UNIVERSITY OF TRENTO

Master of Science in Mechatronics Engineering

Suspension kinematic analysis

Professor:
Francesco Biral

Students:

Andrea Cardone	224836
Sara De Godenz	222217
Francesco Mazzoni	224840

Department of Industrial Engineering

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1 Introduction

In the following report is described the kinematics of the suspension model for a GP2 car. The first step consists into the explanation of how the model is derived. After that are shown the different maps describing the relationship between independent coordinates and the dependent ones of interest. The conclusion of the report shows how the roll axis and the equivalent stiffness can be obtained.

2 Kinematic model of the suspension

The full suspension kinematic model is made of 16 constraint equations. There are 7 constraint equations per wheel, which are derived from the initial quarter car model, for a total of 14 constrain equations. Additionally, the motion of one wheel with respect to the other one via the anti-roll bar mechanism needs to be constrained. In order to do so, needs to be introduced other two constraint equations that link each rocker with one side of the anti-roll bar imposing a constant length between the two edges. Doing this, a total of 19 generalised coordinates split into 3 independent ($s(t)$, $z_r(t)$ and $z_l(t)$) and 16 dependent are obtained. So the values of the dependent coordinates can be computed solving numerically the expressions.

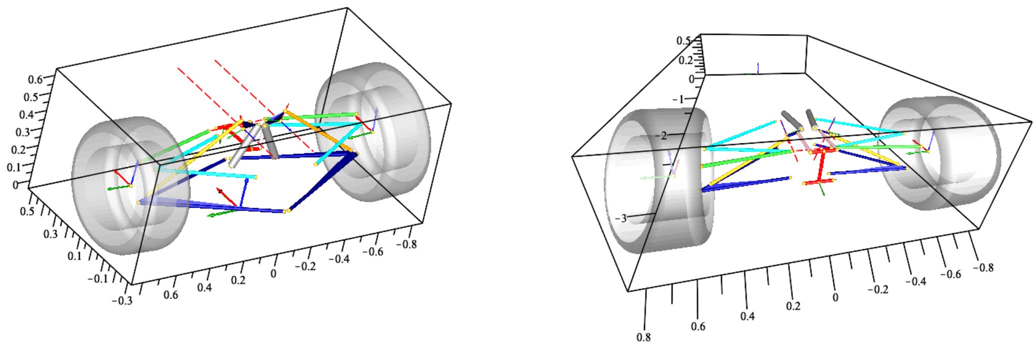

Figure 1: Front and rear suspension model

3 Kinematic maps

In the following section the maps obtained from the model are shown, for their creation 11 breakpoints were used for each input. In particular $z_l(t) \in [-0.0209, 0.0398]$, $z_r(t) \in [-0.0209, 0.0398]$, $s(t) \in [-0.0224, 0.0336]$.

3.1 Wheel hub maps

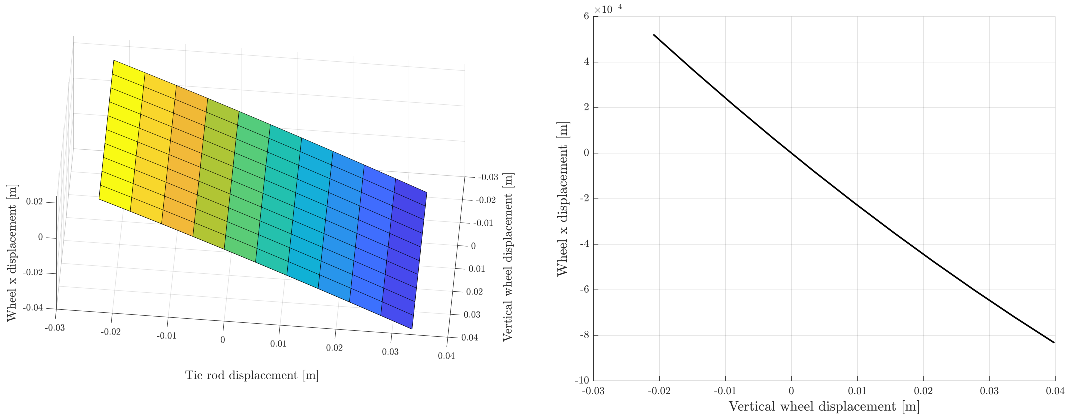

Figure 2: Wheel x displacement, front suspension(left) and rear suspension(right).

