



Time-varying cross-correlation between trading volume and returns in US stock markets

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ABSTRACT

This paper used detrended cross-correlation analysis (DCCA) to study contemporaneous co-movements between trading volume and returns in US stock markets (Dow Jones, Nasdaq and Standard & Poor-500). It was found that cross-correlations are not constant, but exhibit important variations with time and scale (i.e., horizon). It was argued that the complexity of the behavior of cross-correlations is in line with the adaptive market hypothesis (AMH), which states that the behavior of the market participants evolves to adapt to changing market conditions. An interesting result is that cross-correlations are positive for early periods (from 1950 to late 2000s), and shifted to negative cross-correlations in the recent two decades, a transition that may be linked to changes in the long-term risk aversion of market participants.

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1. Introduction

The relationship between trading volume and returns in financial markets is an issue with both theoretical and practical importance. Of particular interest is whether the trading volume carries useful information for predicting future stock returns [1–3]. Besides, the trading volume-returns link is important as it provides valuable insights on the dissemination of information flows into the market system, besides a more accurate understanding of the price formation mechanisms [4]. Further importance of the trading volume relationship relies on the fact that such a potential relationship has significant relevance for future markets. More specifically, price variability has a strong effect on the trading volume of futures contracts [5]. Importantly, the trading volume-returns relationship can also provide valuable insights on the effects of private and public information in the formation of demands by market investors [1].

Empirical and theoretical studies on the relationship between trading volume and prices or returns have a long history in financial fields. A brief review of salient results can be given as follows. Ying [6] postulated that trading volume and stock prices are intrinsically related, such that models intended to describe stock market dynamics should consider both variables. Epps and Epps [7] considered that the logarithmic of price could be described as a mixture of distributions, with the trading volume as the mixing variable. Tauchen and Pitts [8] used joint probability distribution of the returns and trading volume to detect positive cross-correlations in speculative markets. Smirlock and Starcks [9] used Granger causality tests to show a significant causal relationship between absolute returns and trading volume. Besides, the causality relationship becomes stronger in periods closing to earnings announcements. Blume et al. [3] and Suominen [10] characterized the content of information of trading volume, finding that trading volume carries valuable information that price dynamics alone cannot convey to the market. Lee and Swaminathan [11] showed that past trading volume has

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information for forecasting the price behavior. Chen [12] showed that returns have some power for predicting trading volume in bear and bull markets. Rak et al. [13] analyzed the relationship between the returns and corresponding trading volumes using high-frequency data from the Polish stock market, finding that the empirical impact of a given volume in prices can be described by a square-root dependence. In a subsequent report, Rak et al. [14] extended the analysis to the American stock market via multifractal detrended cross-correlation analysis [15] and showed a strong power-law cross-correlation between trading activity and volume traded. Hasan and Salim [16] reported that the link between returns and trading volume of the Indian stock market is determined by negative (i.e., anti-persistent) cross-correlations. Azad et al. [17] detected weak cross-correlations between trading volume and price changes on emerging stock markets. Gupta et al. [18] failed to find strong cross-correlation between stock returns and trading volume. Bucci et al. [19] studied a very large data set of orders executed in the US equity market and confirmed the existence of crossover from a linear (in volume) behavior for small volumes to a square-root behavior for intermediate volumes. Zheng et al. [20] showed that volume is an effective parameter for the prediction of the maximum value of volatility for both same-day and near-future periods in stock markets. Naeem et al. [21] study cryptocurrency markets and showed that extreme returns are associated with extreme trading volumes, whereas tail dependence is stronger when returns and volumes are high than when returns and volume are low.

Studies based on econophysics have shed some light on the trading volume/price relationship issue. In their development of the method for detrended cross-correlation analysis (DCCA), Podobnik and Stanley [22] showed that trading volume and price returns follow a power-law behavior. Yuan et al. [23] used a multifractal version of the DCCA to show that Chinese stock markets exhibit nonlinear cross-correlation between trading volume and prices. Stošić et al. [24] found negative long-term cross-correlations between price and volume changes in stock markets, a result that is in contrast to most results reported in the specialized literature (see Patil and Rastogi [4] for a survey on the issue). Based on DCCA, Sukpitak and Hengpunya [25] reported weak cross-correlation between market efficiency and trading volume in leading (US, Japan and Hong Kong) and emerging (India, Korea and Thailand) stock markets. Similarly, El Alaoui [26] detected multifractal cross-correlation on the Moroccan stock market.

This work used DCCA for further exploration of the cross-correlations between trading volume and returns of the three main US stock markets; namely, Dow Jones, Nasdaq and Standard & Poor-500. The samples employed in the present work consist of daily closing prices and trading volumes. The main point of the study is the characterization of the variations of the cross-correlations for time and scale. While time variations of self-correlations of stock prices have attracted some attention [27–29], time variations of the trading volume–price relationship have been scarcely studied. In particular, the evolution of the cross-correlation index over time shows interesting insights on the market dynamics, such as shifting from positive to negative values.

