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Mixed-Integer Linear Programming Model for Refinery Short-Term Scheduling of Crude Oil Unloading with Inventory Management

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This paper addresses the problem of inventory management of a refinery that imports several types of crude oil which are delivered by different vessels. This problem involves optimal operation of crude oil unloading, its transfer from storage tanks to charging tanks, and the charging schedule for each crude oil distillation unit. A mixed-integer optimization model is developed which relies on time discretization. The problem involves bilinear equations due to mixing operations. However, the linearity in the form of a mixed-integer linear program (MILP) is maintained by replacing bilinear terms with individual component flows. The LP-based branch and bound method is applied to solve the model, and several techniques, such as priority branching and bounding, and special ordered sets are implemented to reduce the computation time. This formulation and solution method was applied to an industrial-size problem involving 3 vessels, 6 storage tanks, 4 charging tanks, and 3 crude oil distillation units over 15 time intervals. The MILP model contained 105 binary variables, 991 continuous variables, and 2154 constraints and was effectively solved with the proposed solution approach.

Introduction

While mathematical programming technologies for long-term, plant-wide refinery planning have been extensively studied and implemented, much less work has been devoted to short-term operation scheduling. Computational tools for refinery planning, such as RPMS (Bonner and Moore, 1979) and PIMS (Bechtel Corp., 1993), rely mostly on linear programming models and, therefore, address longer term planning optimization problems. On the other hand, mathematical programming techniques for scheduling problems require the use of mixed integer optimization models to explicitly model the discrete decisions.

In this paper, we consider the short-term scheduling for crude oil inventory management problem which involves crude oil unloading to storage tanks from crude vessels, crude oil transfer from storage tanks to charging tanks for mixing, and a crude charging schedule for each CDU (crude distillation unit). We assume that the crude supply plan and production rate are determined by the long-term refinery planning, and this paper concerns the on-line optimization problem minimizing the cost involved in operation to meet this resource supply and production planning. Crude supply planning provides information about the arrival date, crude amount, and crude composition data of the shipping vessel, while the CDU charging planning involves both quantity and quality specification of the mixed crude oil as feedstock to CDU. Since the crude resource is supplied intermittently by vessels while CDU charging of the crude oil is continuous, the use of crude storage tanks is required. Moreover, if a refinery uses different kinds of crude oils for CDU charging, crude oil mixing is necessary and tank inventory management to meet the quantity and quality specification as a CDU feedstock is relevant.

Unlike scheduling of batch processes which has received considerable attention in the literature (Reklaitis, 1992; Rippin, 1992), much less work has been reported in the scheduling of continuous multiproduct plants. The special case of a two product mix was addressed by Kella (1991), and an example of the planning of a multiproduct continuous process under resource constraints was presented by Kondili et al. (1993b). A petroleum refinery which has to process different crude oils is one example of continuous multiproduct plants, and many problems involving refinery planning have been studied since the 1950s (Symonds, 1955; Manne, 1956). These problems included long-term supply and production planning of crude oil and the blending schedule for distilled crude oil products. Since the problem addressed in this paper involves transition cost and inventory cost in its objective function, some of the ideas by Sahinidis and Grossmann (1991) and Pinto and Grossmann (1994) on optimal cyclic scheduling of continuous multiproduct plants will also be used. To facilitate the modeling of material balance equations of each oil container, time will be discretized and nonlinear mixing equations will be reformulated into linear ones. State task network (STN) representation introduced by Kondili *et al.* (1993a) also assumes discretization of time, but the major difference is that in the STN the continuous time variables are not handled explicitly while in the proposed model they are introduced to deal with the variable duration of the tasks in modeling the vessel unloading operation. Concerning mixing equations, bilinear terms in mass balance equations are replaced by individual component flow and consequently reformulated into the linear equations. This exact linear reformulation is possible since this scheduling system involves only mixing operation without splitting operation (Quesada and Grossmann, 1995).

The scope of this work is to develop a rigorous mixed-integer linear programming (MILP) optimization model for short-term crude oil unloading, tank inventory management, and CDU charging schedule. The operating cost in the crude inventory management problem includes inventory cost, crude vessel harboring and sea-waiting cost, and changeover cost for charged oil change in each CDU. Constraints involve quality and quantity

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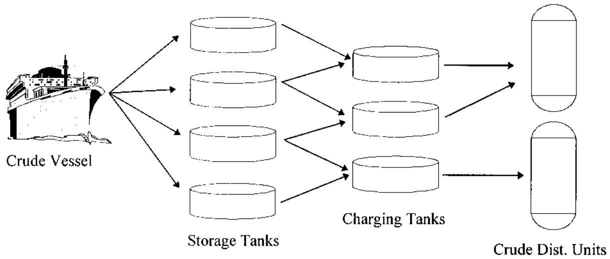

Figure 1. Graphical overview of a crude oil charging system.

specification of a mixed crude oil feed to CDU, mixing equations, mass balance for each tank, and capacity limitations of tanks and pumps.

This paper is organized as follows. First, the problem definition is presented followed by a motivating example where major tradeoff and comparison with conventional operation rules are stated. Next, the general mathematical formulation of this problem is given in MILP form. In this section bilinear terms in mixing equations are substituted by individual component flows to maintain the linearity. As a solution method, an effective branching and bounding method is then proposed that uses special ordered sets (Beale and Tomlin, 1970) and priority branching and bounding. Finally, numerical results of example problems demonstrate the effectiveness of the proposed model and solution method.

Problem Definition

The system configuration of this scheduling problem corresponds to a multistage system consisting of vessels, storage tanks, charging tanks, and CDUs as illustrated in Figure 1. During a given scheduling horizon, crude vessels arrive in the vicinity of the refinery docking station and, if busy, wait for unloading of the preceding vessel in the docking station. At the docking station, crude oil is unloaded into storage tanks. Crude oil is then transferred from storage tanks to charging tanks which are buffers to produce a crude mix, of which component compositions were determined at the planning level. The crude oil mix in each charging tank is then charged into a CDU. Given the configuration of the multistage system as well as the arrival times of vessels, equipment capacity limitations, and key component concentration ranges, the problem is then to determine the following operating variables to minimize operating costs: (a) waiting time of each vessel in the sea, (b) unloading duration time for each vessel, (c) crude unloading rate from vessels to storage tanks, (d) crude oil transfer and mixing rate from storage tank to charging tanks, (e) inventory levels of storage and charging tanks, (f) crude distillation unit charging rates, and (g) sequence of mixed crude to be charged into each CDU.

The following are operating rules that have to be obeyed in this problem: (a) In the scheduling horizon, each vessel should arrive and leave the docking station for unloading. (b) If a vessel does not arrive at the docking station, it cannot unload the crude oil. (c) If a vessel leaves the docking station, it cannot unload the crude oil. (d) The vessel should leave the docking station after its arrival. (e) The vessel cannot arrive at the docking station if the preceding vessel does not leave. (f) While the charging tank is charging CDU, crude cannot be fed into the charging tank and vice versa. (g) Each charging tank can feed at most one CDU at one time interval. (h) Each CDU is charged by only one mixed crude oil at one time interval.

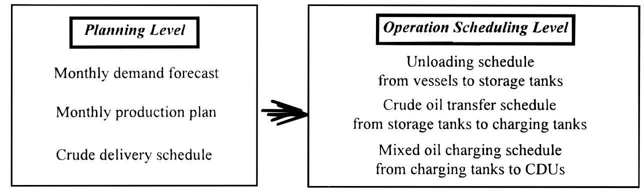

Figure 2. Conceptual framework of the decision-making procedure.

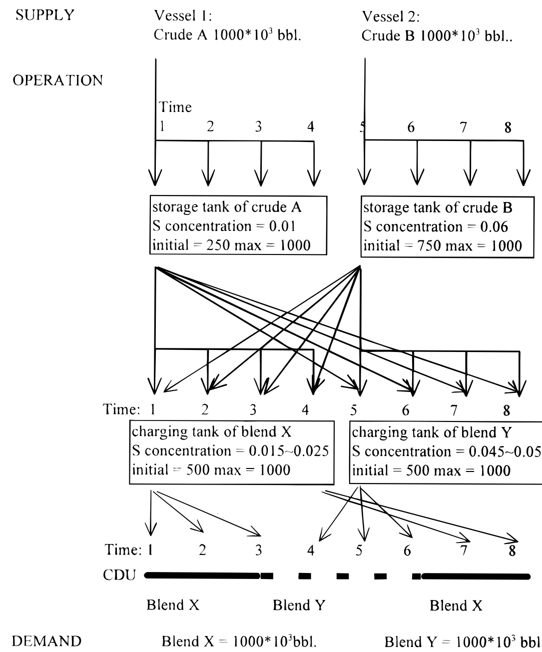

Figure 3. Operation schedule for motivating example.

