N T   I N T E G R A T E D   L O A D S
                  CONFIGURATION = AEROSG2D     XY-SYMMETRY = ASYMMETRIC     XZ-SYMMETRY = SYMMETRIC
                                  MACH = 2.941200E-01                  Q = 6.125000E+03

      CONTROLLER STATE:
      TAIL      =   2.1435E-01        ANGLEA    =   2.2628E-01        URDD3     =   1.0000E+00

      MONITOR POINT NAME = AEROSG2D          COMPONENT =                     CLASS = COEFFICIENT
      LABEL = Full Vehicle Integrated Loads
       CP =         0          X =  0.00000E+00          Y =  0.00000E+00          Z =  0.00000E+00          CD =        0

        AXIS      RIGID AIR       ELASTIC REST.
        ----    -------------    -------------
         CX     0.000000E+00     0.000000E+00
         CY     0.000000E+00     0.000000E+00
         CZ     5.493643E+04     5.493643E+04
         CMX    1.010069E+05     1.010069E+05
         CMY   -7.848062E+04    -7.848062E+04
         CMZ    0.000000E+00     0.000000E+00
```

Computation of integrated loads

```
                                             D I S P L A C E M E N T   V E C T O R

      POINT ID.   TYPE          T1             T2             T3             R1             R2             R3
             1      G      0.0            0.0            0.0            0.0            0.0            0.0
             2      G      0.0            0.0           -2.495972E-05  -4.071082E-05   3.029830E-05   0.0
             3      G      0.0            0.0           -1.008727E-04  -7.849120E-05   5.063143E-05   0.0
             4      G      0.0            0.0           -2.175911E-04  -1.046171E-04   6.193333E-05   0.0
             5      G      0.0            0.0           -3.572742E-04  -1.140837E-04   6.476733E-05   0.0
             6      G      0.0            0.0           -9.810573E-06  -4.071082E-05   3.029830E-05   0.0
             7      G      0.0            0.0           -7.555699E-05  -7.849120E-05   5.063143E-05   0.0
             8      G      0.0            0.0           -1.866244E-04  -1.046171E-04   6.193333E-05   0.0
             9      G      0.0            0.0           -3.248906E-04  -1.140837E-04   6.476733E-05   0.0
            10      G      0.0            0.0            0.0            0.0            0.0            0.0
            11      G      0.0            0.0           -4.010887E-05  -4.071082E-05   3.029830E-05   0.0
            12      G      0.0            0.0            0.0            0.0            0.0            0.0
```

Computation of displacements