2. Data

The present work considered daily variations of price and trading volume in the three main US stock markets. The public website finance.yahoo.com is the source of the price and trading volume data. The Standard & Poor-500 (S&P-500) is the oldest stock market index, dating back to 1923. S&P-500 reflects the stock performance of the largest 500 companies inscribed on the US stock market. Although the S&P-500 index is recorded from January 3, 1928, the trading volume was not recorded until January 3, 1950. Therefore, the analysis of the S&P-500 is considered for the period 1950:Q1-2021:Q2 (17 959 points). The Nasdaq Composite index (NCI) reflects the capitalization of the companies (about 17 588) inscribed in the Nasdaq stock market. This market contains the second largest capitalization, just below the S&P-500, and contains the stock performance of the largest companies dealing with technology development and commercialization. The NCI can be traced back to February 5, 1971, although the trading volume registers date back from November 11, 1984. Hence, the analysis of the NCI will be carried out for the period 1985:Q4-2021:Q2 (9224 points). Finally, Dow Jones Industrial Average (DJIA) is an index that quantifies the performance of 30 large companies with activity on the US stock markets. Records of the DJIA and the corresponding trading volume date back to January 1st, 1930, such that the scrutinized period for the DJIA is 1930:Q1-2021:Q2 (23 943 points).

The daily data for the three stock markets are shown in semi-log format in Fig. 1, and the respective normalized price (r_p) and trading volume (r_v) returns

$$r_p(t) = \frac{p(t) - p(t-1)}{p(t)} \quad (1a)$$

$$r_v(t) = \frac{V(t) - V(t-1)}{V(t)} \quad (1b)$$

are shown in Fig. 2. The descriptive statistics in Table 1 summarizes the pattern of price and trading volume returns. The mean value of all returns is positive, which reflects the long-term positive trend of indices and trading volume. On the other hand, the standard deviation of trading volume returns is quite high, from 23.66% for the Nasdaq stock market to 32.14% for the Dow Jones stock market. This means that the dynamic behavior of the trading volumes is very volatile. The kurtosis is relatively high for indices and trading volumes, which shows that the return time series presents fat-tail characteristics.

Table 1
Descriptive statistics of returns of US stock indices and trading volumes.

Time series	Max	Min	Mean	SD	Skew	Kurt
DJIA index	+15.42	−22.61	2.63×10^{-2}	1.10	−2.43	65.38
DJIA volume	+91.30	−898.21	−3.47	32.14	−9.23	165.37
NAI index	+14.17	−12.32	5.31×10^{-2}	1.39	−0.47	11.27
NAI volume	+90.50	−936.21	−1.80	23.66	−10.52	319.01
S&P-500 index	+9.64	−25.21	2.63×10^{-2}	0.94	−1.45	39.89
S&P-500 volume	+96.36	−2420.12	−1.91	27.85	−39.06	3221.42