Finally, the following are the major operating constraints that must be met: (a) equipment capacity limitations (tank capacity, pumping rate), (b) quality limitations on each mixed crude oil (components in mixed crude oil stream), and (c) demand of each mixed oil to be charged into CDU. The conceptual framework of the decision making procedure of this problem is described in Figure 2.

Motivating Example

In order to provide some insight into the nature of this optimization problem, consider the following small size problem. At the planning stage, two crude vessels are to arrive at days 1 and 5, and unloading for both vessels should be completed by day 8. Vessels 1 and 2 contain 1 million bbl of crude oil A and B, respectively. There is one CDU which has to process 1 million bbl of mixed crude oil X and Y, respectively. The weight fractions of sulfur which determine the quality of crude oil are 0.01 for crude oil A and 0.06 for crude oil B. Two crudes are mixed to make two types of mixtures: crude oil mixes X and Y. The sulfur concentration of X should be in the range of 0.015 and 0.025, while that of Y is between 0.045 and 0.055. The initial volumes of the storage tanks for crude oil A and B are respectively 250 000 and 750 000 bbl, while the initial volumes of the charging tanks for crude mix X and Y are all 500 000 bbl. The costs involved in this problem are inventory cost, vessel harboring cost, vessel sea waiting cost, and CDU changeover cost for crude oil mix charging mode change.

Figure 3 shows the system overview for this example. Each arrow in the figure corresponds to flow transfer.

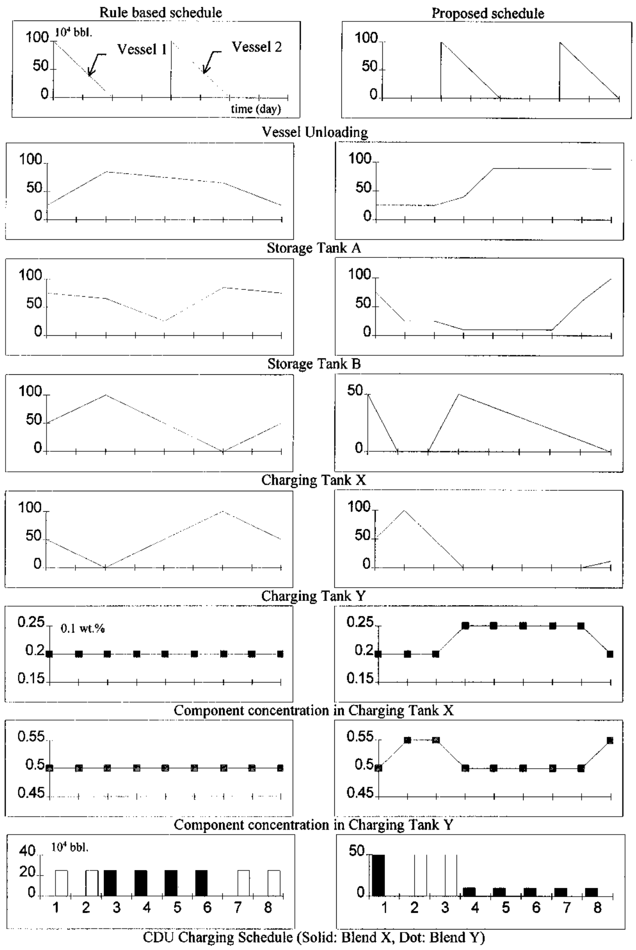

Figure 4. Rule based vs. optimal scheduling.

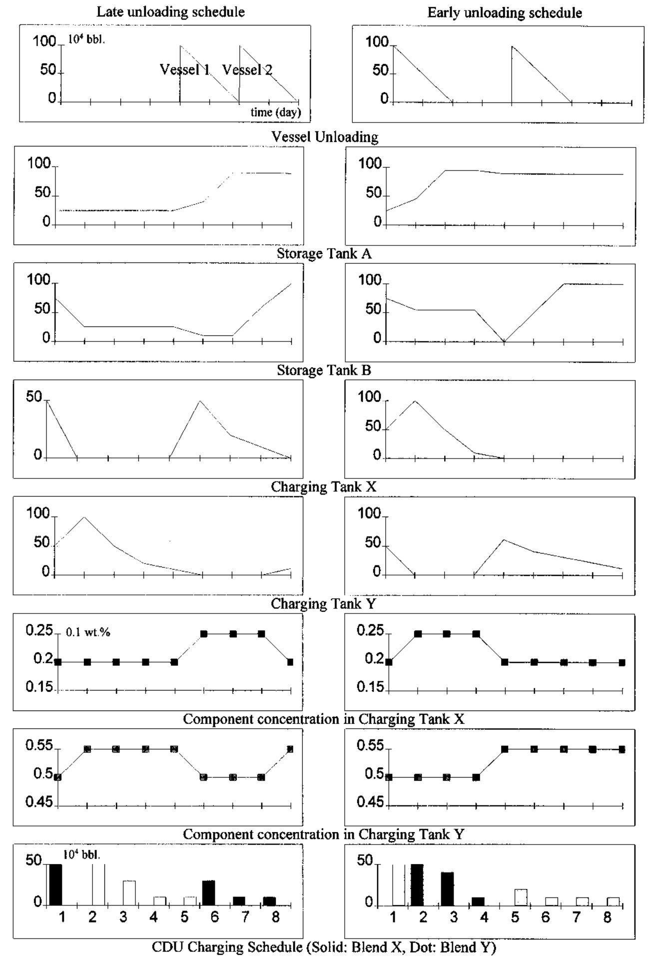

Figure 5. Operation schedule for late and early unloading.

Vessel 1 arrives at the docking station at day 1 and starts to unload crude oil A into storage tank A. It should complete unloading before day 6 and leave the docking station. At every point in time, crude oils can be transferred and mixed into charging tank X and charging tank Y. Mixed crudes then charge the CDU. CDU can process only one type of crude mix at a time, and setup costs are involved each time charged crude oil is switched. Vessel 2 arrives at day 5 and unloads crude oil B into storage tank B. Unit inventory cost for each storage tank and charging tank are 8×10^{-3} and 5×10^{-3} [\$/(day × bbl)], respectively. Changeover cost for crude charging to CDU is 50×10^3\$ each time it occurs. Costs involving the vessels are due to waiting in the sea and harboring for unloading the crude oil. These costs are 5×10^3 and 8×10^3 [\$/day], respectively; i.e., unloading incurs higher costs. Ideal mixing is assumed in the charging tank, and the crude mix cannot be fed into the CDU while crude oil is transferred from storage tanks to charging tanks. Appendix A contains the mathematical formulation for this problem.

Conventional operation is based on heuristic rules which are mainly focused on early unloading and a constant charging rate of crude to CDU. In Figure 4, one example of a schedule generated heuristically is shown. The total operation cost for this scheduling is 246×10^3\$, as seen in Table 1. In Figure 4 inventory profiles of each vessel and tank by optimal operation strategy are shown. A cost comparison of the two operation strategies is given in Table 1 and shows that the operation cost can be decreased by 11.5%, which represents a savings of over US\$28 333 per scheduling horizon /(8 days) or approximately 1.3 million dollars/year. The optimization model involved 36 binary variables, 192 continuous variables, and 331 constraints. The modeling system GAMS (Brooke et al., 1988) was used for setting up the optimization model, and the number of variables and constraints was reduced by considering the data structure of binary variables. The problem was solved by OSL (IBM, 1991) on an IBM RS-6000 in 17.1 s of CPU time.

