UNIVERSITY OF TRENTO

Industrial Engineering Department

Master of Mechatronics Engineering

CONTROL OF A SINNGLE-LINK FLEXIBLE MANIPULATOR

Final Project

Course of Automatic Control

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1. Introduction

In this project will be designed the observer and controller of a single-link flexible manipulator (fig. 1) and then simulated the system reaching a desired position θ with small oscillation of the beam final zero velocity.

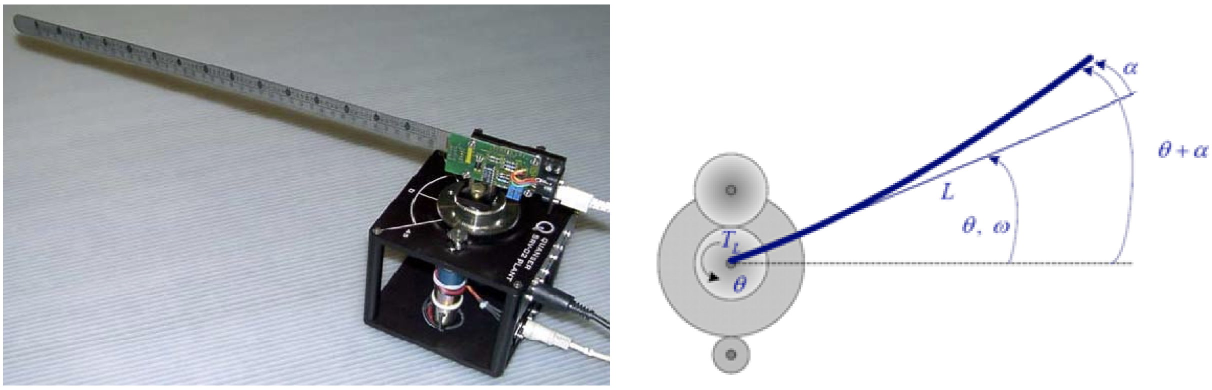

Fig. 1: Experimental setup (left) and system model (right).

The system is described by the plant

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

with

$$x := \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}$$

2. Observability and controllability test

To see if the system is controllable it has been computed its controllability matrix of the system and then its rank is computed. The controllability matrix is full rank and so the system is controllable.
The same is done for the observability matrix, which is also full rank.
Known that the system is both controllable and observable we are able to design a controller and an observer. In fact if the system is controllable we can design a state-feedback regulator with no limitation in the pole-placement. The same statement can be said regarding the design of an observer. So we can design both the controller and the observer theoretically with a convergence rate as fast as we want.
Also the stability test is done, the system is stable showing all eigenvalues with no positive real part.

$$eig(A) = \{0, -11.5449, -11.5449, -12.020\}$$

3. Design of the Luenberger observer

Thanks to the separation principle (LTI systems) we can design separately the L matrix of the observer and the K matrix of the controller.
The output matrix doesn't let us to access to the full state of the system. So we need to design an observer. The synthesis of the observer L gain matrix that ensure a desired convergence speed $\alpha_{obs} = 5$ is done by solving the following LMI and finally checking the feasibility of the solution.

$$W > I$$
$$He(\overline{A}W + \overline{B}X) < -\alpha_{obs}W$$

with

$$\overline{A} = A^T$$
$$\overline{B} = C^T$$
$$\overline{K} = XW^{-1}$$
$$L = K^T$$

The obtained matrix L is:

$$L = 10^5 \begin{bmatrix} 0.0006 \\ -0.0015 \\ -0.9780 \\ 1.4694 \end{bmatrix}$$

with the resulting observer eigenvalues real part:

$$eig(A + LC) = \{-36.6055, -36.6055, -40.0229, -10.5359\}$$

In case of a noisy output y of the state space an arbitrarily large convergence speed may lead to an amplification of the noise with a consequent bad estimation of the state. A good strategy is to choose the right convergence speed such that the observer acts like a low pass filter. This can be done knowing a priori the information about the noise or with a trial an error process. In addition a large convergence speed can cause a large error in the initial part of the transitory of state estimation. Once the right α the L matrix of the Luenberger observer is calculated solving the LMI problem.