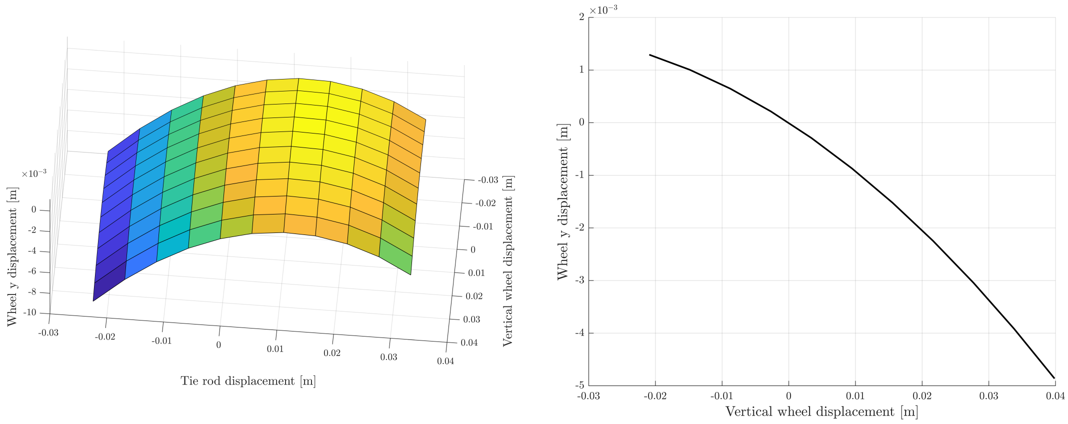

Figure 3: Wheel y displacement, front suspension(left) and rear suspension(right).

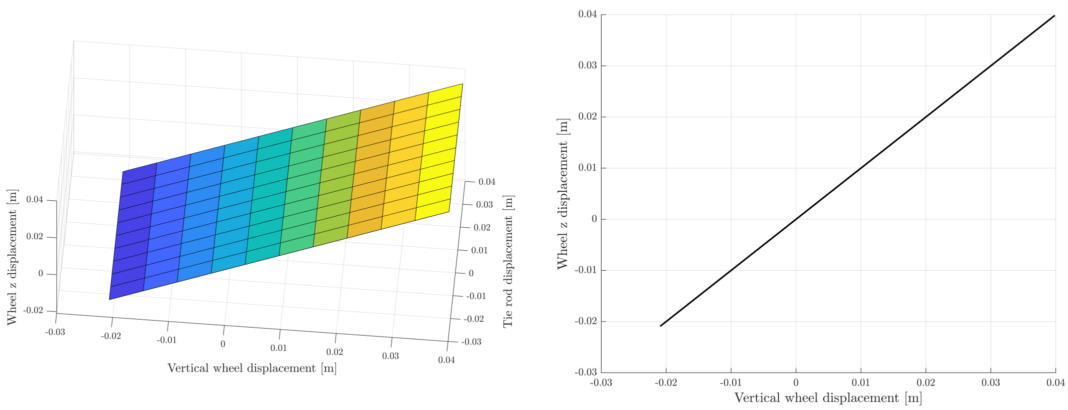

Figure 4: Wheel z displacement, front suspension(left) and rear suspension(right).