Table 1. Cost Comparison between Rule-Based Method and Optimal Scheduling for Motivating Example

costs during scheduling horizon	rule base operation	optimal scheduling
unloading + sea waiting cost	32	52
inventory cost	114	65.667
changeover cost	100	100
total cost	246 ($\times 10^3$\$/8 days)	217.667 ($\times 10^3$\$/8 days)
cost saving by optimization	11.5%	

Figure 5 and Table 2 illustrate the trade-off between unloading cost and inventory cost inherent to this problem. The late unloading schedule was obtained by setting the unloading rate at 500 000 bbl/day starting unloading at days 5 and 7 for vessels 1 and 2, respectively, while the early unloading schedule was solved under the assumption that each vessel starts unloading as soon as it arrives. Since the production amount of each crude oil mix is the same for scheduling horizon, the earlier the crude is unloaded from the vessel, the longer the crude oil should remain in storage and charging tanks, which as a result incurs higher inventory cost. As shown in Table 2, late unloading of crude oil reduces inventory cost, while it causes higher cost

Table 2. Trade-Off between Inventory Cost and Unloading Cost for Motivating Example

costs during scheduling horizon (=8 days)	late unloading with optimal inventory schedule	early unloading with optimal inventory schedule	optimal unloading with optimal inventory schedule
unloading cost	62	32	52
inventory cost	57.667	87.1	65.667
changeover cost	100	100	100
total cost	219.667 ($\times 10^3$\$)	219.1 ($\times 10^3$\$)	217.667 ($\times 10^3$\$)

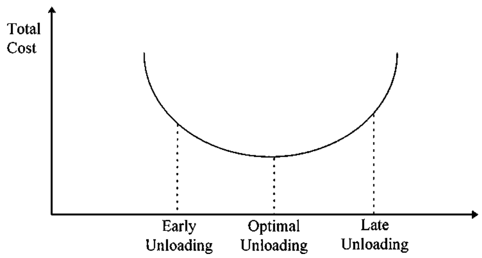

Figure 6. Relationship between optimal unloading and fixed unloading schedule under optimal inventory management.

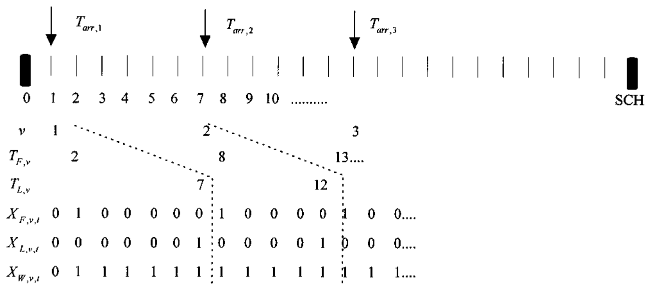

Figure 7. Timing variables to describe the rules for unloading and waiting of crude vessel.

related to the vessel. Note that the close values of the early and late unloading simply mean that the model is not very sensitive to these decisions. However, significant savings can still be obtained when compared to the heuristic solution. This relationship between optimal unloading and two extreme unloading strategies is illustrated in Figure 6. In addition to the economic advantage gained, automation of this operating procedure saves manpower and time by replacing the conventional heuristic operation with this optimization based scheduler. This advantage drastically increases as the size of the problem increases.

This example shows the advantage of an optimization approach over the conventional rule based operation and the complex trade-off between costs involved in this scheduling problem. There exists an optimal operation schedule by the result of interaction between decision variables such as crude unloading rates and duration, oil transfer rates from storage tanks to charging tanks, and CDU charging rates. Therefore, there is a clear incentive for developing systematic techniques to handle this scheduling problem.

General Mathematical Formulation

The following are the assumptions for the model that will be proposed in this paper:

(a) Only one vessel docking station for crude oil unloading is assumed.

(b) Changeover times are neglected. This assumption is, in general, reasonable since the time required for CDU mode change is small compared to the operation horizon.

(c) Perfect mixing is assumed for the charging tank while it is being fed by different crude oils, and additional mixing time is not required before it charges CDU. In the real plant, a mixing time is required to completely mix feed with existing oil in the charging tank, but for the sake of simplicity the mixing time between oil feeding and charging to/from charging tank is ignored.

(d) Only specific key components in crude or mixed oil decide the property of each oil. Usually the property of the mixed oil is judged by one or two key components and the viscosity of the oil. The viscosity and other property of the mixed oil is hard to obtain for its highly nonlinear nature, so it is not considered in the model. The concentration of a key component such as sulfur is described in a bilinear equation, and quality constraints for the mixed oil are developed to bind this key component concentration in the mixed oil.

The proposed scheduling model is based on a uniform discretization of time in the given scheduling horizon. Selection of the time length of each discretized time span involves a trade-off between accurate operation and computational effort. With smaller discretized time sizes more sophisticated operation is possible since there are more operation changes during the scheduling horizon. However, this increases the computational effort due to the increased number of binary variables and constraints. The examples in this paper involve less than 15 time intervals during the scheduling horizon.

The relationships between the timing variables for vessel arrival, unloading, and departure in the proposed model are illustrated in Figure 7. In the figure vessel 1 arrives at time 1, but the actual unloading process begins at time 2. It completes unloading at time 7 and leaves the docking station. The variables $X_{W,l,t}$, which denotes whether unloading of crude oil from vessel 1 to the storage tank is possible or not, have value 1 during the time period from 2 to 7. Also, vessel 2 arrives at 7, unloads its crude oil at 8, and leaves the docking station at 12. Generally speaking, vessel v arrives around the docking station at time $T_{ARR,v}$, and it waits until preceding vessel $v - 1$ leaves the docking station. If the preceding vessel leaves, vessel v starts to unload its crude oil. The binary variable $X_{F,v,t}$ is activated when the unloading of vessel v starts at time t. The variables $X_{W,v,t}$ which are related to the unloading flow of vessel v to storage tanks at time t are activated at time $T_{F,v}$ which is the unloading start time of vessel v at the docking station. The $X_{W,v,t}$ are deactivated at time $T_{L,v}$ when the vessel v completes unloading and leaves the docking station. The binary variable $X_{L,v,t}$ is activated when the unloading of vessel v is completed. In this way, the variables $X_{W,v,t}$ are activated from $T_{F,v}$ to $T_{L,v}$. For each vessel the binary variables $X_{F,v,t}$ and $X_{L,v,t}$ are activated only once throughout the scheduling horizon, and they are used to calculate $T_{F,v}$ and $T_{L,v}$, respectively.

Figure 8 is a representation to help understand the indices, variables, and parameters of the proposed model.

The MILP model for the scheduling problem is as follows.

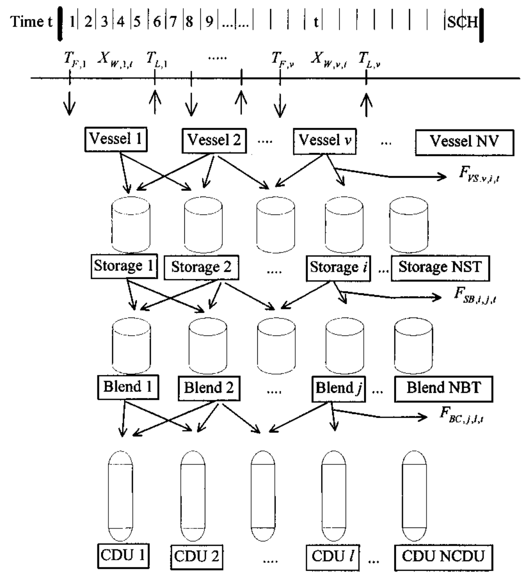

Figure 8. Model representation.

(A) Operating Cost.

