arXiv:0811.0827v3 [astro-ph] 16 Feb 2009

The Effective Theory of Quintessence: the $w < -1$ Side Unveiled

Paolo Creminelli[a], Guido D'Amico[b,c],
Jorge Noreña[b], and Filippo Vernizzi[d,a]

[a] *Abdus Salam International Center for Theoretical Physics*
Strada Costiera 11, 34014, Trieste, Italy

[b] *SISSA/ISAS, via Beirut 2-4, 34014, Trieste, Italy*

[c] *INFN - Sezione di Trieste, 34014 Trieste, Italy*

[d] *CEA, IPhT, 91191 Gif-sur-Yvette cédex, France*
CNRS, URA-2306, 91191 Gif-sur-Yvette cédex, France

Abstract

We study generic single-field dark energy models, by a parametrization of the most general theory of their perturbations around a given background, including higher derivative terms. In appropriate limits this approach reproduces standard quintessence, k-essence and ghost condensation. We find no general pathology associated to an equation of state $w_Q < -1$ or in crossing the phantom divide $w_Q = -1$. Stability requires that the $w_Q < -1$ side of dark energy behaves, on cosmological scales, as a k-essence fluid with a virtually zero speed of sound. This implies that one should set the speed of sound to zero when comparing with data models with $w_Q < -1$ or crossing the phantom divide. We summarize the theoretical and stability constraints on the *quintessential plane* $(1+w_Q)$ vs. speed of sound squared.

1 Introduction

The origin of the present acceleration of the Universe is likely to be the most important theoretical problem in physics today. Given the general reluctance in accepting as explanation an incredibly small cosmological constant and the absence of compelling alternatives, it seems that one should keep an open-minded approach, concentrating on very general theoretical constraints and on observables more than on specific models.

In this paper we study in generality, and focusing on perturbations, dark energy scenarios where the dark sector is described by a single scalar degree of freedom, without direct coupling to matter

(in the Einstein frame). We will often call this general model *quintessence*, although in the literature this name is usually reserved to a scalar field with a canonical kinetic term.

Following [1, 2], we will rewrite the scalar field Lagrangian in order to make explicit what is the most general theory of quintessence perturbations around a given background solution characterized by its pressure p_Q and energy density ρ_Q. The freedom that we have after the unperturbed history is fixed is made clear in this way. This separation is particularly important given that a host of new experiments is going to test dark energy clustering properties [3]. In our formalism, the general theoretical constraints on single field models are also made clear.

In particular, we will study whether a single field model that is safe from ghost and gradient instabilities can have an equation of state $w_Q < -1$, where $w_Q = p_Q/\rho_Q$. In this regime, the stability of the model can be guaranteed by the presence of higher derivative operators, a conclusion already reached in [1], where single field models were studied focusing on the constraints enforced by stability. Here, after reviewing and extending the results of [1], we will concentrate on the behaviour of cosmological perturbations, which are relevant for observations. On cosmological scales we find that these higher derivative terms are irrelevant for the phenomenology, so that a model with $w_Q < -1$ simply behaves as a k-essence fluid with virtually zero speed of sound. Higher derivative terms are relevant for cosmology only when the equation of state gets very close (and experimentally indistinguishable from) a cosmological constant. In this limit our general Lagrangian reduces to the Ghost Condensate theory [4] smoothly connecting quintessence to this theory of modification of gravity. Notice that, as detailed in Appendix A, we are interested in a regime where higher derivative terms do not introduce additional degrees of freedom (contrary to what happens for example in [5]). We find it convenient to summarize our results in the plane $(1 + w_Q)\Omega_Q$ vs. c_s^2, where Ω_Q is the quintessence contribution to the critical density. We dub this plane of parameters the *quintessential plane*.

We also study the issue of whether it is possible to cross the so-called phantom divide $w_Q = -1$ [6, 7]. We find that the speed of sound vanishes exactly at the divide [8, 9] and since quintessence may remain stable for $w_Q < -1$ there is no general pathology associated with the crossing. We show this explicitly with an example. The phantom divide can be crossed with a single scalar degree of freedom, without introducing ghost-like fields.

The paper is organized as follows. In section 2 we study the most general theory of single field quintessence, taking into account higher derivative operators and focusing on the stability constraints following [1]. In section 3 we study the phenomenology in various limits, considering also the gravitational effect of dark matter on quintessence. In section 4 we consider the issue of crossing the phantom divide $w_Q = -1$ and we show explicit examples of the crossing without pathologies. In section 5 we concentrate on another kind of higher derivative operators [1, 2], different from the ones studied for the Ghost Condensate. Although the phenomenology on cosmological scales does not change, the modification of gravity at short distances is quite different. Conclusions are drawn

in section 6.

Several issues concerning our effective theory approach are left to the appendix. Appendix A is devoted to reviewing how higher derivative operators must be treated in the effective field theory approach. In appendix B we write down the stress-energy tensor for the action discussed in the main text. In appendix C we derive the most general action for quintessence perturbations following the approach of [1, 2]. Finally, in appendix D we discuss the modification of gravity induced by the kind of higher derivative operators that were not studied in [4].

2 Effective theory of quintessence

Our aim is to study the most general theory of quintessence perturbations. We will do it step by step, first by considering a model with an action containing at most a single derivative acting on the field. This is known as k-essence [10, 11] and it will be possible to write the action for the perturbations in such a way as to make explicit the dependence on the background energy density and pressure ρ_Q and p_Q. Then we will add higher derivative operators to the k-essence action in such a way as to leave the background invariant. In this section we will consider the kind of operators introduced in the context of ghost condensation. Other higher derivative operators will be discussed later in section 5.

An alternative derivation of the most general action for quintessence perturbations is given in appendix C following the approach of refs. [1, 2], that consists in writing down all the terms preserving the symmetries of the system in a 'unitary' gauge, where the quintessence perturbation is set to zero and appears as a scalar metric degree of freedom. This approach is elegant and straightforward but less pedagogical than the one adopted here. Both approaches lead to the same physical results.

2.1 The limit of k-essence

Let us start with a k-essence action

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X) \ , \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \ . \tag{2.1}$$

We assume a flat Friedmann-Lemaître-Robertson-Walker (FLRW) Universe with metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$. Initially, we will treat this as a fixed background and neglect the perturbations of the metric.

To describe perturbations around a given background solution $\phi_0(t)$, it is useful to write the scalar field as

$$\phi(t, \vec{x}) = \phi_0(t + \pi(t, \vec{x})) \ , \tag{2.2}$$

and expand the action (2.1) in terms of π. In the following, we are going to assume that the function $\phi_0(t)$ is strictly monotonic, $\dot{\phi}_0(t) \neq 0$, to avoid singularities in the relation between ϕ and π.

Using the expansions

$$\phi(t,\vec{x}) = \phi_0 + \dot{\phi}_0\pi + \frac{1}{2}\ddot{\phi}_0\pi^2 + \dots \,, \tag{2.3}$$

$$X(t,\vec{x}) = X_0 + \dot{X}_0\pi + \frac{1}{2}\ddot{X}_0\pi^2 + 2X_0\dot{\pi} + 2\dot{X}_0\pi\dot{\pi} + X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + \dots \,, \tag{2.4}$$

where $X_0 = \dot{\phi}_0^{\,2}$, we have, up to second order in π,

$$S = \int \mathrm{d}^4x\, a^3 \left[P_0 + \dot{P}_0\pi + \frac{1}{2}\ddot{P}_0\pi^2 + 2P_X X_0\dot{\pi} + 2\left(P_X X_0\right)^{\cdot}\pi\dot{\pi} + P_X X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + 2P_{XX}X_0^2\dot{\pi}^2\right] \,, \tag{2.5}$$

where $P_X = \partial P/\partial X|_0$ and $P_{XX} = \partial^2 P/\partial X^2|_0$. The term P_0 does not affect perturbations as it is independent of π, while one can verify that the linear terms cancel using the background equation of motion. Indeed, by integrating by parts the term $\pi\dot{\pi}$ and making use of the background equation of motion, after some manipulations we are left with

$$S = \int \mathrm{d}^4x\, a^3 \left[\left(P_X X_0 + 2P_{XX}X_0^2\right)\dot{\pi}^2 - P_X X_0\frac{(\nabla\pi)^2}{a^2} + 3\dot{H}P_X X_0\,\pi^2\right] . \tag{2.6}$$

We can now rewrite the coefficients of this expansion in terms of the stress-energy tensor of the background solution. From the definition of the stress-energy tensor,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \,, \tag{2.7}$$

one obtains the background energy density and pressure,

$$\rho_Q = 2X_0P_X - P_0 \,, \qquad p_Q = P_0 \,. \tag{2.8}$$

Using these expressions the action above can be cast in the form

$$S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2\right] . \tag{2.9}$$

Here we have defined $M^4 \equiv P_{XX}X_0^2$, where M has the dimension of a mass.

At this stage we can straightforwardly introduce the coupling with metric perturbations. This coupling is particularly simple in synchronous gauge where the metric takes the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j \,. \tag{2.10}$$

Indeed, at quadratic order in the action the coupling with gravity only comes through the perturbed $\sqrt{-g}$ in the action. Replacing a^3 with $a^3(1 + h/2)$ in eq. (2.5) we have

$$S = \int \mathrm{d}^4x\, a^3 \left(1 + \frac{h}{2}\right)\left[\dot{P}_0\pi + 2P_X X_0\dot{\pi} + \dots\right] . \tag{2.11}$$

Integrating by parts and using again the background equation of motion one gets the full action for π,

$$S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi \right] . \quad (2.12)$$

The quadratic Lagrangian is thus specified by the functions $(\rho_Q + p_Q)(t)$ and $M^4(t)$. It is important to stress that these two functions are completely unconstrained. For any choice of these two functions one can in fact construct a Lagrangian $P(\phi, X)$, such that the quadratic Lagrangian around the unperturbed solution has the form (2.12). Let us see explicitly how this works. First of all, given $\rho_Q + p_Q$ (and possibly the other contributions to the total energy density and pressure coming from other components) one can find the two functions $\rho_Q(t)$ and $p_Q(t)$ solving the Friedmann equations.[1] At this point it is easy to check that the Lagrangian

$$P(\phi, X) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi)X + \frac{1}{2}M^4(\phi)(X-1)^2 \quad (2.13)$$

has the solution $\phi = t$, gives the requested pressure and energy density as a function of time, and gives eq. (2.12) as the quadratic action for perturbations. Note that the dimension of ϕ is that of an inverse of a mass. The coefficient M^4 is time dependent and for quintessence we expect that it varies with a time scale of order H^{-1}. Somewhere in this paper, to simplify the calculations we take $M^4 = \text{const}$.

One advantage of the action (2.12) is that the coefficients of all terms are physically measurable quantities – we will see below that M^4 is related to the sound speed of perturbations. This standard form of rewriting the action of quintessence perturbations does not suffer from field redefinition ambiguities. Indeed, there is an infinite number of physically equivalent Lagrangians $P(\phi, X)$ related by field redefinitions $\phi \to \tilde{\phi}(\phi)$ but they all give the same action (2.12). Note that for most purposes the explicit construction of the action in terms of ϕ is irrelevant. Indeed, one is free to choose *any* function $\rho_Q(a)$ to describe the evolution of the quintessence energy density as a function of the scale factor. Then the action (2.12) will describe the perturbations around this background. In particular, one can always make a field redefinition such that $\phi = t$, as we did in eq. (2.13).

Let us see what are the theoretical constraints that we can put on the general form of the action (2.12). A basic requirement that we will impose on our theory is that it is not plagued by ghosts, i.e. that its kinetic energy term is positive,

$$\rho_Q + p_Q + 4M^4 > 0 \; . \quad (2.14)$$

[1] One has to solve the continuity equation for quintessence, $\dot{\rho}_Q + 3H(\rho_Q + p_Q) = 0$, where $\rho_Q + p_Q$ is a known function of time, together with the Friedmann equation, $H^2 = (\rho_Q + \rho_{\rm rest})/3M_{\rm Pl}^2$ where $\rho_{\rm rest}$ includes all the additional sources of energy density in the Universe. These equations can be integrated up to a constant in the initial condition. This ambiguity corresponds to a shift in the cosmological constant which does not enter in the action for perturbations.

The presence of a sector with the "wrong" sign of the energy implies that the Hamiltonian is unbounded from below. If one studies this sector alone, no pathology arises as the sign of the energy is a matter of convention. However, quintessence is (at least) gravitationally coupled to the rest of the world, so that there is the danger of exchanging energy between a healthy sector and a negative-energy one without bound. Classically this is not a problem if quintessence perturbations are very small and remain in the linear regime. Therefore, for a quintessence with negligible clustering (with speed of sound $c_s \sim 1$), there is no obvious classical danger. At the quantum level the situation is more pathological. The vacuum is unstable to the spontaneous decay into positive and negative energy states and the decay rate is UV divergent and strictly infinite in Lorentz invariant theories [12]. Although it has been shown that it is possible to cut-off this instability in a non-Lorentz invariant theory [13], in this paper we take a more conservative approach and forbid the existence of ghosts.

If we set $M^4 = 0$ in the general action (2.12) we reduce to the case of a standard quintessence field with canonical kinetic term $(\partial\phi)^2$. In this case, forbidding the ghost implies $\rho_Q + p_Q > 0$. Thus, as it is well known, a scalar field with minimal kinetic term can violate the null energy condition, equivalent in a cosmological setting to $\rho + p < 0$, only if it is a ghost [14]. In this simple case the speed of sound of scalar fluctuations is $c_s^2 = 1$. When M^4 does not vanish the speed of sound of fluctuations differs from unity [15] and reads

$$c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4} \ . \tag{2.15}$$

One can see that, for $\rho_Q + p_Q > 0$, i.e. for positive c_s^2, $M^4 < 0$ implies that scalar perturbations propagate super-luminally [16, 17]. This is problematic in a theory with a Lorentz invariant UV completion [18].

