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Inversion Using Finite Elements

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Outline

- Formulation of the inversion problem
  - Gravity
  - MT
- Cost Function Gradient
- Solver: BFGS method
- FEM implementation

Inversion Problem

Optimization problem for property function m

$$\text{Find } \underset{m}{arg\,min} \ \ \Phi(m)$$

$$\text{cost function}: \Phi(m) = \underbrace{\Phi_d(m)}_{misfit} + \mu \cdot \underbrace{\Phi_r(m)}_{\text{regularization}}$$

Positive regularization parameter

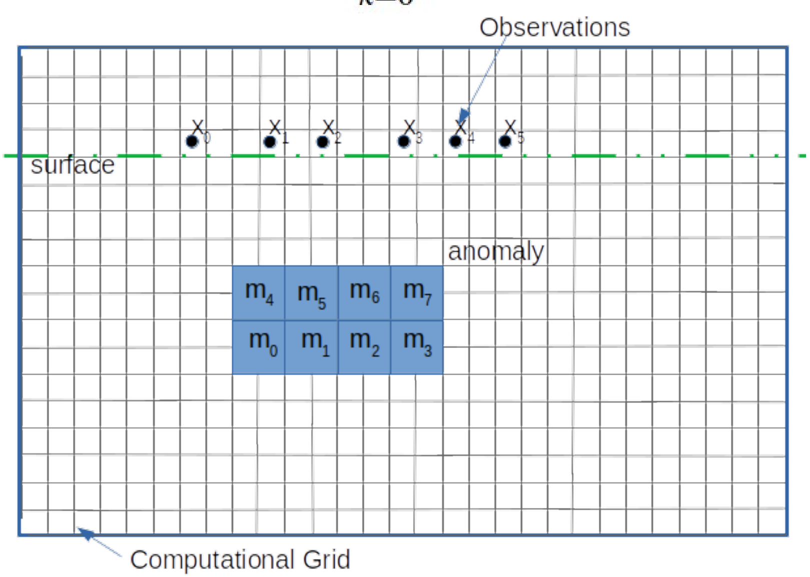
Property Function m

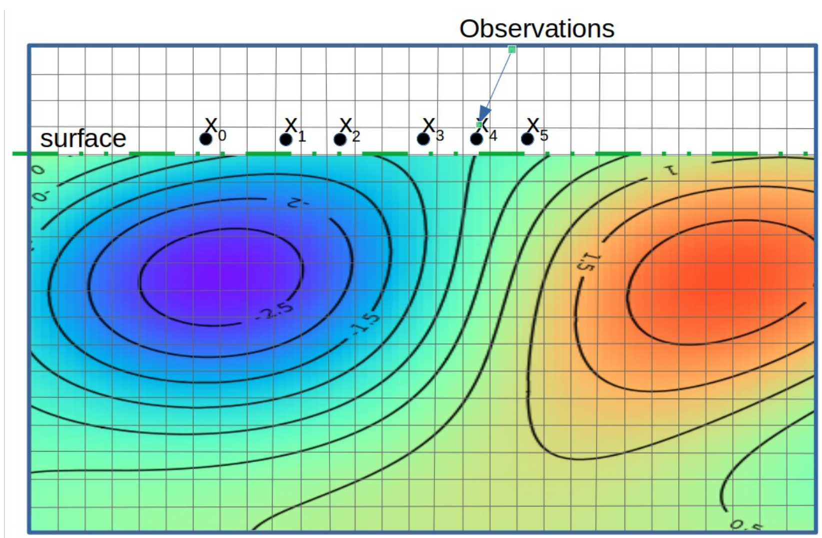
Before:
unknown is a vector of real values

$$\mathbf{m} = [m_0, \ldots, m_{N_p-1}]^T$$

$$m = \sum_{k=0}^{N_p-1} m_k \chi_k$$

Now:
unknown m is a function of $\mathbf{x}=(x_0,x_1)$





Regularization

- Now we use integrals and gradients

$$\Phi_r(m) = \frac{1}{2}\int_{\Omega} \nu_1 \|\nabla m\|^2 + \nu_0 |m|^2 \, d\boldsymbol{x}$$