Minimize

Operating cost =
unloading cost for the crude vessel +
cost for vessel waiting in the sea +
inventory cost for storage and charging tanks +
changeover cost

$$\mathrm{COST} = C_{\mathrm{UNLOAD},v}\sum_{v=1}^{\mathrm{NV}}(T_{L,v} - T_{F,v}) + C_{\mathrm{SEA},v}\sum_{v=1}^{\mathrm{NV}}(T_{F,v} - T_{\mathrm{ARR},v}) + C_{\mathrm{INVST},i}\sum_{i=1}^{\mathrm{NST}}\sum_{t=1}^{\mathrm{SCH}}\left(\frac{V_{S,i,t} + V_{S,i,t-1}}{2}\right) + C_{\mathrm{INVBL},j}\sum_{j=1}^{\mathrm{NBT}}\sum_{t=1}^{\mathrm{SCH}}\left(\frac{V_{B,j,t} + V_{B,j,t-1}}{2}\right) + \sum_{t=1}^{\mathrm{SCH}}\sum_{j=1}^{\mathrm{NBT}}\sum_{j'=1}^{\mathrm{NBT}}\sum_{t=1}^{\mathrm{NCDU}}(C_{\mathrm{SETUP}j,j',l}Z_{j,j',l,t}) \quad (1)$$

subject to:

(B) Vessel Arrival and Departure Operation Rules. Each vessel arrives at the docking station for unloading only once throughout the scheduling horizon.

$$\sum_{t=1}^{\mathrm{SCH}} X_{F,v,t} = 1 \qquad v = 1, ..., \mathrm{NV} \qquad (2a)$$

Each vessel leaves the docking station only once throughout the scheduling horizon.

$$\sum_{t=1}^{\mathrm{SCH}} X_{L,v,t} = 1 \qquad v = 1, ..., \mathrm{NV} \qquad (2b)$$

Equation for unloading initiation time.

$$T_{F,v} = \sum_{t=1}^{\mathrm{SCH}} tX_{F,v,t} \qquad v = 1, ..., \mathrm{NV} \qquad (2c)$$

Equation for unloading completion time.

$$T_{L,v} = \sum_{t=1}^{\mathrm{SCH}} tX_{L,v,t} \qquad v = 1, ..., \mathrm{NV} \qquad (2d)$$

Each crude vessel should start unloading after arrival time set in the planning level.

$$T_{F,v} \geq T_{\mathrm{ARR},v} \qquad v = 1, ..., \mathrm{NV} \qquad (2e)$$

Duration of the vessel unloading is bounded by the initial volume of oil in the vessel divided by the maximum unloading rate.

$$T_{L,v} - T_{F,v} \geq \left\lceil \frac{V_{V,v,0}}{\mathrm{MAX}_i[F_{VS,v,i,\max}]} \right\rceil \qquad v = 1, ..., \mathrm{NV} \qquad (2f)$$

($\lceil\ \rceil$ corresponds to round-up to the next highest integer value.) Vessel in the sea cannot arrive at the docking station for unloading unless the preceding vessel leaves.

$$T_{F,v+1} \geq T_{L,v} \qquad v = 1, ..., \mathrm{NV} \qquad (2g)$$

Unloading is possible between time $T_{F,v}$ and $T_{L,v}$.

$$X_{W,v,t} \leq \sum_{m=1}^{f} X_{F,v,m}, \quad X_{W,v,t} \leq \sum_{m=t}^{\mathrm{SCH}} X_{L,v,m} \qquad v = 1, ..., \mathrm{NV}, \quad t = 1, ..., \mathrm{SCH} \quad (2h)$$

(C) Material Balance Equations for the Vessel. Crude oil in vessel v at time t = initial crude oil in the vessel v − crude oil transferred from vessel v to storage tanks up to time t

$$V_{V,v,t} = V_{V,v,0} - \sum_{i=1}^{\mathrm{NST}}\sum_{m=1}^{t} F_{VS,v,i,m} \qquad v = 1, ..., \mathrm{NV}, \quad t = 1, ..., \mathrm{SCH} \quad (3a)$$

Operating constraints on crude oil transfer rate from vessel v to storage tank i at time t.

$$F_{VS,v,j,\min}X_{W,v,t} \leq F_{VS,v,i,t} \leq F_{VS,v,j,\max}X_{W,v,t} \qquad v = 1, ..., \mathrm{NV}, \quad i = 1, ..., \mathrm{NST}, \quad t = 1, ..., \mathrm{SCH} \quad (3b)$$

The volume of crude oil transferred from vessel v to storage tanks during the scheduling horizon equals the initial crude oil volume of vessel v.

$$\sum_{i=1}^{\mathrm{NST}}\sum_{t=1}^{\mathrm{SCH}} F_{VS,v,i,t} = V_{V,v,0} \qquad v = 1, ..., \mathrm{NV} \qquad (3c)$$

(D) Material Balance Equations for the Storage Tank. Crude oil in storage tank i at time t = initial crude oil in storage tank i + crude oil transferred from vessels to storage tank i up to time t − crude oil transferred from storage tank i to charging tanks up to time t

$$V_{S,i,t} = V_{s,i,0} + \sum_{v=1}^{\mathrm{NV}}\sum_{m=1}^{t} F_{VS,v,i,m} - \sum_{j=1}^{\mathrm{NBT}}\sum_{m=1}^{t} F_{SB,i,j,m} \qquad i = 1, ..., \mathrm{NST}, \quad t = 1, ..., \mathrm{SCH} \quad (4a)$$

Operating constraints on crude oil transfer rate from storage tank i to charging tank j at time t. The term $1 - \sum_{l=1}^{\mathrm{NCDU}} Dj,l,t$ denotes that if charging tank j is charging any CDU, there is no oil transfer from storage tank i to charging tank j.

$$F_{SB,i,j,\min}(1-\sum_{l=1}^{NCDU}D_{j,l,t})\le F_{SB,i,j,t}\le F_{SB,i,j,\max}(1-\sum_{l=1}^{NCDU}D_{j,l,t})\qquad i=1,...,NST,\quad j=1,...,NBT,\quad t=1,...,SCH\quad(4b)$$

Volume capacity limitations for storage tank i at time t

$$V_{S,i,\min}\le V_{S,i,t}\le V_{S,i,\max}\qquad i=1,...,NST,\quad t=1,...,SCH\quad(4c)$$

(E) Material Balance Equations for Charging Tank. Crude oil mix in charging tank j at time t = initial mixed oil in charging tank j + crude oil transferred from storage tanks to charging tank j up to time t − crude oil mix j charged into CDUs up to time t

$$V_{B,j,t}=V_{B,j,0}+\sum_{i=1}^{NST}\sum_{m=1}^{t}F_{SB,i,j,m}=\sum_{l=1}^{NCDU}\sum_{m=1}^{t}F_{BC,j,l,m}\qquad j=1,...,NBT,\quad t=1,...,SCH\quad(5a)$$

Operating constraints on mixed oil transfer rate from charging tank j to CDU l at time t.

$$F_{BC,j,l,\min}D_{j,l,t}\le F_{BC,j,l,t}\le F_{BC,j,l,\max}D_{j,l,t}\qquad j=1,...,NBT,\quad l=1,...,NCDU,\quad t=1,...,SCH\quad(5b)$$

Volume capacity limitations for charging tank j at time t

$$V_{B,j,\min}\le V_{B,j,t}\le V_{B,j,\max}\qquad j=1,...,NBT,\quad t=1,...,SCH\quad(5c)$$

Total production amount of crude oil mix j should meet the demand of crude mix j for the scheduling horizon.

$$\sum_{l=1}^{NCDU}\sum_{t=1}^{SCH}F_{BC,j,l,t}=DM_j\qquad j=1,...,NBT\quad(5d)$$

(F) Material Balance Equations for Component *k* in the Charging Tank. Volume of component k in charging tank j at time t = initial component k in charging tank j + component k in crude oil transferred from storage tanks to charging tank j up to time t − component k in crude oil mix j transferred to CDUs up to time t

$$v_{B,j,t}=v_{B,j,0}+\sum_{m=1}^{t}(\sum_{i=1}^{NST}f_{SB,i,j,m}-\sum_{t=1}^{NCDU}f_{Bc,j,l,m})\qquad j=1,...,NBT,\quad k=1,...,NCE,\quad t=1,...,SCH\quad(6a)$$

Operating constraints on volumetric flow rate of component k from storage tank i to charging tank j

$$f_{SB,i,j,k,t}=F_{SB,i,j,t}\xi_{S,i,k}\qquad\forall i=1,...,NST,\quad j=1,...,NBT,\quad k=1,...,NCE,\quad t=1,...,SCH\quad(6b)$$

Operating constraints on volumetric flow rate of component k from charging tank j to CDU l

$$F_{BC,j,l,t}\xi_{B,j,k\min}\le f_{BC,j,l,k,t}\le F_{BC,j,l,t}\xi_{B,j,k,\max}\qquad j=1,...,NBT,\quad k=1,...,NCE,\quad l=1,...,NCDU,\quad t=1,...,SCH\quad(6c)$$

Volume capacity limitations for component k in charging

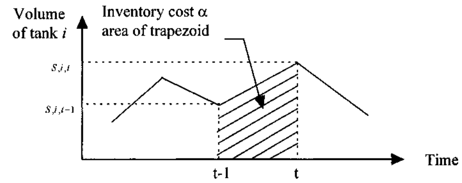

Figure 9. Tank inventory cost calculation in the trapezoidal way.