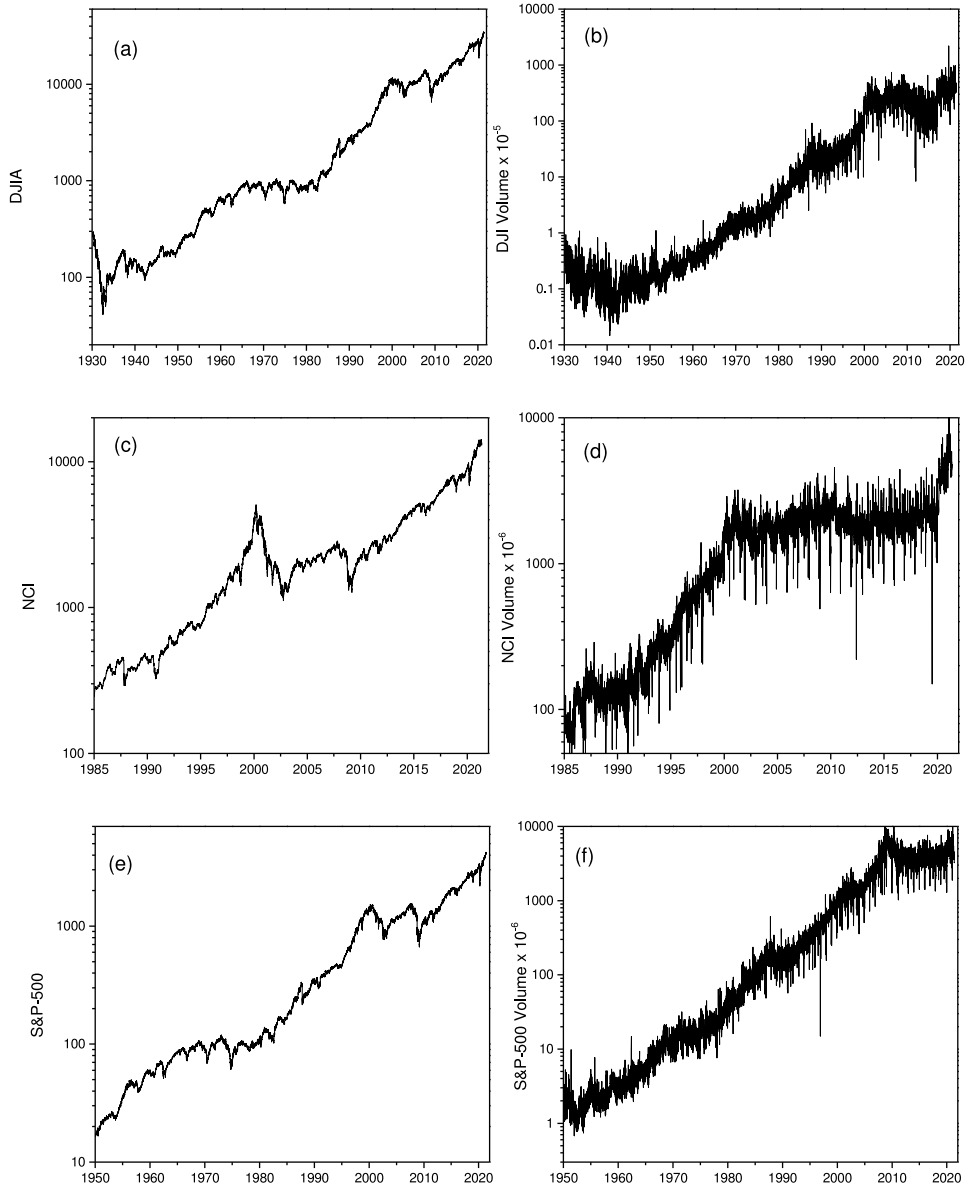


Fig. 1. US stock indices and trading volume dynamics: (a) and (b) Dow Jones Index Average, (c) and (d) Nasdaq, and (e) and (f) Standard & Poor-500.

3. Methodology

The approach followed in this work is based on the cross-correlation detrended analysis (DCCA) proposed by Zebende [30]. For the sake of completeness in presentation, a brief description of the DCCA can be made as follows.

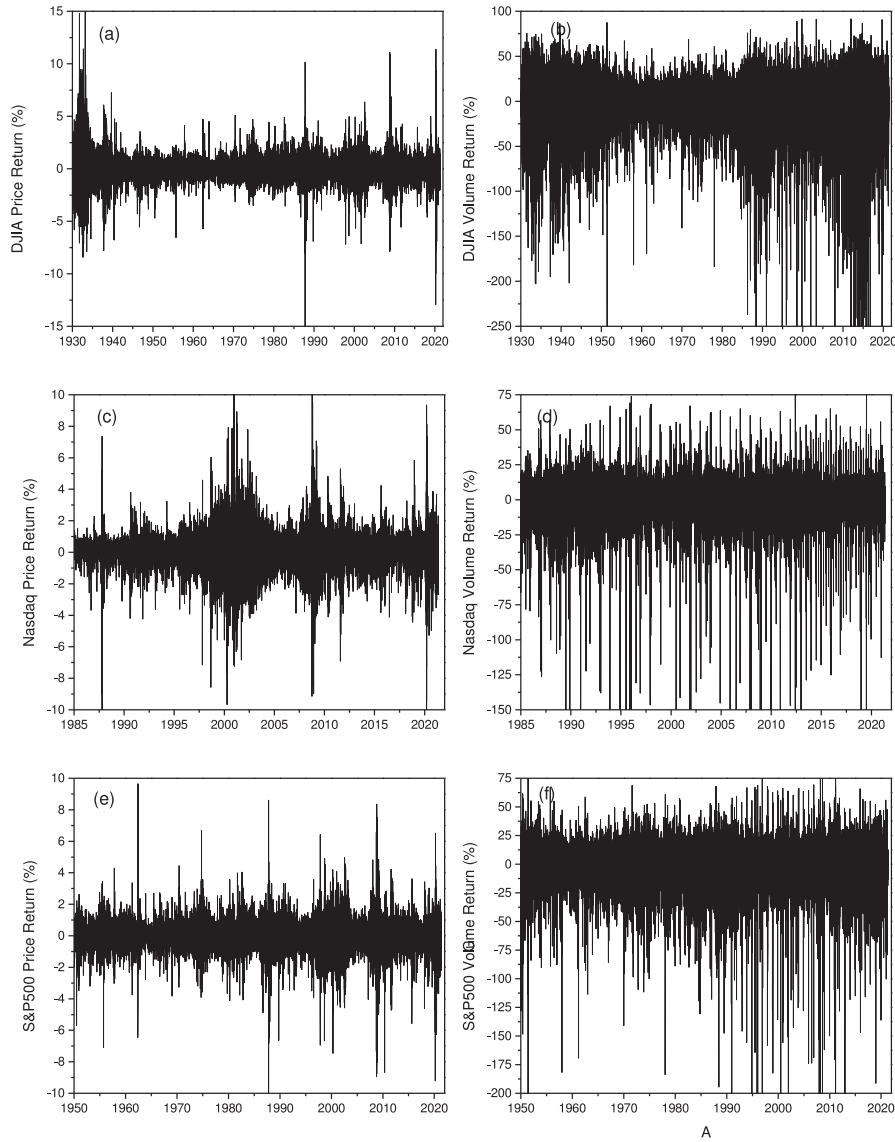


Fig. 2. Normalized price returns (left panel) and trading return differences (right panel): (a) and (b) Dow Jones Index Average, (c) and (d) Nasdaq, and (e) and (f) Standard & Poor-500.