3.2 Wheel hub maps

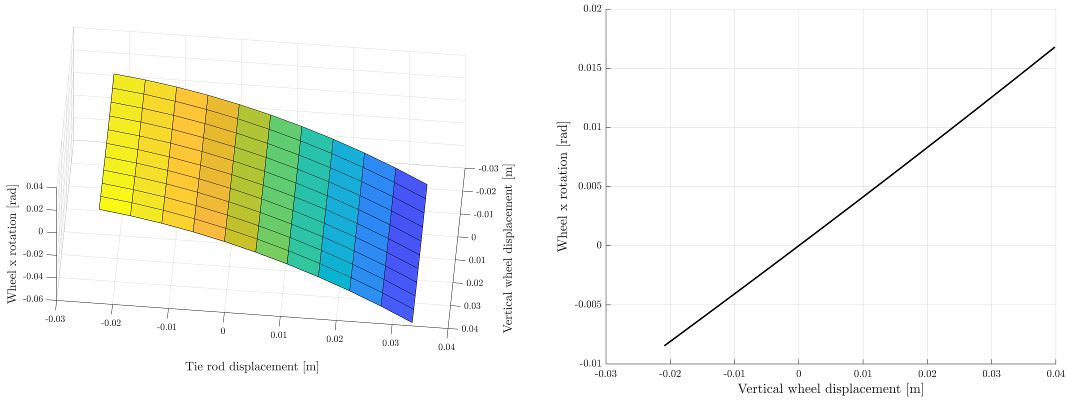

Figure 5: Wheel x rotation, front suspension(left) and rear suspension(right).

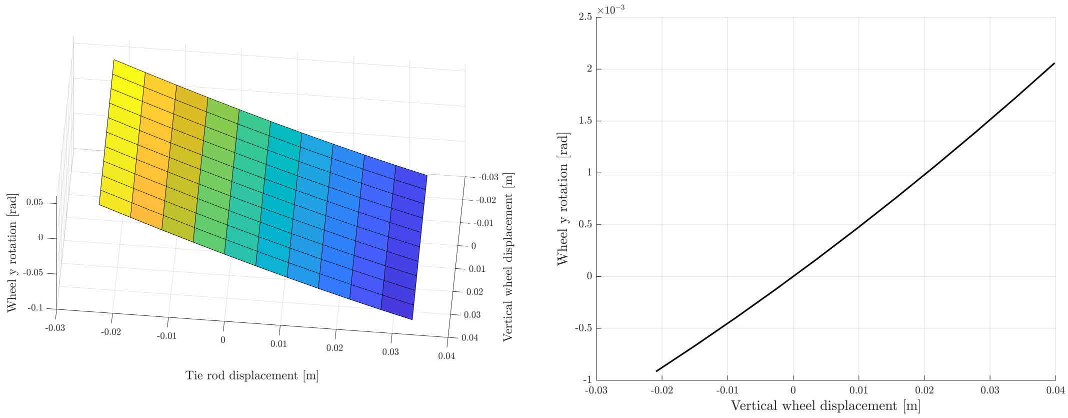

Figure 6: Wheel y rotation, front suspension(left) and rear suspension(right).

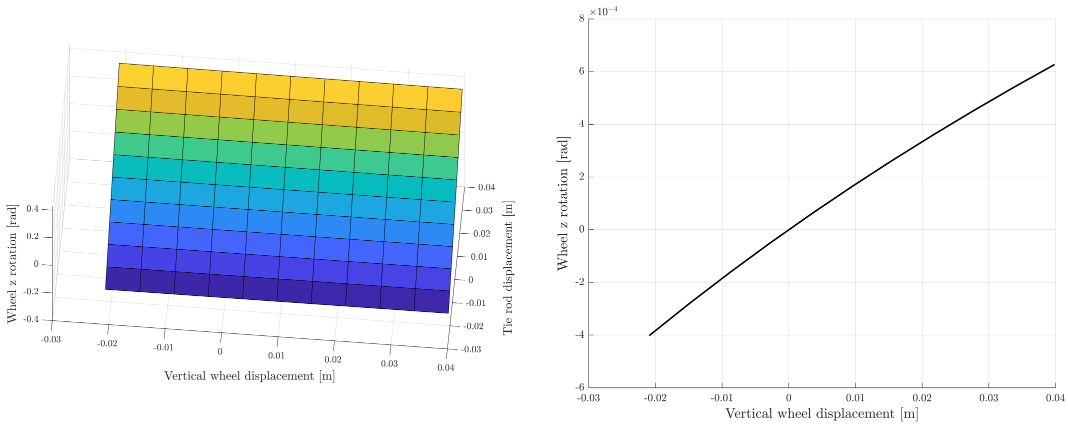

Figure 7: Wheel z rotation, front suspension(left) and rear suspension(right).

