On Manipulating Weight Predictions in Signed Weighted Networks

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Abstract

Adversarial social network analysis studies how graphs can be rewired or otherwise manipulated to evade social network analysis tools. While there is ample literature on manipulating simple networks, more sophisticated network types are much less understood in this respect. In this paper, we focus on the problem of evading FGA—an edge weight prediction method for signed weighted networks by (Kumar et al. 2016). Among others, this method can be used for trust prediction in reputation systems. We study the theoretical underpinnings of FGA and its computational properties in terms of manipulability. Our positive finding is that, unlike many other tools, this measure is not only difficult to manipulate optimally, but also it can be difficult to manipulate in practice.

Introduction

Adversarial social network analysis studies how networks can be rewired or otherwise manipulated to falsify network examination. In particular, many works in this body of research studied how to manipulate classic tools of social network analysis such as centrality measures (Crescenzi et al. 2016; Bergamini et al. 2018; Was et al. 2020), and community detection algorithms (Waniek et al. 2018a; Fionda and Pirro 2017; Chen et al. 2019). Also, a rapidly growing body of works studies adversarial learning on graphs using deep learning (Chen et al. 2020).

While most of the above literature focused on simple networks, in this paper, we consider a more complex model of weighted signed networks. In this class of networks, links are labeled with real-valued weights representing positive or negative relations between the nodes (Leskovec, Huttenlocher, and Kleinberg 2010a,b; Tang et al. 2016). An important application of signed weighted networks is the modelling of trust networks/reputation systems, the goal of which is to avoid transaction risk by providing feedback data about the trustworthiness of a potential business partner (Resnick et al. 2000). As an example, let us consider the cryptocurrency trading platform Bitcoin OTC (Kumar et al. 2016). In this platform, users are allowed to rate their business partners on the scale $\{-10, -9, \ldots, 10\}$, and the rat-

ings are publicly available in the form of a who-trusts-whom network. A 6-node fragment of this network is presented in Figure 1.

A user who thinks of doing a transaction with another user for the first time can use the information from such a who-trust-whom network to predict the potential risk. Technically, given a trust network modeled as a weighted signed network, predicting trust amounts to predicting the weights of potential new edges. A well-known edge weight prediction method, called FGA, was proposed by (Kumar et al. 2016). FGA is based on two measures of node behavior: the goodness that evaluates how much other nodes trust a given node, and the fairness that captures how fair this node is in rating other nodes. Both concepts have a mutually recursive definition that converges to a unique solution. Most importantly, Kumar et al. showed that FGA is effective in predicting edge weights, i.e., the level of trust between unlinked nodes. For example, in Figure 1, the trust of node 1031 towards node 715 is predicted by FGA to be 2.26.

While FGA seems to be an interesting tool to apply in practice, little is known about its resilience to malicious behaviour. In this paper, we present the first study of manipulating the FGA function by a rating fraud (Cai and Zhu 2016; Mayzlin, Dover, and Chevalier 2014). It involves fraudulent raters to strategically underrate or overrate other users for their own benefit. To magnify the strength of the manipulation, the attacker may create and act via multiple fake user identities. Such so called Sybil attacks are especially tempting in environments such as cryptocurrency trading platforms where creating a new identity is affordable. Rating fraud attacks may be direct—when targeted nodes

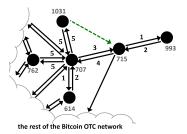


Figure 1: A fragment of the Bitcoin OTC network composed of nodes 993, 715, 707, 614, 1031, 762.

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are rated directly by the attackers—and *indirect*—when the attackers try to manipulate the neighbourhood of the target nodes rather than the target nodes themselves (see Figure 2). It is important to distinguish between direct and indirect manipulations, as in some situations, only indirect ones will be practical. This may be the case on e-commerce platforms such as e-Bay, where nodes rate each other only after completing a transaction. When a retailer of expensive products is the target, the cost of a direct attack can be prohibitive. Hence, an indirect attack becomes an attractive alternative—it may be much cheaper to attack through the clients or business partners of such an expensive retailer (see the next section for an example).

Our contributions can be summarised as follows:

- To analyze the theoretical underpinnings of the FGA measure, we propose the system of basic axioms for both fairness and goodness. We prove that together they uniquely determine the FGA measure;
- Next, we formulate the issue of manipulating the FGA
 measures of some target group of nodes as a set of computational problems. We then prove that all these problems are NP-hard and W[2]-hard, i.e., FGA is, in general, hard to manipulate.
- Given the hardness of attacking a group of nodes, we then focus our analysis on targeting a single node directly or indirectly. We first prove that direct attacks on a single node are easy, i.e., it is easy for an attacker to directly rate the target node to change the sign of her *goodness* value. As for an indirect attack, we show analitycally that for some class of networks (which we call *minimum-k-neighbour graphs*, since we require that every node in this network has *indegree* and *outdegree* at least k), we can bound the strength of indirect attacks. Our positive finding is that, in this case, FGA measure turns out to be rather difficult to manipulate.
- In our experimental analysis, we first evaluate two benchmarks: (a) the strength of the aforementioned direct attack, and (b) the strength of an indirect attack based on a simple greedy approach. The latter one turns out to be very ineffective. Next, we analyse an improved greedy approach by attacking at a larger scale in every step. This approach, although costly, proves to be often effective.

Preliminaries

A Weighted Signed Network (WSN) is a directed, weighted graph G=(V,E,W), where V is a set of users, $E\subseteq V\times V$ is a set of (directed) edges, and $\omega:E\to[1,+1]$ is a weight function that to each $(u,v)\in E$ assigns a value between -1 and +1 that represents how u rates v. For any directed edge $(u,v)\in E$, let us denote by $\overline{(u,v)}$ the edge in the opposite direction, i.e., $\overline{(u,v)}=(v,u)$. For any set of directed edges E, denote by $\overline{E}=\{\overline{e}:e\in E\}$. Furthermore, let P be a set of pairs of nodes of cardinality n, i.e, $P=\{\{u_1,v_1\},...,\{u_n,v_n\}\}$. The domain of P is the set of nodes that make the pairs in P, i.e. $dom(P)=\{u:u\in\{u,v\}\in P\}$. Finally, we write Pred(v) (resp. Succ(v)) to denote the set of Predecessors (resp. Successors) of v (resp.

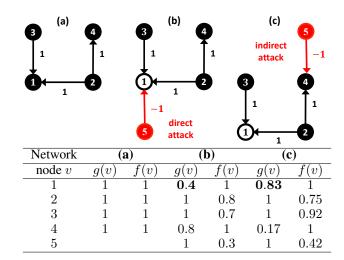


Figure 2: Sample networks and two types of attacks.

u) defined as follows: $Pred(v) = \{u : u \in (u, v) \in E\}$ (resp. $Succ(u) = \{v : v \in (u, v) \in E\}$).

For a square matrix $M^{m\times m}$, we define $||M||_{\infty} = \max_{1\leq i\leq m}\sum_{j=1}^m m_{ij}$, $||M||_1 = \max_{1\leq j\leq m}\sum_{i=1}^m m_{ij}$. It is also known that $||M\times M||_{\infty}\leq ||M||_{\infty}\cdot ||M||_1$ and $||M\times M||_{\infty}\leq ||M||_1\cdot ||M||_1$ (see https://en.wikipedia. org/wiki/Matrix_norm).

Kumar et al. (2016) define a recursive function, FGA, that assigns to each vertex of a weighted directed graph two values: fairness and goodness, (f(v), g(v)). The first one, f(v), assigns a real value from range [0,1] to v that indicates how fair this node is in rating other nodes. The second one, g(v), assigns a value from range [-r,r] to v indicating how much trusted this node is by other nodes (for simplicity we assume that r=1 in this paper). Finally we define an in-degree (indeg(u)) and out-degree (outdeg(u)) of a node $u \in V$. $indeg(u) = |\{(v,u): (v,u) \in E\}|$ and $outdeg(u) = |\{(u,v): (u,v) \in E\}|$. Kumar et al.'s recursive formula for (f(v),g(v)) is as follows:

$$g(v) = \frac{1}{indeg(v)} \sum_{u \in Pred(v)} f(u) \times \omega(u, v)$$
 (1)

$$f(u) = 1 - \frac{1}{outdeg(u)} \sum_{v \in Succ(u)} \frac{|\omega(u, v) - g(v)|}{2}, \quad (2)$$

where g(v) = 1 for $v \in V$ with indeg(v) = 0, and f(v) = 1 for $v \in V$ with outdeg(v) = 0.

Kumar et al. (2016) showed that this function can be computed iteratively starting from $f^{(0)}(u) = g^{(0)}(u) = 1$. Theorem 1 from the aforementioned work states that at each step, t, the estimated values $f^{(t)}(u)$, $g^{(t)}(u)$ get closer to their limits $f^{(\infty)}(u)$, $g^{(\infty)}(u)$, i.e. we have $|f^{(\infty)}(u) - f^{(t)}(u)| < \frac{1}{2^t}$ and $|g^{(\infty)}(u) - g^{(t)}(u)| < \frac{1}{2^{t-1}}$. The FGA function can be used for predicting the weight of some not-yet existing (or unknown) edge $(u,v) \in V \times V \setminus E$ by computing the product: $\omega(u,v) = f(u) \times g(v)$.