Before:

$$\Phi_r(\boldsymbol{m}) = \frac{1}{2}\boldsymbol{m}^T\boldsymbol{m}$$

$$\text{weighting factors: } \nu_1, \nu_0 \geq 0$$

$$\text{recall: } \|\nabla m\|^2 = \left(\frac{\partial m}{\partial x_0}\right)^2 + \left(\frac{\partial m}{\partial x_1}\right)^2 + \left(\frac{\partial m}{\partial x_2}\right)^2$$

Misfit

- Now we use integrals and weighting functions:

$$\Phi_d(m) = \frac{1}{2} \int_\Omega w_d \cdot |d - d^{ops}|^2 \, dx$$

Before:

$$\Phi_d(\mathbf{m}) = \frac{1}{2}(\mathbf{d} - \mathbf{d}^{obs})^T \mathbf{W}_d (\mathbf{d} - \mathbf{d}^{obs})$$

- Forward model: $d=F(m)$
- Weighting function w_d

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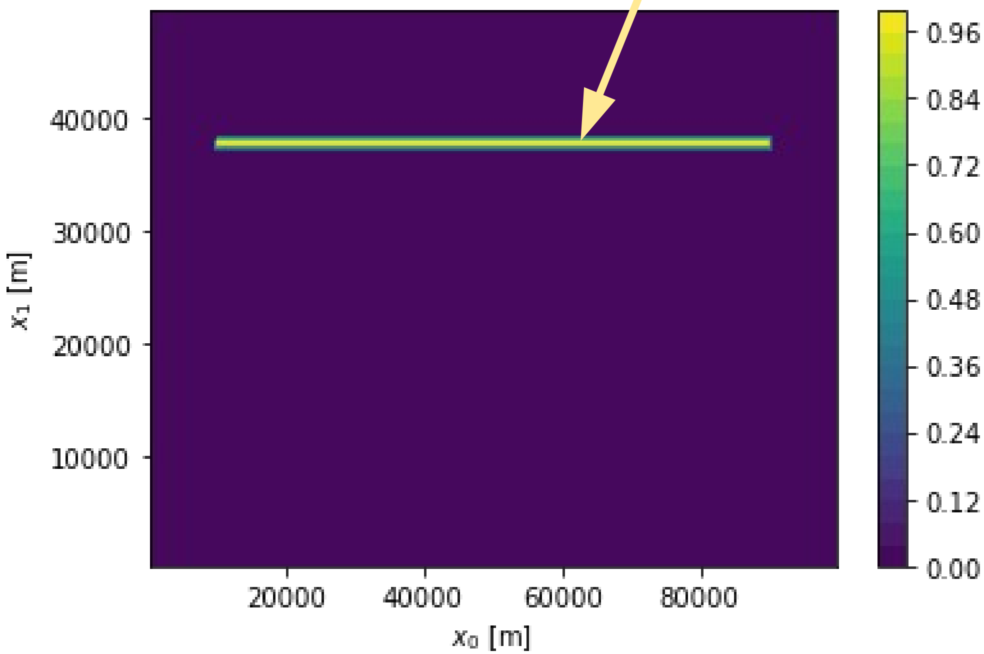
Weighting Function

$$w_d(\boldsymbol{x}) = \begin{cases} >0 & \text{data } d^{obs}(\boldsymbol{x}) \text{ at } \boldsymbol{x} \\ 0 & \text{no data} \end{cases}$$

Data are given here



normalize:

$$\int_{\Omega} w_d \, dx = 1$$

Forward Model: Gravity

- Parametrization of density

$$\rho = \rho' \cdot m + \rho_{ref}$$

- Gravity potential u from PDE:

$$-\nabla^T \nabla u = -4\pi G \rho$$

- Predicted data

$$d = F(m) = g_z = -e_z^T \nabla u$$

Inversion Problem

$$\text{cost function}: \Phi(m) = \Phi_d(m) + \mu \cdot \Phi_r(m)$$

$$\text{Find } m^* = \underset{m}{\arg\min}\ \Phi(m)$$

We need a critical point condition for minimum m^*

Towards a Critical Point Condition

- For any increment δm for the property function:
- The function

$$\varphi_{\delta m}(\alpha) = \Phi(m^* + \alpha \cdot \delta m) \text{ for } \alpha \in \mathbb{R}$$

has a minimum at $\alpha = 0$.

