
Heart Disease Prediction from heart beat audio signals using Machine Learning and Network Analysis

Ligari D. • Alberti A. ¹

¹ Department of Computer Engineering, Data Science, University of Pavia, Italy

Course of Advanced Biomedical Machine Learning

Github page: <https://github.com/DavideLigari01/advanced-biomedical-project>

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1. INTRODUCTION

2. DATASET DESCRIPTION

The dataset for this project was obtained from a Kaggle repository titled *Dangerous Heartbeat Dataset (DHD)* [1], which in turn sources its data from the PASCAL Classifying Heart Sounds Challenge 2011 (CHSC2011) [2]. This dataset comprises audio recordings of heartbeats, categorized into different types of heart sounds. Specifically, the dataset consists of 5 types of recordings: Normal Heart Sounds, Murmur Sounds, Extra Heart Sounds, Extrasystole Sounds, and Artifacts. Data has been gathered from the general public via the iStethoscope Pro iPhone app and from a clinic trial in hospitals using the digital stethoscope DigiScope.

2.1. Data Distribution

Figure 1, show the presence of highly imbalanced classes in the dataset. This poses a challenge for the classification task as the model may not have enough samples to learn from, especially for the 'Extrastole' and 'Extrahls' classes. This problem is tackled trying to augment the data available, both by segmenting the audio files and by using data augmentation techniques. Furthermore, we test the effectiveness of oversampling and undersampling techniques on the model performance. The data is split into training and testing sets, with a 80% - 20% ratio, respectively. The val-

validation set is omitted, due to the low number of samples available.

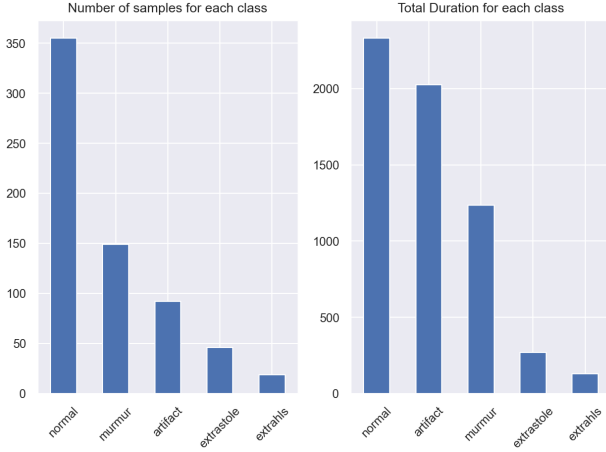


Fig. 1: Number of samples per duration.

2.2. Data Preprocessing

Here is a list of the preprocessing operations performed on the dataset:

Noise Reduction: the audio data was already provided in a clipped format to minimize noise and irrelevant information.

Removal of Corrupted Files: corrupted files were identified and removed from the dataset to ensure data quality.

Resampling: all audio files were resampled to a common frequency of 4000 Hz, which was identified as the optimal sampling rate (see Section 5.3).

Segmentation: the audio data was segmented into 1-second intervals, identified as the optimal extraction interval (see Section 5.3).

Outlier Detection and Removal: we investigated the average duration of each class and found that the 'artifact' class had a significantly larger average duration. This was due to a few lengthy audio recordings (see Figure 2). These recordings were considered as outliers and removed from the dataset, as a large number of samples from the same audio might not be as informative.

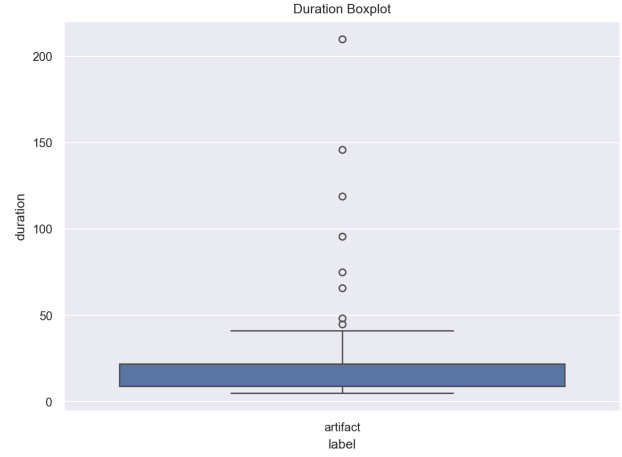


Fig. 2: Outliers in the Artifacts class.