As an example of the FGA function and how it could be attacked, let us consider Figure 2. Network (a) is a bench-

mark, where every node rates others with the highest possible value. In network (b), a new node 5 is used to perform a **direct attack** by rating node 1 with the worst possible value of -1. This decreases the goodness of node 1 to 0.4. However, as argued in the introduction such a direct attack can be prohibitively costly. Nevertheless, given the definition of the FGA function, node 5 can also perform an indirect attack on node 1. This can be done, for instance, by directly attacking node 4. As node 4 has already been rated positively by node 2, an opposite rating introduced by 5 will decrease the fairness of 2. In particular, comparing network (c) to (a) in Figure 2, the fairness of 2 decreased from 1 to 0.75. This lower fairness means that node's 2 ratings are less meaningful in network (c) than in network (a). Hence, the goodness of node 1 decreases to 0.83.

Axiomatization

Our first result is an axiom system that completely characterizes the FGA. Below we present a comprehensive summary, while the details will are available in the appendix of the paper.

We begin with the characterization of the goodness part of the FGA function. Recall that the idea behind the goodness of v is that it should reflect how this node is rated by its predecessors. Moreover, the ratings of the fairer predecessors should count more. We translate these high-level requirements into the following axioms:

- SMOOTH GOODNESS—let all predecessors of a particular node, $v \in V$, be unanimous in how they rate v and let their fairness be the same. Now, let us assume that their fairness increases equally, i.e., intuitively, the nodes that rate v become more trustworthy. Then, we require that this will result in an increase of the goodness of v, and that this increase is proportional to the increase of the fairness of v's predecessors;
- INCREASE WEIGHT—let the predecessors of v be all equally fair and unanimous in how they rate v. Now, let them increase their rating of v equally. Then, we require that the goodness of v increases and that this increase is proportional to the increase in how v is rated;
- MONOTONICITY FOR GOODNESS—the predecessors with higher fairness should have a bigger impact on the goodness of v. Similarly, higher weights should also have a bigger impact;
- GROUPS FOR GOODNESS—let v be rated by k groups of the predecessors and let the nodes in each group be homogeneous and unanimous w.r.t. v. What is then the relationship between the impact these groups have on the goodness of v? In line with the previous axioms, we require that the goodness of v should be equal to the weighted average of the ratings achieved when these groups separately rate v;
- MAXIMAL TRUST—this basic condition requires that any if all the predecessors of v have the highest possible fairness and their ratings are the highest possible, then the goodness of v should be the highest possible;
- BASELINE FOR GOODNESS—a non-rated node has the goodness of 1.

Our first result is that the above axioms uniquely define the goodness part of the FGA function.

Let us now characterise the fairness part of the FGA function. Recall that the idea behind the fairness of v is that it should reflect how the ratings given by this node agree with the ratings given by other nodes, i.e. how erroneous v is. In this respect, we have the following axioms:

- SMOOTH FAIRNESS—this axiom stipulates that the fairness of a node making an average error is an average of the fairness values of nodes making extreme errors;
- MONOTONICITY FOR FAIRNESS—our first axiom stipulates that the fairness of a node that rates more accurately than before should rise;
- GROUPS FOR FAIRNESS—if the nodes rated by v can
 be divided into k groups such that each node in a particular group is rated by v in the same way, then the fairness
 of v should be equal to the weighted average of v's fairness in a setting where v rates these groups separately;
- OBVIOUS FAIRNESS METRIC—here, we stipulate that when a node makes maximal errors when rating all of its neighbors, then its fairness should be 0, and when there is no error, then the fairness is 1;
- BASELINE FOR FAIRNESS—the fairness of a node that rates noone is 1.

The above axioms uniquely define the fairness part of the FGA function. In summary, all the above axioms uniquely define the FGA function.

Theorem 1. The SMOOTH GOODNESS, INCREASE WEIGHT, MONOTONICITY FOR GOODNESS, MAXIMAL TRUST, GROUPS FOR GOODNESS, BASELINE FOR GOODNESS axioms and the SMOOTH FAIRNESS, MONOTONICITY FOR FAIRNESS, OBVIOUS FAIRNESS METRIC, GROUPS FOR FAIRNESS, and BASELINE FOR FAIRNESS axioms uniquely define the FGA function.

Complexity of attack

Let us now study the complexity of manipulating FGA.

Attack models Given $G=(V,E,\omega(E))$, let $A\subseteq V$ be a set of attackers. We define two types of the A's objectives:

• targeting potential links — here, the target set TP is composed of disconnected pairs of nodes from $V\setminus A$:

$$TP \subseteq \{\{u, v\} : u, v \in V \setminus A \land (u, v), (v, u) \notin E\}. \quad (3)$$

Intuitively, the aim is to change the predicted weight of the potential links between the pairs from TP.

• targeting nodes — here, the target set is $T \subseteq V \setminus A$. Intuitively, the goal is to alter the targets' reputation.

The attackers can make the following types of moves:

- edge addition the attackers can add an edge (u,v) to G, where $u \in A, v \in T, (u,v) \notin E$, and with the weight $\omega(u,v) \in [-1,1]$. This corresponds to the attacker $u \in A$ rating node $v \in T$ for the first time.
- weight update an attacker $u \in A$ can update the weight of an existing edge $(u,v) \in G$ to some value $\omega(u,v) \in [-1,1]$. This corresponds to a modification of the existing rating by the attacker.

Algorithm 1: Direct attack

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Data: A,T=\{t\},G for a\in A do \mid add an edge (a,t),\omega(a,t)=-1 to the graph G end
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All the attackers are allowed to make no more than k such moves in total. We will refer to k as a budget.¹

We will now formalize our computational problems. In the first one, the attackers aim at modifying the predicted weights between the pairs of nodes in TP to decrease them below (increase above) a certain threshold. This attack corresponds to breaking potential business connections.

Problem 1 (DECREASE (INCREASE) MUTUAL TRUST, DMT (IMT)). Given a weighted signed network $G = (V, E, \omega)$, a set of attacking nodes $A \subseteq V$, a target set of disconnected pairs of nodes TP as defined in eq. 3, an intermediary set $I \subseteq V$, the budget k, and a threshold $t \in [-1, 1]$, decide for all $\{u, v\} \in TP$ whether it is possible to decrease (increase) the value of either predicted weight $f(u) \times g(v)$ or $f(v) \times g(u)$ to or below (above) the threshold t by making no more than k edge additions or weight updates with the restriction that the attackers $u \in A$ are rating only the nodes from the intermediary set I.

In the second problem, the attackers aim at altering the goodness value of the nodes from a target set T. This attack corresponds to spoiling the reputation of the target nodes.

Problem 2 (DECREASE (INCREASE) NODES RATING, DNR (INR)). Given $WSNG = (V, E, \omega)$, a set of attackers $A \subseteq V$, a target set $T \subseteq V \setminus A$, an intermediary set $I \subseteq V$, the number of possible moves k, and threshold $t \in [-1, 1]$, decide whether it is possible, for all $v \in T$, to decrease (increase) the goodness of each vertex v to or below (above) threshold t by making no more than k edge additions or weight updates with the restriction that the attackers $u \in A$ are rating only the nodes from the intermediary set I.

Hardness Results We first consider DMT (IMT).

Theorem 2. Solving the $DMT(IMT) = (G = (V, E, \omega), A, TP, I, t, k)$ problem is NP-hard.

Theorem 3. Solving the $DNR(INR) = (G = (V, E, \omega), A, T, I, t, k)$ problem is NP-hard.

Proof of the above theorems can be found in the appendix of the paper.

Parametrized complexity The following results, in terms of the *W*-hierarchy for the parameterized algorithms (Cygan et al. 2015), hold:

Theorem 4. DNR(INR) parameterized by k is W[2]-hard.

Theorem 5. DMT(IMT) parameterized by k is W[2]-hard.

Proof of the above theorems can be found in the appendix of the paper.

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Algorithm 2: Indirect attack
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end

Manipulating a node directly

Let us now focus on attacks on attacking a single node. First, we report a result on the scale of manipulability of the FGA function by a direct attack. In particular, the following theorem says that it does not take many edges to change the sign of a single node in the DNR problem when the attacker is able to rate the target directly.

Theorem 6. Let us consider an instance of the DNR problem, where, for an arbitrary $G = (V, E, \omega)$, a single node u_T is attacked with $0 < g(u_T) \le 1$ (thus $T = \{u_T\}$), and the set of attacking nodes $A \subseteq V$ is relatively trusted, $f(v) \ge \frac{1}{2}$ for $v \in A$ and $|A| > \lceil 2 \times g(u_T) \times indeg(u_T) \rceil$. Then, it is trivial to change the sign of the goodness value of u_T (i.e. to achieve the threshold t = 0) if the attackers can attack directly (i.e. $T \subseteq I$ in the DNR problem).

The proof can be found in the appendix of the paper.

