

# Hearing the spacetime curvature in quantum noise

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M. Saravani, S. Aslanbeigi, A. Kempf, Phys. Rev. D **93**, 045026 (2016)

# Background

- Why is quantum gravity hard?

*GR is structurally very different from quantum theories*

- Any bridge between GR and QT desirable.
- Here: build one of those bridges.

# Idea

- Quantum fields fluctuate, have quantum noise
- Spacetime curvature affects that quantum noise

Key question:

- Can we get the curvature back from the quantum noise?
- Is the metric expressible in terms of quantum noise?

# How could this work?

- First in flat spacetime:

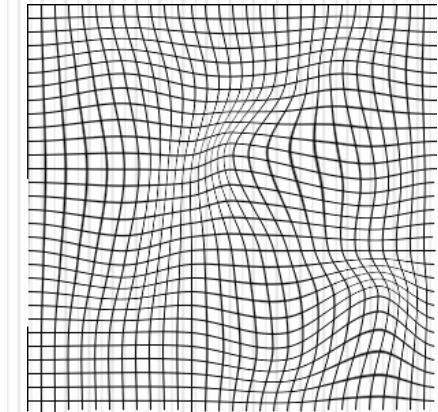
$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = \frac{m^2 c^2}{\hbar^2} \psi$$

- Coupled harmonic oscillators
- Their quantum fluctuations are correlated
- Quantified by: 2-point functions such as the propagator

# In curved spacetime

- Klein Gordon equation now:

$$\left( \frac{1}{\sqrt{g}} \partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu + \frac{m^2 c^2}{\hbar^2} \right) \psi = 0.$$



- Curvature affects the coupling of the oscillators

→ Curvature affects the correlations of quantum noise

→ Curvature affects the propagator

**Does the propagator know all about the curvature?**

# Result

- For dimensions  $D > 2$ , the metric can be expressed in terms of the Feynman propagator:

$$g_{ij}(y) = -\frac{1}{2} \left[ \frac{\Gamma(D/2 - 1)}{4\pi^{D/2}} \right]^{\frac{2}{D-2}} \lim_{x \rightarrow y} \frac{\partial}{\partial x^i} \frac{\partial}{\partial y^j} \left( G(x, y)^{\frac{2}{2-D}} \right)$$

- Proof: covariance, geodesic coordinates, asymptotic behavior
  - Also in paper: worked-out examples
- In principle: Can replace the metric with the propagator!

# Intuitively, why does this work?

- Recall that in 3+1 dimensions:

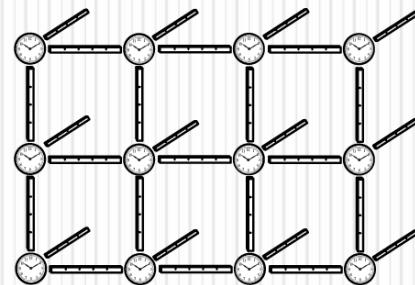
$$\text{curvature} = \text{causal structure} + \text{scalar function}$$

- Propagator knows the causal structure
- But: propagator also indicates effective spacetime distances!

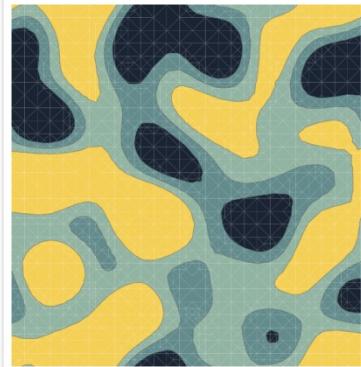
And knowing infinitesimal distances is to know the metric.

# Interpretation

- Einstein built general relativity on rods and clocks



- But no rods and clocks at sub-atomic scales!
- Instead: as distance proxy, use strength of noise correlators:



# Accelerators measure curvature

- Accelerators test Feynman rules, including the propagators
- Measuring the propagator locally is to measure the metric

*With a mobile accelerator one could  
measure the metric anywhere*

# What can we use this tool for ?

- We have a bridge between GR and QFT:  
From the propagator we can calculate the metric.