Consider two time-series designed as $x(t)$ and $y(t)$, $t = 1, \dots, N$. Integrate the time-series to obtain their profiles:

$$X(t) = \sum_{k=1}^t (x(k) - \bar{x}) \quad (2a)$$

$$Y(t) = \sum_{k=1}^t (y(k) - \bar{y}) \quad (2b)$$

for $t = 1, \dots, N$. Here, \bar{x} and \bar{y} stand for the mean values of the corresponding time-series. The integrated time series is divided into $N_S = \text{int}(N/S)$ of length S . The trend of the integrated time series is obtained by means of n th-order polynomial fittings denoted by $P_{n,X}(t; S)$ and $P_{n,Y}(t; S)$. The covariance function relative to the polynomial trends is given by

$$F_{n,XY}(S) = \frac{1}{N_S} \sum_{k=1}^{N_S} (X(t) - P_{n,X}(t; S)) (Y(t) - P_{n,Y}(t; S)) \quad (3)$$

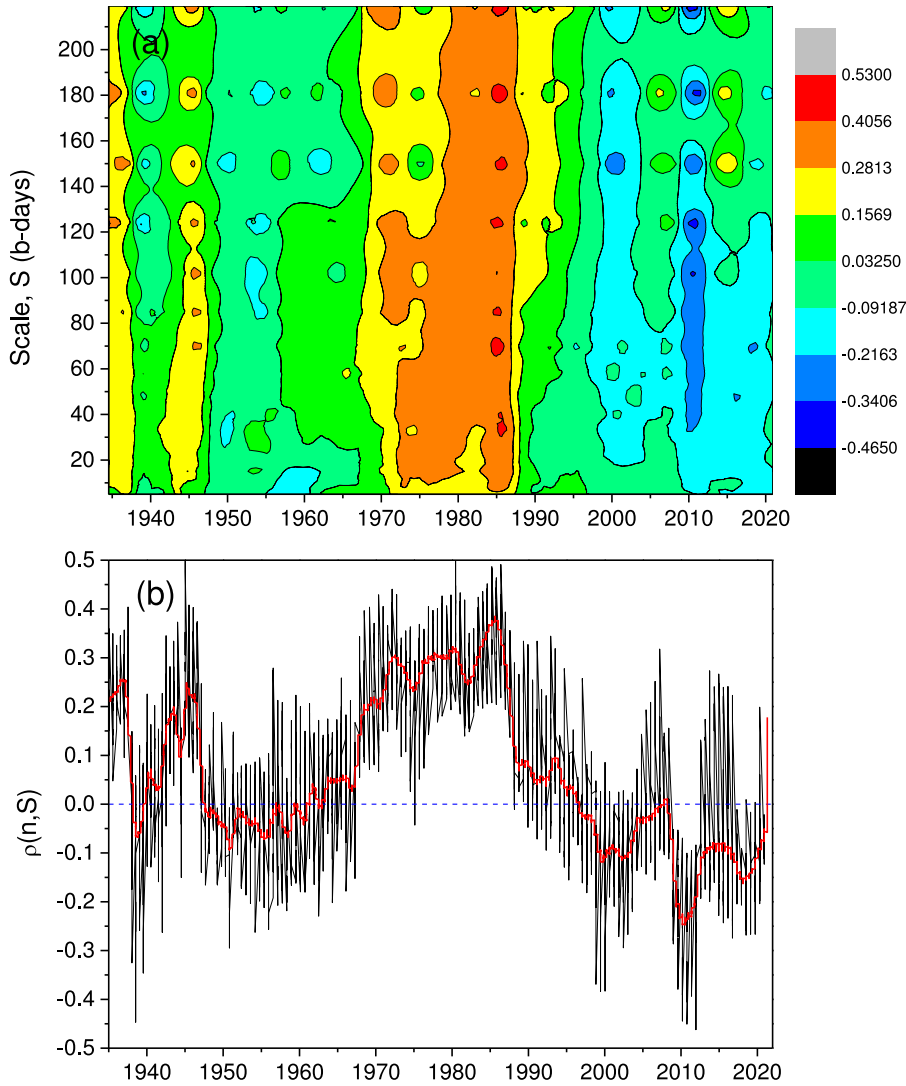


Fig. 3. (a) Scale-time map of the cross-correlation index for the Dow Jones Index Average. (b) Bundle of time variations of the cross-correlation index for different scales. The red line depicts the 100-moving average trend.