Bounding the strength of indirect attacks

In this section, we give bounds on the strength of an indirect Sybil attacks, i.e., the attack in which the attacker creates a new node when adding a new edge. Our results hold for a family of relatively dense networks, $G=(V,E,\omega)$, in which every node has a lower bound on its indegree and outdegree, i.e., $\forall v \in Vindeg(v) \geq k$ & $outdeg(v) \geq k$, and the intermediary nodes, $j \in V$, are relatively weakly rated, i.e. $\sum_{v \in Pred(j)} |\omega_{vj}| \leq k$. We call such networks minimumk-neighbour networks.

Theorem 7 (Indirect Sybil attack). Assume a WSN $G = (V, E, \omega)$, where a new Sybil node s_i is added that rates some intermediary node $i \neq t$. Whenever $\forall_{v \in V} indeg(v) \geq k$ & outdeg $(v) \geq k$, and $\forall_{j \in V} \sum_{v \in Pred(j)} |\omega_{vj}| \leq k$, then $|\Delta g(t)| \leq \frac{2}{(indeg(i)+1) \times k}$.

Proof. Let us define set V' as follows. We begin with $V' = \{t\}$. Next, we iteratively add to V' other nodes $v \in V$ which are indirectly connected to at least one node in V' (i.e., $\exists v' \in V' : \exists (l,v'), (l,v) \in E$). It is easy to see that the intermediary node i has to belong to V' in order to make the indirect attack successful.

 $^{^{1}}$ We place no constraints on how the attackers distribute this budget among themselves. In an extreme case, a single attacker can do all k actions.

²Our analysis also provides some additional bound for a direct Sybil attack.

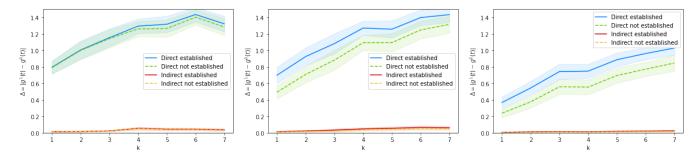


Figure 3: The comparison of direct/indirect, established/non-established attacks for Bitcoin OTC, Bitcoin Alpha and RFA Net networks.

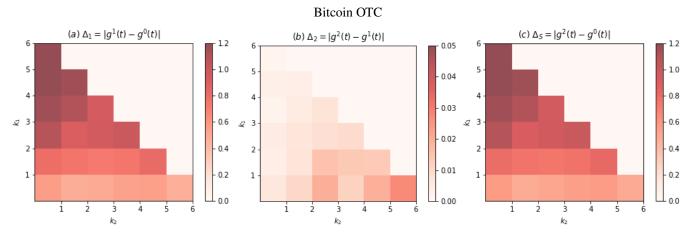


Figure 4: The results for the mixed settings in Bitcoin OTC. The average strength of an indirect attack is small and significantly smaller that the average strength of a direct attack. Δ_1 shows the influence of the attack with k_1 direct edges, Δ_2 shows the influence of the attack with k_1 direct edges and k_2 indirect edges.

We denote by g(l)/f(l) the goodness/fairness value of the node l before the Sybil attack, and by g'(l)/f'(l) the goodness/fairness value after the attack. Let $\Delta g(l) = g'(l) - g(l)$ and $\Delta f(l) = f'(l) - f(l)$.

For the target node, $t \in V'$, we can calculate how its g(t) changes w.r.t. the changes introduced to the goodness of all other nodes. Here, we assume that the Sybil attack is indirect, i.e., the Sybil edge is not added to t.

$$\begin{split} g(t) &= \frac{1}{indeg(t)} \sum_{u \in Pred(t)} f(u) \times \omega(u,t) = \frac{1}{indeg(t)} \times \\ &\sum_{u \in Pred(t)} \omega(u,t) \times \left[1 - \frac{1}{outdeg(u)} \sum_{v \in Succ(u)} \frac{|\omega(u,v) - g(v)|}{2}\right] \end{split}$$

Thus, from the triangle inequality:

$$\begin{split} |\Delta g(t)| & \leq \frac{1}{2 \times indeg(t)} \sum_{u \in Pred(t)} \sum_{v \in Succ(u)} \\ & \frac{1}{outdeg(u)} \times |\omega(u,t)| \times |\Delta g(v)| \leq \\ & \frac{1}{2 \times indeg(t)} \sum_{v: \exists (t,u) \in E \ \& \ (u,v) \in E} \sum_{u \in Succ(v)} \frac{|\Delta g(v)|}{outdeg(v)} \end{split}$$

And because indeq(t) > k, then:

$$|\Delta g(t)| \le \frac{\sum_{v:\exists (t,u) \in E \& (u,v) \in E} |\Delta g(v)|}{2 \times k}.$$
 (4)

Let us calculate $\Delta g(l)$ for all $l \in V'$. We can see that whenever we introduce a new node, s_i , that aims at node i in the network, then:

$$\begin{split} |\Delta g(i)| &= \Big| \frac{-1 + \sum_{u \in Pred(i)} f'(u) \times \omega(u, i)}{indeg(i) + 1} - \\ &\qquad \frac{\sum_{u \in Pred(i)} f(u) \times \omega(u, i)}{indeg(i)} \Big| \leq \\ \Big| \frac{\sum_{u \in Pred(i)} \Delta f(u) \times \omega(u, i)}{indeg(i)} - \frac{1}{indeg(i + 1)} - \\ &\qquad \sum_{u \in Pred(i)} \frac{f'(u) \times \omega(u, i)}{indeg(i)(indeg(i) + 1)} \Big| \leq \\ \Big| \frac{\sum_{u \in Pred(i)} \Delta f(u) \times \omega(u, i)}{indeg(i)} \Big| + \frac{2}{indeg(i) + 1} \end{split}$$

For the other nodes, $l \in V'$, that are not targeted by s_i :

$$|\Delta g(l)| = \left| \frac{\sum_{u \in Pred(l)} \Delta f(u) \times \omega(u, l)}{indeg(l)} \right|$$

For all $l \in V'$, we can write:

$$\begin{split} \Big| \sum_{u \in Pred(l)} \frac{\Delta f(u) \times \omega(u, l)}{indeg(l)} \Big| \leq \\ \frac{1}{2 \times indeg(l)} \sum_{v \in Pred(l), u \in Succ(v) \backslash \{l\}} \frac{|\omega_{vl}| \times |\Delta g(v)|}{outdeg(v)} = \\ \frac{1}{2 \times indeg(l)} \sum_{i \in V} \sum_{(v, l), (v, i) \in E} \frac{|\omega_{vi}|}{outdeg(v)} |\Delta g(i)| \end{split}$$

In the matrix form, we thus have:

$$Q \leq M \times Q + \begin{bmatrix} 0 & \frac{2}{indeg(i)+1} & 0 & 0 & 0 \end{bmatrix}^T,$$

where Q is a vector of length |V'| which on the l'th position has $\Delta g(l)$ for $l \in V'$. And M is a matrix of size $|V'| \times |V'|$, and its coefficients are filled according to Equation 5. Note

$$\frac{1}{2 \times indeg(l)} \sum_{v \in Pred(l), u \in Succ(v) \setminus \{l\}} \frac{|\omega_{vl}|}{outdeg(v)} \leq \frac{1}{2} \quad (5)$$

This implies that $||M||_{\infty} \leq \frac{1}{2}$. On the other hand, for a given column j in the matrix M:

$$\sum_{l \in V} \frac{1}{2 \times indeg(l)} \sum_{(v,l),(v,j) \in E} \frac{|\omega_{vj}|}{outdeg(v)} \le \frac{1}{2k} \sum_{l \in V} \sum_{(v,l),(v,j) \in E} \frac{|\omega_{vj}|}{outdeg(v)} \le \frac{1}{2k} \sum_{v \in Pred(j)} |\omega_{vj}| \le \frac{1}{2}$$

whenever $\sum_{v \in Pred(j)} |\omega_{vj}| \leq k$, which implies that $||M||_1 \leq \frac{1}{2}.$ The values of $\Delta g(l)$ achieve maximum when:

$$Q = M \times Q + \begin{bmatrix} 0 & \frac{2}{indeg(i) + 1} & 0 & 0 & 0 \end{bmatrix}^T$$

But in this case, we can solve the equation system with:

$$Q = \frac{1}{I-M} \times \begin{bmatrix} 0 & \frac{2}{indeg(i)+1} & 0 & 0 & 0 \end{bmatrix}^T$$

Matrix M is indeed invertible due to appropriately selected nodes $l \in V'$. What is more, since $||M||_{\infty} \leq \frac{1}{2}$, then we can write $\frac{1}{I-M} = I + M + M^2 + \ldots$ (Turnbull 1930). Finally, the above quality and $||M||_1 \leq \frac{1}{2}$ imply that $|\sum_{l \in V} \Delta g(l)| \leq \frac{1}{2}$

Now, because Equation 4 holds for the target node, t, and $|\sum_{l \in V} \Delta g(l)| \leq \frac{4}{indeg(i)+1}$, then:

$$|\Delta g(t)| \le \frac{2}{(indeg(i)+1) \times k}.$$

The above theorem shows that in a minimum-k-neighbour network, the indirect attack is at least k times weaker than the direct attack. That is, when we modify the goodness value of some node i by Δ , then the value of the target node t is modified by at most $\frac{\Delta}{k}$.