3. HEART SOUND CATEGORIES

Heart sounds can be categorized into different classes based on their acoustic characteristics and clinical significance. Accurate classification of these sounds is essential for diagnosing and treating a variety of cardiac conditions. The primary categories include Normal heart sounds, Murmurs, Extra Heart Sounds, Artifacts, and Extra Systoles. Understanding the distinct features and clinical implications of each class is a crucial step before building a machine learning model to classify heartbeats. This phase is particularly important for the identification of patterns that are characteristic of specific classes, which in turn guides the selection of features to extract from the audio. This knowledge aids in identifying specific patterns and anomalies within the heart sounds, leading to more precise and reliable model predictions.

3.1. Normal

The Normal category includes recordings of typical, healthy heart sounds. These sounds exhibit the characteristic “lub-dub, lub-dub” pattern, where “lub” (S1) represents the closing of the atrioventricular valves and “dub” (S2) signifies the closing of the semilunar valves. In a normal heart, the time interval between “lub” and “dub” is shorter than the interval from “dub” to the next “lub,” especially when the heart rate is below 140 beats per minute. Most normal heart rates at rest fall between 60 and 100 beats per minute, though rates can vary from 40 to 140 beats per minute based on factors such as age and activity level. Recordings may include background noises like traffic or radio sounds and may capture incidental noises such as breathing or microphone contact with clothing or skin. It contains both clean and noisy normal recordings, the latter featuring significant background noise or distortion, which simulates real-world conditions.

Figure 3 shows a sample of a normal heart beat audio. The characteristic “lub-dub, lub-dub” pattern can be observed, where the peaks represent the “lub” (S1) and “dub” (S2) sounds of a healthy heart.

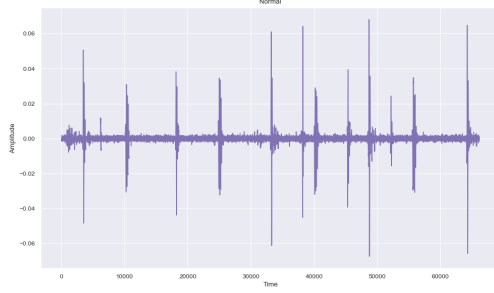


Fig. 3: Sample of normal heart beat audio.

3.2. Murmur

Heart murmurs are abnormal sounds during the heartbeat cycle, such as a “whooshing, roaring, rumbling, or turbulent fluid” noise, heard between the “lub” and “dub” (systolic murmur) or between “dub” and “lub” (diastolic murmur). These murmurs are typically indicative of turbulent blood flow in the heart and can signal various heart conditions, some of which may be serious. It is crucial to distinguish murmurs from the normal “lub-dub” sounds since they occur between the primary heart sounds and not concurrently with them. It also includes noisy murmur data, which mimics real-world recording scenarios by incorporating significant background noise and distortions. Figure 4 shows a sample of a murmur heart beat audio. The presence of additional sounds between the “lub” and “dub” peaks can be observed, indicating the characteristic “whooshing, roaring, rumbling, or turbulent fluid” noise typical of heart murmurs.

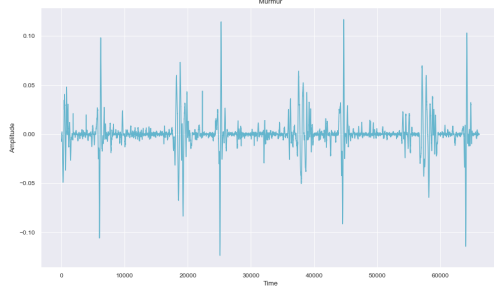


Fig. 4: Sample of murmur heart beat audio.

3.3. Extra Heart Sound

Extra heart sounds are characterized by an additional sound in the cardiac cycle, producing patterns such as “lub-lub dub” or “lub dub-dub”. These sounds can arise from physiological or pathological conditions. For example, a third heart sound (S3) may indicate heart failure or volume overload, while a fourth heart sound (S4) can be associated with a stiff or hypertrophic ventricle. Detecting these extra sounds is important for identifying potential heart diseases early, allowing for timely intervention and management. Figure 5 shows a sample of an extra heart sound audio. The presence of additional peaks within the normal “lub-dub” pattern indicates extra heart sounds, which can be critical for diagnosing various heart conditions.