4. Design of the controller

Now the design of the gain matrix K of the feedback control law with a convergence speed $\alpha_{ctrl} = 10$ and with minimum actuator effort is achieved is achieved solving the following LMI eigenvalue problem.

$$\min_{W,X,\kappa} \kappa, \quad \textit{subject to}$$

$$W > I$$

$$He(\overline{A}W + \overline{B}X) < -\alpha_{ctrl}W$$

$$\begin{bmatrix} \kappa I & X^T \\ X & \kappa I \end{bmatrix} > 0$$

The obtained matrix K is:

$$K = \begin{bmatrix} -5.6876 & 1.3241 \cdot 10^{-5} & -0.47351 & -0.3115 \end{bmatrix}$$

with the resulting closed-loop eigenvalues real part:

$$eig(A + KB) = \{-11.5449, -11.5449, -10.0008, -12.0199\}$$

and with a bound on the K matrix norm:

$$|K| < 5.7158$$

In this case an arbitrarily large convergence speed α may lead to an arbitrarily large feedback gain K, that can cause an excessive use of the actuators. So due to the physical limits of the actuators authority and the possibility to have an overshoot in the transitory reaching an equilibrium position (especially if the observer is not fast enough), we can’t choose arbitrarily large convergence speed.

5. Simulation

Our controller guarantees an asymptotic convergence to a specific angle θ with no overshoot, the oscillation of the tip also converges exponentially to zero reaching the equilibrium configuration.

In the following figures are reported the Simulink model of the system (fig. 2), the state evolution compared with its estimation for 0.5 s (fig. 3) and the output of the plant (fig. 4) .

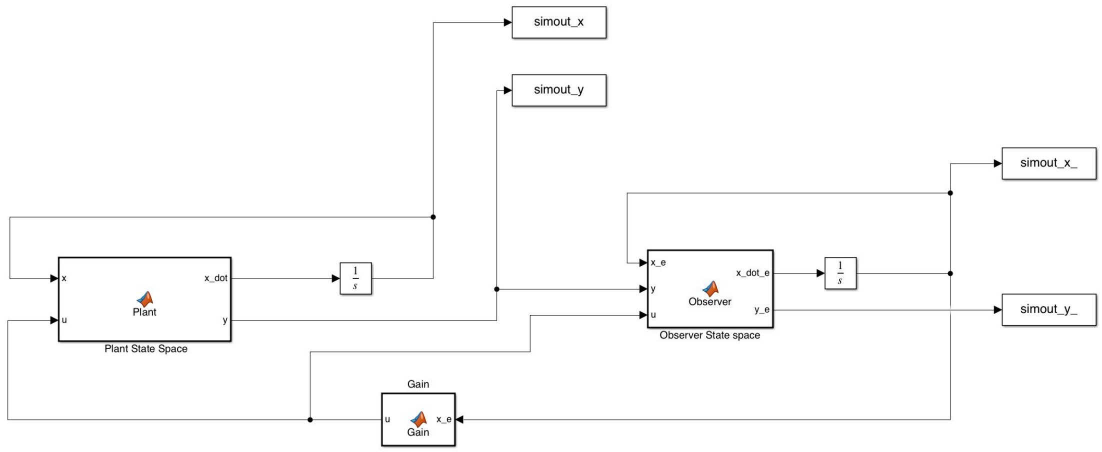

Fig. 2: Simulink system model.