Building upon the above reasoning, we can show that the following result also holds (the proof in the appendix of the paper).

Theorem 8 (Direct Sybil attack). Assuming in a WSN G = (V, E, ω) where one adds a new Sybil node rating directly some target node t, then the goodness value of the target node t decreases by at most $|\Delta g(t)| \leq \frac{2}{indeg(t)}$

Simulations

We conduct a series of simulations on the Bitcoin OTC, Bitcoin Alpha, and RFA Net datasets studied by Kumar et al. (2016). They consist of weighted signed networks with $|V| = \ge 3,700, |E| \ge 24,000$ each, where the proportion of positively weighted edges is > 84%. A vast majority of the nodes in each network, i.e., more than 76%, have an indegree up to 10. Furthermore, most of the users in the networks are evaluated as fair by the FGA function— $f(v) \ge 0.7$ for 100% of the nodes (with the mean f(v) equal to 0.94). As for goodness, only less than 4% of users have a strongly positive score of more than 0.5, and in the Bitcoin OTC network 8% of users have negative score of less than -0.3, whereas in Bitcoin Alpha 3, 8\% have goodness below -0.3.

We focus on the attacks that lower the goodness of the nodes, as in the DNR problem. In particular, each experiment was conducted on the set of attacking nodes A of size $k = \{1, \dots, 7\}$ and the target set $T = \{t\}$ of size 1. The target, $t \in T$, was chosen randomly from those nodes that have relatively high goodness $(q(t) \ge 0.50)$ and a low indegree (0 < indeg(t) < 10). We study two types of the attackers:

- not-established attackers chosen from relatively newly created nodes with 0 < indeg < 10 and outdeg =0). This allows for studying Sybil-style attacks; and
- established attackers chosen from the nodes with outdeg(v) > 5 (and iteratively choosing nodes with fairness f(v) > 0.7). This allows for studying attacks by the nodes whose standing in the network has been built for some time.

We simulate three types of attacks:

- direct attacks a set of attackers A of size k rates directly the target node $t \in T$. The pseudocode is presented in Algorithm 1. Each attacker rates t using weight -1;
- **indirect attacks** the attackers set A of size k rates the neighbors of the neighbors of the target node, to minimize the goodness part of the FGA of the target node by manipulating fairness of the targets' neighbors. The pseudocode of the attack is presented in the Alorithm 2. More precisely, the algorithm implements a greedy approach, where each new edge is used to minimize the goodness of the target node t by minimizing (or maximizing) the fairness of one of the targets' neighbors by directly rating the successor of the target's neighbor with an edge of weight 1 or -1. The algorithm performs calculations iteratively on the attackers sorted by the value of their fairness value.
- mixed attack k_1 attacking nodes perform a direct attack and k_2 perform an indirect one, where $k_1 + k_2 = k$.

The results in Figure 3 are presented with a 95% confidence interval (marked with the opaque region around the solid/dashed lines). They show how a direct/indirect attack by established/not established nodes influences the goodness

Algorithm 3: Modified indirect attack

```
 \begin{aligned} & \textbf{Data: } A, T = \{t\}, G \\ & \text{sort nodes in } A \text{ by their fairness score} \\ & \textbf{while } i < len(sorted(A)) \textbf{ do} \\ & a \leftarrow sorted(A)[i] \\ & N_1 \leftarrow Pred(t) \\ & \text{find a neighbor } n_2 \in Succ(n_1) \setminus \{t\} \text{ of a neighbor } \\ & n_1 \in N_1 \text{ that minimizes the goodness value of } t, \text{ when adding an edge } (a, n_2) \text{ with weight } \omega(a, n_2) = 1 \text{ or } \\ & \omega(a, n_2) = -1 \\ & edges\_len \qquad \leftarrow & min(SCALE \quad * \\ & len(indegree(n_2), MAX, len(A) - i) \\ & \text{add } edges\_len \text{ edges to the graph } G \\ & i \leftarrow i + edges\_len \end{aligned}
```

| | B. OTC | B. Alpha | RFA Net |
|-----------------|--------|----------|---------|
| $\max indeg$ | 10 | 13 | 10 |
| $\min goodness$ | 0.8 | 0.5 | 0.5 |
| num of samples | 20 | 30 | 27 |
| num of edges | 20 | 20 | 20 |

Table 1: The parameters used to search weak target nodes in the test sets. "B." stands for "Bitcoin".

of the target node (Δ). For Bitcoin OTC and Bitcoin Alpha, and RFA Net in both cases (direct and indirect attacks), there is no significant difference between the established and not established results (solid and dashed lines).

In Figure 4 (see the full version in Figure 13), we present results for a mixed setting. The individual cells of the heatmaps show: (a) Δ_1 —the absolute change of the goodness of the target node introduced by the k_1 direct edges; (b) Δ_2 —the absolute change of the goodness of the target node introduced by the k_2 indirect edges; and (c) Δ_S —the total change, i.e., $\Delta_S = \Delta_1 + \Delta_2$. The results show that the average strength of a direct attack varies between 0.2 and 1.2 for different k, and the average strength of the indirect attack is lower than 0.05, i.e., significantly smaller than the average strength of a direct attack. Theorem 6 proved in the appendix of the paper provides some intuition why the direct attacks have such a strong impact, whereas Theorem 11 gives another intuition why the indirect attack is weaker than the direct attack.

Better heuristic for indirect attacks

We conduct additional tests to analyze the strength of the indirect attacks. In Algorithm 3, instead of adding only a single edge in each iteration, as in Algorithm 2, we add a series of new edges. In more detail, we add SCALE=5 times more Sybil edges than the indegree of the target node (but at most some predefined maximum MAX=10). We take this approach to scale up the effect of manipulating the goodness value of the target nodes.

We attack only nodes with bounded *indegree*, and the goodness value bigger than some threshold. We believe these nodes are more easily manipulable than an average

node. See Table 1 for the details.

| | B. OTC | B. Alpha | RFA Net |
|--------------------|--------|----------|---------|
| average change | 0.081 | 0.085 | 0.030 |
| standard deviation | 0.089 | 0.085 | 0.028 |
| min change | 0.010 | 0.008 | 0.009 |
| max change | 0.300 | 0.298 | 0.131 |
| median | 0.053 | 0.042 | 0.021 |
| 0.75-quantile | 0.111 | 0.121 | 0.041 |

Table 2: The results of Algorithm 3 on different datasets.

The analysis of the data in Table 2 shows that the attack using the Algorithm 3 may (but rarely does) achieve relatively strong results in some cases. To be more precise, the maximum change of the goodness value of the target node introduced by the indirect attack in the Bitcoin OTC and Bitcoin Alpha detasets reached the barrier of 0.3. This shows that in general networks (unlike the minimum-k-neighbour ones described in Theorem 7) do not have strong resistance property against indirect attacks. In most cases however the attack gives rather weak results (75%-quantile on all datasets is at most 0.12 with low median of at most 0.05). The minimum strength of the attack in all datasets achieves 0.01.

Conclusions

In this paper, we axiomatized the FGA measure with respect to, among others, the properties of homogeneously and unanimously rated nodes and with respect to the properties of the rating nodes that achieve constant rating error. Furthermore, we presented the hardness results on the manipulability problems. We also derived analytical results concerning the strength of the direct attacks and weakness of the indirect attacks in the networks in which each node has minimum k neighbours (in and out). Finally, we visualised experimentally the strength of direct attacks and analysed two different greedy algorithms for indirect attacks. This showed that FGA might be manipulated indirectly in nonminimum-k-neighbour networks. Overall, a higher-level insight from our analysis is that FGA is generally more difficult to manipulate compared to other social network analysis tools (e.g., centrality measures). In particular, while worstcase hardness results are common in the literature, various other tools turned out to be easily manipulable in practice by well-crafted heuristics (Bergamini et al. 2018; Waniek et al. 2018b, 2019, e.g.). The FGA measure turns out to be more resilient, which provides a good argument for using it in practice.

In future it would be plausible to compare other candidate measures existing in the literature in the terms of manipulability and try to derive a more general approach for analyzing the manipulability of weighted ranking functions. We also encourage studies on the axiomatization of the ranking functions, which would result in a better understanding of their properties.

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An appendix to "Predicting Weights in Signed Weighted Networks is Difficult to Manipulate":

Axiomatization

Goodness axiomatization

Below we formally define the axioms and present the uniqueness proofs.

We begin with the characterization of goodness part of the FGA function. Let $v \in V$ have all the predecessors $u_i \in Pred(v)$ homogenous and unanimous w.r.t. v, i.e. they all have the same fairness f_0 and they rate v with the same rating ω_0 . Now, let us assume that f_0 of all the predecessors gets increased by the same amount, Δ . We require that the goodness of the rated node v should rise proportionally to Δ (see Figure 5). To formalize this axiom, let us denote the goodness of v in such a setting by $g^{\phi_\omega,\phi_f}(v)$, where ϕ_ω indicates the value of weight of the edges (u_i,v) , and ϕ_f indicates the value of the fairness of all $u_i \in Pred(v)$.