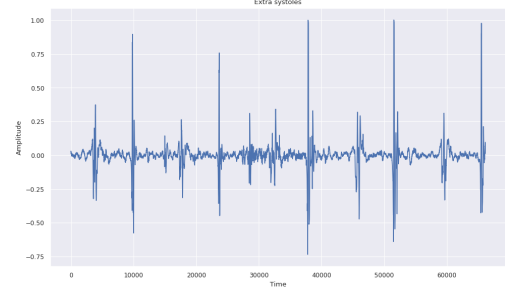


Fig. 5: Sample of extra heart sound audio.

3.4. Artifact

The Artifact category consists of recordings with non-cardiac sounds, including feedback squeals, echoes, speech, music, and various types of noise. These recordings generally lack discernible heart sounds and do not exhibit the temporal periodicity typical of heartbeats at frequencies below 195 Hz. Accurately identifying artifacts is essential to avoid misinterpreting non-cardiac sounds as pathological heart sounds, ensuring that data collection efforts focus on genuine heart sounds. Figure 6 shows a sample of an artifact heart beat audio, there can be observed that there is not a clear pattern in the audio.

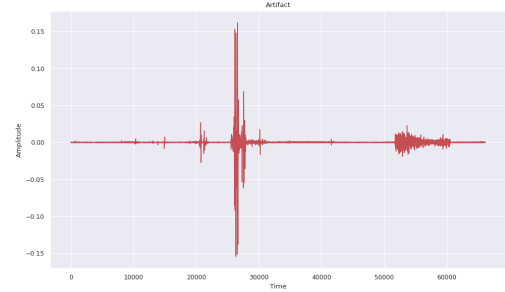


Fig. 6: Sample of artifact heart beat audio

3.5. Extra systoles

Extra systoles refers to extra or skipped heartbeats, resulting in irregular patterns such as “lub-lub dub” or “lub dub-dub”. Unlike the regular extra heart sounds, extra systoles are sporadic and do not follow a consistent rhythm. These premature beats can occur in healthy individuals, particularly children, but they may also be associated with various heart diseases. Identifying extra systoles is crucial as they can be early indicators of cardiac conditions that might require medical attention if they occur frequently or in certain patterns.

In the audio signal depicted in Figure 7, irregularities within the normal “lub-dub” pattern are evident. These irregularities manifest as additional peaks or skipped beats, indicating extra systoles.

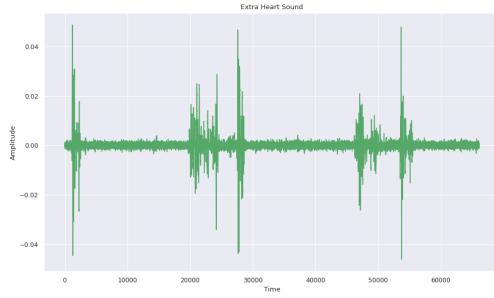


Fig. 7: Sample of extra systoles heart beat audio

3.6. Comparison of Heart Sounds

In Figure 8, a comparison of the different classes of heart sounds can be observed.

As we can see, the “Artifact” signal appears erratic with no consistent pattern, likely representing noise or interference rather than true heart sounds.

The “Murmurs” signal shows irregular fluctuations in amplitude, which could indicate turbulent blood flow typically associated with murmurs.

The signal for “Extra Heart Beat Sound” has occasional spikes in amplitude that stand out from the baseline.

The “Normal” signal appears more uniform and regular compared to the others, reflecting the expected rhythm of a healthy heartbeat. Finally, the signal for “Extra Systoles” shows extra spikes at irregular intervals, indicating unexpected contractions of the heart muscle (systoles) occurring outside the normal rhythm.

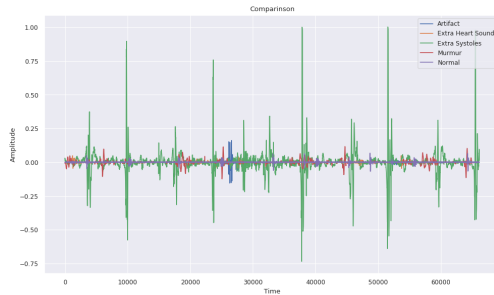


Fig. 8: Comparison of the different classes of heart sounds.

4. GOALS

5. FEATURE EXTRACTION

As demonstrated by [4] and [3], MFCCs are highly effective features for heartbeat classification. In addition to MFCCs, we incorporated other features to capture various characteristics of heart sounds, enhancing the classification accuracy. The features used are explained in the following section.

5.1. Features Type

MFCC

Mel-Frequency Cepstral Coefficients (MFCCs) are representations of the short-term power spectrum of sound.