From the action (2.12) we see that in the presence of M^4 there is no generic connection between the violation of the null energy condition and a wrong sign of the time-kinetic term. The coefficient in front of $\dot{\pi}^2$ can be positive also when $\rho_Q + p_Q < 0$. On the other hand, $\rho_Q + p_Q$ fixes the sign of the term in front of the spatial kinetic term $(\nabla\pi)^2$ [19]. Thus, in absence of ghosts, the violation of the null energy condition implies a negative speed of sound squared. The constraints that we have derived are summarized in the *quintessential plane* $1 + w_Q$ vs. c_s^2, represented in figure 1.

An imaginary speed of sound ($c_s^2 < 0$) represents a gradient instability of the system. Taking $M^4 \gg |\rho_Q + p_Q|$ the gradient term is suppressed and the instability is irrelevant for scales of cosmological interest in the sense that the instability time is much longer than the age of the Universe. Still, the instability is relevant for short wavelengths so that it seems difficult to make sense of the $w_Q < -1$ region [8]. However, this conclusion is too hasty. Indeed, when the term $(\nabla\pi)^2$ is suppressed, higher derivative operators may become relevant as we will discuss in the next section.

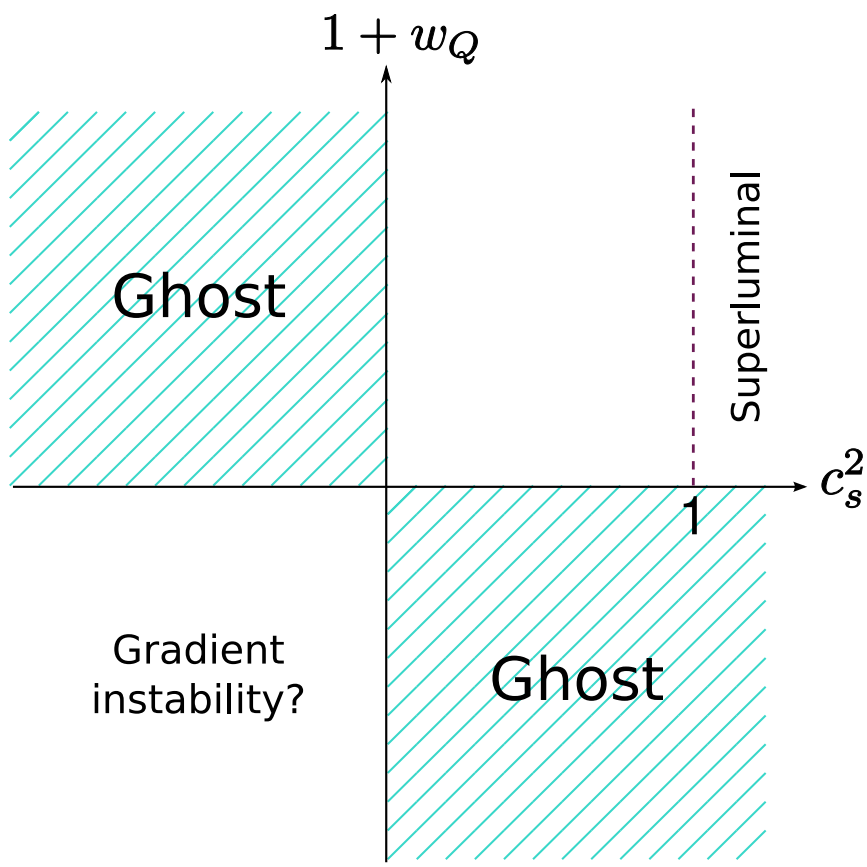

Figure 1: *The quintessential plane* $1 + w_Q$ *vs.* c_s^2 *in the case of* k*-essence. If we require the absence of ghosts, the sign of the spatial kinetic term is fixed to be the same as* $1 + w_Q$*, so that one has to worry about gradient instabilities for* $1 + w_Q < 0$*. For* $1 + w_Q > 0$ *one has superluminal propagation for* $M^4 < 0$*.*

2.2 Higher derivative terms and stability

In the previous section we saw that in the limit in which the quintessence speed of sound goes to zero, the standard spatial kinetic term vanishes. Obviously the action (2.1) is not the end of the story: the full Lagrangian will contain higher derivative operators such as $Q(X)R(\Box\phi)$ and these will give rise to the leading higher derivative spatial kinetic term. A generic higher derivative operator will however change the background solution of (2.1), while here we want to study the effect of higher derivative operators on quintessence perturbations around a given background, as we did in eq. (2.13). To keep the background unchanged, let us add to the Lagrangian (2.13) the operator

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}(\Box\phi + 3H(\phi))^2 \ . \tag{2.16}$$

$-\frac{\partial \rho}{\partial v} \neq 0$ $\quad p = \omega \rho \quad$ $-\omega \frac{\partial \rho}{\partial v} =$

For reasons that will become clear later, we need $\bar{M}^2 > 0$.[2] This term does not alter the background evolution $\phi = t$, $\rho_Q(t)$ and $p_Q(t)$. Indeed, $\Box\phi + 3H(\phi)$ vanishes on the background so that the operator is explicitly quadratic in the perturbations. At quadratic order this operator reads (neglecting for the moment metric perturbations)

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right)^2 . \tag{2.17}$$

One may worry about the presence of terms with higher time derivatives, as these would naïvely be associated with additional solutions of the equation of motion. However, if one compares $\bar{M}^2\ddot{\pi}^2$ with the standard time kinetic term $M^4\dot{\pi}^2$ of eq. (2.12) – assuming $\bar{M} \sim M$ – the former is always suppressed with respect to the latter for frequencies below the scale M. In general, we expect that for frequencies $\omega \sim M$ all operators containing higher time derivatives become important, so that the scale $M \sim \bar{M}$ sets the maximum energy scale for which the theory makes sense: it is the energy cutoff. This is the standard situation in an effective field theory: higher derivative terms become important for energies of the order of the cutoff and at lower energies they must be treated perturbatively. In particular, there is no physical meaning in the new solutions that arise from taking higher and higher time derivatives. We postpone a complete discussion about this point to appendix A. Notice that the same argument cannot be used for the operator $-\bar{M}^2(\nabla^2\pi)^2/2$. Indeed, in the limit of small $\rho_Q + p_Q$ there is no spatial kinetic term of the form $(\nabla\pi)^2$ so that $-\bar{M}^2(\nabla^2\pi)^2/2$ is the leading spatial kinetic term. At short scales we have a non-relativistic dispersion relation of the form $\omega \simeq k^2/M$ which implies that energy and momentum behave very differently (as we will see in appendix A they have different *scaling dimensions*). In particular, when comparing the first and last terms in the brackets in eq. (2.17) we have $\nabla^2\pi \sim M\dot{\pi} \gg \ddot{\pi}$, for energies below the cutoff. This means that we can drop $\ddot{\pi}$ altogether from eq. (2.17).

It is important to stress that there is no fine tuning in the limit $|\rho_Q + p_Q| \ll M^4$ – or equivalently $|c_s^2| \ll 1$ – as this limit is "technically natural", i.e., there is a symmetry that is recovered in the limit $\rho_Q + p_Q = 0$. Indeed, as shown below, in this limit we obtain the Ghost Condensate theory [4], which is invariant under the shift symmetry $\phi \to \phi + \lambda$. In the presence of this symmetry the expansion of the Universe drives the background solution to $\rho_Q + p_Q = 0$. Thus, models with very small speed of sound should be thought of as small deformations of the Ghost Condensate limit [20, 1, 2].

Let us come back to the issue of stability for $\rho_Q + p_Q < 0$ including the new higher gradient term (2.17) and follow the discussion in [1]. As we discussed in the previous section, the only dangerous modes are those on scales much smaller than the Hubble radius, as their instability rate can be arbitrarily large; we thus concentrate on $k/a \gg H$. In this regime the operator (2.17) further simplifies as the second and third terms are negligible with respect to $\nabla^2\pi$. Considering the action

[2] As M^4 also $\bar{M}^2$ can have a time dependence on a time scale of order H^{-1}. For simplicity, in the following we assume that it is constant.

(2.12) with the addition of the only remaining operator $-\bar{M}^2(\nabla^2\pi)^2/2$, the dispersion relation of π is thus modified to

$$\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0\,. \tag{2.18}$$

For $\rho_Q + p_Q < 0$ the k^2 gradient term has the unstable sign, but in the presence of the new operator this instability is confined to sufficiently large scales. In particular the fastest rate of instability is given by

$$\omega^2_{\rm grad} \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4}\,, \tag{2.19}$$

where we have taken $M^4 \gg |\rho_Q + p_Q|$. This gradient instability is not dangerous when it is slower than the Hubble rate, i.e., when

$$-\frac{\rho_Q + p_Q}{\bar{M}M^2} \lesssim H\,. \tag{2.20}$$

It is clear that a larger $\bar{M}$ makes the gradient instability slower.

However, a large $\bar{M}$ sources another form of instability, which contrarily to the gradient instability is already present for $\rho_Q + p_Q = 0$ and was originally discussed for the Ghost Condensate theory in [4]. Indeed, when the coupling with gravity is taken into account, the system shows a sort of Jeans instability similarly to a standard fluid. To see this, let us introduce metric perturbations and consider the limit $\rho_Q + p_Q = 0$. In this case, the complete Lagrangian reads, neglecting the expansion of the Universe,

$$S = \int {\rm d}^4x \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(\frac{\dot{h}}{2} - \nabla^2\pi\right)^2\right]\,. \tag{2.21}$$

We see that a large $\bar{M}$ enhances the mixing of π with gravity, i.e. the Jeans instability.

Thus, the equation of motion for π reads, in this case,

$$\ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = \frac{\bar{M}^2}{8M^4}\nabla^2\dot{h}\,. \tag{2.22}$$

The gravitational perturbation h is sourced by the perturbations of π through Einstein equations. In synchronous gauge, h satisfies [21]

$$\ddot{h} = -\frac{1}{M^2_{\rm Pl}}(\delta\rho_Q + 3\delta p_Q)\,, \tag{2.23}$$

where we have neglected the expansion of the Universe, and we have introduced the reduced Planck mass $M^2_{\rm Pl} \equiv (8\pi G)^{-1}$. The stress-energy tensor can be straightforwardly derived by varying the action (the complete expression of the stress-energy tensor is given in appendix B) and the leading term is $\delta\rho_Q = 4M^4\dot{\pi}$ while the pressure perturbation is negligible.[3] The solution of eq. (2.23) can

[3]Indeed, we have

$$\delta\rho_Q = 4M^4\dot{\pi} + \bar{M}^2\left(\frac{\ddot{h}}{2} - \nabla^2\dot{\pi}\right)\,, \tag{2.24}$$

$$\delta p_Q = \bar{M}^2\left(\frac{\ddot{h}}{2} - \nabla^2\dot{\pi}\right)\,. \tag{2.25}$$

be plugged back into eq. (2.22). This yields the equation of motion of π taking into account its gravitational back-reaction,

$$\ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = -\frac{\bar{M}^2}{2M_{\rm Pl}^2}\nabla^2\pi\,. \tag{2.26}$$

Mixing with gravity induces an unstable k^2 term in the dispersion relation, similarly to the gradient instability discussed above. We can compute again the fastest instability rate,

$$\omega^2_{\rm Jeans} \simeq -\left(\frac{\bar{M}M^2}{M_{\rm Pl}^2}\right)^2\,. \tag{2.27}$$

As expected, in this case the instability gets worse for large $\bar{M}^2$, i.e., when the mixing with gravity is enhanced. By imposing that this instability rate is smaller than the Hubble rate[4] we obtain

$$\frac{\bar{M}M^2}{M_{\rm Pl}^2} \lesssim H\,. \tag{2.28}$$

Requiring that both stability conditions (2.20) and (2.28) are satisfied we get the window [1]

$$-(1+w_Q)\Omega_Q \lesssim \frac{\bar{M}M^2}{HM_{\rm Pl}^2} \lesssim 1\,. \tag{2.29}$$

In conclusion, considering higher derivative terms, a quintessence model with $w_Q \leq -1$ can be completely stable inside the window of parameters (2.29). On the other hand, eq. (2.29) indicates that it is difficult to avoid instabilities when $(1+w_Q)\Omega_Q \ll -1$. These stability constraints were already obtained, for $\Omega_Q = 1$, in [1].

3 Phenomenology on the quintessential plane

3.1 k-essence vs. Ghost Condensate

Coming back to the quintessential plane of figure 1, in the previous section we have learned an important lesson: *the gradient instabilities for $w_Q < -1$ can be made harmless by higher derivative operators.* Thus, part of the lower left quadrant of the quintessential plane is allowed.

To discuss the phenomenology of these models (for a related discussion see [23]) let us write the full action for perturbations including the higher derivative operator (2.16):

$$\begin{aligned} S \;=\; & \int {\rm d}^4x\, a^3 \Bigg[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q+p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q+p_Q)\pi^2 - \frac{1}{2}(\rho_Q+p_Q)\dot{h}\pi \\ & -\frac{\bar{M}^2}{2}\left(3H\dot{\pi} - 3\dot{H}\pi + \frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2 \Bigg]\,. \end{aligned} \tag{3.1}$$

For $k \ll M$, $\bar{M}^2\nabla^2\dot{\pi} \ll M^4\dot{\pi}$. Moreover, eq. (2.23) shows that also the $\ddot{h}$ terms can be neglected in front of $M^4\dot{\pi}$, so that the operator proportional to $\bar{M}^2$ gives a negligible contribution to the stress-energy tensor.

[4] A more careful analysis [22] indicates that this condition is very conservative and much larger instability rates can be experimentally allowed.