Axiom 1 (SMOOTH GOODNESS). Let $v \in V$, such that $\forall u_i \in Pred(v) \ f(u_i) = f_0 \land \ \omega(u_i, v) = \omega_0$. Then $\forall \Delta \in \mathbb{R}$:

$$g^{\omega_0, f_0 + \Delta}(v) = g^{\omega_0, f_0}(v) + g^{\omega_0, \Delta}(v).$$

Next, let us consider an analogous situation, but now the weight ω_0 of the edges from the predecessors Pred(v) to v increases by Δ while their fairness f_0 remains the same (see Figure 6). This leads to the following axiom:

Axiom 2 (INCREASE WEIGHT). Let $v \in V$, such that $\forall u_i \in Pred(v) f(u_i) = f_0 \wedge \omega(u_i, v) = \omega_0$. Then, $\forall \Delta \in \mathbb{R}$: $a^{\omega_0 + \Delta, f_0}(v) = a^{\omega_0, f_0}(v) + a^{\Delta, f_0}(v)$.

Next, we require that nodes with higher fairness have a higher impact on the goodness of the rated nodes. Similarly, higher weights should result in a better rating of the target node (see Figure 7).

Axiom 3 (MONOTONICITY FOR GOODNESS). Let u_1 and u_2 be two nodes rated by unanimous and homogeneous sets of predecessors S_1 , S_2 . S_i consisting of nodes with identical fairness f_i who rate u_i with identical ω_i . Then, if $f_1 = f_2$ and $\omega_1 > \omega_2$, then $g(u_1) \geq g(u_2)$. Also, if $\omega_1 = \omega_2$ and $f_1 > f_2$, then $g(u_1) \geq g(u_2)$ as well.

MONOTONICITY FOR GOODNESS is a weaker version of the Goodness Axiom proposed by Kumar et al. (2016). While the Goodness Axiom concerns any predecessors, MONOTONICITY FOR GOODNESS focuses on unanimous and homogeneous sets of them.

Next, any node, $v \in V$, that has the best possible rating given by each of its predecessors and all its predecessors have the highest possible fairness, then v should have maximal possible goodness (see Figure 7).

Axiom 4 (MAXIMAL TRUST). For any $v \in V$ such that $\forall u_i \in Pred(v) \ f(u_i) = 1 \ \land \ \omega(u_i, v) = 1$, it holds that, $\forall \Delta \in \mathbb{R}, \ g(v) = 1$.

The following axiom states that, when $v \in V$ is rated by k groups, where the nodes in each group are homogeneous and unanimous w.r.t. v, then the goodness of v should be equal to the *weighted average* of the ratings achieved when these groups separately rate v.

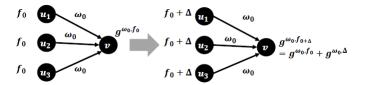


Figure 5: Axiom 1 says that the increase in the homogenous *fairness* of the unanimous predecessors results in the proportional increase of the *goodness* of the rated node.

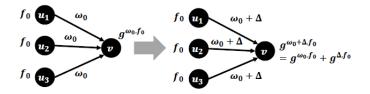


Figure 6: Axiom 2 says that the increase in the homogenous *weight* of the unanimous predecessors results in the proportional increase of the *goodness* of the rated node.

Axiom 5 (GROUPS FOR GOODNESS). Given $v \in V$, let $\{S_1, \ldots, S_k\}$ be a partition of Pred(v) such that $\forall i \in [k]$ there exists $f_i, \omega_i : \forall_{u_j \in S_i} f(u_j) = f_i \wedge \omega(u_j, v) = \omega_i$. Then, it holds that:

$$g(v) = \frac{\sum_{i \in [k]} (|S_i| \times g_i(v))}{\sum_{i \in [k]} |S_i|},$$

where $g_i(v)$ denotes the rating of the node v rated only by the homogeneous and unanimous predecessors from group i.

Finally, we have the following baseline:

Axiom 6 (BASELINE FOR GOODNESS). Any v with indeg(v) = 0 has g(v) = 1.

We will now show that the above axioms uniquely define the goodness part of the FGA function.

Theorem 9. For any fixed fairness function f(u), the SMOOTH GOODNESS, INCREASE WEIGHT, MONOTONICITY FOR GOODNESS, MAXIMAL TRUST, GROUPS FOR GOODNESS, and BASELINE FOR GOODNESS axioms uniquely define goodness function (1).

Proof. It is easy that the goodness function (1) meets the conditions of the above axioms. Now, let us define some $g_i^{w_0,f_0}(v)$ for a node v rated by homogeneous and unanimous nodes w.r.t. v, i.e. all $u \in Pred(v)$ have the same fairness $f(u) = f_0$ and they rate v with the same rating $\omega(u,v) = \omega_0$, From SMOOTH GOODNESS and MONOTONICITY FOR GOODNESS and the Cauchy's equation (Small 2007), we know that $g_i(v)$ is linearly dependant on f(u) when $\omega(u,v)$ is fixed to some ω_0 , i.e. $g_i^{\omega_0,f(u)}(v) = a_{\omega_0} \times f(u)$ for some constant $a_{\omega_0} \in \mathbb{R}$. Again, from INCREASE WEIGHT, MONOTONICITY FOR GOODNESS, and the Cauchy's equation we know that $g_i(v)$ is linearly dependant on $\omega(u,v)$ when f(u) is fixed to some f_0 , i.e. $g_i^{\omega(u,v),f_0}(v) = \omega(u,v) \times b_{f_0}$. The two equations above

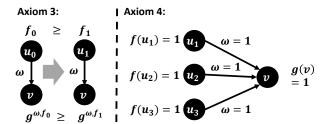


Figure 7: Axiom 3 says that fairer nodes should have a bigger impact on the goodness of the rated nodes. In the figure, we assume that $\omega > 0$ and node u_0 has bigger fairness than node u_1 . Axiom 4 says that a fully trusted node should have the goodness value equal to 1.

imply that for a set of homogeneous and unanimous predecessors, $g_i^{\omega(u,v),f(u)}(v)=g_i^{\omega,f}(v)=a_\omega\times f=b_f\times\omega.$ Since the function $g_i^{\omega(u,v),f(u)}(v)$ is defined for all $\omega,f\in\mathbb{R}$, this equality implies that $a_\omega=\frac{b_f}{f}\times\omega.$ Furthermore, since a_ω is not dependant on f by definition, then $a_\omega=c\times\omega$ for some $c\in\mathbb{R}$. We conclude that $g_i^{\omega,f}(v)=c\times f\times\omega.$ From MAXIMAL TRUST, we get that $g^i(v)=f(u)\times\omega(u,v).$ Now, when a node does not have unified predecessors, we can divide its predecessors to groups with fixed $(f_i,\omega_i).$ From GROUPS FOR GOODNESS, we get:

$$g(v) = \frac{\sum_{i \in [k]} (|S_i| \times g_i(v))}{\sum_{i \in [k]} |S_i|} = \frac{\sum_{i \in [k]} (|S_i| \times f_i \times \omega_i)}{\sum_{i \in [k]} |S_i|} =$$
$$= \frac{1}{in(v)} \sum_{u \in Pred(v)} f(u) \times \omega(u, v).$$

From BASELINE FOR GOODNESS, g(v) = 1 for v with indeg(v) = 0.

Fairness axiomatization

In this section, we present the axiomatization of the fairness part of the FGA function. The fairness axiomatization is defined with respect to the rating error of nodes. We define the error of node v rating the node u as $d = |\omega(v, u) - g(u)|$.

Our first axiom stipulates that the fairness of a node that makes an average error when rating other nodes is equal to the average of the fairness values of nodes in extreme cases.

Axiom 7 (SMOOTH FAIRNESS). Assume a node v rates a set of its successors S with equal error $d=|g(u)-\omega(v,u)|$ for $u\in S$ in one setting, and with an error D in another setting, then $f^{\frac{d+D}{2}}(v)=\frac{f^d(v)+f^D(v)}{2}$.

The following axiom states that fairness of the nodes that rate more accurately should rise (see Figure 8).

Axiom 8 (MONOTONICITY FOR FAIRNESS). Let u_1 and u_2 be two nodes rating their sets of successors S_1 , S_2 . S_i consists of nodes v_i rated by u_i with identical error $d_i = |g(u_i) - \omega(u_i, v_i)|$. If $d_1 > d_2$, then $f(u_1) \leq f(u_2)$.

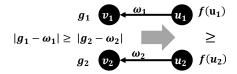


Figure 8: Axiom 8 says that the fairness of a node should rise when the node gives more precise ratings.

This is a weaker version of the Fairness Axiom in (Kumar et al. 2016). In our case, it is defined only for a set of successors S_i rated with *equal* rate by the node u_i .

Next, we stipulate that when a node makes maximal errors when rating all of its neighbors, then its fairness should be 0, and when it always agrees with the actual goodness value of its rated nodes, then its fairness is 1 (see Figure 9).