*Assume some arbitrary model for the Planck scale.*

*How does bridge hold up **when approaching** the Planck scale ?*

- We can calculate the corresponding impact on the metric!

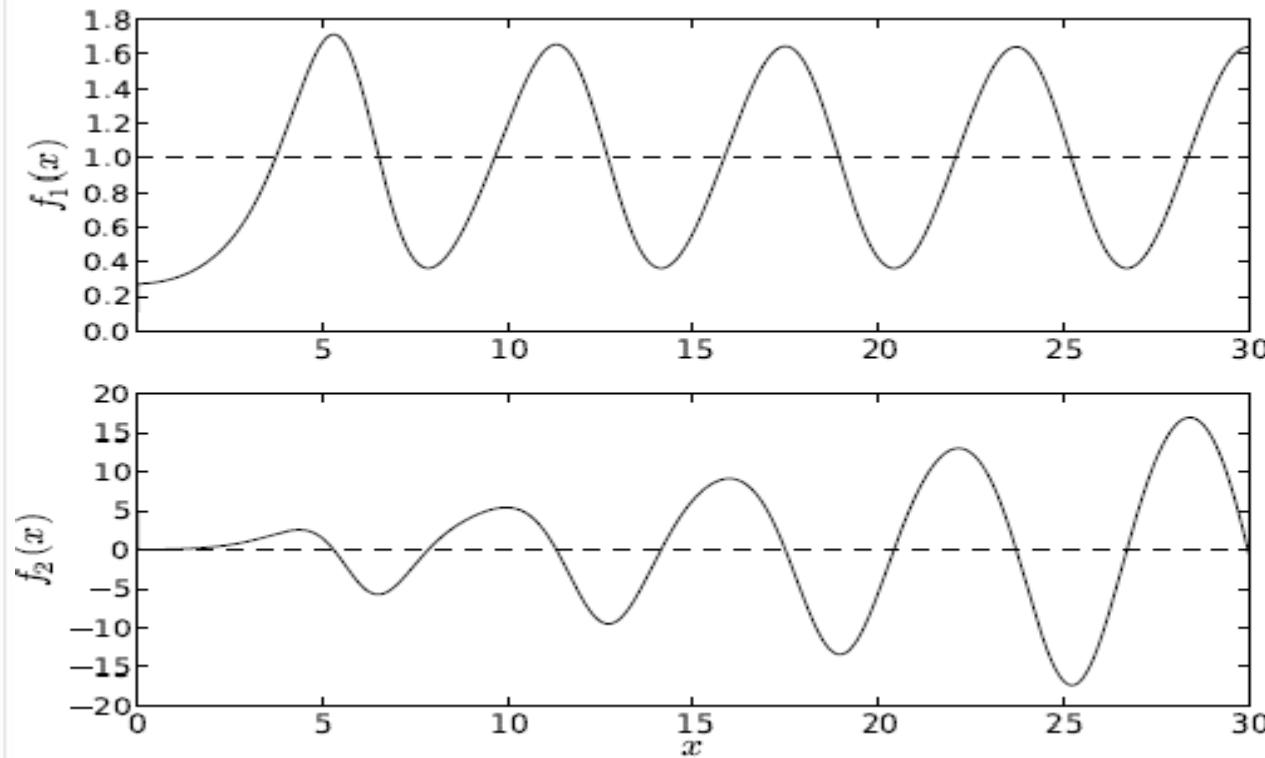
# Example: sharp natural UV cutoff

$$g_{ij}^{\Lambda}(y) \equiv -\frac{1}{2} \left[ \frac{\Gamma(D/2 - 1)}{4\pi^{D/2}} \right]^{\frac{2}{D-2}} \lim_{x \rightarrow y} \frac{\partial}{\partial x^i} \frac{\partial}{\partial y^j} G_{\Lambda}(x, y)^{\frac{2}{2-D}}$$

- Notice: the UV limits  $x \rightarrow y$  and  $\Lambda \rightarrow \infty$  compete !
- Do they commute?
- It's more subtle: only “on average” they do