3.3 Spring and damper maps

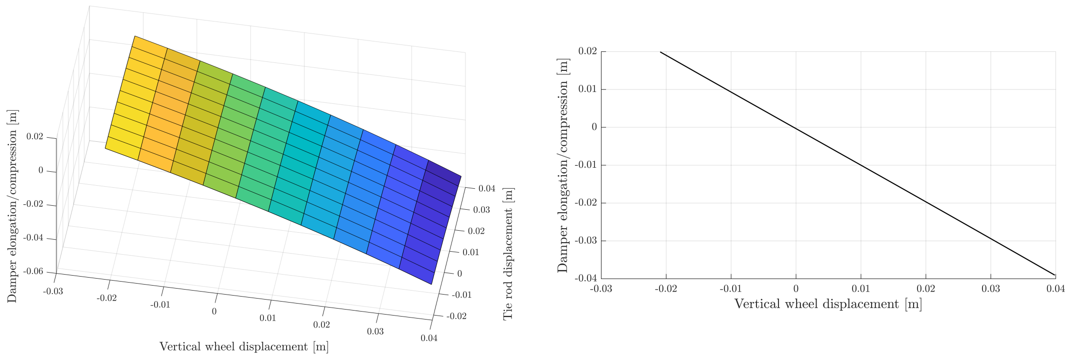

Figure 8: Damper elongation/compression, front suspension(left) and rear suspension(right).

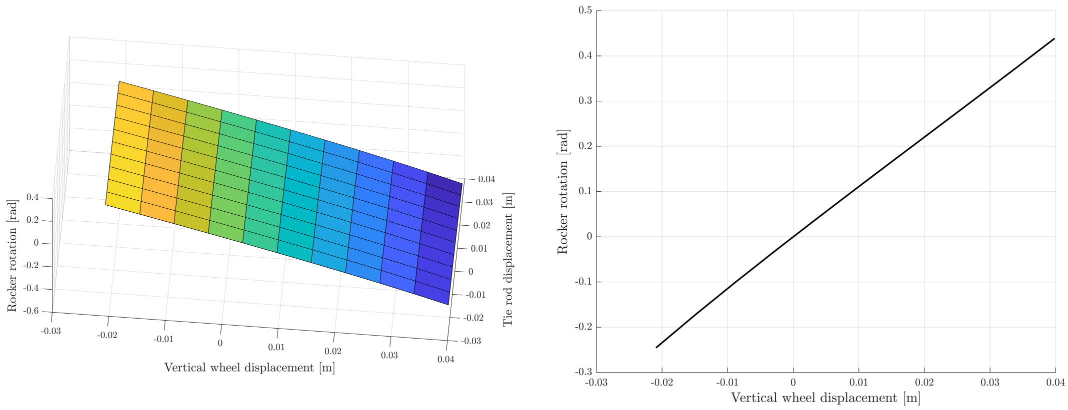

Figure 9: Rocker rotation, front suspension(left) and rear suspension(right).

3.4 Anti roll bar maps

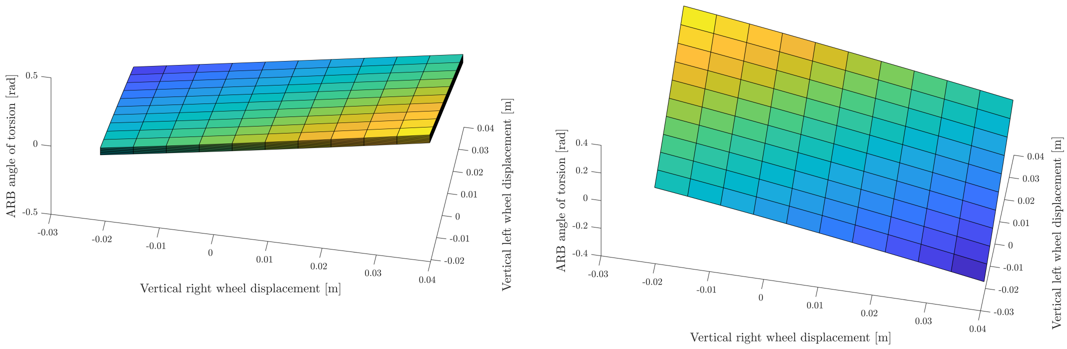

Figure 10: ARB torsion, front suspension(left) and rear suspension(right).