Compute the variances of the integrated time series:

$$F_{n,XX}(S) = \frac{1}{SN_S} \sum_{k=1}^{SN_S} |X(t) - P_{n,X}(t; S)|^2 \quad (4a)$$

$$F_{n,YY}(S) = \frac{1}{SN_S} \sum_{k=1}^{SN_S} |Y(t) - P_{n,Y}(t; S)|^2 \quad (4b)$$

In this way, the correlation index is given by

$$\rho(n, S) = \frac{F_{n,XY}(S)}{\sqrt{F_{n,XX}(S)}\sqrt{F_{n,YY}(S)}} \quad (5)$$

The index $\rho(n, S)$ varies in the domain $[-1, +1]$, with $\rho(n, S) = 0$ for time series that are not cross-correlated. Besides, $\rho(n, S) > 0$ for time series that are positively cross-correlated (i.e., the increase of a time series is statistically followed by the increase of the other time series and vice versa), and $\rho(n, S) < 0$ for time series that are negatively cross-correlated (i.e., the increase of a time series is statistically followed by the decrease of the other time series and vice versa). The cross-correlation index $\rho(n, S)$ depends on both the order of the detrending polynomial n and the scale S . Following Ruan

et al. [31], time scales in the range $10 < S < N/5$ will be considered. On the other hand, as used in most reports dealing with DCCA, first-order (i.e., linear) polynomial detrending will be used in the sequel.

The DCCA analysis was carried for the normalized price and trading volume returns given by Eq. (1) and exhibited in Fig. 2. The method was implemented over a rolling window of size 1056 observations (about 4 calendar years), and a sliding of 25 observations. The units of the scale variable S are business days (b-days in short), such that 20, 60 and 265 b-days correspond respectively with monthly, quarterly and yearly scales.

4. Results and discussion

Fig. 3.a presents a scale-time map of the cross-correlation index $\rho(1, S)$ between returns of the index and the trading volume for the DJIA signal. The index is not constant as important variations with time and scale were exhibited. Patches of positive and negative cross-correlation index are observed. The period from 1972–1987 showed a patch high positive cross-correlation with values as high as 0.53. On the other hand, a sharp transition from positive to negative values by 2000–2001 was exhibited. The lowest (negative) cross-correlations were shown by 2010–2011. The complex variation of the cross-correlation index is in line with the notion that stock markets are evolutionary systems with mechanisms to deal with exogenous shocks and to adapt to changing environments [32,33]. A more compact view of the variation of the cross-correlation index with time is given by Fig. 3.b, which shows the bundle of time variations for the whole scales in Fig. 3.a. The red line corresponds to a 100-neighbor moving average of the complete collection. In the period 1985–1998, the cross-correlation index moved in the positive value region. By the late 1990s, the average index started a long-term decreasing pattern to achieve negative values as low as -0.28 by 2010. Only recently, in the last months of 2019, the index exhibited a sharp increase to positive values.

The cross-correlation index of the Nasdaq stock market presents a pattern similar to that of the Dow Jones stock market (Fig. 4.a). The main difference relies on the year where the cross-correlation index changed from long-term positive to negative values (Fig. 4.b). For the period 1985–2008, the cross-correlation index varied primordialily in the region of positive values. Coinciding with the surge of the 2008 Great Recession, the cross-correlation index changed to negative values to reach values as low as -0.4 by 2018:Q1. It is not clear whether the index for the Nasdaq stock market will change to long-term positive values as it was done by the index of the Dow Jones stock market (Fig. 3.b).

The analysis of the Standard & Poor-500 stock market spans a longer period, from 1950 to 2019, giving the opportunity of evaluating the variation of the cross-correlation index for a longer historical period (Fig. 5.a). The cross-correlation index varied in the positive value region for the period 1950–1999. This means that an increase (resp., decrease) of the index return was accompanied likely by an increase (resp., decrease) of the trading volume, and vice versa. The year 1999 witnessed the shift from positive to negative values, as the index reached values of about -0.35 by 2017:Q3. Although the index has exhibited an increasing behavior in 2019–2021, the transition to positive values cannot be expected as the average index has shown an oscillatory pattern with a period of about 6.0–6.5 years.

The results shown in Figs. 3 to 5 were obtained for a rolling window of 1045 observations (about 4 calendar years). This window size is intended to capture the cross-correlation patterns for scales of up to one year. Fig. 6 shows that the pattern of time variation of the cross-correlation index is obtained for rolling windows of different sizes. The shift from positive to negative cross-correlation was obtained for the S&P-500 for three different values of the window size. This means that the time variation pattern observed in Figs. 3 to 5 is an intrinsic property of the trading volume and price signals, and not an artifact of the size of the rolling window.