tank j at time t

$$V_{B,j,t}\xi_{B,j,k,\min}\le v_{B,j,k,t}\le V_{B,j,t}\xi_{B,j,k,\max}\qquad j=1,...,NBT,\quad k=1,...,NCE,\quad l=1,...,NCDU,\quad t=1,...,SCH\quad(6d)$$

(G) Operating Rules for Crude Oil Charging. Charging tank j can charge at most one CDU at any time t.

$$\sum_{l=1}^{NCDU}D_{j,l,t}\le 1\qquad j=1,...,NBT,\quad t=1,...,SCH\quad(7a)$$

CDU l can be charged only by one charging tank at any time t.

$$\sum_{j=1}^{NBT}D_{j,l,t}=1\qquad l=1,...,NCDU,\quad t=1,...,SCH\quad(7b)$$

If CDU l is charged by crude oil mix j at time $t-1$ and charged by j' at time t, changeover cost is involved.

$$Z_{j,j',l,t}\ge D_{j',l,t}+D_{j,l,t-1}-1\qquad j,j'\ (j\ne j')=1,...,NBT,\quad l=1,...,NCDU,\quad t=2,...,SCH\quad(7c)$$

As shown in eq 1, the operation cost consists of unloading cost, vessel waiting cost, tank inventory cost, and changeover cost for CDU mode change. Unloading cost for the crude vessel is proportional to the unloading duration of vessels at the docking station: $T_{L,v}-T_{F,v}$. Waiting of vessel in the sea also incurs cost, and it is proportional to the time difference between the time when $X_{F,v,t}$ is activated and $T_{ARR,v}$, that is to say, $T_{F,v}-T_{ARR,v}$. Tank inventory costs are calculated according to the trapezoid area, as illustrated in Figure 9. Changeover cost for CDU charging mode change is proportional to the number of the crude mix switch. Whenever crude mix changeover occurs, variable Z gets value 1 by the constraint (7c), and summation of Z results in the total number of changeovers during the scheduling horizon of concern.

Constraints (2a)−(2g) describe the rules for unloading and waiting of a crude vessel. Constraints (2a) and (2b) are equations which state that arrival and departure of a vessel to/from the docking station for unloading take place only once throughout the scheduling horizon. Equation 2c states that $T_{F,v}$ is equal to the time when $X_{F,v,t}$ is activated, while eq 2d states that $T_{L,v}$ is equal to the time when $X_{L,v,t}$ is activated. Equation 2e clearly states that the vessel should unload its crude oil at the docking station after its arrival. Constraint (2f) states that for each vessel arrival time should precede departure time, and constraint (2g) describes the relationship between the succeeding vessel and the preceding vessel. Inequality (2h) states that $X_{W,v,t}$ is activated between the time when $X_{F,v,t}$ is activated and the time when $X_{L,v,t}$ is activated. Constraints (3a)−(5c) involve material balance equations for each oil container (vessel and tanks), its inflow and outflow bounds, and capacity limitations. In each material balance equation, the

Table 3. Computational Results[a]

example	variables	0−1 vars.	constraints	LP soln.	MILP soln.	nodes	iterations	CPU time
				Without SOS1, and Priority				
1	192	36	331	150.25	217.667	817	9228	73.4
2	456	70	825	211.15	352.55	10525	331493	4158.8
3	581	84	1222	180.24	296.56	>8993	>515541	>7744
				With SOS1, and Priority				
1	192	36	331	150.25	217.667	208	1695	17.1
2	456	70	825	211.15	352.55	904	21148	287.9
3	581	84	1222	180.24	296.56	2519	60663	1089.4
4 (0%)	991	105	2154	307.67	420.99	5011	157883	4372.8
4 (1%)	991	105	2154	307.67	420.99	4314	152871	4054.3
4 (5%)	991	105	2154	307.67	420.99	2727	123427	3051.9
4 (10%)	991	105	2154	307.67	420.99	1327	78455	1827.3

[a] Example 3 without SOS1, and priority stopped iteration because of resource limitation. Example 4 without them could not be solved.

Table 4. System Information for Example 2

Scheduling Horizon (# of unit time)			10	
Number of Vessel Arrivals			3	
	Arrival Time	Amount of Crude	Component 1	Component 2
Vessel 1	1	100	0.01	0.04
Vessel 2	4	100	0.03	0.02
Vessel 3	7	100	0.05	0.01
Number of Storage Tanks			3	
Storage Tanks	Capacity	Initial Oil Amount	Component 1	Component 2
Tank 1	100	20	0.01	0.04
Tank 2	100	50	0.03	0.02
Tank 3	100	70	0.05	0.01
Number of Charging Tanks			3	
Charging Tanks	Capacity	Initial Oil Amount	Initial Comp. 1 (min, max)	Initial Comp. 2 (min, max)
Tank 1	100	30	0.0167(0.01, 0.02)	0.0333(0.03,0.038)
Tank 2	100	50	0.03(0.025, 0.035)	0.023(0.018,0.027)
Tank 3	100	30	0.0433(0.04,0.048)	0.0133(0.01,0.018)
Number of CDU			2	
Unit costs involved in vessel operation			Unloading cost: 8, Sea waiting cost: 5	
Tank inventory unit costs			Storage tank: 0.05, Charging tank: 0.08	
Unit changeover cost for charged oil switch			50 (independent of sequence and CDU)	
Demand of mixed oils by CDUs			Oil mix 1:100, Oil mix 2:100, Oil mix 3:100	

Table 5. System Information for Example 3

Scheduling Horizon (# of unit time)			12
Number of Vessel Arrivals			3
	Arrival Time	Amount of Crude	Concentration of key component
Vessel 1	1	50	0.01
Vessel 2	5	50	0.085
Vessel 3	9	50	0.06
Number of Storage Tanks			3
Storage Tanks	Capacity	Initial Oil Amount	Initial comp. concentration (min~max)
Tank 1	100	20	0.02 (0.01~0.03)
Tank 2	100	20	0.05 (0.04~0.06)
Tank 3	100	20	0.08 (0.07~0.09)
Number of Charging Tanks			3
Charging Tanks	Capacity	Initial Oil Amount	Initial comp. concentration (min, max)
Tank 1	100	30	0.03 (0.025~0.035)
Tank 2	100	50	0.05 (0.045~0.065)
Tank 3	100	30	0.08 (0.075~0.085)
Number of CDU			2
Unit costs involved in vessel operation			Unloading cost: 10, Sea waiting cost: 5
Tank inventory unit costs			Storage tank: 0.04, Charging tank: 0.08
Unit changeover cost for charged oil switch			50 (independent of sequence and CDU)
Demand of mixed oils by CDUs			Oil mix 1: 50, Oil mix 2: 50, Oil mix 3: 50

volume of each tank at each discrete interval is described as the initial volume plus the summation of inflows subtracted by the summation of outflows up to each time interval. Constraint (5d) states that the sum of crude mix j charged into CDUs throughout the scheduling horizon should be met by the demand of each crude mix j by CDUs. Constraints (6a)−(6d) involve the material balance equations for key components in the charging tank and bounding inequalities for each variable in the equations. Equation 6a is a mass balance for component k in charging tank j at time t. Constraints (6b) and (6c) bound the mass inflow and outflow of component k to/from charging tank j at time t. Constraint (6d) is for volume capacity limitation of component k in charging tank j at time t. Note that the concentration of component k is not directly used.