- Therefore

$$\left. \frac{d\varphi_{\delta m}}{d\alpha} \right|_{\alpha=0} = 0 \text{ for all } \delta m$$

Gradient of cost function

- Directional derivative of Φ at m along δm:

$$\langle \nabla \Phi(m) | \delta m \rangle = \left. \frac{d\Phi(m + \alpha \cdot \delta m)}{d\alpha} \right|_{\alpha=0}$$

  - Linear function of δm
- Gradient of cost function Φ at m: $\nabla \Phi(m)$

Critical Point Condition

- Find m^* with

$$\langle \nabla \Phi(m^*) | \delta m \rangle = 0$$

for any increment δm

We need to calculate this gradient!?

Increments

- We can see $\delta\Phi$ as a change of Φ due to a \`tiny\` change δm *to* m*:*

$$\delta\Phi = \langle \nabla\Phi(m) | \delta m \rangle$$

- Idea is to track the changes through from the property function m all the way to the cost function Φ

m→ density→ potential → gravity → misfit → cost function

← Derive in backwards direction

Increments (cont)

- Some simple rules:

$$f = f(m) \qquad : \delta f = f' \cdot \delta m = \frac{df}{dm} \cdot \delta m$$

$$f = f(g(m)) \qquad : \delta f = f' \cdot \delta g = f' \cdot g' \cdot \delta m$$

$$\boldsymbol{f} = \nabla g \qquad : \delta \boldsymbol{f} = \nabla \delta g$$

Gradient Of Regularization

$$\Phi_r(m) = \frac{1}{2}\int_{\Omega} \nu_1 \|\nabla m\|^2 + \nu_0 m^2 \, d\boldsymbol{x}$$

$$\delta \Phi_r = \frac{1}{2}\int_{\Omega} \nu_1 \cdot \delta \|\nabla m\|^2 + \nu_0 \cdot \delta m^2 \, d\boldsymbol{x}$$

$$\delta m^2 = 2 m \cdot \delta m$$

$$\delta \|\nabla m\|^2 = \delta\left(\nabla^T m \nabla m\right) = 2 \nabla^T m \nabla \delta m$$

$$\delta \Phi_r = \langle \nabla \Phi_r(m) | \delta m \rangle = \int_{\Omega} \left(\nu_1 \cdot \nabla^T m \nabla \delta m + \nu_0 \cdot m \cdot \delta m\right) d\boldsymbol{x}$$

Representation of the Gradient

- Identify coefficients:

$$\langle \nabla \Phi_r(m) | \delta m \rangle = \int_\Omega \left(\underbrace{\nu_1 \cdot \nabla^T m}_{=X} \nabla \delta m + \underbrace{\nu_0 \cdot m}_{=Y} \cdot \delta m \right) d\boldsymbol{x}$$

- Represent gradient by ***X*=*X(m)***, *Y*=***Y(m)***:

$$\nabla \Phi_r(m) \Leftrightarrow (\boldsymbol{X}, Y)$$

- Then

$$\langle \nabla \Phi_r(m) | \delta m \rangle = \int_\Omega \left(\boldsymbol{X}^T \nabla \delta m + Y \cdot \delta m \right) d\boldsymbol{x}$$

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Gradient of Misfit

- This is a bit harder!

$$\Phi_d = \frac{1}{2}\int_{\Omega} w_d \cdot |d - d^{ops}|^2 \, dx$$

$$\delta\Phi_d = \frac{1}{2}\int_{\Omega} w_d \cdot \delta |d - d^{ops}|^2 \, dx = \int_{\Omega} w_d \cdot (d - d^{ops}) \cdot \delta d \; dx$$

$$d = g_z = -e_z^T \nabla u \qquad \longrightarrow \qquad \delta d = \delta g_z = -e_z^T \nabla \delta u$$

$$\delta\Phi_d = \int_{\Omega} w_d \cdot (d^{ops} - d) \cdot e_z^T \nabla \delta u \; dx$$

We need a δm here!