They are derived by taking the Fourier transform of a signal, mapping the powers of the spectrum onto the mel scale, taking the logarithm, and then performing a discrete cosine transform. MFCCs are effective in capturing the timbral texture of audio and are widely used in speech and audio processing due to their ability to represent the envelope of the time power spectrum. In heartbeat classification, MFCCs can reflect the different perceived quality of heart sounds, such as the presence of murmurs or other anomalies.

Chroma STFT

Chroma features represent the 12 different pitch classes of music (e.g., C, C#, D, etc.). They are particularly useful for capturing harmonic and melodic characteristics in music. By mapping audio signals onto the chroma scale, these features can identify pitches regardless of the octave, making them useful for analyzing harmonic content in heart sounds.

RMS

Root Mean Square (RMS) measures the magnitude of varying quantities, in this case, the amplitude of an audio signal. It is a straightforward way to compute the energy of the signal over a given time frame. RMS is useful in audio analysis for detecting volume changes and can help identify different types of heartbeats based on their energy levels. For example, in a given timeframe the RMS may be altered by the presence of a murmur with respect to a normal heart sound.

ZCR

Zero-Crossing Rate (ZCR) is the rate at which a signal changes sign, indicating how often the signal crosses the zero amplitude line. It is particularly useful for detecting the noisiness and the temporal structure of the signal. In heartbeat classification, ZCR can help differentiate between normal and abnormal sounds by highlighting changes in signal periodicity.

CQT

Constant-Q Transform (CQT) is a time-frequency representation with a logarithmic frequency scale, making it suitable for musical applications. Since it captures more detail at lower frequencies, it may be useful for analyzing the low-frequency components of heart sounds.

Spectral Centroid

The spectral centroid indicates the center of mass of the spectrum and is often perceived as the brightness of a sound. It is calculated as the weighted mean of the frequencies present in the signal, with their magnitudes as weights. In heart sound analysis, a higher spectral centroid can indicate sharper, more pronounced sounds, while a lower centroid suggests smoother sounds.

Spectral Bandwidth

Spectral bandwidth measures the width of the spectrum

around the centroid, providing an indication of the range of frequencies present. It is calculated as the square root of the variance of the spectrum. This feature helps in understanding the spread of the frequency components in the heart sounds, which can be indicative of different heart conditions.

Spectral Roll-off Spectral roll-off is the frequency below which a certain percentage of the total spectral energy lies. It is typically set at 85% and helps distinguish between harmonic and non-harmonic content. In heartbeat classification, spectral roll-off can be used to differentiate between sounds with a concentrated energy distribution and those with more dispersed energy.

5.2. Methodology

The features were extracted from the audio signals using the *Librosa* library in Python. It is worth to underline four main aspects in the extraction methodology having a direct impact on the results:

- **Normalization:** the audio are loaded using the *torchaudio.load()* function, which normalized the audio signals in the range $[-1, 1]$. This is important to ensure that the features are on the same scale and to prevent the model from being biased towards features with larger values.
- **Audio Clipping:** to extract features, we divided the audio recordings into chunks of a given length (in seconds). This segmentation allowed us to analyze the impact of different extraction intervals on model performance, additionally it allow for augmenting the data available.
- **Sampling Rate:** while literature on spoken language often suggests that 16000 Hz is sufficient, it was necessary to assess the best sampling rate for heartbeat sounds specifically. We evaluated two sampling rates to determine the optimal rate for heartbeat sounds.
- **Hop and Window Size:** the hop size determines the number of samples between successive windows, while the window size determines the number of samples considered. Each feature was extracted using the same window length and hop length facilitating a fair assessment of each feature's contribution to the classification task.

The library used for the extraction is *Librosa*, which is a Python package for music and audio analysis.

5.3. Sampling Rate and Interval Selection

To determine the optimal sampling rate and extraction interval for heartbeat classification, a series of experiments was conducted. We trained various models (Random Forest, Support Vector Machine, Logistic Regression) with different features (Chroma, MFCC_30, MFCC_120, CQT_30, CQT_70), sampling rates (mixed, 4000 Hz), and extraction intervals (0.5s, 1s, 2s, 3s). Features were used as extracted without any additional processing. The models were trained on 80% of the data and tested on the remaining 20%.

5.3.1. Findings on Sampling Rate

Our experiments revealed no clear advantage of using a mix of frequencies over a fixed sample rate, independently of the metric used. Specifically, a fixed sampling rate of 4000 Hz was found to reduce the risk of bias introduction, improve efficiency, and enable the use of a wider variety of features. This sampling rate provided a consistent and reliable basis for feature extraction, as demonstrated in Figure 9.