First of all, note that it is not possible to switch off quintessence perturbations for $\rho_Q + p_Q \neq 0$; doing it by hand would give gauge dependent unphysical results. On the other hand, the converse is not true: even for $\rho_Q + p_Q = 0$ perturbations may still be present, as in the Ghost Condensate case.

We saw that the operator in the second line of eq. (3.1) allows the stabilization of the short scale gradient instability; on the other hand, for cosmological purposes we are interested in very large scales. Let us see whether this operator is relevant for scales of the order of the Hubble radius (although our discussion will extend to all scales of cosmological interest). We want to show that when

$$|\rho_Q + p_Q| \gg \bar{M}^2 H^2 \tag{3.2}$$

the higher derivative operator can be neglected when discussing the cosmological clustering of quintessence. In this case we reduce to a standard k-essence model, with the only difference that there are no short-scale instabilities even for $w_Q < -1$. On the other hand in the opposite case,

$$|\rho_Q + p_Q| \ll \bar{M}^2 H^2 \ , \tag{3.3}$$

all the terms in the action (3.1) proportional to $\rho_Q + p_Q$ can be neglected. In this case the model reduces to the Ghost Condensate theory.

Verifying the existence of these two regimes is quite straightforward. For instance the dispersion relation at $k/a \sim H$ is dominated either by $(\rho_Q + p_Q)(\nabla\pi)^2$ or by $\bar{M}^2(\nabla^2\pi)^2$ depending on the hierarchy between $|\rho_Q + p_Q|$ and $\bar{M}^2 H^2$. The same applies for the operators involving the metric perturbation, $(\rho_Q + p_Q)\dot{h}\pi$ and $\bar{M}^2\dot{h}\nabla^2\pi$. This check can be done for all the other operators, by taking $\nabla/a \sim H$ and considering that time derivatives are at most of order H. The existence of these two regimes can also be seen by looking at the stress-energy tensor (see appendix B).

Now we can go back and complete our quintessential plane. When w_Q is close to -1,

$$-\frac{\bar{M}^2}{M_{\rm Pl}^2} \lesssim (1 + w_Q)\Omega_Q \lesssim \frac{\bar{M}^2}{M_{\rm Pl}^2} \ , \tag{3.4}$$

the model behaves as the Ghost Condensate. We can estimate the width of this region by imposing the absence of Jeans instability, eq. (2.28). Assuming $M \sim \bar{M}$ one gets a rough upper bound: $\bar{M} \lesssim 10$ MeV [4]. A more accurate analysis shows that this limit is much too conservative and it can be relaxed to $\bar{M} \lesssim 100$ GeV [22]. Even in this case the window above is extremely tiny:

$$|1 + w_Q|\Omega_Q \lesssim 10^{-34} \ . \tag{3.5}$$

We can therefore draw an important conclusion: only for values of w_Q which are observationally indistinguishable from the cosmological constant, does quintessence behave as the Ghost Condensate on cosmological scales. This regime corresponds to the strip around the horizontal axis $w_Q = -1$ in figure 2. Notice that in this region the dispersion relation is of the form $\omega \propto k^2$, so that the speed of sound c_s^2 is not well defined, i.e. it becomes scale dependent.

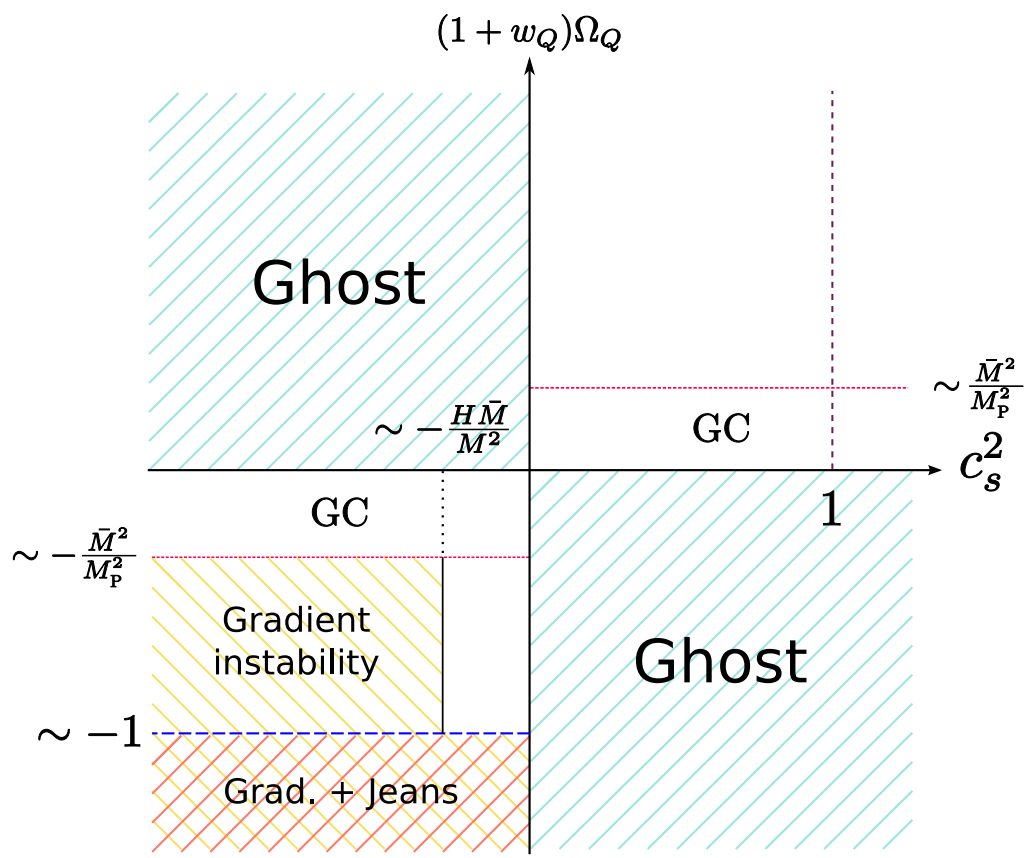

Figure 2: *On the quintessential plane, we show the theoretical constraints on the equation of state and speed of sound of quintessence, in the presence of the operator $\bar{M}$. Instability regions are dashed. Where $1+w_Q$ and c_s^2 have opposite sign we have a ghost-like instability corresponding to negative kinetic energy. For $w_Q < -1$, the dashed regions in the left-lower panel is unstable by gradient ($c_s^2 \lesssim -H\bar{M}/M^2$) and Jeans ($(1+w_Q)\Omega_Q \lesssim -1$) instabilities, while the strip close to the vertical axis corresponds to the stability window (2.29). Furthermore, the strip around the horizontal axis given in eq. (3.4) corresponds to the Ghost Condensate. Above this region, in the right-upper panel, we find standard k-essence.*

On the other hand, for any value of w_Q which is appreciably different from the one of the cosmological constant, the model reduces to k-essence as higher derivative terms are cosmologically irrelevant. Their only role is to stabilize the short scale gradient instabilities for $w_Q < -1$. Although in practice not relevant, note however that w_Q cannot be made arbitrarily negative. This is shown by eq. (2.29) and, in the quintessential plane, it excludes the bottom shaded region of the lower-left quadrant.

Let us now constrain the values of the speed of sound c_s^2. For $w_Q > -1$ there are no constraints, besides the possible limit $c_s^2 \leq 1$ already discussed. For $w_Q < -1$ the speed of sound is negative and

very small, as it is constrained by the absence of gradient instability, eq. (2.20),

$$-c_s^2 \simeq -\frac{\rho_Q + p_Q}{4M^4} \lesssim \frac{H\bar{M}}{M^2}\,. \tag{3.6}$$

We can numerically constrain the right-hand side of this equation by considering that the scales $M \sim \bar{M}$ represent the cutoff of our effective field theory. By requiring this cutoff to be larger than the minimum scale at which gravity has been proved, i.e., $M \gtrsim 10^{-3}$eV, and using in eq. (3.6) the value of the Hubble parameter today, $H_0 \sim (10^{-3}\text{eV})^2/M_{\text{Pl}}$, we obtain

$$-c_s^2 \lesssim \left(\frac{H_0}{M_{\text{Pl}}}\right)^{1/2} \sim 10^{-30}\,. \tag{3.7}$$

Thus, for all practical purposes the speed of sound can be taken to be exactly zero. On the quintessential plane in figure 2, in the lower-left quadrant, we can only live in a tiny strip along the vertical axis. Notice however that there is no fine tuning in keeping c_s^2 extremely small. Indeed, as we discussed, in the limit of Ghost Condensate c_s^2 vanishes exactly for symmetry reasons. Thus, the speed of sound remains small for tiny deviations from this limit.

3.2 Including dark matter

After the discussion about the stability constraints, we would like to understand the dynamics of quintessence perturbations and their impact on cosmological observations. In order to do this, we will now study quintessence in the presence of cold dark matter, which gravitationally sources quintessence perturbations. A thorough analysis of the phenomenology of these models is beyond the scope of this paper. Here we want to focus on the main qualitative features in the various limits.

Let us start from the Ghost Condensate limit (3.3). It is known that the Ghost Condensate affects only short scales, i.e., π perturbations induce a modification of the Newtonian potential at scales parametrically smaller than the Hubble scale [4]. Therefore, we expect to have extremely small effects on cosmological scales. To verify that this is the case, we can study the action (3.1) in the limit of $\rho_Q + p_Q = 0$. This reads

$$S = \int \mathrm{d}^4x\, a^3 \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(3H\dot{\pi} - 3\dot{H}\pi + \frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2 \right]. \tag{3.8}$$

For simplicity, let us momentarily disregard the first two terms in parentheses,

$$S = \int \mathrm{d}^4x\, a^3 \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2 \right]. \tag{3.9}$$

Notice that this is the action used in the Ghost Condensate paper [4]. The equation of motion for the π perturbations is given by

$$\ddot{\pi} + 3H\dot{\pi} + \frac{\bar{M}^2}{4M^4}\frac{\nabla^4\pi}{a^4} = \frac{\bar{M}^2}{8M^4}\frac{\nabla^2\dot{h}}{a^2}\,. \tag{3.10}$$

The gradient term on the left hand side can be neglected on cosmological scales. Indeed, the time derivatives will be at least of order $\sim \bar{M}k^2/(aM)^2$, so that the friction term in the previous equation will always dominate the gradient term for $k/a \sim H$. As we want to show that Ghost Condensate perturbations remain small, we assume that the dark matter dominates the perturbed Einstein equations. The validity of this assumption can be checked *a posteriori*.

In a matter dominated Universe $\dot{h} = -2\dot{\delta}_m$ [21], with $\delta_m \equiv \delta\rho_m/\rho_m$, which, using $\dot{\delta}_m = H\delta_m$ and the background Friedmann equation, leads to

$$\dot{h} = -\frac{2}{3H}\frac{\delta\rho_m}{M_{\rm Pl}^2}\,. \tag{3.11}$$

We can now replace this as the source of Ghost Condensate perturbations on the right-hand side of eq. (3.10). This yields, neglecting the gradient term,

$$\ddot{\pi} + 3H\dot{\pi} \simeq -\frac{\bar{M}^2}{12M^4M_{\rm Pl}^2}\frac{\nabla^2\delta\rho_m}{a^2H}\,. \tag{3.12}$$

If we now assume that the initial quintessence perturbations are small so that the homogeneous solutions are sub-dominant, similarly to what happens in standard quintessence [24], this equation can be solved to give

$$\dot{\pi} = -\frac{\bar{M}^2}{24M^4M_{\rm Pl}^2}\frac{\nabla^2\delta\rho_m}{a^2H^2}\,. \tag{3.13}$$

As we discussed in the previous section, the energy density and pressure perturbations of the Ghost Condensate are dominated by the M^4 operator so that $\delta\rho_Q \simeq 4M^4\dot{\pi}$ and $\delta p_Q \simeq 0$. Thus, on cosmological scales,

$$\delta\rho_Q \sim \frac{\bar{M}^2}{M_{\rm Pl}^2}\delta\rho_m\,. \tag{3.14}$$

From the simple estimate of $\bar{M}$ below eq. (3.4), we conclude that quintessence perturbations are negligibly small with respect to dark matter perturbations, $\delta\rho_Q \lesssim 10^{-34}\delta\rho_m$. It is straightforward to generalize this analysis including the two terms in parentheses of eq. (3.8) previously neglected and verify that eq. (3.14) remains valid. The conclusion of eq. (3.14) is quantitatively consistent with the (small) modification of the Newton law derived in [4], as one can check for example in their eq. (7.11) for $k/a \sim H$ and $\omega \sim H$.

Close to the $w_Q = -1$ line, we saw that there are no appreciable effects of perturbations on cosmological scales which can help in distinguishing quintessence from a cosmological constant; all the interesting dynamics is limited to short scales. As we move away from the $w_Q = -1$ line we enter in the k-essence regime, as we pointed out in the previous section. The case $w_Q > -1$ is well studied in the literature (see, for instance, [25, 26]). The case $w_Q < -1$ is much less studied: here we have a negative speed of sound squared that is so small – see eq. (3.7) – that can be taken to be zero for all practical purposes. With a small speed of sound we expect quintessence to cluster on scales shorter than the Hubble radius driven by dark matter gravitational potential wells. To study

this, let us repeat the calculation we just did in the Ghost Condensate case for a k-essence with $c_s^2 = 0$.