Gradient of Misfit

- PDE needs to be solved to connect u with m.
- We solve this in weak form:

$$\rho = \rho' \cdot m + \rho_{ref} \quad \longrightarrow \quad \delta\rho = \rho' \cdot \delta m$$

$$\int_{\Omega} \nabla^T v \nabla u \, d\boldsymbol{x} = -4\pi G \int_{\Omega} v \rho \, d\boldsymbol{x}$$

$$\downarrow \delta\text{PDE}$$

$$\int_{\Omega} \left(\nabla^T v \nabla \delta u \right) d\boldsymbol{x} = \int_{\Omega} \gamma \left(v \, \delta m \right) d\boldsymbol{x}$$

$$\gamma = -4\pi G \rho'$$

$$\rho' = \frac{d\rho}{dm}$$

Adjoint problem

- The magic idea: Solve the adjoint equation for the adjoint potential u^*
- How to get the adjoint equation?
  - collect all δu terms of δPDE
  - Set this equal $\delta \Phi_d$
  - Replace $v \rightarrow u^*, \delta u \rightarrow v$
  - Solve adjoint equation for any v to get u^*

Adjoint problem (cont)

- collect all δu terms of δPDE: $\int_\Omega \left(\nabla^T v \nabla \delta u\right) d\boldsymbol{x}$
- Set this equal $\delta\Phi_d$

$$\int_\Omega \left(\nabla^T \underbrace{v}_{\to u^*} \nabla \underbrace{\delta u}_{\to v}\right) d\boldsymbol{x} = \int_\Omega w_d \cdot \left(d^{ops} - d\right) \cdot e_z^T \nabla \underbrace{\delta u}_{\to v}\, dx$$

- Replace $v \to u^*$, $\delta u \to v$:

$$\int_\Omega \left(\nabla^T u^* \nabla v\right) d\boldsymbol{x} = \int_\Omega w_d \cdot \left(d^{ops} - d\right) \cdot e_z^T \nabla v \; dx \;\; \text{for all } v$$

What is the gradient of ϕ_d then?

- Adjoint problem: set $v=\delta u$:

$$\int_{\Omega} \left(\nabla^T u^* \nabla \delta u\right) d\boldsymbol{x} = \int_{\Omega} w_d \cdot \left(d^{ops} - d\right) \cdot e_z^T \nabla \delta u \, dx = \delta \Phi_d$$

- in δPDE: set $v=u^*$:

$$\int_{\Omega} \left(\nabla^T u^* \nabla \delta u\right) d\boldsymbol{x} = \int_{\Omega} \gamma \left(u^* \delta m\right) d\boldsymbol{x}$$

- So we have: $\delta \Phi_d = \int_{\Omega} \gamma \left(u^* \delta m\right) d\boldsymbol{x}$

Gradient of cost function

- We put this all together:

$$\langle \nabla \Phi | \delta m \rangle = \int_{\Omega} \left(\nu_1 \cdot \nabla^T m \nabla \delta m + \left(\gamma u^* + \nu_0 \cdot m \right) \cdot \delta m \right) d\boldsymbol{x}$$

- Representation for an implementation:

$$\nabla \Phi = (\boldsymbol{X}, Y) = (\mu \nu_1 \cdot \nabla m, \gamma u^* + \mu \nu_0 \cdot m)$$

How to find the minimum?

- Quasi-Newton Scheme
  - Iterative process creating sequence of property function approximations m^k $k=1,2,3,4,\ldots$
  - second-order approximation of cost function at m^k

$$\Phi(m^k+p) \approx \Phi(m^k) + \langle \nabla \Phi(m^k) | p \rangle + \langle B^k p | p \rangle$$

  - with an approximation B^k of the Hessian of Φ
- Apply gradient with respect to p:

$$\nabla \Phi(m^k+p) = \nabla \Phi(m^k) + B^k p$$

Iterative Scheme

- Iterative scheme:
  - Solve for search direction p^k:

$$B^k p^k = -\nabla\Phi(m^k)$$

  - Apply line search $\alpha_k = \underset{\alpha}{arg\,min}\,\Phi(m^k + \alpha\cdot p^k)$
  - Update solution: $m^{k+1} = m^k + s^k = \text{ with } s^k = \alpha^k\cdot p^k$
  - Calculate $g^k = \nabla\Phi(m^{k+1}) - \nabla\Phi(m^k)$
  - Update Hessian: such that: $B^{k+1} s^k = g^k$
  - Next step: $k \leftarrow k+1$

Inversion of Hessian

- Gradient and hence difference of gradients are represented by tuple $g=(\boldsymbol{X},Y)$:

$$\text{solving } B^k p = g \text{ means :}$$

$$\langle B^k p | q \rangle = \int_{\Omega} \left(\boldsymbol{X}^T \nabla q + Y \cdot q \right) d\boldsymbol{x} \text{ for all } q$$