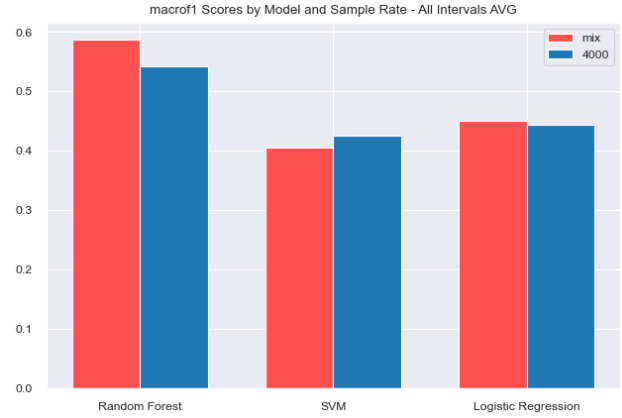


Fig. 9: Comparison of the macro F1 score for different sampling rates.

5.3.2. Findings on Extraction Interval

The choice of extraction interval had a significant impact on the number of samples and the class distribution, as shown in Figure 10. To address the high class imbalance, we used the macro F1 score and Matthews Correlation Coefficient (MCC) as evaluation metrics. Our results indicated that a 2-second interval yielded the best performance. However, this choice also reduced the number of samples, potentially causing issues during training and testing, especially with more complex models.

As a compromise, we recommend using a 1-second interval. This interval offers a good balance between the number of samples and the class distribution, ensuring robust model performance while maintaining a sufficient dataset size. Figures 11 and 12 illustrate the impact of different extraction intervals on the macro F1 score and MCC, respectively.

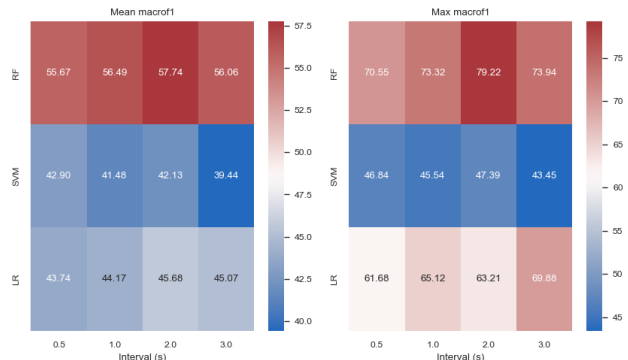


Fig. 11: Comparison of the macro F1 score for different extraction intervals.

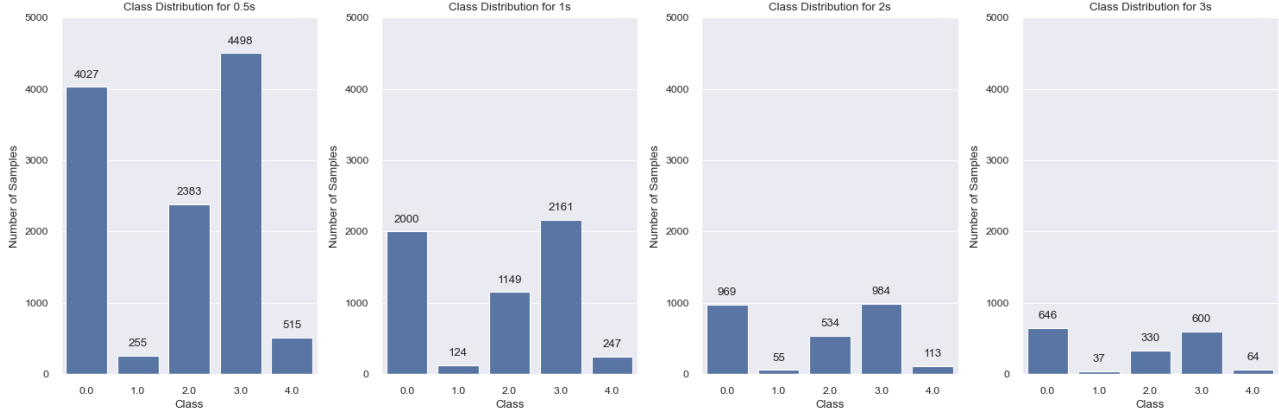


Fig. 10: Class distribution for different extraction intervals.