Table 6. System Information for Example 4

Scheduling Horizon (# of unit time)		15	
Number of Vessel Arrivals		3	
	Arrival Time	Amount of Crude	Concentration of key component
Vessel 1	1	60	0.03
Vessel 2	6	60	0.05
Vessel 3	11	60	0.65
Number of Storage Tanks		6	
Storage Tanks	Capacity	Initial Oil Amount	Initial comp. concentration (min~max)
Tank 1	10~90	60	0.031 (0.25~0.38)
Tank 2	10~110	10	0.03 (0.2~0.4)
Tank 3	10~110	50	0.05 (0.04~0.06)
Tank 4	10~110	40	0.065 (0.06~0.07)
Tank 5	10~90	30	0.075 (0.07~0.08)
Tank 6	10~90	60	0.075 (0.07~0.08)
Number of Charging Tanks		4	
Charging Tanks	Capacity	Initial Oil Amount	Initial comp. concentration (min, max)
Tank 1	80	5	0.0317 (0.03~0.035)
Tank 2	80	30	0.0483 (0.043~0.05)
Tank 3	80	30	0.0633 (0.06~0.065)
Tank 4	80	30	0.075 (0.071~0.08)
Number of CDU		3	
Unit costs involved in vessel operation		Unloading cost: 7, Sea waiting cost: 5	
Tank inventory unit costs		Storage tank: 0.05, Charging tank: 0.06	
Unit changeover cost for charged oil switch		30 (independent of sequence and CDU)	
Demand of mixed oils by CDUs		Oil mix 1: 60, Oil mix 2: 60, Oil mix 3: 60, Oil mix 4: 60	

Otherwise, we would generate nonconvex bilinear equations which are hard to solve (Quesada and Grossmann, 1995). In that respect, constraints (6c) and (6d) were linearly reformulated from the following nonlinear ones:

$$\xi_{B,j,k,\min} \le \xi_{B,j,k,t} \le \xi_{B,j,k,\max} \qquad j = 1, ..., \mathrm{NBT}, \quad k = 1, ..., \mathrm{NCE}, \quad t = 1, ..., \mathrm{SCH} \quad (8a)$$

$$f_{BC,j,l,k,t} = \xi_{B,j,k,t} F_{BC,j,l,t} \qquad j = 1, ..., \mathrm{NBT}, \quad k = 1, ..., \mathrm{NCE}, \quad l = 1, ..., \mathrm{NCDU}, \quad t = 1, ..., \mathrm{SCH} \quad (8b)$$

$$v_{B,j,k,t} = \xi_{B,j,k,t} V_{B,j,t} \qquad j = 1, ..., \mathrm{NBT}, \quad k = 1, ..., \mathrm{NCE}, \quad t = 1, ..., \mathrm{SCH} \quad (8c)$$

Inequality (8a) is a quality constraint for the concentration of component k in charging tank j. Equations 8b and 8c are bilinear equations for the volumetric flow rate of component k in the charging stream and volume of component k in charging tank j at time t, respectively. Constraint (6c) is obtained by multiplying inequality (8a) by $F_{BC,j,l,t}$ and replacing the bilinear term with $f_{BC,j,l,k,t}$ by eq 8b. Likewise, constraint (6d) is obtained by multiplying eq 8a by $V_{B,j,t}$ and replacing the bilinear term with $v_{B,j,l,k,t}$ in eq 8c. Constraints (7a) and (7b) describe the operating rules for crude charging. Constraint (7c) accounts for changeovers in the CDUs for changeover cost calculation in the objective function. The implication from which this inequality is derived is as follows. "If crude mix j charges CDU l at time $t-1$ and if j' charges CDU l at time t, then a changeover from j to j' is involved." This rule can be translated into implication form (9). It is straightforward to derive inequality (7c) from eq 9.

$$D_{j,l,t-1} \wedge D_{j',l,t} \Rightarrow Z_{j,j',l,t} \qquad j, j' \; (j \neq j') = 1, ..., \mathrm{NBT}, \quad l = 1, ..., \mathrm{NCDU}, \quad t = 2, ..., \mathrm{SCH} \quad (9)$$

Solution Method

The branch and bound code OSL (IBM, 1991) was used to solve this problem. In order to reduce the computational expense, several techniques have been employed. First, by specifying a priority to the binary variables according to their contributions to the cost function, the number of nodes in the branch and bound enumeration has been reduced. There are three sets of binary variables ($X_{F,v,t}$, $X_{L,v,t}$, and $D_{j,l,t}$) in this MILP model. Giving higher priority to $D_{j,l,t}$ which contribute to changeover costs reduced the size of the search tree. Second, by defining binary variables as special ordered sets (Beale and Tomlin, 1970), considerable reduction in the enumeration has been achieved. Binary variables $X_{F,v,t}$ and $X_{l,v,t}$ are defined as SOS1 variables according to constraints (2a) and (2d). Likewise, $D_{j,l,t}$ is also defined as SOS1 according to constraint (7b). For the large-size problem a finite value for the relative optimality criterion (OPTCR) was used to reduce the CPU time. Table 3 in the next section shows the advantage of using these techniques. The modeling system GAMS (Brooke et al., 1988) was used in order to implement the scheduling model and its solution method.

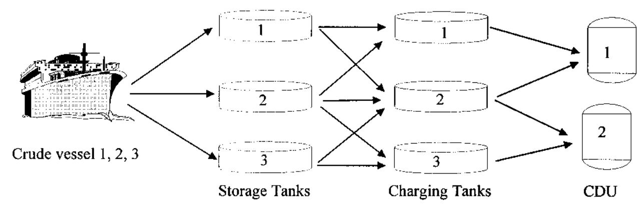

Figure 10. Oil transfer network of Example 2.

Examples

Three examples are presented. Example 2 presents a problem with three storage tanks, three charging tanks, and two distillation units. In Example 2 two key component concentrations are considered. Example 3 deals with the case when there are more crude oils imported than the number of storage tanks. In this case the storage tank is modeled as a mixing tank as it was in the charging tank, and more constraints are involved in this example. Example 4 deals with an industry size problem. Table 3 shows the computational results for each examples. For the sake of simplicity units have been omitted throughout the examples.

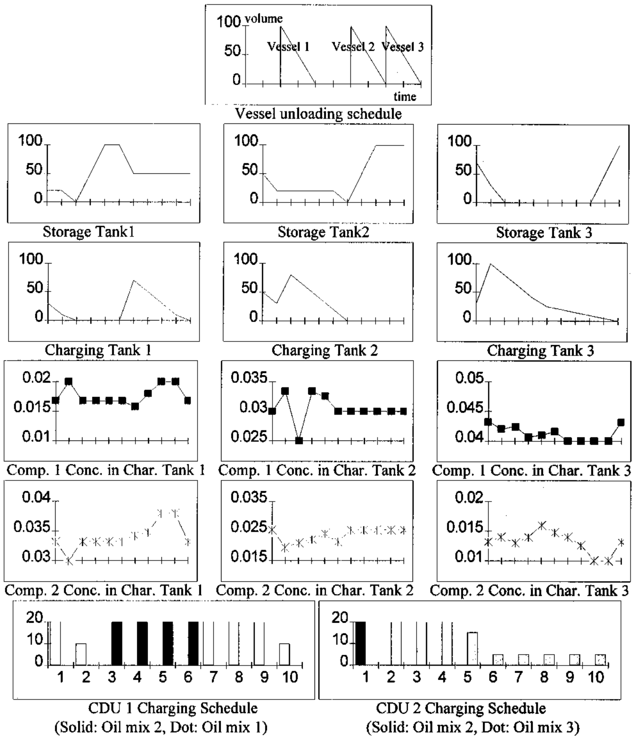

Figure 11. Optimal operation schedule for example 2.

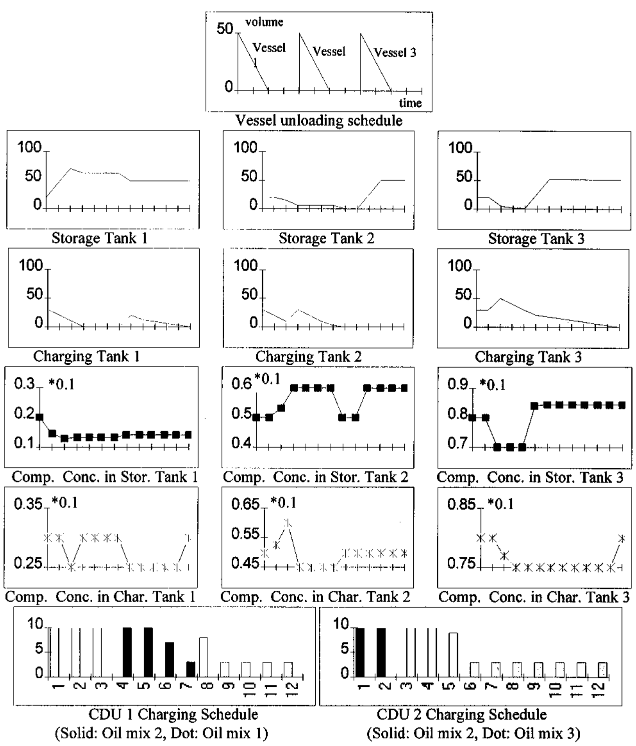

Figure 12. Optimal operation schedule for example 3.