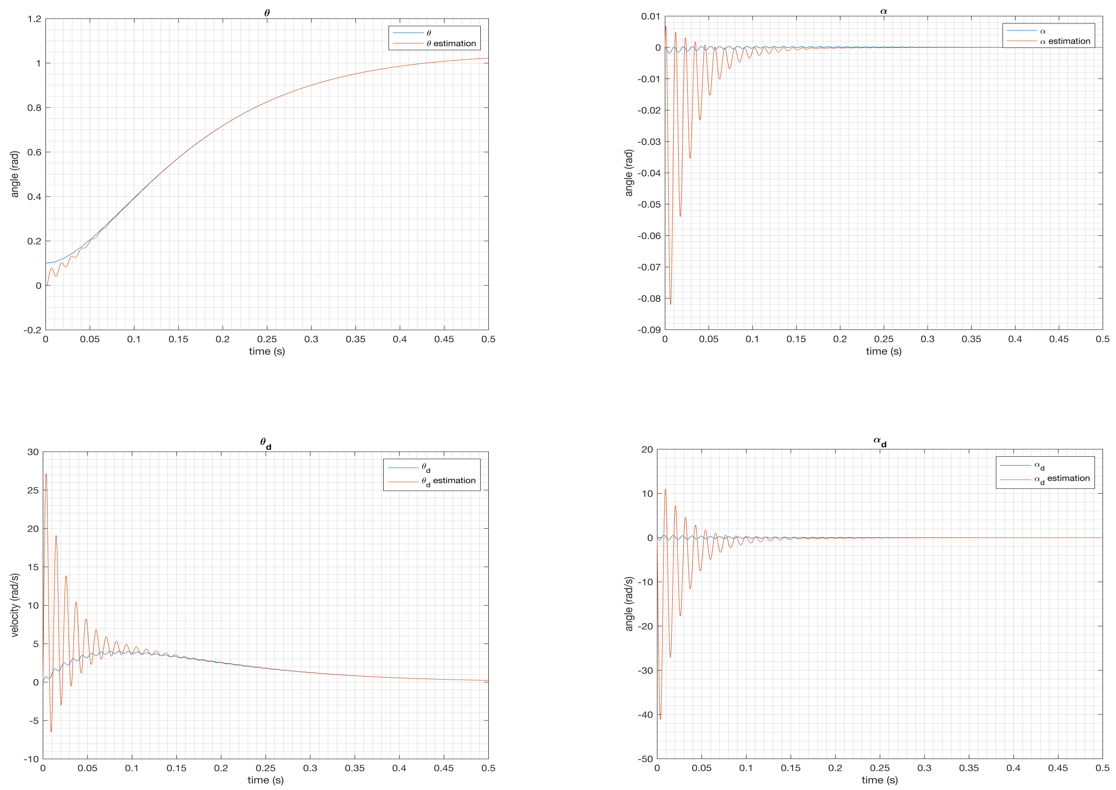

Fig. 3: Evolution of the state compared to the observer estimation.

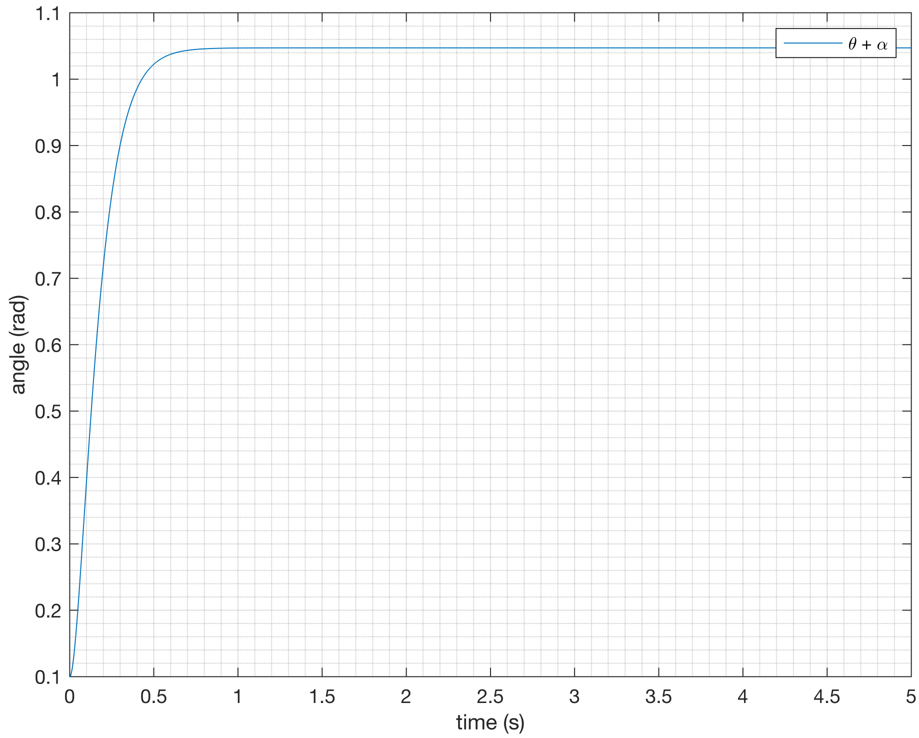

Fig. 4: Output of the system.