For simplicity, let us assume for the moment that M is constant. Varying the action (2.12) we get the equation of motion for π,

$$4M^4(\ddot{\pi} + 3H\dot{\pi}) - (\rho_Q + p_Q)\frac{\nabla^2}{a^2}\pi - 3\dot{H}(\rho_Q + p_Q)\pi = -\frac{1}{2}(\rho_Q + p_Q)\dot{h} \ . \tag{3.15}$$

Small $|c_s^2|$ is equivalent to $|\rho_Q + p_Q| \ll M^4$ so that the gradient and mass term in this equation can be neglected. Using again $\dot{h} = -2\dot{\delta}_m$ [21], we thus have

$$4M^4(\ddot{\pi} + 3H\dot{\pi}) = (\rho_Q + p_Q)\dot{\delta}_m \ . \tag{3.16}$$

One can verify that, neglecting decaying modes, the solution of this equation is

$$\delta_Q = \frac{1 + w_Q}{1 - 3w_Q}\delta_m \ . \tag{3.17}$$

It is easy to show that this equation holds, for constant w_Q, for a general time dependent speed of sound which satisfies $|c_s^2(t)| \ll 1$.

Equation (3.17) describes quintessence perturbations both for positive and negative $1 + w_Q$. When $w_Q > -1$ quintessence energy density clusters in the dark matter potential wells, while in the opposite case $w_Q < -1$ it escapes from them [27]. However, clustering of quintessence remains small compared to dark matter as the coupling with gravity is suppressed by $1 + w_Q$. For very small values $|1 + w_Q| \sim \bar{M}^2/M_{\rm Pl}^2$ we smoothly enter in the Ghost Condensate regime. Indeed it is easy to see that eq. (3.17) smoothly matches eq. (3.14) in the intermediate regime.

It is important to stress that for $w_Q < -1$ the speed of sound is constrained to be so small that quintessence effectively clusters on all scales. It would be interesting to understand the effect of the short scale clustering on structure formation. We will come back to this point in the conclusion, section 6.

In this section we have studied the phenomenology of quintessence in various regimes of $1 + w_Q$. Quintessence perturbations smoothly turn off when we approach the cosmological constant limit $w_Q = -1$ from both sides. This suggests that in general there is no pathology in crossing the $w_Q = -1$ line, as we discuss in the next section.

4 Crossing the phantom divide

It has been claimed that, during its evolution, single field quintessence cannot cross the $w_Q = -1$ line as perturbations become pathological. For this reason this line has been dubbed "phantom divide" [7]. However, there is no real pathology in crossing this line, besides the fact that for $w_Q < -1$ short-scale gradient instabilities must be stabilized [8, 9]. If one does not take into account higher

derivative terms, a negative c_s^2 leads to catastrophic instabilities at short scales. Once instabilities are cured as we discussed in the previous sections, crossing the phantom divide becomes trivial.[5]

Indeed, the Lagrangian (2.13) gives an explicit way to construct a model which crosses the phantom divide. If one assumes for simplicity that quintessence is the only component in the Universe,[6] the crossing of the phantom divide corresponds to a change of sign of $\dot{H}$. In particular, considering only quintessence, one can use Friedmann equations to recast the Lagrangian (2.13) in the form (higher derivative operators will be considered later)

$$P(X,\phi) = -3M_{\rm Pl}^2 H^2(\phi) - M_{\rm Pl}^2 \dot{H}(\phi)(X+1) + \frac{1}{2}M^4(\phi)(X-1)^2 \ . \tag{4.1}$$

This is similar to what happens in inflation, where the inflaton is the only relevant component in the Universe [2].

As an example, we consider the case where $\dot{H}$ evolves linearly in time and changes sign from negative to positive,

$$\dot{H}(t) = \frac{\mu^4}{M_{\rm Pl}^2}(mt-1) \ . \tag{4.2}$$

This implies that $H(t)$ will be a parabola of the form

$$H(t) = \frac{\mu^4}{M_{\rm Pl}^2}\left(\frac{m}{2}t^2 - t\right) + H_* \ , \tag{4.3}$$

as shown in figure 3 (left panel). Using the general expression (4.1), we deduce that the Lagrangian

$$P(\phi,X) = -3\left[\frac{\mu^4}{M_{\rm Pl}}\left(\frac{m}{2}\phi^2 - \phi\right) + M_{\rm Pl}H_*\right]^2 + \mu^4(m\phi - 1)\left[(\partial\phi)^2 - 1\right] + \frac{1}{2}M^4(\phi)\left[(\partial\phi)^2 + 1\right]^2 \tag{4.4}$$

admits the background solution $\phi = t$ and the cosmological evolution (4.2) and (4.3). Note that there are no theoretical limitations on the choice of the background evolution $H(t)$. Indeed, we can cross the phantom divide as many times as we want. For example, choosing $\dot{H}(t) \propto \sin(mt)$ the cosmological evolution keeps oscillating up and down around $w_Q = -1$!

We can now study perturbations around a solution crossing $w_Q = -1$ to show that no pathology arises. The evolution equation derived from the action (2.12) reads

$$\left(\rho_Q + p_Q + 4M^4\right)\ddot{\pi} + \frac{1}{a^3}\partial_t\left[a^3(\rho_Q + p_Q + 4M^4)\right]\dot{\pi} - 3\dot{H}(\rho_Q + p_Q)\pi - (\rho_Q + p_Q)\frac{\nabla^2\pi}{a^2} = -\frac{1}{2}(\rho_Q + p_Q)\dot{h} \ . \tag{4.5}$$

At the phantom divide, the last three terms of this equation vanish but the equation is clearly non-singular. In our approach, it is manifest that the speed of sound squared changes sign at $w_Q = -1$. In the example above c_s^2 is given by

$$c_s^2 = \frac{\mu^4(1-mt)}{\mu^4(1-mt) + 2M^4} \ , \tag{4.6}$$

[5]Models that cross the phantom divide have been found in $f(R)$ theories of gravity. However, for $w_Q < -1$ these models are equivalent to scalar fields with negative kinetic energy, i.e. to ghosts. Thus, according to our stability constraints, they are forbidden.

[6]The conclusions drawn in this section also hold when including dark matter.

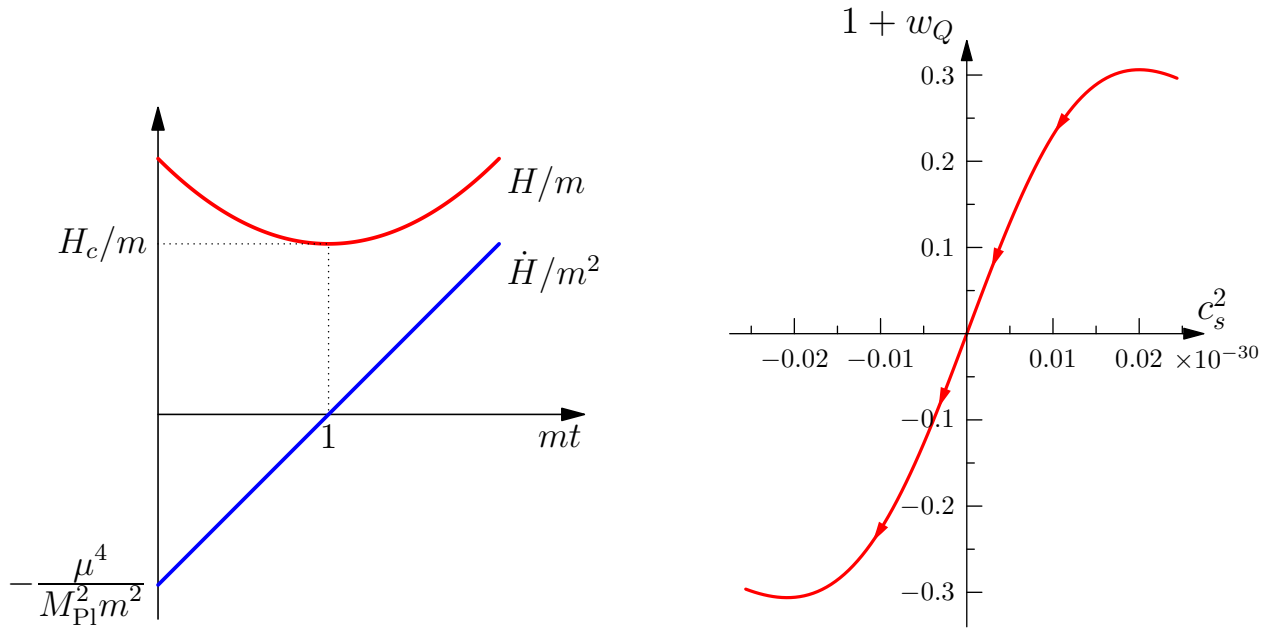

Figure 3: *Example of phantom divide crossing, as given by eqs. (4.2) and (4.3), where we have defined* $H_c \equiv H_* - \mu^4/(2mM_{Pl}^2)$. *Left figure: behavior of* H *and* $\dot{H}$*; the crossing of* $w_Q = -1$ *takes place at* $t = m^{-1}$ *when* $\dot{H} = 0$ *and* $H = H_c$. *Right figure: trajectory on the quintessential plane.*

and the trajectory of the crossing on the quintessential plane is shown in figure 3 (right panel). The stability in the $w_Q < -1$ region requires that $|c_s^2|$ remains extremely small so that it is mandatory to have a hierarchy between μ and M, $\mu \ll M$. As we discussed, this hierarchy is naturally realized.

Another approach to study perturbations is using a fluid description, as k-essence is equivalent to a perfect fluid. In synchronous gauge, denoting with a prime the derivative with respect to the conformal time $d\eta \equiv dt/a$ and defining $\mathcal{H} = aH$, the fluid equations read, in Fourier space, (see for example [28])

$$\delta_Q' + 3\mathcal{H}(c_s^2 - w_Q)\delta_Q = -(1+w_Q)\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta_Q}{k^2} - (1+w_Q)\frac{h'}{2}\,, \tag{4.7}$$

$$\frac{\theta_Q'}{k^2} + \mathcal{H}(1-3c_s^2)\frac{\theta_Q}{k^2} = \frac{c_s^2}{1+w_Q}\delta_Q\,, \tag{4.8}$$

where θ_Q is the divergence of the velocity field of quintessence, $\theta_Q \equiv ik^i T_i^0/(\rho_Q + p_Q)$ [21], c_s^2 is $\delta p_Q/\delta\rho_Q$ calculated in a velocity orthogonal gauge ($T_i^0 = 0$) [29] and it corresponds to the speed of sound squared that can be deduced from the π Lagrangian[7], to be distinguished from the adiabatic speed of sound squared, $c_a^2 \equiv \dot{p}_Q/\dot{\rho}_Q = w_Q - \dot{w}_Q/(3H(1+w_Q))$. The absence of pathologies at $w_Q \to -1$ can also be shown in this formalism. Indeed, in the continuity equation the divergence of

[7]The velocity orthogonal condition $T_i^0 = 0$ is equivalent to the condition $\pi = 0$. As ϕ is unperturbed in this gauge, the perturbations of pressure and energy density only come from fluctuations of X, i.e. $\delta p_Q/\delta\rho_Q = p_{Q,X}/\rho_{Q,X} = P_X/(2P_{XX}X + P_X)$, which is the speed of sound c_s^2 which appears in the π Lagrangian (2.6).

c_a^2 is compensated by the prefactor in front of the squared brackets, while the $1 + w_Q$ term at the denominator in the Euler equation is harmless as c_s^2 also vanishes for $w_Q \to -1$. Thus both δ_Q and θ_Q are continuous through the divide. This is not surprising as θ_Q is just the Laplacian of the scalar perturbation π, $\pi = a\,\theta_Q/k^2$.

At this point the reader may be puzzled: in the previous sections we stressed that close to the $w_Q = -1$ line quintessence behaves like the Ghost Condensate on cosmological scales, while eq. (4.5) as well as the fluid eqs. (4.7) and (4.8) do not contain higher derivative terms. Let us see why these additional terms are irrelevant in realistic cases of crossing the phantom divide. With these new terms, the equation of motion for π derived from the full action (3.1) is obviously still continuous so that also in this case the line $w_Q = -1$ can be crossed smoothly. The operator proportional to $\bar{M}^2$ dominates in the Ghost Condensate strip around the $w_Q = -1$ line. However, this happens only in the extremely narrow range $|1 + w_Q| \lesssim \bar{M}^2/M_{\rm Pl}^2 \lesssim 10^{-34}$. The equation of state parameter w_Q will stay in this range only for a time much smaller than H^{-1}, unless its evolution is tremendously slow. Thus π has no time to evolve in the Ghost Condensate regime, so that for all practical purposes one can totally neglect this strip around the $w_Q = -1$ line on cosmological scales.

We have seen that k-essence can be described with the fluid equations (4.7) and (4.8). Even including higher derivative terms, quintessence remains a perfect fluid (see appendix B) but does not satisfy the fluid equations (4.7) and (4.8) as these assume a linear dispersion relation. However, as we discussed, higher derivative terms are phenomenologically irrelevant on cosmological scales, so that one can still use the fluid description above when comparing with observations.

From a practical point of view we conclude that, when comparing with observations a dark energy model which crosses the phantom divide, it is consistent and theoretically motivated to set $c_s^2 = 0$. On the other hand, it is inconsistent to turn off perturbations as sometimes done in the literature.

5 Additional higher derivative operators

As we discussed, higher derivative operators become relevant when the speed of sound is very close to zero. This regime is particularily interesting when $w_Q < -1$ so that in the following we will consider mostly the case $\rho_Q + p_Q < 0$.