4.1. Discussion

Several studies in the 1960–1990s reported a positive relationship between the change of US stock indices and trading volume [1,5–7,34]. For instance, Cornell [5] reported positive cross-correlations between changes in price and changes in trading volume over two-month scales for at least 17 Standard & Poor-500 contracts. Besides, he found that the index–volume relationship was contemporaneous as led–lag effects were statistically insignificant. Also, Harris [35] reported positive cross-correlation between trading volume and price change for 479 common stocks. The results reported in the present study corroborate the findings reported in the aforementioned studies. Figs. 3 to 5 showed that a positive cross-correlation between a price change and trading volume variation was dominating the dynamics of the US stock markets. In general, the cross-correlation relationship was not constant but exhibited important variations with time and scale (i.e., time horizon). The positive price/volume relationship spanned over a long period of the last century, ending by the late 1990s for Dow Jones and Standard & Poor-500 stock markets. The Nasdaq stock market exhibited a longer period of positive cross-correlations, as long-term negative values surged by 2007. Theoretical efforts have been devoted to explaining the positive price/volume cross-correlation. The importance of the issue relies on the fact that the presence or not of cross-correlations is linked to the flow of information in the market system. In particular, the lack of cross-correlations would indicate that the trading volume is a variable independent of the stock price, and so it should contain valuable information to build up, e.g., investment decisions. Maybe, the positive cross-correlations between price and volume simply reflect the effect of a supply/demand mechanism where prices increase when the demand (i.e., high volume) increases [36]. In this case, the price behavior is mostly driven by bull traders with low risk-aversion that increase prices by increasing the demand for stocks.

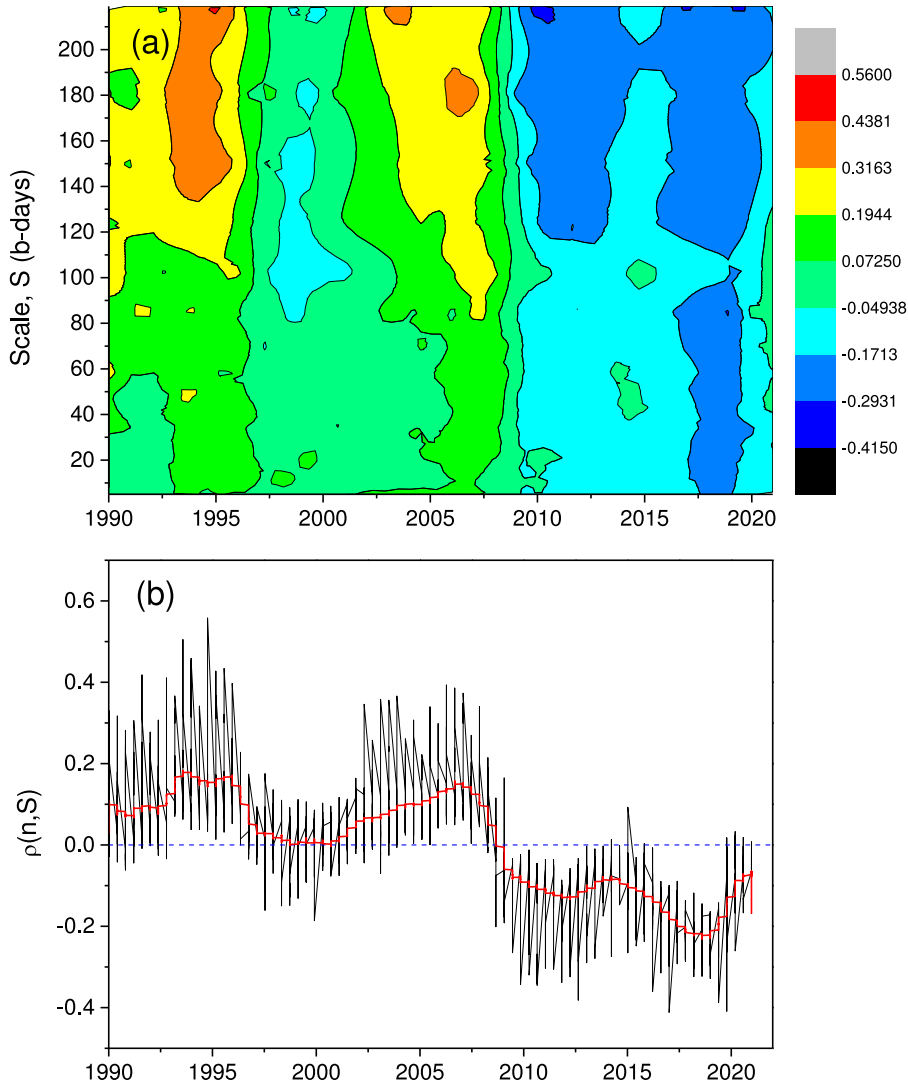


Fig. 4. (a) Scale-time map of the cross-correlation index for the Nasdaq stock market. (b) Bundle of time variations of the cross-correlation index for different scales. The red line depicts the 100-moving average trend.