Axiom 9 (OBVIOUS FAIRNESS METRIC). Assume node v rates all its successor nodes S with distance $d=|g(u)-\omega(v,u)|=0$, for $u\in S$, then f(v)=1. Assume a node v rates all its successor nodes S with distance $d=|g(u)-\omega(v,u)|=2$, for $u\in S$, then f(v)=0.

Also, when $v \in V$ rates its neighbors that can be divided to k such groups that each node in a group is rated by v with the same distance as other nodes in this group, then the fairness of v should be equal to the *weighted average* of its fairness in a setting where v rates these groups separately.

Axiom 10 (GROUPS FOR FAIRNESS). Given $v \in V$, let $\{S_1, \ldots, S_k\}$ be a partition of Succ(v) such that $\forall i \in [k]$ there exists $d_i : \forall_{u_i \in S_i} | g(u_j) - \omega(v, u_j) | = d_i$. Then:

$$f(v) = \frac{\sum_{i \in [k]} (|S_i| \times f_i(v))}{\sum_{i \in [k]} |S_i|},$$

where $f^{i}(v)$ is the fairness of v rating group i.

Finally, a baseline for node v with outdeq(v) = 0 is:

Axiom 11 (BASELINE FOR FAIRNESS). A node v with outdeg(v) = 0 has f(v) = 1.

Theorem 10. For fixed goodness function g(n), the SMOOTH FAIRNESS, MONOTONICITY FOR FAIRNESS, OBVIOUS FAIRNESS METRIC, GROUPS FOR FAIRNESS, and BASELINE FOR FAIRNESS axioms uniquely define fairness function (2).

Proof. It is easy that the fairness function (2) meets the conditions of the above axioms. Now, let us define $f_i(v)$ for a node v and a group of nodes with some fixed error $d_i = |g(u) - \omega(v, u)|$, From SMOOTH FAIRNESS, MONOTONICITY FOR FAIRNESS and the Jensen's equation (Small 2007), we know that $f_i(v)$ is linearly dependant on $d_i = |g(u) - \omega(v, u)|$, i.e. $f_i(v) = b + a \times d_i$ for some $a, b \in \mathbb{R}$. From OBVIOUS FAIRNESS METRIC we get that $f_i(v) = 1 - d_i/2 = 1 - |g(u) - \omega(v, u)|/2$. Now when a node does not have unified successors, we can divide its successors to groups with fixed $|g(u_j) - \omega(v, u_j)| = d_i$. From GROUPS FOR FAIRNESS:

$$f(v) = \frac{\sum_{i \in [k]} (|S_i| \times f_i(v))}{\sum_{i \in [k]} |S_i|} = \frac{\sum_{i \in [k]} (|S_i| \times (1 - d_i/2))}{\sum_{i \in [k]} |S_i|} =$$

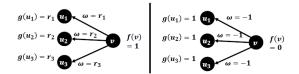


Figure 9: Axiom 9 says that a node that rates with perfect accuracy (its rating error d=0 for all the nodes that it rates) should have maximal fairness equal to 1, and a node that makes the biggest errors (its rating error d=2 for all the nodes that it rates) should have minimal fairness equal to 0.

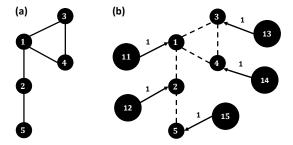


Figure 10: (a) The original VC problem (k=2) with vertices $V=\{1,2,3,4,5\}$ and edges as in the picture. This set can be covered with 2 nodes - 1 and 5. (b) The corresponding DMT instance with number of moves k=2, threshold t=-1, the set of attacked edges H, as in the original problem, and a new set of attacking nodes $\{11,12,13,14,15\}$ marked with big circles for which attacking edges were created $\{(11,1),(12,2),(13,3),(14,4),(15,5)\}$ - each with the weight of 1. To solve this problem we need to modify the values of the edges $\{(11,1),(15,5)\}$ to -1.

$$=1-\frac{1}{out(v)}\sum_{u\in Succ(v)}|g(u)-\omega(v,u)|/2.$$

Finally, from BASELINE FOR FAIRNESS we get that f(v)=1 for nodes with outdeg(v)=0.

FGA axiomatization

The above results imply the final axiomatization result:

Theorem 1. The SMOOTH GOODNESS, INCREASE WEIGHT, MONOTONICITY FOR GOODNESS, MAXIMAL TRUST, GROUPS FOR GOODNESS, BASELINE FOR GOODNESS axioms and the SMOOTH FAIRNESS, MONOTONICITY FOR FAIRNESS, OBVIOUS FAIRNESS METRIC, GROUPS FOR FAIRNESS, and BASELINE FOR FAIRNESS axioms uniquely define the FGA function.

Proof. The proof follows from the proofs of Theorems 9 and 10. $\hfill\Box$

Ommitted complexity proofs

Theorem 2. Solving the $DMT(IMT) = (G = (V, E, \omega), A, TP, I, t, k)$ problem is NP-hard.

Proof of Theorem 2. We reduce from the VERTEX COVER (VC) problem. In the VC problem we are given a parameter

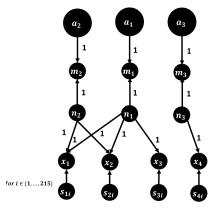


Figure 11: The corresponding DNR instance with the set of targets $\{1,2,3,4\}$, the set of attackers $\{a_1,a_2,a_3\}$. The intermediary nodes created for every set from \mathcal{S} , i.e. $\{n_1,n_2,n_3,m_1,m_2,m_3\}$, 215 stabilising nodes s_{ji} for each node x_j , k=2, threshold $t=1-\epsilon$.

k and a graph G=(V,E) and we need to decide whether there exists a set of vertices, $U\subseteq V$, $|U|\le k$, that "cover" the set of the edges of this graph, i.e., every edge from E is adjacent to at least one node in U.

Given the VC problem (G,k), where G=(V,E), we create an instance of our problem $DMT=(G'=(V',E',\omega),A,TP,I,t,k')$, by adding, for every node $v\in V$, an attacking node a_v with an edge (a_v,v) with weight=1. We set the target threshold as follows: t=-1. We observe that $A=\{a_v:v\in V\},V'=V\cup A,E'=\{(a_v,v):v\in V\}$. Finally, the set of intermediary vertices is I=V, and the set of attacked edges TP is E, i.e., the set of the edges from G. We set k'=k. See Figure 10 for an example.

We now need to show that the reduction is correct. Firstly, given a graph G=(V,E) and its vertex cover of size k, i.e., $U\subseteq V$ with $U=\{x_1,...,x_k\}$, we show that its corresponding problem, DMT, as outlined above can be solved. To this end, we modify all of the k edges (a_{x_i},x_i) , where $x_i\in U$, by setting each of them to -1. Now:

- due to the fact that computing (f(u), g(u)) = FGA(G, u) of a node u adjacent only to a single directed edge results in g(u) being equal to the weight of this single edge, then for all $x_i \in U$ we have $g(x_i) = -1$;
- from the definition of the FGA function, the fairness of nodes with out-degree of 0 is equal to 1. Hence, for all $u \in V$ we have that f(u) = 1.

From these we conclude that all of the connections in the target set TP are decreased to the threshold t=-1, i.e. either $f(u)\times g(v)=1\times (-1)=-1$ or $f(v)\times g(u)=-1$ for every pair $u,v\in TP$.

For the other direction, assume that we have a solution to our problem DMT that was created as outlined above. Recall that the set of attacking nodes is the set of newly created nodes for the DMT instance, i.e., $A = \{a_v : v \in V\}$, and each of them is connected with a single edge directed towards its corresponding node from V, i.e., (a_v, v) for $v \in V$ and weight of these edges equals 1. We observe that, from

the definition of FGA and the construction of the DMT instance, it follows that to modify the values of the predicted connections between pairs $\{u,v\} \in TP$ (i.e. either $f(u) \times g(v)$ or $f(v) \times g(u)$) one needs to modify the goodness of the nodes in dom(TP). This is because there are no outgoing edges from the nodes in TP; thus, fairness of the nodes in dom(TP) is constant and equal to 1.

The goodness of the nodes in dom(TP) can be modified by changing (a_v,v) , where $a_v \in A$, or by adding some new edges between the attackers and the nodes from dom(TP). However, to attack a single connection $\{u,v\} \in TP$, we have to obtain either g(u) = -1, or g(v) = -1. To this end, since reaching g(v) = -1 is only possible when all of the edges pointed at v have value -1, it is always necessary to modify the value of the existing edge (a_v,v) as well. Specifically, whenever we can reach one of the nodes in dom(TP) with a modified edge, we are, in fact, "marking" all of the edges pointing at this node. Each of these edges corresponds to a pair in TP. If all the pairs are marked, then both the DMT and VC problems are solved.

The proof for the *IMT*-hardness is analogous with the opposite signs of the weights of the created/modified edges.

Theorem 3. Solving the $DNR(INR) = (G = (V, E, \omega), A, T, I, t, k)$ problem is NP-hard.