Theories with very small c_s^2 should be thought of as tiny deformations of the Ghost Condensate theory [20, 1, 2]: in this limit one recovers the shift symmetry $\phi \to \phi + \lambda$, so that a small deviation from the Ghost Condensate is technically natural. In the Ghost Condensate limit there is an additional symmetry that one can impose, i.e. the parity symmetry $\phi \to -\phi$. The background $\phi = t$ in Minkowski space breaks this parity symmetry and the time reversal symmetry to the composition of the two; the theory of perturbations is thus invariant under $\pi \to -\pi$, $t \to -t$ [4]. This symmetry is present only when the background metric is Minkowski: in de Sitter there is a preferred time direc-

tion singled out by the expansion. In this case terms violating the symmetry will be proportional to H, and thus tipically suppressed by H/M. In this paper we have considered small departures from the Ghost Condensate limit, i.e., tiny breakings of the shift symmetry. These also generate terms which are not invariant under parity $\pi \to -\pi$, $t \to -t$, as for instance the mass term in eq. (2.12). These terms will be of the same order of magnitude, i.e. suppressed by H/M as we are assuming that H^{-1} is the typical time scale of evolution of the operators.

However, one can also consider the case when the parity symmetry $\phi \to -\phi$ is absent in the Ghost Condensate limit [1]. This happens for example if we add to the k-essence Lagrangian (2.13) the operator

$$\mathcal{L}_{\hat{M}} = -\frac{\hat{M}^3}{2}(\Box\phi + 3H)(X-1) \ , \tag{5.1}$$

which again does not change the background evolution as it starts quadratic in the perturbations. For simplicity we assume that $\bar{M} = 0$ and a constant $\hat{M}$.[8] In synchronous gauge at quadratic order this operator is

$$\mathcal{L}_{\hat{M}} = \hat{M}^3\dot{\pi}\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{\dot{h}}{2}\right) \ . \tag{5.2}$$

The first two terms in the parentheses contribute (after an integration by parts) to the time kinetic term. Assuming $\hat{M} \sim M$ they can be neglected in comparison with $2M^4\dot{\pi}^2$. The third term gives a mass term that is parametrically smaller than H and can thus be neglected.

To discuss the stability and phenomenology of this model, let us write the full action for perturbations, assuming $|\rho_Q + p_Q| \ll M^4$,

$$S = \int \mathrm{d}^4x\, a^3 \left[2M^4\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi + \hat{M}^3\dot{\pi}\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)\right] , \tag{5.3}$$

where we have neglected the mass terms. This equation is analogous to eq. (3.1) for the $\bar{M}$ operator (2.16). Analogously to what we have done in sections 2.2 and 3.2 for the operator proportional to $\bar{M}^2$, we will now study the stability and phenomenology on cosmological scales with the operator $\mathcal{L}_{\hat{M}}$. In appendix D we briefly study the effect of this operator at short distances, i.e. the modification of gravity induced by it.

5.1 Stability constraints with $\hat{M}$

Let us first study the stability of the system neglecting other sources of gravity. The equation of motion for π derived varying (5.3) reads

$$\ddot{\pi} + 3H\dot{\pi} - \frac{\rho_Q + p_Q}{4M^4}\frac{\nabla^2\pi}{a^2} - \frac{\hat{M}^3 H}{4M^4}\frac{\nabla^2\pi}{a^2} = -\frac{\rho_Q + p_Q}{8M^4}\dot{h} - \frac{\hat{M}^3}{8M^4}(\ddot{h} + 3H\dot{h}) \ . \tag{5.4}$$

[8] This assumption can be relaxed by having a time dependence with time scale of order H^{-1}, as in the case of $\bar{M}^2$.

Notice that the operator $\hat{M}^3\dot{\pi}\nabla^2\pi/a^2$ induces a spatial kinetic term for π proportional to H. Indeed, this operator is a total derivative in Minkowski spacetime. Choosing $\hat{M} > 0$, the spatial kinetic term has the "healthy" sign and can be chosen sufficiently large to cure the gradient instability associated to $\rho_Q + p_Q < 0$, giving a positive and very small c_s^2. This also allows us to neglect the first term on the right hand side in eq. (5.4). To complete the stability analysis one has to take into account the mixing with gravity, i.e., solve for h in terms of the quintessence stress-energy tensor using the Einstein equation (2.23), and plug the result back in the right hand side of eq. (5.4). This, similarly to what happens for the Ghost Condensate, will give rise to a Jeans-like instability.

The contribution to the stress-energy tensor of the operator (5.1) is (see appendix B)

$$\delta\rho_Q \supset \hat{M}^3\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right) , \tag{5.5}$$

$$\delta p_Q \supset -2\hat{M}^3\left(\ddot{\pi} + 3H\dot{\pi}\right) . \tag{5.6}$$

Given the small speed of sound, time derivatives are much smaller than the spatial ones and the pressure perturbation is negligible, $\delta p_Q \ll \delta\rho_Q$. Concentrating on frequencies much larger than the Hubble rate one can neglect the terms containing $H\dot{\pi}$ and $H\dot{h}$ in eq. (5.4). A further simplification comes from disregarding the standard k-essence contribution to the energy density perturbation $\delta\rho_Q$, i.e. $4M^4\dot{\pi}$, in comparison with $\hat{M}^3\nabla^2\pi/a^2$. Indeed, from eq. (5.4) we have $\hat{M}^3\nabla^2\pi/a^2 \sim M^4\ddot{\pi}/H \gg M^4\dot{\pi}$. Moreover, as we will see, the absence of Jeans instability will impose $\hat{M}^3 \lesssim M_{\rm Pl}^2 H$. This implies that the term with $\dot{h}$ in eq. (5.5) is negligible with respect to $\ddot{h}$ in the Einstein equation (2.23), that becomes

$$\ddot{h} = \frac{\hat{M}^3}{M_{\rm Pl}^2}\frac{\nabla^2\pi}{a^2}. \tag{5.7}$$

Plugging this into the right hand side of eq. (5.4) we finally find

$$\ddot{\pi} - \left(\frac{\rho_Q + p_Q}{4M^4} + \frac{\hat{M}^3 H}{4M^4} - \frac{\hat{M}^6}{8M^4 M_{\rm Pl}^2}\right)\frac{\nabla^2\pi}{a^2} = 0 . \tag{5.8}$$

This same result would have been found using a more rigorous approach, as done in [1]. Again, as in the Ghost Condensate case, mixing with gravity induces a Jeans-like instability, represented by the last term in this equation. Thus, for $\rho_Q + p_Q < 0$ we need to cure both the gradient and the Jeans instabilities. This is possible for

$$-(1 + w_Q)\Omega_Q \lesssim \frac{\hat{M}^3}{M_{\rm Pl}^2 H} \lesssim 1 . \tag{5.9}$$

This stability window [1] is analogous to the one discussed in the Ghost Condensate case, eq. (2.29).

We conclude that with the inclusion of the operator $\mathcal{L}_{\hat{M}}$ we can have a dispersion relation $\omega \propto k$ with positive speed of sound squared; thus, there is no sign of instability even for $\rho_Q + p_Q < 0$.

5.2 Including dark matter

Analogously to what done in section 3.2, to study the phenomenology induced by the $\hat{M}$ operator we study quintessence perturbations generated by the coupling with dark matter. For simplicity we assume matter dominance. The action (5.3) gives the equation of motion for π sourced by the dark matter perturbation δ_m,

$$\ddot{\pi} + 3H\dot{\pi} - \frac{H\hat{M}^3}{4M^4}\frac{\nabla^2\pi}{a^2} = \frac{5}{8}\frac{\hat{M}^3}{M^4}H^2\delta_m + \frac{\rho_Q + p_Q}{4M^4}H\delta_m \ , \tag{5.10}$$

where we have used $\dot{h} = -2H\delta_m$ and neglected the gradient term proportional to $\rho_Q + p_Q$ that is subdominant. Since the speed of sound $c_s^2 = H\hat{M}^3/(4M^4)$ is very small and we are interested in cosmological scales, one would naïvely neglect the term with $\nabla^2\pi$. This gives the solution

$$\dot{\pi} = \frac{H\hat{M}^3}{4M^4}\delta_m + \frac{\rho_Q}{4M^4}\frac{1+w_Q}{1-3w_Q}\delta_m \ . \tag{5.11}$$

However, the approximation of neglecting the gradients is not good. Indeed, when we plug this expression into the energy density perturbation

$$\delta\rho_Q = 4M^4\dot{\pi} + \hat{M}^3\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right) \ , \tag{5.12}$$

there is a cancellation of terms proportional to $\hat{M}^3$ up to gradient terms. Thus one is forced to go back to the equation of motion (5.10) and keep the term proportional to the speed of sound.

Once we do that, we obtain

$$\delta_Q = -\frac{\hat{M}^6}{24M^4M_{\rm Pl}^2\Omega_Q}\frac{\nabla^2}{H^2a^2}\delta_m + \frac{1+w_Q}{1-3w_Q}\delta_m \ . \tag{5.13}$$

This equation displays the existence of two different regimes, in strict analogy with what happens in the Ghost Condensate case. For large enough $|1+w_Q|$, the second term on the right hand side dominates and one recovers eq. (3.17), where the system behaves as standard k-essence. Note however that even for $\rho_Q + p_Q < 0$ there are no stability problems in the stability window (5.9). The dynamics of the system is dominated by the $\hat{M}$ operator only when we are very close to $w_Q = -1$, i.e., for

$$-\frac{\hat{M}^6}{M^4M_{\rm Pl}^2} \lesssim (1+w_Q)\Omega_Q \lesssim \frac{\hat{M}^6}{M^4M_{\rm Pl}^2} \ . \tag{5.14}$$

This region is the analog of the Ghost Condensate strip around the horizontal axis of figure 2. In this range the first term in eq. (5.13) dominates and δ_Q remains extremely small with respect to δ_m. Although the dispersion relation is of the form $\omega \propto c_s k$, quintessence does not follow the simple fluid equations (4.7) and (4.8) because of the presence of higher derivative operators. However, as in the Ghost Condensate case, given the narrowness of the strip (5.14), for all practical purposes we can always use the fluid equations with $c_s^2 = 0$, even when crossing the phantom divide. In conclusion,

the addition of the operator $\hat{M}$ can stabilize k-essence in the phantom case $w_Q < -1$, and the phenomenology of the model is the same as for a k-essence with $c_s^2 = 0$. This general conclusion will hold even when considering both operators $\bar{M}$ and $\hat{M}$ at the same time, and can be extended to all the possible higher derivative operators, included in the general action in appendix C.

6 Conclusion and outlook

In this paper we have studied the most generic action describing the perturbations of a single field dark energy – here called quintessence – around a given background. We have constructed the action by adding to the k-essence Lagrangian higher derivative operators that leave the background evolution invariant. Using this action, we have reproduced the results of [1] concerning the theoretical constraints on the equation of state parameter w_Q as a function of the speed of sound squared c_s^2, by the requirement that perturbations are ghost-free – i.e., that their kinetic energy is positive – and that there are no gradient-like instabilities. These constraints have been conveniently represented on the quintessential plane $(1 + w_Q)\Omega_Q$ vs. c_s^2, in figures 1 and 2.

In particular we have considered the case $w_Q < -1$, which is commonly believed to be unstable, and we have shown that for very small c_s^2 both the gradient and the Jeans instabilities can be avoided and perturbations stabilized [1]. Higher derivative operators are crucial for the stabilization. Indeed, it is important to stress that taking an extremely small c_s^2 does not represent a fine tuning, as in the limit $c_s^2 \to 0$ we recover the Ghost Condensate theory which is protected by the shift symmetry $\phi \to \phi + \lambda$. Thus, for $w_Q < -1$ quintessence should be thought of as a small deformation of the Ghost Condensate limit [20, 1, 2]. When the higher order terms containing k^4 dominate over the spatial kinetic term $c_s^2 k^2$ the phenomenology reduces to that of the Ghost Condensate. This always happens on small scales, where the higher order gradients must dominate to stabilize the perturbations, but on cosmological scales this only occurs for values of w_Q extremely close to the one of the cosmological constant, i.e., for $|1 + w_Q|\Omega_Q \lesssim 10^{-34}$. Away from this tiny strip – i.e., for all practical purposes – the behavior on cosmological scales is very different from that of the Ghost Condensate: higher derivative terms are irrelevant so that the phenomenology of the $w_Q < -1$ side of quintessence reduces to that of a k-essence fluid with $c_s^2 = 0$.

Furthermore, we have studied the behavior of quintessence perturbations when crossing the so-called phantom divide $w_Q = -1$. By restricting the analysis to k-essence perturbations around a given background crossing the phantom divide, we have shown that, as the speed of sound vanishes exactly at the divide, perturbations remain finite during the crossing. For $w_Q < -1$, higher derivative terms, while irrelevant on cosmological scales, are essential to stabilize the short scale gradient instabilities. We conclude that no pathology arises during the crossing: the phantom divide can be crossed without the addition of new degrees of freedom. We have illustrated this with an example shown in figure 3. An important thing to retain is that a consistent and theoretically motivated way

of comparing with data a dark energy evolution which crosses the phantom divide is to set to zero the speed of sound of perturbations.

Our study motivates the possibility that quintessence has a virtually vanishing speed of sound, especially when $w_Q < -1$. Such a quintessence can be detected through its effects on structure formation. The speed of sound defines the sound horizon $\ell_Q \equiv a \int c_s dt/a$, which sets the characteristic length scale of smoothness of the perturbations. In the matter dominated era $\ell_Q = 2c_s H_0^{-1}$ for a constant c_s. Hence, for $c_s = 1$ – corresponding to the speed of sound of a scalar field with a canonical kinetic term – quintessence can cluster only on scales larger than the Hubble radius while for $c_s = 0$ it clusters on all scales, thus affecting the gravitational potential and the formation of structures of dark matter and galaxies. The effect of a clustering quintessence can be measured with the cosmic microwave background [26, 27, 28, 30], galaxy redshift surveys [31], large neutral hydrogen surveys [32], or by cross-correlating the integrated Sachs-Wolfe effect in the cosmic microwave background with large scale structures [33, 34]. For instance, in [33, 31] it was found that with future surveys it will be possible to measure a zero speed of sound of dark energy if $w_Q \gtrsim -0.95$. As these analysis were restricted to positive values of $1 + w_Q$ only, it would be interesting to repeat them for negative values. Notice that for a vanishing speed of sound, dark energy will actively participate to the formation of non-linear objects, affecting the halo bias. It would be interesting to evaluate this effect, which gives additional signatures of these models.