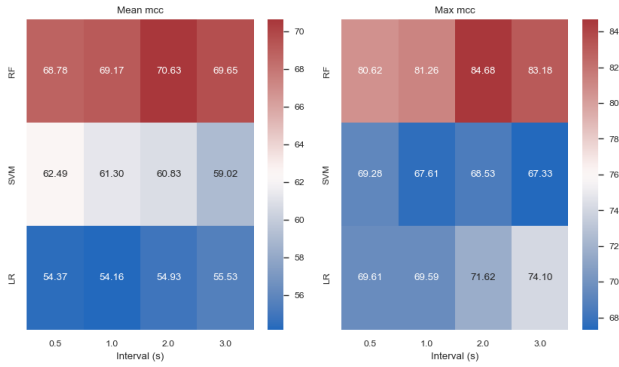


Fig. 12: Comparison of the MCC for different extraction intervals.

5.4. Results

6. FEATURE SELECTION

In this section, the focus is on identifying the optimal number of features for different types of audio features (MFCC, Chroma, CQT, etc.). Each type of feature can consist of a varying number of individual features, and it is crucial to determine the optimal number to maximize the model's performance. Proper feature selection is vital because it directly impacts the efficiency, accuracy, and generalizability of the machine learning model. Including too many features can lead to overfitting, increased computational costs, and degraded performance due to the curse of dimensionality. On the other hand, using too few features can result in underfitting, where the model fails to capture the necessary patterns in the data. Therefore, the goal of this study is to identify the optimal number of features for each type to ensure the model is both robust and efficient.

6.1. Determining the Optimal Number of Features

To maximize the model's performance, it is crucial to determine the optimal number of features for each type. This was achieved using a 'One Model per Feature' approach, which involved the following steps:

Figure 13 shows the results obtained for each type of feature with the Random Forest model, which significantly outperformed the other classifiers.

Algorithm 1 Feature Optimization Process

- 1: **Step 1:** Extract feature sets with varying sizes: 12, 20, 30, 40, 60, 70, 90, and 120 features.
- 2: **Step 2:** Train a separate model for each feature type and feature set size.
- 3: **Step 3:** For each feature set, train three different models using three classifiers: SVM, Random Forest, and Logistic Regression.
- 4: **Step 4:** Evaluate the performance of each model.

6.2. Results and Analysis

MFCC: The MFCC features consistently achieved the highest F1 scores, peaking around 0.7. This indicates their effectiveness for the classification task. Interestingly, the number of MFCC features (ranging from 30 to 120) did not drastically affect performance, suggesting that even a smaller set of MFCC features can be highly informative.

CQT: The CQT features showed moderate performance, with F1 scores around 0.4 to 0.5. The optimal number of features was around 70, beyond which there was no significant improvement.

RMS: RMS features exhibited F1 scores ranging from 0.4 to 0.5, with optimal performance achieved with around 70 features.

ZCR, Spectral Centroid (SC), Spectral Bandwidth (SB), and Spectral Roll-off (SR): The F1 scores for these features generally stabilized around 0.4 to 0.5. Increasing the number of features beyond 40 did not result in significant performance gains and could even degrade the model's performance. This suggests that adding too many features, especially those without strong predictive power, can confuse the model and degrade its performance.

6.3. Optimal Number of Features

Based on the results, the optimal number of features for each type is shown in Table 1.

7. CORRELATION ANALYSIS

Given the large number of features (338 in total), it was necessary to identify and remove features that are poorly