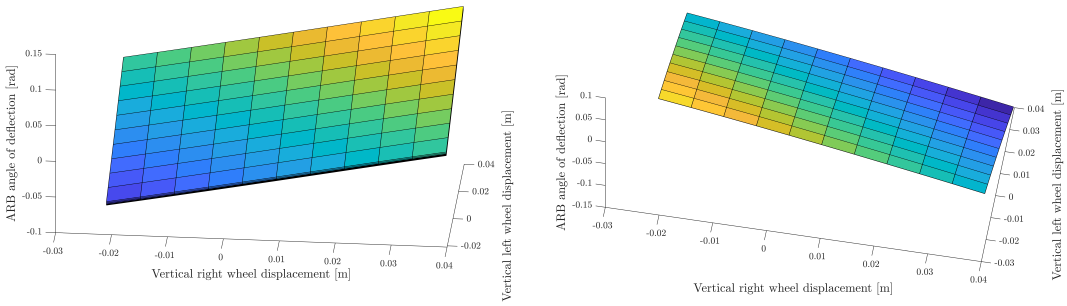

Figure 11: ARB deflection, front suspension(left) and rear suspension(right).

4 Extra analysis

In this section we describe the methods and the results regarding the roll axis - and the equivalent stiffness calculation. Before starting with the calculations, it is necessary to find the equilibrium configuration of the mechanism.

4.1 Equilibrium configuration

The method we implemented to find the equilibrium static configuration is the Principle of Virtual Work. The virtual work of a vertical load F_z on a single tyre has to balance the equivalent work of the elastic element, i.e. a torsional bar for the front - and a spring for the rear axle. Since we are in equilibrium, we will neglect the action of the damper and the one of the anti-roll bar. For the front axle we consider the torsional moment of a bar given the rotation of the rocker θ.

$$-F_z \cdot \delta_z + M_{tor}\delta_\theta = 0 \tag{1}$$

$$F_z \cdot \delta_z = k_{tor} \cdot (\theta_{eq} - \theta_0) \cdot \delta_\theta \tag{2}$$

$$(\theta_{eq} - \theta_0) = \frac{F_z}{k_{tor} \cdot \tau_{\theta z}} \tag{3}$$

Since we found that the initial configuration of the rocker θ_0 is negligible we can write

$$\theta_{eq} = \frac{F_z}{k_{tor} \cdot \tau_{\theta z}} \tag{4}$$

For what concerns the rear axle we consider the action of the spring, that is coaxial with the damper.

$$-F_z \cdot \delta_z + F_{spring}\delta_{l_d} = 0 \tag{5}$$

$$F_z \cdot \delta_z = k_{spring} \cdot (l_{d_{eq}} - l_{d_0}) \cdot \delta_{l_d} \tag{6}$$

$$l_{d_{eq}} = l_{d_0} + \frac{F_z}{k_{spring} \cdot \tau_{l_d z}} \tag{7}$$

The value of F_z is assumed to be $4500N$, l_{d_0} is the length at equilibrium of the damper found from the initial configuration of the mechanism imposing $z_l(t) = 0$. Moreover $k_{tor} = 1621.5Nm/rad$, $k_{spring} = 1.5767 \cdot 10^5 N/m$. What we obtain is that the height of the wheel center changes its relative rotation wrt the chassis. With this new position we can calculate the roll axis position and the equivalent stiffness.

4.2 Roll axis

The roll axis is calculated using a step by step method.

1. Find the straight line which links the two points of the wishbone which are linked to the chassis. This is needed because they have a different height with respect to the ground.
2. Find the slope and the intercept of the straight line passing through the point of the wishbone linked to the wheel and which needs to be perpendicular to the straight found in point 1.
   This is need to be made for both the wishbones: upper and lower.
3. Calculate the intersection point of the two straights found in this way.
4. Find the contact point between the tyre and the road surface.
5. Find the straight passing through the point of intersection found in point 3. and the point found in point 4.
6. Repeat all these point for the other wheel.
7. Find the point obtained by the interception of the two straights obtained in point 5. for the left and right wheel.