- See weak form of PDE template

BFGS

- Broyden–Fletcher–Goldfarb–Shanno algorithm
  - Special update of approximate Hessian

$$B^{k+1} = B^k + \frac{1}{a_k} \; |g^k\rangle\langle g^k| - \frac{1}{b_k} \; |B^k s^k\rangle\langle B^k s^k|$$

$$a_k = \langle g^k | s^k \rangle = \int_\Omega \left(\boldsymbol{X}_g^T \nabla s^k + Y_g \cdot s^k \right) d\boldsymbol{x} \text{ with } g^k = (\boldsymbol{X}_g, Y_g)$$

$$b_k = \langle B^k s^k | s^k \rangle = \int_\Omega \left(\boldsymbol{X}_B^T \nabla s^k + Y_B \cdot s^k \right) d\boldsymbol{x} \text{ with } B^k s^k = (\boldsymbol{X}_B, Y_B)$$

  - Inverse $(B^k)^{-1}$ of B^k can be calculated explicitly.

Remarks

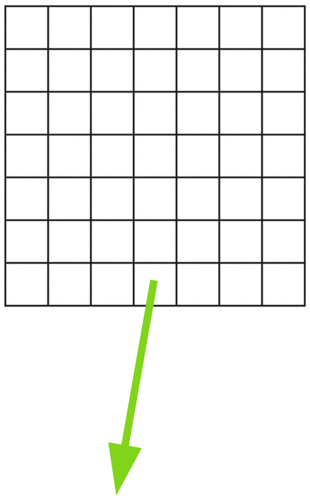
- Implemented with
  - truncation of terms in the Hessian
  - restart to through away bad approximations
- Inverse of the initial Hessian approximation B^0 is required
  - In inversion: Use the Hessian of the regularization

get $p=(B^0)^{-1} g$ by solving

$$\int_\Omega \left(\nu_1 \nabla^T q \nabla p + \nu_0 q p\right) d\boldsymbol{x} = \int_\Omega \left(\boldsymbol{X}_g^T \nabla q + Y_g \cdot q\right) d\boldsymbol{x} \text{ for all } q$$

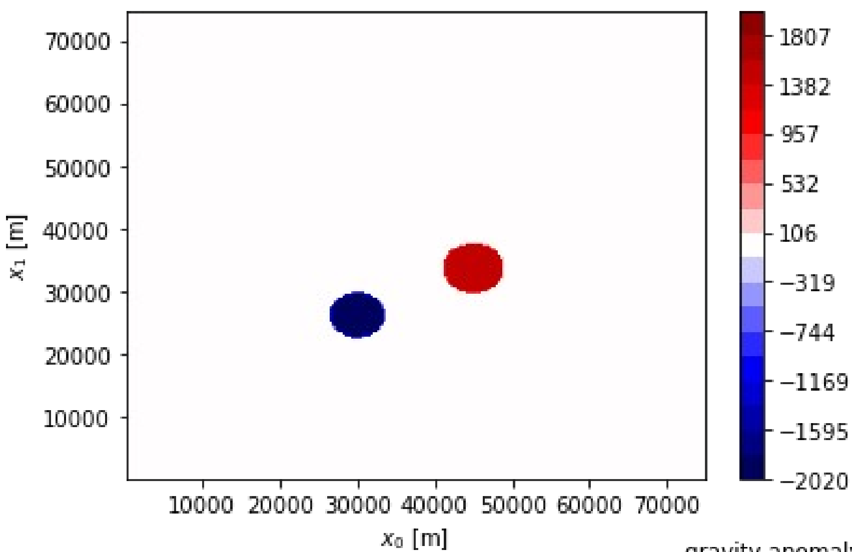
FEM implementation

- Use FEM to solve:
  - Forward problems
  - Adjoint problems
  - Hessian approximation
- Values
  - Potential $u \rightarrow$ FEM nodes
  - Adjoint solution $u^* \rightarrow$ FEM nodes
  - Cost function gradients $(\mathbf{X},Y) \rightarrow$ integration points