Proof of Theorem 3. We reduce from the SET COVER (SC) problem. In the SC problem, we are given a set of sets \mathcal{S} , a target set T, and a parameter k. We need to decide whether it is possible to cover the target set T with at most k sets from \mathcal{S} , i.e. whether there exists a subset $S \subseteq \mathcal{S}$ of size at most k, such that for all $t \in T$, there exists $S_i \in S$, such that $t \in S_i$. Given an SC problem (\mathcal{S}, T, k) , we create an instance of the DNR problem as follows:

- the set of target nodes in the DNR problem is the set T from the SC problem;
- For every set S_i from S we create two intermediary nodes n_i and m_i one link (n_i, m_i) and |S_i| links (n_i, t) for t ∈ S_i, each of them with weight 1. We denote the set of all intermediary nodes n_i which point at the nodes m_i as N_{int};
- for each $S_i \in \mathcal{S}$ we create an attacking node a_i and a link (a_i, m_i) with weight 1; and
- we set the intermediary set I in the DNR problem to the set of m_i nodes,
- given $d_{max} = max \{outdeg(n_i) : n_i \in \mathcal{N}_{int}\}$, we add $l = 8d_{max}^3 d_{max} + 1$ stabilising nodes s_i to each target node $x_j \in T$, they are required in the reduction to ensure that the change in goodness of the target nodes will not affect the goodness of some other nodes too much,
- we set the target threshold in the DNR problem to be $1-\epsilon$, where $\epsilon=\frac{1}{4*d_{max}(d_{max}+l)},$
- we set the budget to k.

In Figure 11, we present a sample DNR construction for the SC problem in which $T = \{1, 2, 3, 4\}$ has to be covered with at most k = 2 sets from $S = \{\{1, 2\}, \{1, 2, 3\}, \{4\}\}$.

We now need to show that the reduction is correct. Firstly, let us consider an SC problem (\mathcal{S},T,k) and its set cover of size k, consisting of sets $S_i \in \mathcal{S}$ with indexes $i \in \{x_1,\ldots,x_k\} = U$. We will show that our corresponding DNR problem created as in the instructions above can be solved. To this end, we modify the value of each link (a_{x_i},m_{x_i}) for $i \in U$ to -1.

In this case, the goodness of the intermediary node decreases to a value bounded by the factor introduced by $\omega((a_{x_i},m_{x_i})) \times f(a_{x_i}) = -1 \times f(a_{x_i}) \leq 0$ and the factor introduced by $\omega((n_{x_i},m_{x_i})) \times f(n_{x_i}) \leq 1$, resulting in a value $g(m_{x_i}) \leq \frac{1}{2}$. This implies the decrease in the fairness value of the intermediary node n_{x_i} , resulting in $f(n_{x_i}) \leq 1 - \frac{1-\frac{1}{2}}{2*outdeg(n_{x_i})} \leq 1 - \frac{\frac{1}{2}}{2*d_{max}}$. Finally this decreases the goodness values of all nodes v_j rated by n_{x_i} to a value less or equal to $g(v_j) \leq \frac{indeg(v_j)-1}{indeg(v_j)} + \frac{1}{indeg(v_j)}(1-\frac{\frac{1}{2}}{2*d_{max}}) \leq 1 - \frac{1}{4*d_{max}*indeg(v_j)} \leq 1 - \frac{1}{4*d_{max}*(l+d_{max})} \leq 1 - \epsilon$. Since T is covered by the sets indexed by indices in U, then modifying the value of the links (a_{x_i}, m_{x_i}) decreases the rating of all of the target nodes in the DNR problem below or to the threshold.

For the other direction, assume that we have a solution to our corresponding problem DNR. In fact, since the only allowed actions are edge additions and weight updates to the nodes from $I=\{m_i\}_i$, the only way of modifying the goodness of the target nodes is by modifying the fairness of the intermediary nodes n_i . Either modifying an edge (a_i,m_i) or adding an edge (a_j,m_i) marks a set $S_i \in \mathcal{S}$ and sets the value of the goodness of the nodes $k \in S_i$ below the threshold $1-\epsilon$. One needs to see that it is necessary to rank the node m_i to mark the node $v_j \in S_i$, otherwise its goodness value will stay above threshold (i.e. $goodness(v_j) > 1-\epsilon$. We achieve this result by introducing l stabilising nodes for every $v_j \in T$. From the properties of the given construction one may conclude that marking nodes in the DNR problems implies marking sets in the set cover problem.

We will use an intermediary Theorem 11. It shows that for a node x when fairness of its k rating nodes is decreased by Δ , and there are l stabilising nodes rating it with 1, then the goodness value of the node x does not change too much - i.e. $g(x) \geq 1 - 2\frac{k}{l+k} \times \Delta$.

Using this result we may see that even in an edge case the nodes in the target set do not have their *goodness* value changed below the threshold if they are not marked properly as mentioned before. In the edge case a node v_i may be indirectly influenced by a set of k_1 nodes (denoted \mathcal{K}), which have their fairness value indirectly changed because they rate at most k_2 nodes (denoted \mathcal{L}) which are marked by at most k_3 intermediary nodes which change their fairness value by at most $\Delta = 1$. Note that $k_1, k_2, k_3 \leq d_{max}$.

In this case the goodness value of the nodes in $\mathcal L$ can be bounded by the above theorem $g(v_j) \geq 1 - 2\frac{d_{max}}{d_{max}+l}$. This implies that the fairness of the nodes in $\mathcal K$ falls to a value not less that $f(n_m) \geq 1 - \frac{d_{max}}{d_{max}+l}$. This fairness modification will further influence the target nodes, but since in any scenario also for the target nodes we have $g(v_i) \geq 1 - 2\frac{d_{max}}{d_{max}+l}$, then the fairness value of the inter-

mediary nodes will not fall below $1-\frac{d_{max}}{d_{max}+l}$. Finally we can conclude that the nodes in the target set are influenced by at most $g(v_i) \geq 1-2\frac{d_{max}}{d_{max}+l}\frac{d_{max}}{d_{max}+l}$. We can see that when l is big enough, this value never reaches the threshold ϵ , i.e. $1-2\frac{d_{max}}{d_{max}+l}\frac{d_{max}}{d_{max}+l} > 1-\frac{1}{4*d_{max}(d_{max}+l)}$ when $l>8d_{max}^3-d_{max}$.

The proof for the INR-hardness is analogous with the opposite signs of the weights of the created/modified edges.

Theorem 11. We have a node x that is rated by k influencing nodes n_i (for $i \in [k]$). What is more this node is rated by l other stabilising nodes s_j (for $j \in [l]$). All rates are of value 1. Suppose the fairness value of the influencing nodes decreases by at most Δ (after all modifications in the network), then the goodness value of the node x decreases by at most $2\frac{k}{l+k}\Delta$.

Proof. By MONOTONICITY FOR GOODNESS we know that the goodness value of the node x will decrease maximally when we decrease the fairness value of all influencing nodes n_i by exactly Δ . We can estimate how the fairness value of the stabilising nodes $(f^{(t)}(s_i))$ and the goodness value of the rated node $(g^{(t)}(x))$ will change in the next iterations of the FGA function computation.

By the FGA definition, the stabilising nodes s_i which rate only one node x have $f^{(0)}(s_i)=1$ and $f^{(t)}(s_i)=1-\frac{|1-g^{(t-1)}|}{2}$ for $t\geq 1$. What is more since all ratings are of value 1, we know that $g(x)\geq 0$, then $f^{(t)}(s_i)=1-\frac{1-g^{(t-1)}}{2}$ for $t\geq 1$. The goodness value of the node x rated by k nodes n_i with decreased fairness and l stabilising nodes s_i , can be bounded as follows $g^{(0)}(x)=1$ and $g^{(2t)}(x)\geq \frac{k}{k+l}(1-\Delta)+\frac{l}{l+k}\times(1-\frac{1-g^{(2t-2)}}{2})$ for $t\geq 1$. We prove by induction that for $2t\geq 2$ we have $g^{(2t)}(x)\geq 1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{l}{2(l+k)}]^{2i}$. The statement trivially holds for 2i=0. Let's assume it holds for 2t, then for 2t+2 we have $g^{(2t+2)}(x)\geq \frac{k}{k+l}(1-\Delta)+\frac{l}{l+k}\times(1-\frac{1-g^{(2t)}}{2})\geq \frac{k}{k+l}(1-\Delta)+\frac{l}{l+k}\times(1-\frac{1-[1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{l}{2(l+k)}]^{2i}})=1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t}[\frac{l}{2(l+k)}]^{2i}$ what proves the induction. We may also further bound this sum $g^{(2t)}(x)\geq 1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{l}{2(l+k)}]^{2i}\geq 1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{1}{2}]^{2i}\geq 1-\frac{k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{1}{2}]^{2i}\geq 1-\frac{2k\Delta}{k+l}\sum_{2i=0}^{2t-2}[\frac{1}{2}]^{2i}\geq 1-2\Delta\frac{k}{k+l}$ It is also easy to see that $g^{(2t)}(x)=g^{(2t-1)}(x)$, thus we can conclude that $g(x)\geq 1-2\Delta\frac{k}{k+l}$.