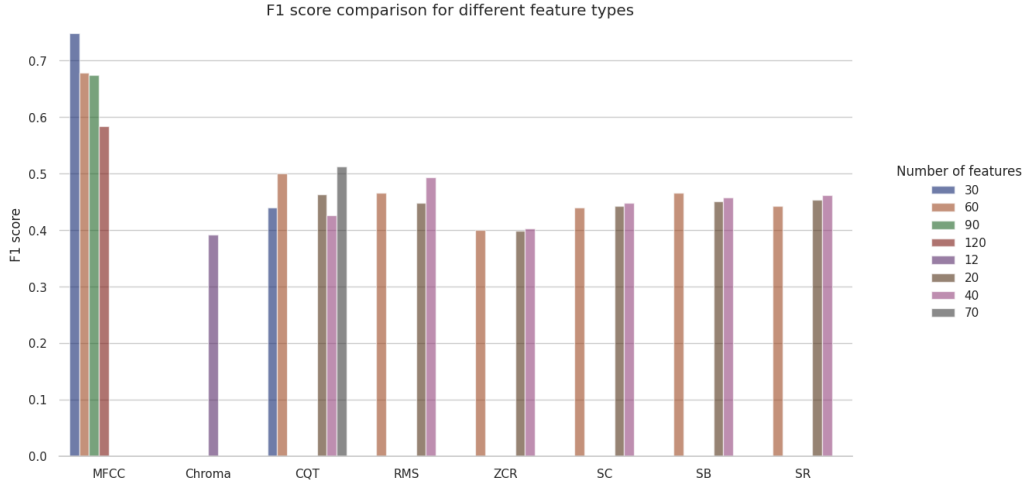


Fig. 13: F1 score per number of features

| Feature Type | Optimal Number of Features |
|--------------------|----------------------------|
| MFCC | 30 |
| Chroma | 12 |
| CQT | 70 |
| RMS | 40 |
| ZCR | 40 |
| Spectral Centroid | 40 |
| Spectral Bandwidth | 60 |
| Spectral Roll-off | 40 |

Table 1: Optimal number of features for each type

correlated with the target variable as well as those that are highly correlated with each other. Due to the high number of features, a visual approach, such as a correlation matrix, was not feasible. Instead, two filters were applied to select the most relevant features using the Spearman correlation coefficient, as the normality test failed.

The first filter is based on the correlation between the features and the target variable. Features with a correlation below a certain threshold with the target variable are removed.

The second filter focuses on the correlation among the features themselves. It counts, for each feature, the number of other features with which it has a correlation above a certain threshold. Features with a number of correlations above a specified threshold are then removed.

7.1. Threshold Selection and Model Evaluation

Threshold values were chosen empirically and the filters were applied using the combinations shown in Table 2. Using the filtered data, Random Forest models were trained and evaluated, as Random Forest was found to be the best performing model. The optimal combination of thresholds was found to be: threshold 1 = 0, threshold 2 = 0.6 and number of features = 30, resulting in 30 features.

With threshold 1 = 0, the filter on the correlation between the features and the target variable was effectively bypassed. However, with threshold 2 = 0.6, a stringent filter was applied on the correlation among the features themselves, removing features that had a correlation above 0.6

Algorithm 2 Feature Selection Process

- Step 1:** Compute the normal test (D'Agostino Pearson).
- Step 2:** Compute the Spearman correlation coefficient for each feature with the target variable.
- Step 3:** Apply the first filter to remove features with a correlation below a certain threshold with the target variable.
- Step 4:** Compute the correlation matrix among all features.
- Step 5:** Apply the second filter to remove features that have a high number of correlations (above a certain threshold) with other features.
- Step 6:** Choose threshold values empirically and apply the filters using various combinations of these thresholds.
- Step 7:** Train Random Forest models on the filtered data to evaluate performance and select the best combination of thresholds.

| Threshold | Values |
|-------------|---------------------------------|
| THRESHOLD 1 | 0 - 0.1 - 0.2 - 0.3 - 0.4 - 0.5 |
| THRESHOLD 2 | 0.6 - 0.7 - 0.8 - 0.9 - 1 |
| N° FEATURES | 5 - 10 - 15 - 20 - 25 - 30 - 40 |

Table 2: threshold values

with at least 30 other features. This indicates that having features highly correlated with each other is more detrimental to the model than having features poorly correlated with the target variable.

Figure 14 shows the results obtained with the model trained on filtered features compared to the model trained on all features. As demonstrated, the model trained on filtered features performs significantly better.

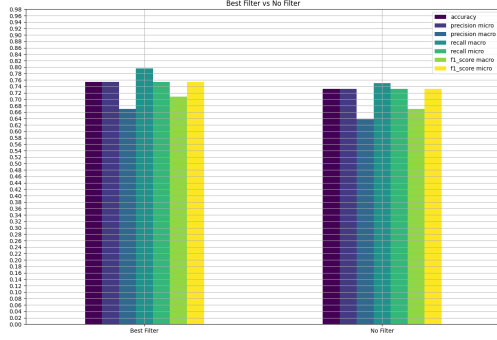


Fig. 14: Comparison of different metrics between the model on all features and the model on the filtered ones

7.2. Selected Features and Correlation Matrix

From this analysis, 41 features remained: 28 MFCC, 12 Chroma, and 1 ZCR. The correlation matrix of the filtered features is shown in Figure 15. This matrix illustrates the pairwise correlation between the selected features, with the color intensity indicating the strength and direction of the correlation. Dark red cells represent high positive correlations, while dark blue cells indicate high negative correlations.