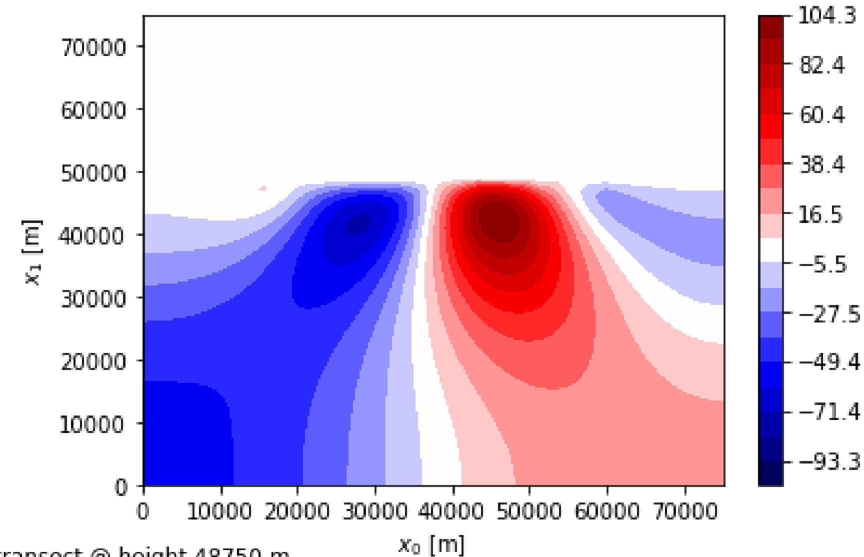


Gravity

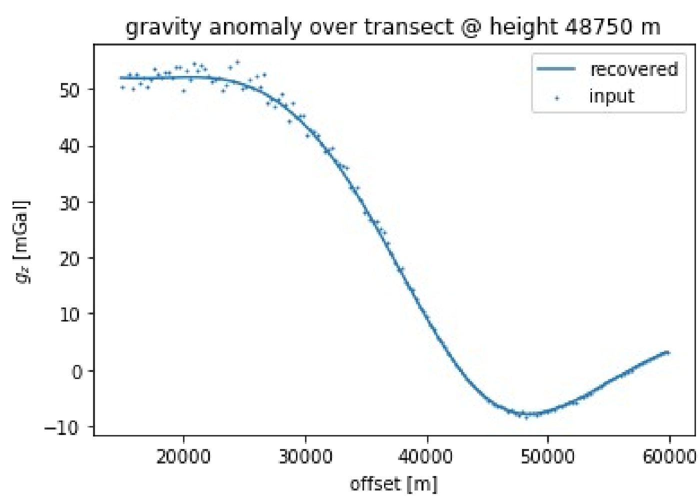
True Density



Recovered



$$\mu = 10^{-6}$$



FEM inversion

Inversion MT-TE

- Conductivity model (needs to be non negative)

$$\sigma = \sigma_{ref} e^{m}$$

- Electric field Ex solution of PDE

$$-\nabla^T \nabla E_x + j \mu_0 \omega \sigma E_x = 0$$

- Predicted data

$$d = F(m) = Z_{xy} = \boldsymbol{j} \omega \mu_0 E_x \cdot \left(\frac{\partial E_x}{\partial x_1} \right)^{-1}$$

Gradient of MT Cost Function?

$$\Phi_d = \frac{1}{2}\int_{\Omega} w_d \cdot |Z - Z^{ops}|^2 \, dx$$

$$\delta |a|^2 = \delta(\bar{a}\, a) = a\, \overline{\delta a} + \delta a\, \bar{a} = 2\,\Re[\delta a\, \bar{a}]$$

$$\delta \Phi_d = \int_{\Omega} w_d \cdot \Re[\overline{(Z - Z^{ops})}\, \delta Z]\, dx$$

$$Z = \boldsymbol{j}\,\omega\,\mu_0\, u \cdot \left(\frac{\partial u}{\partial x_1}\right)^{-1}$$

$$\delta Z = \boldsymbol{j}\,\omega\,\mu_0 \left(\delta u \frac{\partial u}{\partial x_1} - u \frac{\partial\, \delta u}{\partial x_1}\right) \cdot \left(\frac{\partial u}{\partial x_1}\right)^{-2}$$

$$\delta \Phi_d = \Re \int_{\Omega} w_d \cdot (Y_d\, \delta u + \boldsymbol{X}_d^T \nabla \delta u)\, d\boldsymbol{x} \ \text{ with}$$

$$Y_d = \boldsymbol{j}\,\omega\,\mu_0 \overline{(Z - Z^{ops})} \left(\frac{\partial u}{\partial x_1}\right)^{-1} \quad \boldsymbol{X}_d = -\boldsymbol{j}\,\omega\,\mu_0 \overline{(Z - Z^{ops})} \cdot u \cdot \left(\frac{\partial u}{\partial x_1}\right)^{-2} \boldsymbol{e}_1$$