Theorem 4. DNR(INR) parameterized by k is W[2]-hard.

Proof of Theorem 4. The Set Cover problem parameterized by the number of sets k is a W[2]-hard problem in the Whierarchy. Since the reduction in the proof of Theorem 3 runs polynomial time, the budget of the DNR(INR) problem is k, this reduction is also a parameterized reduction (Cygan et al. 2015).

Theorem 5. DMT(IMT) parameterized by k is W[2]-hard.

Proof of Theorem 5. One needs to see that a slight modification of the reduction in the proof of Theorem 3 allows to create a parameterized reduction from the Set Cover problem parameterized by the number of sets k to DMT(IMT) parameterized by the budget k. In fact, for a Set Cover problem we can create an instance $DMT(IMT) = (G = (V, E, \omega),$ A, TP, I, t, k) as in the DNR(INR) reduction, but for each node x from the target set T in the corresponding DNRproblem we add a vertex x', and we set $TP = \{\{x, x'\}:$ $x \in T$. In this case since all of the new nodes are disconnected from the graph, the only way to break the connections between the $\{x, x'\}$ links below the given threshold t is to lower the goodness value of the nodes $x \in T$ below the given threshold. Again, the reduction runs polynomial time, the budget of the DMT(IMT) problem is k, this reduction is also a parameterized reduction (Cygan et al. 2015).

Manipulating a node directly

Theorem 6. Let us consider an instance of the DNR problem, where, for an arbitrary $G = (V, E, \omega)$, a single node u_T is attacked with $0 < g(u_T) \le 1$ (thus $T = \{u_T\}$), and the set of attacking nodes $A \subseteq V$ is relatively trusted, $f(v) \ge \frac{1}{2}$ for $v \in A$ and $|A| > \lceil 2 \times g(u_T) \times indeg(u_T) \rceil$. Then, it is trivial to change the sign of the goodness value of u_T (i.e. to achieve the threshold t = 0) if the attackers can attack directly (i.e. $T \subseteq I$ in the DNR problem).

Proof of Theorem 6. We provide a successful strategy for the attackers. A subset S of size $\lceil 2 \times g(u_T) \times indeg(u_T) \rceil$ of the nodes in A creates a new edge between each of them and the attacked node with $\omega(v,u_T)=-1$. Since the attackers want to achieve $g(u_T)<0$, and $f(v)>\frac{1}{2}$ for every $v\in A$ before the attack, then after a successful attack $f'(v)>\frac{1}{2}$ for every $v\in A$ as well. Before the attack we have $g(u_T)=\frac{\sum_{v\in in(u_T)}f(v)\times\omega(v,u_T)}{indeg(u_T)}$. We need to show that $k=2\times indeg(u_T)+1$ edges are enough to change the $g(u_T)$ to a value $g'(u_T)$ lower than 0. First we observe that since $g'(u_T)<0$ after the attack, the $f'(v)\geq f(u)$ for $v\in in(u_T)$ if $\omega(v,u_T)\geq 0$, otherwise $f'(v)\leq f(u)$ for $v\in in(u_T)$ if $\omega(v,u_T)<0$. This implies that $\sum_{v\in in(u_T)}(f(v)\times\omega(v,u_T))\geq \sum_{v\in in(u_T)}(f'(v)\times\omega(v,u_T))$.

In conclusion, after adding k edges we obtain:

$$\frac{1}{indeg(u_T) + k} \left[\sum_{v \in in(u_T)} (f(v) \times \omega(v, u_T)) - \frac{k}{2} \right] < 0$$

if and only if

$$k > 2 \sum_{v \in in(u_T)} (f(v) \times \omega(v, u_T) =$$

$$2\sum_{v \in in(u_T)} (f(v) \times \omega(v, u_T) \times \frac{indeg(u_T)}{indeg(u_T)} \le$$

$$2 \times q(u_T) \times indeq(u_T)$$

Indirect Sybil Attack

We below provide an additional proof omitted in Section Indirect Sybil Attack.

Theorem 8 (Direct Sybil attack). Assuming in a WSN $G = (V, E, \omega)$ where one adds a new Sybil node rating directly some target node t, then the goodness value of the target node t decreases by at most $|\Delta g(t)| \leq \frac{2}{indeg(t)}$

Proof of Theorem 8. Note that in the Equation 5 in the proof of Theorem 7, one could assess the value of $\Delta g(i=t)$ by selecting $indeg_{max}(i)$ such that $\Delta g(i)$ is maximized. In this case $\Delta g(i) \leq \left|\frac{\sum_{u \in Pred(i)} \Delta f(u) \times \omega(u,i)}{indeg_{max}(i)}\right| + \frac{1}{indeg_{max}(i)}$. The metric $||M||_{\infty}$ is still bounded by $\frac{1}{2}$ as in the Equation 5 which implies the result.

Datasets' basic statistics

Figure 12 presents the histogram of the nodes sorted per indegree. Table 3 shows how many random samples were used to simulate direct, indirect attacks for all sizes of the attacking sets considered. For mixed attacks, for each $k1, k2 \in \{1, \ldots, 6\}, \geq 26$ samples were used for Bitcoin OTC, ≥ 12 samples were used for Bitcoin Alpha, and ≥ 17 samples were used for RFA Net.

| k | Bitcoin OTC | Bitcoin Alpha | RFA Net |
|---|-------------|---------------|---------|
| | | | |
| 1 | 24 | 24 | 25 |
| 2 | 21 | 24 | 25 |
| 3 | 26 | 24 | 25 |
| 4 | 25 | 24 | 25 |
| 5 | 24 | 24 | 25 |
| 6 | 23 | 24 | 25 |
| 7 | 22 | 24 | 25 |

Table 3: Number of samples used to simulate direct/indirect established/not established attacks.

| Statistic | Bitcoin OTC | Bitcoin Alpha | RFA Net |
|----------------------------|-------------|---------------|---------|
| Size | 5881 | 3783 | 9654 |
| Edges | 35592 | 24186 | 104554 |
| Positive edges | 89,90% | 93,64% | 84% |
| Small in-degree $< 10\%$ | 87,40% | 85,70% | 76% |
| Fair nodes ≥ 0.95 | 99,45% | 99,65% | 99,99% |
| Fair nodes ≥ 0.7 | 100% | 100% | 100% |
| Goodness score ≥ 0 | 85.85% | 92.36% | 93% |
| Goodness score ≥ 0.5 | 2.2% | 3.3% | 67% |
| Goodness score ≤ -0.3 | 8.2% | 3.8% | 0.2% |

Table 4: Statistics of the networks used for simulations.

Code

The code package that allows running simulations presented in this paper is available under this link https://github.com/irtomek/WeightPredictionsCode.

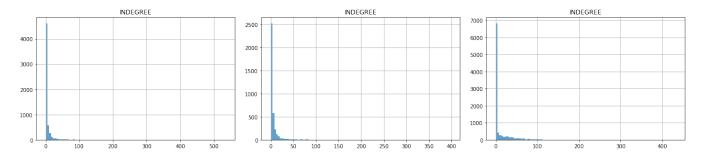


Figure 12: Histograms of indegree of the nodes of the Bitcoin OTC, Bitcoin Alpha, RFA Net networks.

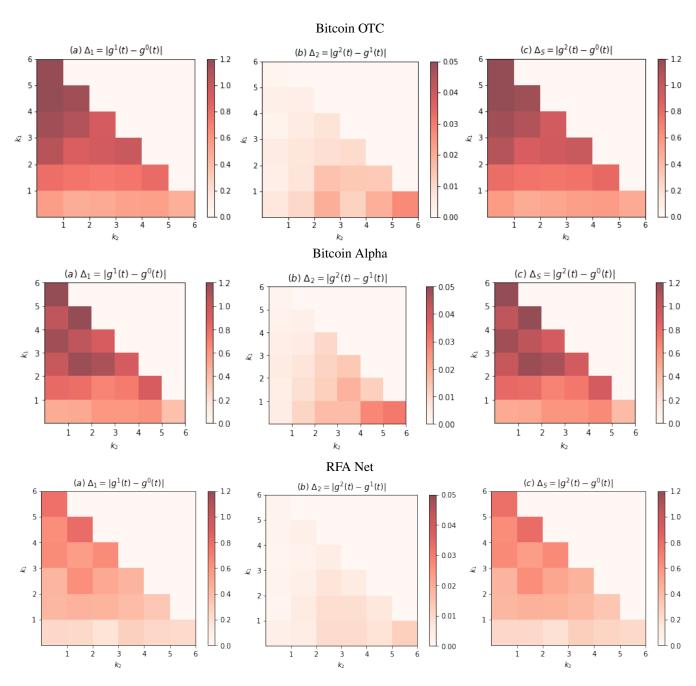


Figure 13: The results for the mixed settings in Bitcoin OTC, Bitcoin Alpha, RFA Net. The average strength of an indirect attack is small and significantly smaller that the average strength of a direct attack. Δ_1 shows the influence of the attack with k_1 direct edges, Δ_2 shows the influence of the attack with k_2 indirect edges, Δ_s shows the influence of the attack with k_1 direct edges and k_2 indirect edges.