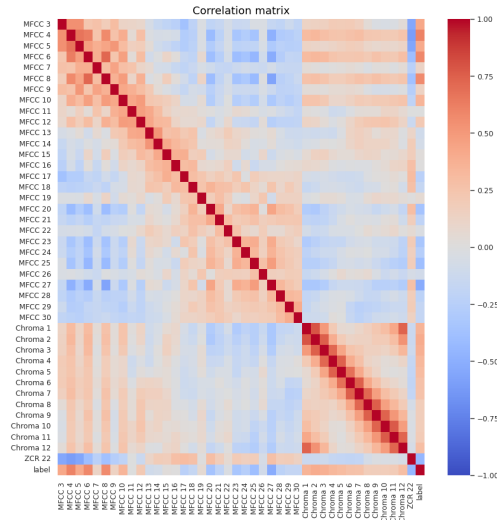


Fig. 15: Correlation matrix of the filtered features

The matrix demonstrates that the remaining features have low correlations with each other, as evidenced by the predominantly light colors away from the diagonal. This implies that the features are relatively uncorrelated, preventing multicollinearity issues and enhancing the robustness of the model. The high diagonal values indicate that each feature is perfectly correlated with itself, which is expected. However, the off-diagonal values being close to zero for most feature pairs confirm that the filtering process was effective in selecting features that do not exhibit high inter-correlations.

8. MODEL SELECTION

In the large majority of medical problems,

8.1. Models Overview

8.1.1. LightGBM

LightGBM (Light Gradient Boosting Machine) is a gradient boosting framework that builds decision trees in a leaf-wise manner. Unlike level-wise growth used in other boosting algorithms, LightGBM grows trees leaf-wise, leading to better optimization.

The objective function to minimize is:

$$L = \sum_{i=1}^n l(y_i, \hat{y}_i) + \sum_k \Omega(f_k)$$

where l is the loss function (e.g., mean squared error for regression), y_i is the true label, \hat{y}_i is the predicted label, and Ω is the regularization term to avoid overfitting.

8.1.2. XGBoost

XGBoost (Extreme Gradient Boosting) enhances the gradient boosting technique by optimizing the loss function using second-order Taylor expansion. The objective function is:

$$L(\theta) = \sum_{i=1}^n l(y_i, \hat{y}_i(t)) + \sum_k \Omega(f_k)$$

where l is the loss function, y_i is the true value, $\hat{y}_i(t)$ is the prediction at the t -th iteration, and $\Omega(f_k)$ is the regularization term. The regularization term is given by:

$$\Omega(f) = \gamma T + \frac{1}{2} \lambda \sum_{j=1}^T w_j^2$$

where T is the number of leaves, w_j are the leaf weights, and γ and λ are regularization parameters.

8.1.3. CatBoost

CatBoost (Categorical Boosting) is designed to handle categorical features natively. It uses ordered boosting to avoid overfitting and provides efficient handling of categorical data without extensive preprocessing. The model learns using the following objective function:

$$L = \sum_{i=1}^n l(y_i, \hat{y}_i) + \lambda \sum_{j=1}^m ||w_j||^2$$

where l is the loss function, y_i is the true label, \hat{y}_i is the predicted label, λ is the regularization parameter, and w_j are the model weights.

8.1.4. Random Forest

Random Forest is an ensemble method that builds multiple decision trees using bootstrap samples and random feature subsets. The final prediction is made by averaging the predictions (regression) or majority voting (classification). The prediction for a regression problem is:

$$\hat{y} = \frac{1}{T} \sum_{t=1}^T h_t(x)$$

where T is the number of trees, h_t is the prediction of the t -th tree, and x is the input feature vector.

8.1.5. MLP (*Multilayer Perceptron*)

MLP (Multilayer Perceptron) is a type of feedforward neural network with one or more hidden layers. Each neuron computes a weighted sum of inputs, applies an activation function, and passes the result to the next layer. The output of a neuron in layer l is:

$$a_i^{(l)} = f \left(\sum_{j=1}^{n^{(l-1)}} w_{ij}^{(l)} a_j^{(l-1)} + b_i^{(l)} \right)$$

where $a_i^{(l)}$ is the activation of the i -th neuron in layer l , $w_{ij}^{(l)}$ is the weight between neuron j in layer $l - 1$ and neuron i in layer l , $b_i^{(l)}$ is the bias term, and f is the activation function (e.g., ReLU, sigmoid).