The shift from positive to negative cross-correlations in the three US stock markets is puzzling and has been rarely documented. Campbell et al. [37] found that stock auto-correlations show a tendency to decline with trading volume. They proposed an explanation for the feature, proposing that excess in the trading volume appear when shifts in the stock demand of liquidity traders are managed by risk-averse market makers. Within this scenario, the market dynamics are dominated by risk-aversion investors that are prompted to sell in the surge of risky endogenous and exogenous conditions. Chordia and Swaminathan [38] found evidence of negative cross-correlation between trading volume and average stock returns for the period 1963–1996. In turn, the result indicated that the trading volume plays an important role in the rate at which prices adjust to information flows. In this way, the negative cross-correlation between trading volume and price would imply that stocks made visible by high trading volume tend to depreciate, maybe by the effect of risk-aversion by stock investors. Overall, the results in Figs. 3 to 5 showed that the US stock market underwent a shift in the trading volume/price relationship in the late 1990s and early 2000s, an effect that could be caused by changes in the risk-aversion behavior of the market agents.

An important issue is the efficiency of stock markets. In its simple notion, the efficient market hypothesis (EMH) states that markets are efficient if prices always reflect the whole information available in the market [39]. In this way, information is instantaneously available to all market participants and price dynamics reflect the information immediately. A test for the weak form of the EMH consists in showing that price predictability is not possible as new information drives prices in a random form. The weak form of the EMH has been tested and extensively debated over the decades. In terms of the price return/trading volume relationship, the EMH would imply that trading volume and price return are

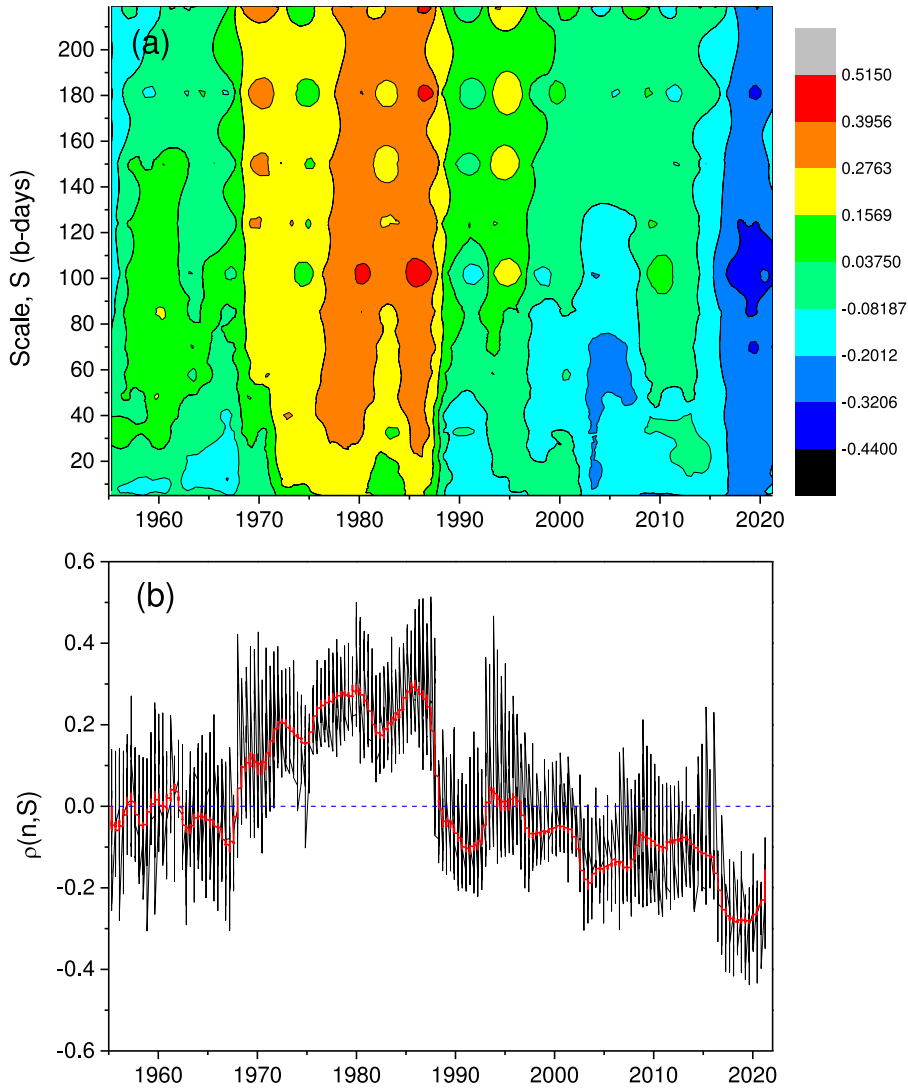


Fig. 5. (a) Scale-time map of the cross-correlation index for the Standard & Poor-500 stock market. (b) Bundle of time variations of the cross-correlation index for different scales. The red line depicts the 100-moving average trend.