Quintessence is perturbed by the presence of sources and thus modifies gravity as any other kind of matter. When this modification happens on scales much smaller than the Hubble radius one can properly talk about a theory of infrared modification of gravity. This happens when quintessence is close to the Ghost Condensate limit; in this case the modification of gravity is due to the Jeans instability induced by higher derivative operators, and persists even in Minkowski spacetime. Modifications of gravity induced by the Ghost Condensate have been studied in details in [4, 22]. In this paper we have considered also the additional operator proportional to $\hat{M}^3$ [1]; it would be interesting to investigate the deviation from General Relativity induced by this operator and its possible observational consequences, extending the preliminary analysis of appendix D, where the treatment has been restricted to linear perturbations in a Minkowski spacetime.

Acknowledgments

It is a pleasure to thank Carlo Baccigalupi, Pier Stefano Corasaniti, Ruth Durrer, Steen Hannestad, Wayne Hu, David Langlois, Alberto Nicolis, Uroš Seljak, and Martin Sloth for useful discussions and comments. We are particularly in debt with Leonardo Senatore for collaboration in the early stages of the work.

Appendix

A Higher derivative operators in effective field theories

In this appendix we want to study the Ghost Condensate and its deformations from an effective field theory point of view. In particular we want to show that, although the operator $(\nabla^2\pi)^2$ dominates the dynamics, operators containing higher time derivatives such as $\ddot{\pi}^2$ must be treated perturbatively.

In the Ghost Condensate limit, the free π action is

$$S = \frac{M^4}{2}\int d^3x dt \left[\dot{\pi}^2 - \frac{(\nabla^2\pi)^2}{M^2}\right] , \tag{A.1}$$

where we neglected the mixing with gravity – as we are interested in the high energy behavior of the theory – and for simplicity we assumed that there is a single scale M ($M \simeq \bar{M}$). This action is manifestly invariant under the energy scaling [4]

$$E \to sE , \qquad t \to s^{-1}t , \qquad x \to s^{-1/2}x , \qquad \pi \to s^{1/4}\pi . \tag{A.2}$$

As the theory is not Lorentz invariant, time and space behave differently under rescaling, and π does not scale as s^1 as in a Lorentz invariant theory. (See for instance [35] for an introduction to scaling in non-Lorentz invariant field theories.)

What is the physical meaning of this scaling transformation? Assuming that the theory is weakly coupled, the free action gives the leading contribution to the correlation functions, so that these will be invariant under the scaling above. For instance, a relativistic massless scalar has scaling dimension 1. Thus, the two-point function satisfies[9]

$$\langle\phi\phi\rangle(x-y) = s^{-2}\langle\phi\phi\rangle\left(\frac{x-y}{s}\right) \qquad \Rightarrow \qquad \langle\phi\phi\rangle \propto \frac{1}{|x-y|^2} . \tag{A.3}$$

In the case of the action (A.1) above, the scaling transformation (A.2) yields

$$\langle\pi\pi\rangle(\Delta t, \Delta\vec{x}) = s^{-1/2}\langle\pi\pi\rangle\left(\frac{\Delta t}{s}, \frac{\Delta\vec{x}}{s^{1/2}}\right) . \tag{A.4}$$

Not only the scaling transformation gives information on the free theory, but, more importantly, it allows one to estimate the effect of different operators added to the free action. In particular, in the Ghost Condensate case, one can check that all additional operators allowed by symmetries have positive scaling dimensions, so that their importance is suppressed by E/M elevated to a positive power [4]. This implies that at low energy the theory is perturbative. For instance, the leading irrelevant operator is $\dot{\pi}(\nabla\pi)^2$, which has scaling dimension 1/4.

[9] Note that the scaling dimension has nothing to do with the mass dimension of the field ϕ. Indeed, eq. (A.3) remains the same if we choose a non-conventional normalization of the action such that ϕ has not mass dimension 1.

Operators containing higher time derivatives have positive scaling dimensions so that they must be treated perturbatively. For instance $\ddot{\pi}^2$ has scaling dimension 2, so that at low energy it is negligible. Additional time derivatives naïvely suggest the existence of more and more solutions of the equations of motion. However, these solutions are non-perturbative in the expansion parameter E/M, and there is no reason to expect that they have any physical meaning. For example, taking seriously these solutions would imply that the Minkowski vacuum is unstable when considering higher order corrections to the Einstein-Hilbert action [36]. The correct way of treating these terms is perturbatively, i.e., evaluating them using the lower order equations of motion [37]. Following this logic, the additional solutions studied in the context of the Ghost Condensate theory in [38] are non-physical, as already pointed out in [39].

As we discussed in this paper, in certain regimes quintessence behaves as a deformation of the Ghost Condensate theory. The free action (A.1) is deformed by the addition of a $(\nabla\pi)^2$ term,

$$S = \frac{M^4}{2}\int d^3x dt \left[\dot{\pi}^2 - c_s^2(\nabla\pi)^2 - \frac{(\nabla^2\pi)^2}{M^2}\right] . \tag{A.5}$$

In these cases the dispersion relation is not exactly $\omega \sim k^2/M$ but it contains also a linear term $\omega \sim c_s k$, with $c_s \ll 1$. (For this discussion we assume that c_s^2 is positive.)

The situation is now trickier than before because one cannot find a scaling transformation which leaves the full action (A.5) invariant. On the other hand, one can separate two regimes, depending on which of the gradient terms dominates, as illustrated in figure 4.

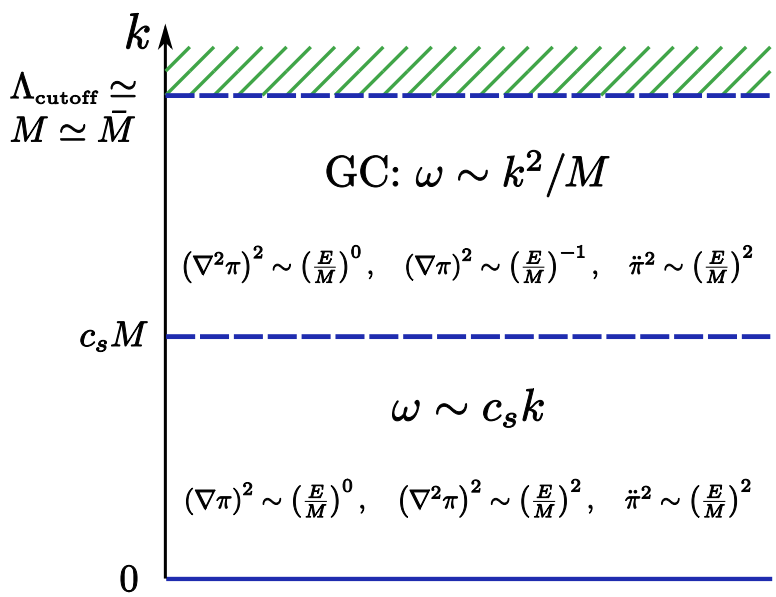

Figure 4: *The two scaling regimes as a function of the momentum k together with the scaling dimensions of some of the operators. Above: the Ghost Condensate regime where $\omega \sim k^2/M$; below: the regime where $\omega \sim c_s k$.*

For $k \gg c_s M$, the dispersion relation is dominated by $(\nabla^2\pi)^2$ and the theory behaves as the Ghost

Condensate. In this regime the scaling of all additional operators can be obtained from eq. (A.2). Notice that now there is a relevant operator, $(\nabla\pi)^2$, that becomes more and more important at low momenta (and energies). The coefficient of this operator is however suppressed by the small deformation parameter $c_s \ll 1$. Thus, it can be treated perturbatively as long as $k \gg c_s M$.

On the other hand, when $k \ll c_s M$ the $(\nabla\pi)^2$ operator dominates the free action. In this regime the scaling becomes the same as in the relativistic case. Now both $\ddot{\pi}^2$ and $(\nabla^2\pi)^2$ are irrelevant operators with the same scaling dimension 2. However, time and spatial derivatives are still on a different footing because the time derivatives are suppressed by c_s with respect to the spatial ones, $\omega \sim c_s k$. Thus $\ddot{\pi}^2 \sim c_s^4(\nabla^2\pi)^2$ for $k \ll c_s M$. In the intermediate regime $k \sim c_s M$ this suppression can also be obtained from the Ghost Condensate limit. Indeed, at high momenta the operator $\ddot{\pi}^2$ scales like $(k/M)^4$, so that it is suppressed by c_s^4 for $k \sim c_s M$.

Even though these theories make perfect sense as effective field theories, it is not clear whether one can find a UV completion. In particular, the violation of the null energy condition may be problematic in the context of black hole thermodynamics [40].

B Stress-energy tensor and fluid quantities

Here we compute the stress-energy tensor for the k-essence action with the addition of the two higher derivative operators $\mathcal{L}_{\bar{M}}$ and $\mathcal{L}_{\hat{M}}$, i.e.,

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[P(\phi, X) - \frac{\bar{M}^2}{2}(\Box\phi + 3H(\phi))^2 - \frac{\hat{M}^3}{2}(\Box\phi + 3H(\phi))(X-1) \right] . \tag{B.1}$$

We start by computing, using eq. (2.7) the stress-energy contribution of the k-essence action, which can be written as

$$(T_{\mu\nu})_P = 2P_X(\phi, X)\partial_\mu\phi\partial_\nu\phi + P(\phi, X)g_{\mu\nu} . \tag{B.2}$$

It is well known that this stress-energy tensor can be put in the perfect fluid form,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} , \tag{B.3}$$

by defining the k-essence contribution to the energy density and pressure respectively as

$$\rho_P \equiv 2XP_X - P , \tag{B.4}$$

$$p_P \equiv P , \tag{B.5}$$

and the unit four-velocity of an observer comoving with the fluid as

$$u_\mu \equiv \frac{\partial_\mu\phi}{\sqrt{-(\partial\phi)^2}} . \tag{B.6}$$

The stress-energy tensor (B.2) can be expanded in perturbations around the background $\phi(t) = t$. In synchronous gauge one finds

$$(T_{00})_P = \rho_Q + \dot{\rho}_Q \pi + (\rho_Q + p_Q + 4M^4)\dot{\pi}\,, \tag{B.7}$$

$$(T_{0i})_P = (\rho_Q + p_Q)\partial_i \pi\,, \tag{B.8}$$

$$(T_{ij})_P = a^2 \delta_{ij}\left[p_Q + \dot{p}_Q \pi + (\rho_Q + p_Q)\dot{\pi}\right] + a^2 p_Q h_{ij}\,. \tag{B.9}$$

One can do the same for the operators proportional to $\bar{M}^2$ and $\hat{M}^3$ and show that they can also be written in the perfect fluid form (B.3), by noting that u^μ defined in eq. (B.6) can be written in several different ways,

$$u_\mu = \frac{\partial_\mu X}{\sqrt{-(\partial X)^2}} = \frac{\partial_\mu \Box\phi}{\sqrt{-(\partial\Box\phi)^2}} = \frac{\partial_\mu(\Box\phi + 3H(\phi))}{\sqrt{-[\partial(\Box\phi + 3H)]^2}}\,. \tag{B.10}$$

Let us first compute the contribution to the stress-energy tensor of the operator proportional to $\bar{M}^2$, which reads[10]

$$(T_{\mu\nu})_{\bar{M}^2} = \bar{M}^2\left[-2\partial_{(\mu}(\Box\phi+3H)\partial_{\nu)}\phi + \frac{1}{2}(\Box\phi-3H)(\Box\phi+3H)g_{\mu\nu} + g^{\alpha\beta}\partial_\alpha(\Box\phi+3H)\partial_\beta\phi g_{\mu\nu}\right], \tag{B.11}$$

where $_{(\mu\nu)}$ denotes symmetrization in μ and ν. The perfect fluid form (B.3) can be obtained by recognizing its contribution to the energy density and pressure, defined as

$$\rho_{\bar{M}^2} \equiv \bar{M}^2\left[g^{\mu\nu}\partial_\mu(\Box\phi + 3H)\partial_\nu\phi - \frac{1}{2}(\Box\phi - 3H)(\Box\phi + 3H)\right], \tag{B.12}$$

$$p_{\bar{M}^2} \equiv \bar{M}^2\left[g^{\mu\nu}\partial_\mu(\Box\phi + 3H)\partial_\nu\phi + \frac{1}{2}(\Box\phi - 3H)(\Box\phi + 3H)\right]. \tag{B.13}$$

In order to expand in perturbations let us first compute the Laplace-Beltrami operator,

$$\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = -3H - \ddot{\pi} - 3H\dot{\pi} + \frac{\nabla^2\pi}{a^2} - \frac{1}{2}\dot{h}\,. \tag{B.14}$$

Then the stress-energy tensor becomes

$$\begin{aligned}(T_{\mu\nu})_{\bar{M}^2} = \bar{M}^2\Bigg[&(\partial_t + 3H)\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right)g_{\mu\nu} \\ &+ 2\delta_{0(\nu}\partial_{\mu)}\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right)\Bigg]\,.\end{aligned} \tag{B.15}$$

We now perform the same procedure for the operator proportional to $\hat{M}^3$. Its stress-energy tensor is

$$(T_{\mu\nu})_{\hat{M}^3} = \hat{M}^3\left[-(\Box\phi + 3H)\partial_\mu\phi\partial_\nu\phi - \partial_{(\mu}X\partial_{\nu)}\phi - \frac{3H}{2}g_{\mu\nu}(X-1) + \frac{1}{2}g_{\mu\nu}\partial^\alpha X\partial_\alpha\phi\right], \tag{B.16}$$