Example 2. Three Vessels, Three Storage Tanks, Three Charging Tanks, and Two CDUs. Figure 10 describes the system for example 2. The scheduling horizon for this problem is 10 time intervals, and three crude vessels arrive at times 1, 4, and 7, respectively. Crude vessels 1, 2, and 3 respectively deliver crude 1, 2, and 3 at the amount of 100. Charging tank 1 contains crude 1 and crude 2 in its crude mix 1. Crude mix 2 is composed of 1, 2, and 3, while charging tank 3 contains crudes 2 and 3. CDU 1 is fed by charging tanks 1 and 2, while CDU 2 is fed by charging tanks 2 and 3. The demand by CDUs for the given scheduling horizon is 100 for crude mix 1, 100 for crude mix 2, and 100 for

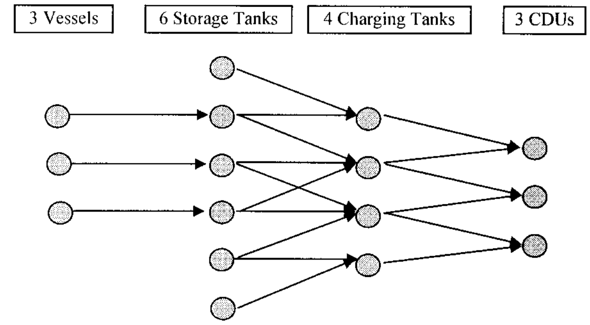

Figure 13. Oil flow network for example 4.

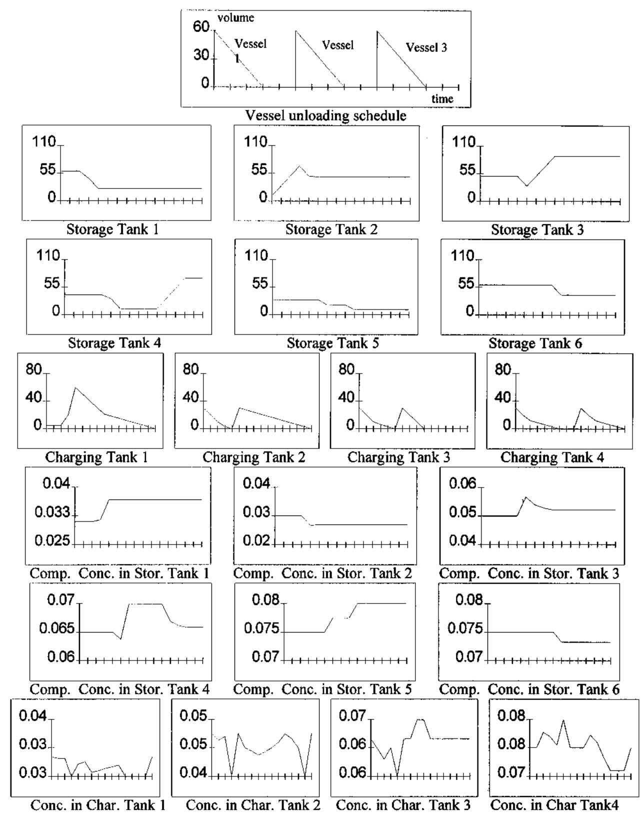

Figure 14. Optimal inventory schedule for example 4.

crude mix 3. The data involved in this problem appear in Table 4. Figure 11 shows the optimal operation schedule obtained.

Example 3. Three Vessels, Three Storage Tanks Which Involve Crude Oil Mixing, Three Charging Tanks, and Two CDUs. Since real industry problems involve more crude oils imported than the existing number of storage tanks, it is unavoidable to mix several crude oils into one storage tank. If mixing is involved in storage tanks, operation rules which were applied to charging tanks are additionally required to model storage tanks. The additional constraints for this example include a material balance equation for components in the storage tank and operation rule: "Oil transfer from a storage tank to charging tanks occurs when there is no crude oil unloading from the vessel to the storage tank." Table 5 shows the data for this problem. Figure 12 illustrates the predicted optimal operation schedule.

Example 4. Industry Size Problem: Three Vessels, Six Storage Tanks Which Involve Crude Oil Mixing, Four Charging Tanks, and Three CDUs. In Figure 13 oil flow network for this problem is illustrated. The data are given in Table 6, and the results are displayed in Figures 14 and 15. As shown

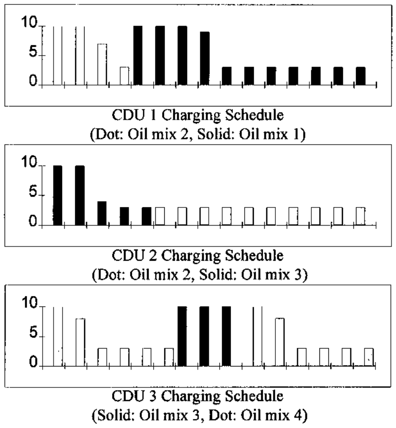

Figure 15. Optimal charging schedule for example 4.

in Table 3, this problem required more than 1 h of CPU time to be solved to 0% optimality. If this problem is solved with 1% tolerance to the optimum, the CPU time is reduced by 7.3%; with 5% it is reduced by 30.2%; and with 10% by 58.2%.

Conclusions

The refinery short-term scheduling problem of crude oil unloading with inventory management has been discussed in this paper. This problem can be modeled as a large scale MILP model in which time is uniformly discretized and mass balance equations are applied at each time interval. The bilinear mixing equations were avoided by introducing individual component flows. The LP-based branch and bound method was applied to solve the model, and several techniques have been implemented for the large-size problem. Finally, this MILP model has been applied to four examples to show the economic potential and trade-offs involved in the optimization of this problem.

Acknowledgment

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Nomenclature

(a) Indices and Sets

$i = 1, ..., \mathrm{NST}$ = crude oil storage tank and the crude oil in it

$j, j' = 1, ..., \mathrm{NBT}$ = crude oil charging tank and the crude oil mix in the charging tank

$k = 1, ..., \mathrm{NCE}$ = key component of crude oil

$l = 1, ..., \mathrm{NCDU}$ = crude distillation unit

$t = 1, ..., \mathrm{SCH}$ = time interval

$v = 1, ..., \mathrm{NV}$ = crude vessel

(b) Variables

$D_{j,l,t}$ = 0−1 variable to denote if the crude oil mix in charging tank j charges CDU l at time t

$X_{F,v,t}$ = 0−1 variable to denote if vessel v starts unloading at time t

$X_{L,v,t}$ = 0−1 variable to denote if vessel v completes unloading at time t

$f_{SB,i,j,k,t}$ = volumetric flow rate of component k from storage tank i to charging tank j at time t

$f_{BC,j,l,k,t}$ = volumetric flow rate of component k from charging tank j to CDU l at time t

$F_{VS,v,j,t}$ = volumetric flow rate of crude oil from vessel v to storage tank i at time t

$F_{SB,i,j,t}$ = volumetric flow rate of crude oil from storage tank i to charging tank j at time t

$F_{BC,j,l,t}$ = volumetric flow rate of crude oil mix from charging tank j to CDU l at time t

$T_{F,v}$ = vessel v unloading initiation time

$T_{L,v}$ = vessel v unloading completion and departure time

$v_{B,j,k,t}$ = volume of component k in charging tank j at time t

$V_{V,v,t}$ = volume of crude oil in crude vessel v at time t

$V_{S,i,t}$ = volume of crude oil in storage tank i at time t

$V_{B,j,t}$ = volume of mixed oil in charging tank j at time t

$X_{W,v,t}$ = 0−1 continuous variable to denote if vessel v is unloading its crude oil at time t

$Z_{j,j',l,t}$ = 0−1 continuous variable to denote transition from crude mix j to j' at time t in CDU l

(c) Parameters

$C_{\mathrm{UNLOAD},v}$ = unloading cost of vessel v per unit time interval

$C_{\mathrm{SEA},v}$ = sea waiting cost of vessel v per unit time interval

$C_{\mathrm{INVST},i}$ = inventory cost of storage tank i per unit time per unit volume

$C_{\mathrm{INVBI},j}$ = inventory cost of charging tank j per unit time per unit volume

$C_{\mathrm{SETup},j,j',l}$ = changeover cost for transition from crude mix j to j' in CDU l

DM_j = demand of crude mix j by CDUs during the scheduling horizon

$F_{VS,v,j,\min}$ = minimum crude oil transfer rate from vessel v to storage tank i