Adjoint Problem

- Weak form of forward model:

$$\int_\Omega \nabla^T v \nabla u + \boldsymbol{j} \omega \sigma u v \, d\boldsymbol{x} = 0 \ \text{ for all } v$$

- Apply δ:

$$\int_\Omega \underline{\nabla^T v \nabla \delta u} + \boldsymbol{j} \omega \mu_0 (u \delta \sigma + \underline{\sigma \delta u}) v \, d\boldsymbol{x} = 0 \ \text{ for all } v$$

- Adjoint problem: collect δu:

$$\int_\Omega (\nabla^T u^* \nabla v + j \mu_0 \omega \sigma u^* v) \, d\boldsymbol{x} = \int_\Omega w_d \cdot (Y_d v + \boldsymbol{X}_d^T \nabla v) \, d\boldsymbol{x} \ \text{ for all } v$$

Gradient MT-TE Mode

- Finally we have:

$$\delta\Phi_d = -\Re\int_\Omega \boldsymbol{j}\mu_0\omega u u^* \delta\sigma \, d\boldsymbol{x} = -\Re\int_\Omega (\boldsymbol{j}\mu_0\omega u u^* \sigma')\delta m \, d\boldsymbol{x}$$

$$\delta\sigma = \frac{d\sigma}{dm}\delta m = \sigma'\delta m \qquad = \int_\Omega \underbrace{(-\Re\,\boldsymbol{j}\mu_0\omega u u^* \sigma')}_{=Y}\delta m \, d\boldsymbol{x}$$

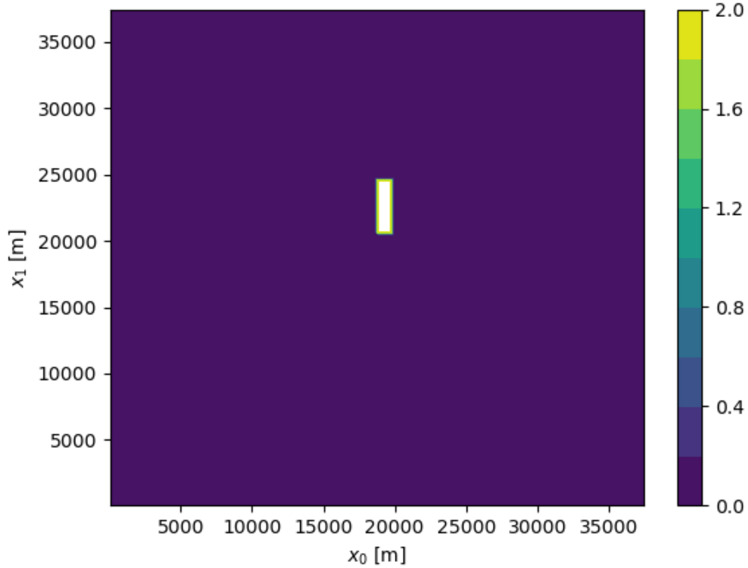
Property function m is real

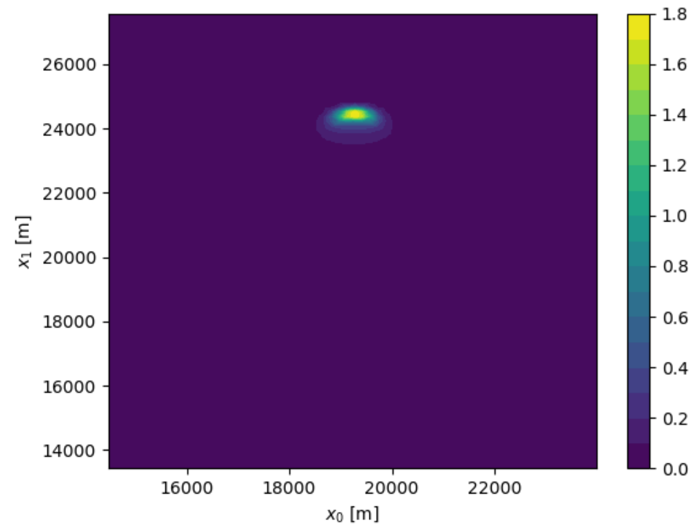
- Put is all together:

$$\langle\nabla\Phi|\delta m\rangle = \int_\Omega \left(\nu_1 \cdot \nabla^T m \nabla\delta m + (\nu_0 \cdot m - \Re\,\boldsymbol{j}\mu_0\omega u u^* \sigma')\cdot\delta m\right) d\boldsymbol{x}$$