# Example D=3 flat space:

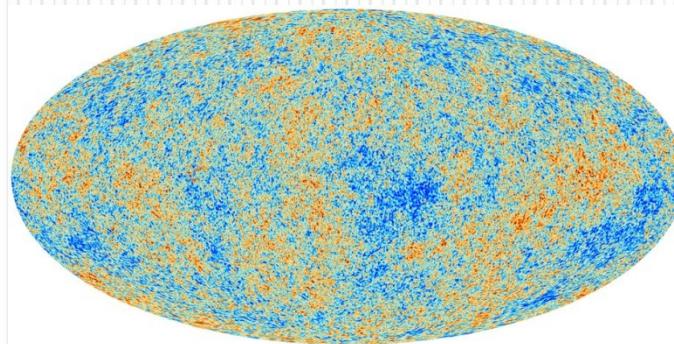
$$g_{\alpha\beta}(x, y) = \delta_{\alpha\beta} f_1(\Lambda r_{xy}) + \frac{(x_\alpha - y_\alpha)(x_\beta - y_\beta)}{r_{xy}^2} f_2(\Lambda r_{xy})$$



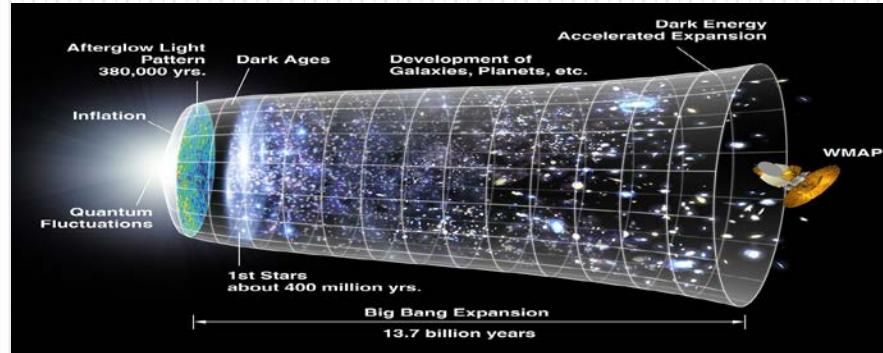
- Oscillations. We recover usual metric “on average”

# Oscillations visible in the CMB ?

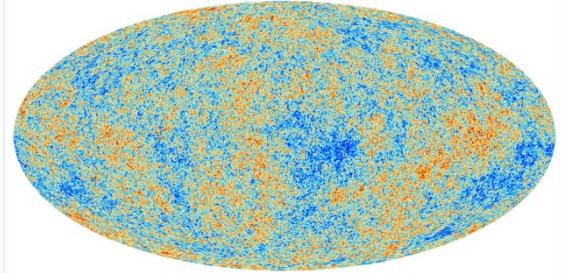
CMB's structure originated close to Planck scale



Hubble scale in inflation only about 5 orders from Planck scale.



# Natural UV cutoffs in inflation



Multiple groups have non-covariant predictions for CMB.

- Effect could be first or second order in (Planck length/Hubble length)  
i.e. could be say  $O(10^{-5})$  or  $O(10^{-10})$

**Big challenge was:**

Predictions with local Lorentz covariant bandlimit cutoff!