$F_{VS,v,j,\max}$ = maximum crude oil transfer rate from vessel v to storage tank i

$F_{SB,i,j,\min}$ = minimum crude oil transfer rate from storage tank i to charging tank j

$F_{SB,i,j,\max}$ = maximum crude oil transfer rate from storage tank i to charging tank j

$F_{BC,j,l,\min}$ = minimum crude oil charging rate from charging tank j to CDU l

$F_{BC,j,l,\max}$ = maximum crude oil charging rate from charging tank j to CDU l

$T_{\mathrm{ARR},v}$ = crude vessel v arrival time around the docking station

$V_{V,v,0}$ = initial volume of crude oil in crude vessel v

$V_{S,j,\min}$ = minimum crude oil volume of storage tank i

$V_{S,j,\max}$ = maximum crude oil volume of storage tank i

$V_{S,j,0}$ = initial crude oil volume of storage tank i

$V_{B,j,\min}$ = minimum mixed crude oil volume of charging tank j

$V_{B,j,\max}$ = maximum mixed crude oil volume of charging tank j

$V_{B,j,0}$ = initial mixed crude oil volume of charging tank j

$\xi_{S,i,k}$ = concentration of component k in the crude oil of storage tank i

$\xi_{B,j,k,\min}$ = minimum concentration of component k in the crude mix of charging tank j

$\xi_{B,j,k,\max}$ = maximum concentration of component k in the crude mix of charging tank j

$\xi_{B,j,k,0}$ = initial concentration of component k in the crude mix of charging tank j

Appendix. MILP Model for the Motivating Example

Note that each constraint is ordered in the same manner as in the General Mathematical Formulation section.

Minimize

$$\mathrm{Cost} = 8(T_{L,A} - T_{F,A} + T_{L,B} - T_{F,B} + 2) + 5(T_{F,A} - 1 + T_{F,B} - 5) + 0.05\sum_{i=1}^{8}\left(\frac{V_{S,A,i} + V_{S,A,i-1} + V_{S,B,i} + V_{S,B,i-1}}{2}\right) + 0.08\sum_{i=1}^{8}\left(\frac{V_{B,X,i} + V_{B,X,j-1} + V_{B,Y,i} + V_{B,Y,i-1}}{2}\right) + 50\sum_{i=1}^{8}(Z_{XY,i} + Z_{YX,i}) \quad (A1)$$

s.t.

$$\sum_{t=1}^{8} X_{F,A,t} = 1, \qquad \sum_{t=1}^{8} X_{F,B,t} = 1 \quad (A2a)$$

$$\sum_{t=1}^{8} X_{L,A,t} = 1, \qquad \sum_{t=1}^{8} X_{L,B,t} = 1 \quad (A2b)$$

$$T_{F,A} = \sum_{t=1}^{8} tX_{F,A,t}, \qquad T_{F,B} = \sum_{t=1}^{8} tX_{F,B,t} \quad (A2c)$$

$$T_{L,A} = \sum_{t=1}^{8} tX_{L,A,t}, \qquad T_{L,B} = \sum_{t=1}^{8} tX_{L,B,t} \quad (A2d)$$

$$T_{F,A} \geq 1, \qquad T_{F,B} \geq 5 \quad (A2e)$$

$$T_{L,A} - T_{F,A} \geq 1, \qquad T_{L,B} - T_{F,B} \geq 1 \quad (A2f)$$

$$T_{F,B} \geq T_{L,A} \quad (A2g)$$

$$X_{W,A,t} \leq \sum_{m=1}^{t} X_{F,A,m}, \qquad X_{W,A,t} \leq \sum_{m=t}^{8} X_{L,A,m} \quad t = 1, ..., 8$$

$$X_{W,B,t} \leq \sum_{m=1}^{t} X_{F,B,m}, \qquad X_{W,B,t} \leq \sum_{m=t}^{8} X_{L,B,m} \quad t = 1, ..., 8 \quad (A2h)$$

$$V_{V,A,t} = 100 - \sum_{i=1}^{t} F_{VS,AA,i}, \qquad V_{V,B,t} = 100 - \sum_{i=1}^{t} F_{VS,BB,i} \quad t = 1, ..., 8 \quad (A3a)$$

$$0 \leq F_{VS,AA,t} \leq 50X_{W,A,t}, \qquad 0 \leq F_{VS,BB,t} \leq 50X_{W,B,t} \quad t = 1, ..., 8 \quad (A3b)$$

$$\sum_{t=1}^{8} F_{VS,AAt} = 100, \qquad \sum_{t=1}^{8} F_{VS,BB,t} = 100 \quad (A3c)$$

$$V_{S,A,t} = 25 + \sum_{i=1}^{t}(F_{VS,AA,i} - F_{SB,AX,i} - F_{SB,Ay,i}), \quad V_{S,B,t} = 75 + \sum_{i=1}^{t}(F_{VS,BB,i} - F_{SB,BX,i} - F_{SB,BY,i}) \quad t - 1, ..., 8 \quad (A4a)$$

$$0 \leq F_{SB,AX,t}, F_{SB,BX,t} \leq 50(1 - D_{X,t}), \qquad 0 \leq F_{SB,Ay,t}, F_{SB,BY,t} \leq 50(1 - D_{Y,t}) \quad t = 1, ..., 8 \quad (A4b)$$

$$0 \leq V_{S,A,t}, V_{S,B,t} \leq 100 \quad t = 1, ..., 8 \quad (A4c)$$

$$V_{B,X,t} = 50 + \sum_{i=1}^{t}(F_{SB,AX,i} + F_{SB,BX,\mathrm{i}} - F_{BC,X,t}), \qquad V_{B,Y,t} = 50 + \sum_{i=1}^{t}(F_{SB,BY,i} + F_{SB,BY,i} - F_{BC,Y,i}) \quad t = 1, ..., 8 \quad (A5a)$$

$$5D_{X,t} \leq F_{BC,X,t} \leq 50D_{X,t}, \qquad 5D_{Y,t} \leq F_{BC,Y,t} \leq 50D_{Y,\mathrm{t}} \quad t = 1, ..., 8 \quad (A5b)$$

$$0 \leq V_{B,X,t}, V_{B,Y,t} \leq 100 \quad t = 1, ..., 8 \quad (A5c)$$

$$\sum_{t=1}^{8} F_{BC,X,t} = 100, \qquad \sum_{t=1}^{8} F_{BC,Y,t} = 100 \quad (A5d)$$

$$v_{B,X,t} = 10 + \sum_{i=1}^{t}(f_{SB,AX,i} + f_{SB,BX,i} - f_{BC,X,t}), \qquad v_{B,Y,t} = 30 + \sum_{i=1}^{t}(f_{SB,BY,j} + f_{SB,BY,i} - f_{BC,Y,i}) \quad t = 1, ..., 8 \quad (A6a)$$

$$f_{SB,AX,t} = F_{SB,AX,t}\xi_{S,A,t}, \qquad f_{SB,AY,t} = F_{SB,AY,t}\xi_{S,A,t}$$
$$f_{SB,BX,t} = F_{SB,BX,t}\xi_{S,B,t}, \qquad f_{SB,BY,t} = F_{SB,BY,t}\xi_{S,B,t} \quad t = 1, ..., 8 \quad (A6b)$$

$$F_{BC,X,t}\xi_{B,X,\min} \leq f_{BC,X,t} \leq F_{BC,X,t}\xi_{B,X,\max}, \qquad F_{BC,Y,t}\xi_{B,Y,\min} \leq f_{BC,Y,t} \leq F_{BC,Y,t}\xi_{B,Y,\max} \quad t = 1, ..., 8 \quad (A6c)$$

$$V_{B,X,t}\xi_{B,X,\min} \leq v_{B,X,t} \leq V_{B,X,t}\xi_{B,X,\max}, \qquad V_{B,Y,t}\xi_{B,Y,\min} \leq v_{B,Y,t} \leq V_{B,Y,t}\xi_{B,Y,\max} \quad t = 1, ..., 8 \quad (A6d)$$

$$D_{X,t} + D_{Y,t} = 1 \quad t = 1, ..., 8 \quad (A7b)$$

$$Z_{X,Y,t} \geq D_{Y,t} + D_{X,t-1} - 1, \qquad Z_{Y,X,t} \geq D_{X,t} + D_{Y,t-1} - 1 \quad t = 1, ..., 8 \quad (A7c)$$

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