not cross-correlated. If cross-correlations are present, then the trading volume can be used as a proxy for carrying out quantitative prediction of the evolution of the price return. For instance, Blume et al. [3] showed that the dynamics of the trading volume may be considered as an indicator of the quality of information revealed by prices. Our results showed the presence of cross-correlations between trading volume and prices, which in principle prescribe the fulfillment of the EMH. However, the cross-correlation was not constant but showed important variations with time and horizon (i.e., scale). In the same context, Alvarez-Ramirez et al. [27] showed that the deviations from efficiency in US stock markets present important variations with time. These results are in line with the adaptive market hypothesis (AMH) [32,33], which states that deviations from the EMH are a consequence of adaptation of the market to changing environmental conditions, including several competitors, profit opportunities, and socio-political factors [4]. The result showed that important stock market events (e.g., Bretton-Woods end, 1987 Crash, and 2008 Recession) are reflected in the traded volume–price return dynamics. For instance, the shift from positive to negative cross-correlation showed by the Nasdaq stock market (see Fig. 4) appears as a consequence of the 2008 Great Recession, which likely changed the relationship between market conditions and participants. In this way, the time-varying traded volume–price return cross-correlation index can be used as an indicator of drastic changes in the stock market. Drożdż et al. [40] postulated that super-bubble dynamics appearing before a market crash can be described by a log-periodic self-similar phenomenon. Grech and Mazur [41] showed that the local Hurst exponent has some ability to anticipate market crashes. The sharp decrease of the cross-correlation index for the DJIA by 1986:Q4 (Fig. 3.b) anteceded the 1987 Black Monday crash. Also, a shift from positive to negative cross-correlation values in the Nasdaq index (Fig. 4.b) occurred by 2007:Q4 and preceded the 2008 Great Recession. In this

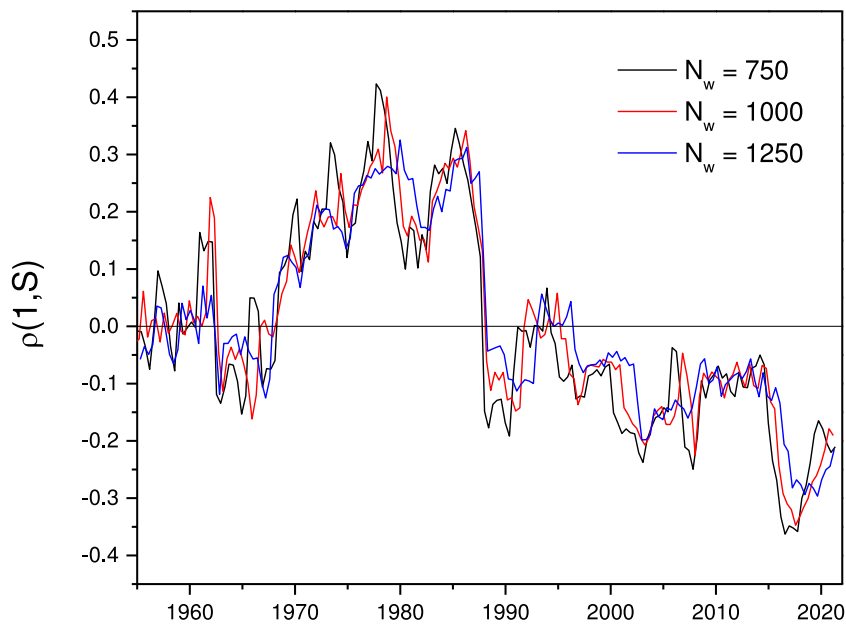


Fig. 6. Effect of the size of the rolling window in the average behavior of the cross-correlation index for the S&P-500 stock market.

way, one can suggest that a shift of the impact of the trading volume impact in the American stock market indices could conduct to the deterioration of the stock market operation. Results in this line were reported by Drożdż et al. [42] by showing that stock market crashes are preceded by an increase of the multifractal spectrum asymmetry. Overall, in the spirit of the postulate by Grech and Mazur [41], the observation and analysis of local cross-correlation between traded volume and price return is a valuable tool for the design of investment strategies, besides other technical indicators like moving average or momentum.

5. Conclusions

In this paper, we found cross-correlations between trading volume and price returns of the US stock markets. However, such cross-correlations are not constant but show important variations with time and scale. In the early phase, the cross-correlations were positive with interlaced periods of low and high values. However, cross-correlations shifted to negative values by the late 1990s and early 2000s, an effect that was likely caused by important changes in the market conditions (e.g., the dot-com bubble burst and the 2008 Great Recession). It was argued that the presence of trading volume/price cross-correlations prescribe the fulfillment of the EMH, although the results are in line with the adaptability of the market participants to changing environments (e.g., exogenous shocks).

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CRediT authorship contribution statement

E. Rodriguez: Visualization, Investigation, Writing - review & editing. **J. Alvarez-Ramirez:** Writing - original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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