[10] Here we take for simplicity $\bar{M}$ and $\hat{M}$ to be constant.

and once again it can be written in the perfect fluid form by defining

$$\rho_{\hat{M}^3} \equiv \hat{M}^3 \left[\frac{1}{2}g^{\mu\nu}\partial_\mu X \partial_\nu \phi - (\Box\phi + 3H)X + \frac{3}{2}H(X-1)\right] , \tag{B.17}$$

$$p_{\hat{M}^3} \equiv \hat{M}^3 \left[\frac{1}{2}g^{\mu\nu}\partial_\mu X \partial_\nu \phi - \frac{3}{2}H(X-1)\right] . \tag{B.18}$$

When expanded to first order in the perturbations, the stress-energy tensor for the operator proportional to $\hat{M}^3$ takes the form

$$(T_{00})_{\hat{M}^3} = \hat{M}^3 \left(6H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right) , \tag{B.19}$$

$$(T_{0i})_{\hat{M}^3} = -\hat{M}^3 \partial_i \dot{\pi} , \tag{B.20}$$

$$(T_{ij})_{\hat{M}^3} = -\hat{M}^3 a^2 \delta_{ij}(\partial_t + 3H)\dot{\pi} . \tag{B.21}$$

Using the notation of [21], the stress-energy tensor of a perfect fluid in synchronous gauge can be written as

$$T_{00} = \rho_Q + \delta\rho_Q , \tag{B.22}$$

$$T_{0i} = -a\,(\rho_Q + p_Q)\partial_i \nabla^{-2}\theta_Q , \tag{B.23}$$

$$T_{ij} = (p_Q + \delta p_Q)a^2\delta_{ij} + a^2 p_Q h_{ij} . \tag{B.24}$$

Notice that we defined θ_Q in the main text, below eq. (4.8), using conformal time [21]. Here we are in cosmic time and this introduces the factor of a in the equation for T_{0i} above. Thus, from equations (B.7)–(B.9), (B.15), and (B.19)–(B.21) one gets, for our complete stress-energy tensor,

$$\begin{aligned}\delta\rho_Q =& \dot{\rho}_Q\pi + (\rho_Q + p_Q + 4M^4)\dot{\pi} \\ &+ \bar{M}^2\left[(\partial_t - 3H)\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right)\right] \\ &+ \hat{M}^3\left(6H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right) ,\end{aligned} \tag{B.25}$$

$$a\,\theta_Q = -\nabla^2\pi + \frac{\bar{M}^2}{\rho_Q + p_Q}\nabla^2\left(-\ddot{\pi} - 3H\dot{\pi} + 3\dot{H}\pi + \frac{\nabla^2\pi}{a^2} - \frac{1}{2}\dot{h}\right) + \frac{\hat{M}^3}{\rho_Q + p_Q}\nabla^2\dot{\pi} , \tag{B.26}$$

$$\begin{aligned}\delta p_Q =& \dot{p}_Q\pi + (\rho_Q + p_Q)\dot{\pi} \\ &+ \bar{M}^2(\partial_t + 3H)\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} + \frac{1}{2}\dot{h}\right) - \hat{M}^3(\partial_t + 3H)\dot{\pi} .\end{aligned} \tag{B.27}$$

C General action in unitary gauge

We wish to write the general action for the perturbations of a single quintessence field minimally coupled to gravity with no direct couplings to other fields. Following ref. [1, 2], we choose a gauge where the scalar field perturbation is set to zero but it appears as a scalar metric degree of freedom.

In this gauge the constant time hypersurfaces are equivalent to the uniform field hypersurfaces. In the fluid language, this implies that the velocity is orthogonal to the constant time surfaces, $T^0_i = 0$, that is why this gauge is called 'velocity orthogonal' [29]. Using a particle physics terminology we can also call it 'unitary gauge', as all the degrees of freedom are in the metric. Notice that one can always parameterize the perturbations of the scalar field as

$$\phi(t, \vec{x}) = \phi_0(t + \pi(t, \vec{x}))\,, \tag{C.1}$$

where π is the difference between the constant time and uniform field hypersurfaces, so that the unitary gauge corresponds to $\pi = 0$.

In order to find the effective action for quintessence, gravity and other matter components described by the Lagrangian $\mathcal{L}_m$, we write down all the terms that preserve the symmetries of the system. Our choice of gauge breaks time diffeomorphism invariance while preserving invariance under spatial diffeomorphisms. Thus, we include linear and quadratic combinations of generic functions of time t, the time-time component of the inverse metric g^{00} and the extrinsic curvature of the constant time hypersurfaces. The effective action up to second order in perturbations is [1, 2]:

$$\begin{aligned} S = \int \mathrm{d}^3x\,\mathrm{d}t\,\sqrt{-g}\Bigg[& \frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_m + c(t)g^{00} - \Lambda(t) + \frac{M^4(t)}{2}(g^{00}+1)^2 - \frac{\bar{M}^2(t)}{2}\delta K^2 \\ & - \frac{\tilde{M}^2(t)}{2}\delta K^i_{\ j}\delta K^j_{\ i} - \frac{\hat{M}^3(t)}{2}\delta K(g^{00}+1)\Bigg]\,, \end{aligned} \tag{C.2}$$

where R is the Ricci scalar and K_{ij} is the extrinsic curvature of constant t hypersurfaces, which at linear order reads

$$K_{ij} = \frac{1}{2}\sqrt{-g^{00}}\left(\partial_0 g_{ij} - \partial_i g_{0j} - \partial_j g_{i0}\right)\,, \tag{C.3}$$

and we have defined $\delta K_{ij} \equiv K_{ij} - a^2 H\delta_{ij}$, and $\delta K \equiv K^i_{\ i} - 3H$.

There are other operators that are invariant under spatial diffeomorphisms that one would naïvely include in this action. However, these operators are irrelevant at energy scales below the cutoff $M \sim \bar{M} \sim \hat{M}$. For instance, one could include the operator $(\dot{g}^{00})^2$. This would give the term $\ddot{\pi}^2$ in the final action, which indeed also appears as part of the operator (2.16), once expanded in the perturbations, eq. (2.17). However, as explained in section 2.2, for frequencies smaller than the cutoff $\ddot{\pi}^2$ is negligible with respect to $(\nabla^2\pi)^2$ so that it can be ignored in the action. For simplicity we will also ignore the operator proportional to $\tilde{M}^2$ as it leads to terms qualitatively similar to those proportional to $\bar{M}^2$.

One can easily fix the coefficients $c(t)$ and $\Lambda(t)$ by computing the background stress-energy tensor. This gives

$$\rho_Q = \Lambda(t) - c(t)\,, \tag{C.4}$$

$$p_Q = -c(t) - \Lambda(t)\,. \tag{C.5}$$

Finally, using these relations, we can write the action (C.2) in unitary gauge in terms of the background quantities $\rho_Q(t)$ and $p_Q(t)$,

$$S = \int \mathrm{d}^4x \sqrt{-g} \Bigg[\frac{M_{\rm Pl}^2}{2} R + \mathcal{L}_m + p_Q - \frac{1}{2}(\rho_Q + p_Q)(g^{00}+1) + \frac{M^4(t)}{2}(g^{00}+1)^2 \\ - \frac{\bar{M}^2(t)}{2}\delta K^2 - \frac{\hat{M}(t)^3}{2}\delta K (g^{00}+1) \Bigg] . \tag{C.6}$$

Now we want to rewrite this action in a gauge-invariant form. This can be done by performing the following time-coordinate transformation,

$$t \to \tilde{t} = t + \pi(x) \qquad x^i \to \tilde{x}^i = x^i \,, \tag{C.7}$$

that reintroduces π defined in eq. (C.1). The action for π reads, up to second order,

$$\begin{aligned} S = \int \mathrm{d}^4x \sqrt{-g} \Bigg\{ & p_Q + \dot{p}_Q \pi + \frac{1}{2}\ddot{p}_Q \pi^2 \\ & - \frac{1}{2}(\rho_Q + p_Q) \left[(g^{00}+1) - 2\dot{\pi} + 2(g^{00}+1)\dot{\pi} - \dot{\pi}^2 + 2g^{0i}\partial_i \pi + \frac{(\nabla\pi)^2}{a^2} \right] \\ & - \frac{1}{2}(\dot{\rho}_Q + \dot{p}_Q)\pi \left[(g^{00}+1) - 2\dot{\pi} \right] + \frac{M^4(t)}{2} \left[(g^{00}+1) - 2\dot{\pi} \right]^2 \\ & - \frac{\bar{M}^2(t)}{2} \left(\delta K - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} \right)^2 - \frac{\hat{M}(t)^3}{2} \left(\delta K - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} \right) \left[(g^{00}+1) - 2\dot{\pi} \right] \Bigg\} , \end{aligned} \tag{C.8}$$

while the part of the action containing R and $\mathcal{L}_m$ is invariant under general diffeomorphisms. We now choose to work in the synchronous gauge, which is defined by the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j \,. \tag{C.9}$$

Using the notation of [21], the two scalar degrees of freedom of h_{ij} are its trace $h \equiv \delta^{ij}h_{ij}$ and η which is defined by $\nabla^2 h_{ij} \equiv \partial_i\partial_j h + 6(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\eta$. Using this metric, after integrating by parts and using the background continuity equation $\dot{\rho}_Q + 3H(\rho_Q + p_Q) = 0$, the action (C.8) takes the form

$$\begin{aligned} S = \int \mathrm{d}^4x\, a^3 \Bigg\{ & p_Q + \frac{1}{2}(\rho_Q + p_Q) \left[\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2} \right] + 2M^4\dot{\pi}^2 + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi \\ & - \frac{\bar{M}^2}{2} \left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} \right)^2 + \hat{M}^3\dot{\pi} \left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2} \right) \Bigg\} . \end{aligned} \tag{C.10}$$

We can now compute the stress-energy tensor of quintessence using the action (C.8). Expanding

in the perturbations, its components read

$$
\begin{aligned}
T_{00} &= \rho_Q + \left(\rho_Q + p_Q + 4M^4\right)\dot{\pi} + \dot{\rho}_Q \pi \\
&\quad - 3H\bar{M}^2\left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2}{a^2}\pi\right) + \hat{M}^3\left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2}{a^2}\pi + 3H\dot{\pi}\right), \qquad \text{(C.11)} \\
T_{0i} &= (\rho_Q + p_Q)\partial_i\pi + \bar{M}^2\partial_i\left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right) - \hat{M}^3\partial_i\dot{\pi}\,, \qquad \text{(C.12)} \\
T_{ij} &= p_Q a^2\delta_{ij} + \left[\dot{p}_Q\pi + (\rho_Q + p_Q)\,\dot{\pi}\right] a^2\delta_{ij} + p_Q a^2 h_{ij} \\
&\quad + 2\bar{M}^2 a^2\delta_{ij}(\partial_0 + 3H)\left(\frac{1}{2}\dot{h} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right) - 2\hat{M}^3 a^2\delta_{ij}(\partial_0 + 3H)\dot{\pi}\,. \qquad \text{(C.13)}
\end{aligned}
$$

In the main body of this paper we have constructed the action for π using a different procedure from the one presented here. Indeed, we started from the action of k-essence, eq. (2.6), and we added the two ϕ-dependent higher derivative operators $-\bar{M}^2(\phi)[\Box\phi + 3H(\phi)]^2/2$ and $-\hat{M}^3(\phi)[\Box\phi + 3H(\phi)](X-1)/2$, that do not change the background equations of motion. Also the action (C.10) can be constructed similarly. First of all, note that the first line of eq. (C.10) is the action for k-essence. Indeed, it is equivalent to the action (2.12), which was found by expanding the k-essence action (2.1) in terms of π. The second line of eq. (C.10) can be constructed by noting that the extrinsic curvature of the hypersurfaces of constant ϕ, defined as

$$
K^\mu{}_\nu \equiv -(g^\rho{}_\nu + u^\rho u_\nu)\nabla_\rho u^\mu\,, \qquad \text{(C.14)}
$$

where u^μ is the unit vector orthogonal to ϕ defined in (B.6), can be rewritten as

$$
K^\mu{}_\nu = \frac{1}{\sqrt{-(\partial\phi)^2}}\left[\nabla^\mu\partial_\nu\phi + \frac{\partial^\mu\phi\partial^\rho\phi}{-(\partial\phi)^2}\nabla_\rho\partial_\nu\phi\right]. \qquad \text{(C.15)}
$$

When expanded around the background solution $\phi = t$, then $\delta K \equiv K^\mu_\mu - 3H(\phi)$ reads[11]

$$
\delta K = -3\dot{H}\pi - \frac{\nabla^2}{a^2}\pi + \frac{1}{2}\dot{h}\,. \qquad \text{(C.16)}
$$

Thus the action (C.10) can be constructed by simply adding to the k-essence action the two operators $-\bar{M}^2(\phi)\delta K^2/2$ and $-\hat{M}^3(\phi)\delta K(g^{00}+1)/2$, that do not change the background solution.

D Modification of gravity with $\hat{M}$

In the main text we studied the effects of the operator $\hat{M}$, focusing on stability and on the phenomenology at cosmological scales. In analogy to what happens for the Ghost Condensate, we expect

[11] One may wonder why terms of the form $\dot{\pi}\nabla^2\pi$ do not appear in δK^2, while they do appear in $(\Box\phi + 3H)^2$. As seen in eq. (C.15), a time diffeomorphism does not change the extrinsic curvature of constant ϕ hypersurfaces. Thus, δK does not contain $\dot{\pi}$ and $\dot{\pi}\nabla^2\pi$ is not generated by δK^2.

that this operator will also be relevant at short scales, inducing a modification of gravity. In this appendix we perform a preliminary analysis, restricted to linear perturbations only, although the non-linear dynamics has been shown to be relevant and quite rich in the Ghost Condensate case (see for example [22]). To simplify the analysis we set $\rho_Q + p_Q = 0$ and $\bar{M} = 0$. Although the background quintessence stress-energy tensor is the one of the cosmological constant, there is still a propagating scalar degree of freedom. Its mixing with gravity induces a deviation from General Relativity; indeed the Ghost Condensate was originally proposed as a consistent modification of gravity in the infrared. The simplest setting to study this modification of gravity is in the Newtonian regime $\omega^2 \ll k^2$ around Minkowski spacetime, where the new scalar degree of freedom modifies the Newtonian potential Φ. For this purpose we will closely follow the discussion done in [4] for the Ghost Condensate case, i.e., for the operator $\bar{M}$.