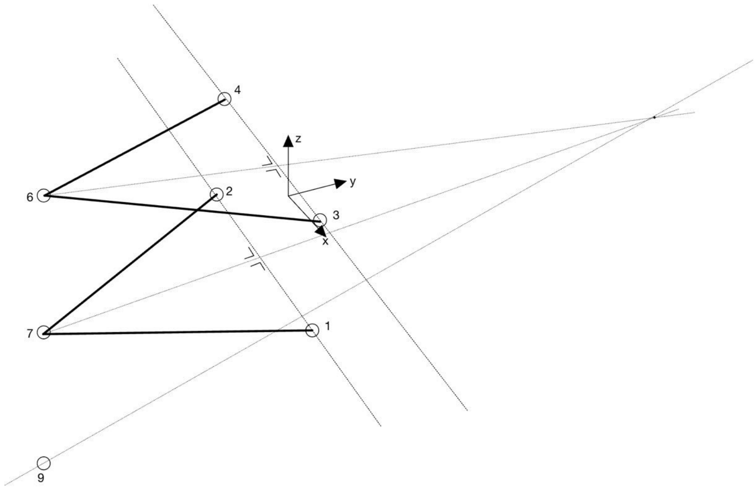

Figure 12: Graphical view of

This type of analysis is made for both front and rear and the points obtained in this way are:

- front: $[z = -0.0165, y = 0]$
- rear: $[z = -0.0191, y = 0]$

Those points are calculated with respect to chassis reference frame.
As we expected the y coordinate for both the axis is $y = 0$ since the system is symmetric. Consider that the coordinates of the points attached to the wheel are obtained by using the dependent coordinates with $z_l(t)$ find at the equilibrium (see the subsection before).

4.3 Equivalent stiffness

The equivalent stiffness we are looking for is the one for a quarter car model, with no vertical motion and of torsional type. In order to find it we apply again the principle of virtual work of the mechanism considering, for the sake of simplicity, only the action of the elastic element and the anti-roll bar. The equations of interest are for the front and for the rear respectively:

$$M_{tor} \cdot \delta_{\theta_{rck}} - k_{arb} \cdot \delta\theta_{arb} = F_{eq} \cdot \delta_z \tag{8}$$

$$-k_{spring} \cdot l_d(z) \cdot \delta_{l_d} + k_{arb} \cdot \delta\theta_{arb} = F_{eq} \cdot \delta_z \tag{9}$$

where $F_{eq} = \frac{k_{eq} \cdot \phi}{W_f/2}$ for the front axle and $F_{eq} = \frac{k_{eq} \cdot \phi}{W_f/2}$ for the rear axle. Since we know the heave motion we can express the roll angle as $\phi = arctan(\frac{z}{W_f/2})$ for the front and $\phi = arctan(\frac{z}{W_r/2})$ for the rear. If we invert the equations of the PLW and substitute the values of roll angle and equivalent force we get

$$k_{eq} = \frac{-k_{tor} \cdot \theta_{rck}(z) \cdot \tau_{\theta_{rck}z} + k_{arb} \cdot \theta_{arb}\tau_{\theta_{arb}z}}{arctan(fraczW_f/2)} \frac{W_f}{2} \tag{10}$$

$$k_{eq} = \frac{-k_{spring} \cdot l_d(z) \cdot \tau_{l_dz} + k_{arb} \cdot \theta_{arb}\tau_{\theta_{arb}z}}{arctan(fraczW_r/2)} \frac{W_r}{2} \tag{11}$$

k_{spring} and k_{tor} are the same used for finding the equilibrium point. $W_f = 1.430m$, $W_r = 1.404$ and $k_{arb} = 652.3650Nm$ at the front and $k_{arb} = 237.6309Nm$. If we use the values of the dependent coordinates found before for the equilibrium we get $k_{eq} = 674858.3396Nm$ at the rear and $k_{eq} = 84658.99843Nm$ at the front.