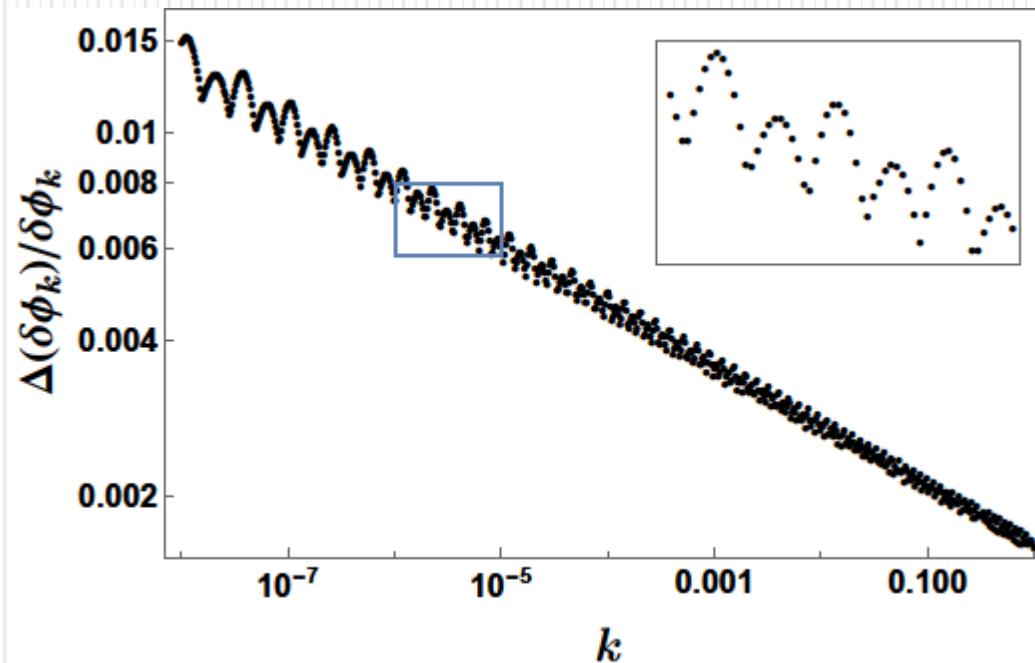
**Recent paper with former students:**

Aidan Chatwin-Davies, (CalTech) and Robert Martin (U. Cape Town)

- ACD, RTM, AK, Phys. Rev. Lett. **119**, 031301 (2017)

# Results for sharp covariant cutoff

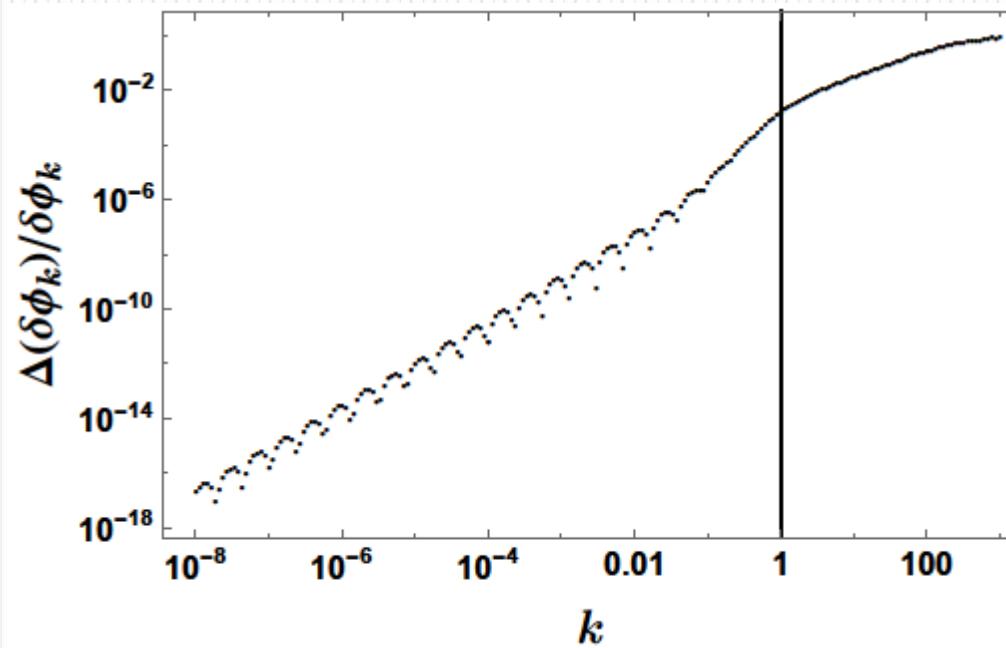
- Power law inflation: relative change in (tensor) spectrum



- Effect is linear in (Planck length/Hubble length)
- Plus characteristic oscillations.

# Results for sharp covariant cutoff

- However, this was assuming decoherence at horizon crossing!



- If decoherence at re-heating, then spectrum is very robust.

# Summary

- Quantum noise knows all about curvature
- Metric expressible through Feynman propagator
- Covariant natural UV cutoff →

Oscillations in inflationary predictions for CMB