Working in Newtonian gauge with $\Psi = \Phi$, the metric is $\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + (1-2\Phi)\mathrm{d}\vec{x}^2$ and the quadratic Lagrangian for π and Φ reads

$$\mathcal{L} = -M_{\rm Pl}^2(\nabla\Phi)^2 + 2M^4(\Phi - \dot{\pi})^2 + \hat{M}^3(\Phi - \dot{\pi})(4\dot{\Phi} - \ddot{\pi} + \nabla^2\pi) \ . \tag{D.1}$$

Let us first assume that $\hat{M}$ is time independent. Dropping total derivatives and terms which are negligible in the limit $\omega^2 \ll k^2$, we are left with

$$\mathcal{L} = -M_{\rm Pl}^2(\nabla\Phi)^2 + 2M^4(\Phi - \dot{\pi})^2 + \hat{M}^3\pi\nabla^2\Phi \ . \tag{D.2}$$

In terms of the canonically normalized fields $\pi_c \equiv 2M^2\pi$ and $\Phi_c \equiv \sqrt{2}M_{\rm Pl}\Phi$, the Lagrangian in Fourier space can be written as

$$\mathcal{L} = \frac{1}{2}\begin{pmatrix}\pi_c & \Phi_c\end{pmatrix}\mathcal{M}\begin{pmatrix}\pi_c \\ \Phi_c\end{pmatrix} \ , \tag{D.3}$$

with

$$\mathcal{M} \equiv \begin{pmatrix} \omega^2 & -i\omega\sqrt{2}M^2/M_{\rm Pl} - k^2\hat{M}^3/(2\sqrt{2}M^2M_{\rm Pl}) \\ i\omega\sqrt{2}M^2/M_{\rm Pl} - k^2\hat{M}^3/(2\sqrt{2}M^2M_{\rm Pl}) & -k^2 + 2M^4/M_{\rm Pl}^2 \end{pmatrix} \ . \tag{D.4}$$

Setting to zero the determinant of this matrix gives the dispersion relation

$$\omega^2 = -\frac{\hat{M}^6}{8M^4M_{\rm Pl}^2}k^2 \ , \tag{D.5}$$

which reproduces the Jeans instability already shown in eq. (5.8). The Jeans instability arises from the non-diagonal (mixing) term and it is thus proportional to $\hat{M}^6$ instead of $\hat{M}^3$.

To study the corrections to the Newtonian theory, one can look at the propagator of Φ that is the $\langle\Phi,\Phi\rangle$ entry of $\mathcal{M}^{-1}$. This can be written as

$$-\frac{1}{k^2}\cdot\left[1 - \frac{k^2\hat{M}^6}{8M^4M_{\rm Pl}^2}\cdot\frac{1}{\omega^2 + k^2\hat{M}^6/(8M^4M_{\rm Pl}^2)}\right] \ , \tag{D.6}$$

where the term $-1/k^2$ is simply the standard Newtonian propagator. As expected, a substantial deviation requires, for a given distance k^{-1}, a sufficient time ω^{-1} for the Jeans instability to develop, i.e.,

$$\omega^2 \lesssim \frac{\hat{M}^6}{8M^4 M_{\rm Pl}^2} k^2 \ . \tag{D.7}$$

We can now consider the case of a time dependent $\hat{M}$. In this way we introduce new terms in the action that were previously dropped because total derivatives. The same happens if we had considered a time dependent spatial metric, but here we stick to Minkowski for simplicity. The matrix $\mathcal{M}$ becomes

$$\mathcal{M} \equiv \begin{pmatrix} \omega^2 - k^2 H \hat{M}^3/4M^4 & -i\omega\sqrt{2}M^2/M_{\rm Pl} - k^2\hat{M}^3/(2\sqrt{2}M^2 M_{\rm Pl}) \\ i\omega\sqrt{2}M^2/M_{\rm Pl} - k^2\hat{M}^3/(2\sqrt{2}M^2 M_{\rm Pl}) & -k^2 + 2M^4/M_{\rm Pl}^2 - 2H\hat{M}^3/M_{\rm Pl}^2 \end{pmatrix} \ , \tag{D.8}$$

where H is the typical rate of variation of $\hat{M}^3$, $\dot{\hat{M}}^3 = H\hat{M}^3$ (if the time dependence is induced by the metric this becomes the Hubble rate). Computing the determinant and restricting to frequencies much larger than H we get the dispersion relation

$$\omega^2 = \frac{H\hat{M}^3}{4M^4} k^2 - \frac{\hat{M}^6}{8M^4 M_{\rm Pl}^2} k^2 \ , \tag{D.9}$$

which correctly matches eq. (5.8). The value of $\hat{M}$ can be chosen to avoid the Jeans instability and have a healthy dispersion relation. The propagator becomes

$$-\frac{1}{k^2} \cdot \left[1 - \frac{k^2 \hat{M}^6}{8M^4 M_{\rm Pl}^2} \cdot \frac{1}{\omega^2 - k^2\hat{M}^3 H/(4M^4) + k^2 \hat{M}^6/(8M^4 M_{\rm Pl}^2)} \right] \ . \tag{D.10}$$

The scalar degree of freedom induces a $1/r$ force which adds to the Newton law: this force, however, propagates at a very small speed

$$c_s^2 \approx \frac{\hat{M}^3 H}{4M^4}. \tag{D.11}$$

Given the absence of a Jeans instability, the modification of gravity induced by $\hat{M}$ is very different at linear and non-linear level with respect to the Ghost Condensate case [4, 22]. More work is needed to understand the constraints on $\hat{M}$ coming from the modifications of gravity that it produces.

References

[1] P. Creminelli, M. A. Luty, A. Nicolis and L. Senatore, "Starting the universe: Stable violation of the null energy condition and non-standard cosmologies," JHEP **0612** (2006) 080 [arXiv:hep-th/0606090].

[2] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, "The Effective Field Theory of Inflation," JHEP **0803** (2008) 014 [arXiv:0709.0293 [hep-th]].

[3] A. Albrecht *et al.*, "Report of the Dark Energy Task Force," arXiv:astro-ph/0609591.

[4] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, "Ghost condensation and a consistent infrared modification of gravity," JHEP **0405** (2004) 074 [arXiv:hep-th/0312099].

[5] M. z. Li, B. Feng and X. m. Zhang, "A single scalar field model of dark energy with equation of state crossing -1," JCAP **0512**, 002 (2005) [arXiv:hep-ph/0503268].

[6] B. Feng, X. L. Wang and X. M. Zhang, "Dark Energy Constraints from the Cosmic Age and Supernova," Phys. Lett. B **607**, 35 (2005) [arXiv:astro-ph/0404224].

[7] W. Hu, "Crossing the phantom divide: Dark energy internal degrees of freedom," Phys. Rev. D **71** (2005) 047301 [arXiv:astro-ph/0410680].

[8] A. Vikman, "Can dark energy evolve to the phantom?," Phys. Rev. D **71**, 023515 (2005) [arXiv:astro-ph/0407107].

[9] R. R. Caldwell and M. Doran, "Dark-energy evolution across the cosmological-constant boundary," Phys. Rev. D **72** (2005) 043527 [arXiv:astro-ph/0501104].

[10] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, "k-inflation," Phys. Lett. B **458**, 209 (1999) [arXiv:hep-th/9904075].

[11] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, "A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration," Phys. Rev. Lett. **85**, 4438 (2000) [arXiv:astro-ph/0004134].

[12] J. M. Cline, S. Jeon and G. D. Moore, "The phantom menaced: Constraints on low-energy effective ghosts," Phys. Rev. D **70**, 043543 (2004) [arXiv:hep-ph/0311312].

[13] B. Holdom, "Accelerated expansion and the Goldstone ghost," JHEP **0407**, 063 (2004) [arXiv:hep-th/0404109].

[14] R. R. Caldwell, "A Phantom Menace?," Phys. Lett. B **545**, 23 (2002) [arXiv:astro-ph/9908168].

[15] J. Garriga and V. F. Mukhanov, "Perturbations in k-inflation," Phys. Lett. B **458**, 219 (1999) [arXiv:hep-th/9904176].

[16] C. Bonvin, C. Caprini and R. Durrer, "Superluminal motion and closed signal curves," arXiv:0706.1538 [astro-ph].

[17] E. Babichev, V. Mukhanov and A. Vikman, "k-Essence, superluminal propagation, causality and emergent geometry," JHEP **0802**, 101 (2008) [arXiv:0708.0561 [hep-th]].

[18] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, "Causality, analyticity and an IR obstruction to UV completion," JHEP **0610**, 014 (2006) [arXiv:hep-th/0602178].

[19] S. D. H. Hsu, A. Jenkins and M. B. Wise, "Gradient instability for $w < -1$," Phys. Lett. B **597**, 270 (2004) [arXiv:astro-ph/0406043].

[20] L. Senatore, "Tilted ghost inflation," Phys. Rev. D **71**, 043512 (2005) [arXiv:astro-ph/0406187].

[21] C. P. Ma and E. Bertschinger, "Cosmological perturbation theory in the synchronous and conformal Newtonian gauges," Astrophys. J. **455**, 7 (1995) [arXiv:astro-ph/9506072].

[22] N. Arkani-Hamed, H. C. Cheng, M. A. Luty, S. Mukohyama and T. Wiseman, "Dynamics of gravity in a Higgs phase," JHEP **0701**, 036 (2007) [arXiv:hep-ph/0507120].

[23] S. Mukohyama, "Accelerating universe and cosmological perturbation in the ghost condensate," JCAP **0610**, 011 (2006) [arXiv:hep-th/0607181].

[24] R. Dave, R. R. Caldwell and P. J. Steinhardt, "Sensitivity of the cosmic microwave background anisotropy to initial conditions in quintessence cosmology," Phys. Rev. D **66**, 023516 (2002) [arXiv:astro-ph/0206372].

[25] J. K. Erickson, R. R. Caldwell, P. J. Steinhardt, C. Armendariz-Picon and V. F. Mukhanov, "Measuring the speed of sound of quintessence," Phys. Rev. Lett. **88**, 121301 (2002) [arXiv:astro-ph/0112438].

[26] S. DeDeo, R. R. Caldwell and P. J. Steinhardt, "Effects of the sound speed of quintessence on the microwave background and large scale structure," Phys. Rev. D **67**, 103509 (2003) [Erratum-ibid. D **69**, 129902 (2004)] [arXiv:astro-ph/0301284].

[27] J. Weller and A. M. Lewis, "Large Scale Cosmic Microwave Background Anisotropies and Dark Energy," Mon. Not. Roy. Astron. Soc. **346**, 987 (2003) [arXiv:astro-ph/0307104].

[28] R. Bean and O. Dore, "Probing dark energy perturbations: the dark energy equation of state and speed of sound as measured by WMAP," Phys. Rev. D **69**, 083503 (2004) [arXiv:astro-ph/0307100].

[29] H. Kodama and M. Sasaki, "Cosmological Perturbation Theory," Prog. Theor. Phys. Suppl. **78** (1984) 1.

[30] S. Hannestad, "Constraints on the sound speed of dark energy," Phys. Rev. D **71**, 103519 (2005) [arXiv:astro-ph/0504017].

[31] M. Takada, "Can A Galaxy Redshift Survey Measure Dark Energy Clustering?," Phys. Rev. D **74**, 043505 (2006) [arXiv:astro-ph/0606533].

[32] A. Torres-Rodriguez and C. M. Cress, "Constraining the Nature of Dark Energy using the SKA," Mon. Not. Roy. Astron. Soc. **376**, 1831 (2007) [arXiv:astro-ph/0702113].

[33] W. Hu and R. Scranton, "Measuring Dark Energy Clustering with CMB-Galaxy Correlations," Phys. Rev. D **70**, 123002 (2004) [arXiv:astro-ph/0408456].

[34] P. S. Corasaniti, T. Giannantonio and A. Melchiorri, "Constraining dark energy with cross-correlated CMB and Large Scale Structure data," Phys. Rev. D **71**, 123521 (2005) [arXiv:astro-ph/0504115].

[35] J. Polchinski, "Effective Field Theory And The Fermi Surface," arXiv:hep-th/9210046.

[36] J. Z. Simon, "The Stability of flat space, semiclassical gravity, and higher derivatives," Phys. Rev. D **43**, 3308 (1991).

[37] J. Z. Simon, "Higher derivative Lagrangians, non-locality, problems and solutions," Phys. Rev. D **41**, 3720 (1990).

[38] R. Kallosh, J. U. Kang, A. Linde and V. Mukhanov, "The New Ekpyrotic Ghost," JCAP **0804** (2008) 018 [arXiv:0712.2040 [hep-th]].

[39] S. Weinberg, "Effective Field Theory for Inflation," Phys. Rev. D **77**, 123541 (2008) [arXiv:0804.4291 [hep-th]].

[40] N. Arkani-Hamed, S. Dubovsky, A. Nicolis, E. Trincherini and G. Villadoro, "A Measure of de Sitter Entropy and Eternal Inflation," JHEP **0705**, 055 (2007) [arXiv:0704.1814 [hep-th]].