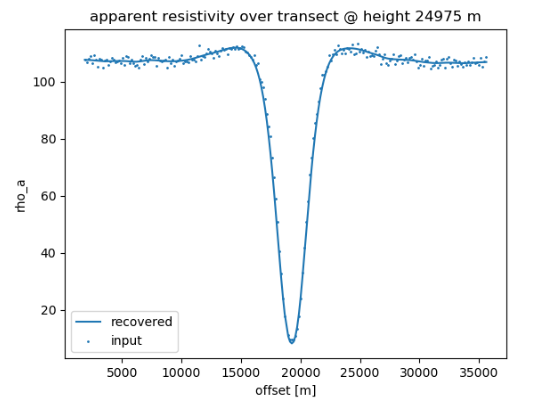
- Ready for the BFGS

MT Example





f=5Hz, $\mu=10^{-2}$



Joint Inversion

- Idea: invert for two or more physical properties simultaneously.
  - Reduce the `degree of ill-posedness` - really???
  - Create consistent distributions of physical properties.
- How to connect two inversions?
  - For instance Gravity and MT.

Constitutive Relationships

- Explicit relationships are known
  - typically empirical
  - for instance: density as function of electric conductivity

$$\rho = F(\sigma)$$

Some lateritic soil

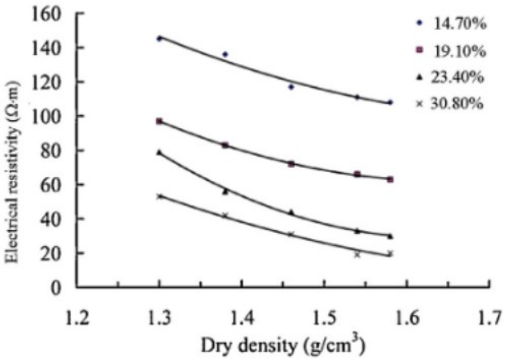


Fig. 12. Relationship between dry density and electrical resistivity at different water contents.

https://doi.org/10.1016/j.jrmge.2013.07.003

How to build the inversion?

- Unknown: property function m
  - Density and conductivity distribution are both parameterized with m:

$$\sigma = \sigma_{ref} e^{m}$$
$$\rho = F\left(\sigma_{ref} e^{m}\right)$$

- Cost function to minimize:

$$\Phi(m) = \alpha^{(1)} \underbrace{\Phi_d^{(1)}(m)}_{\text{gravity}} + \alpha^{(2)} \underbrace{\Phi_d^{(2)}(m)}_{\text{MT}} + \mu \cdot \underbrace{\Phi_r(m)}_{\text{regularization}}$$

Tweak factors to adjust balance : $\alpha^{(1)}$, $\alpha^{(2)}$

- BFGS ready!

Without Constitutive Relationships?

- Difficulties: It is an optimistic approach!
  - Relationships are unknown
    - Or can only obtained for shallow regions
  - Relationships may be valid in certain regions but the extends of the regions are unknown
- Assumption: There is a relationship between density and conductivity but the actual relationship is unknown.

$$\rho = F(\sigma) \text{ with unknown } F$$

Joint Inversion Set-up

- Unknowns: $\boldsymbol{m}=(m_0, m_1)$ $\quad m_0=\rho \quad m_1=\ln\left(\frac{\sigma}{\sigma_{ref}}\right)$
- If there is relationship between σ and ρ then also for the respective property functions m_0, m_1

$$G(m_1, m_2)=0$$

- Problem: Function G is not known!
- Let's assume it not depending on location $\mathbf{x}$

Gradient Correlation

- Apply spatial gradient:

$$a_0 \cdot \nabla m_0 + a_2 \nabla m_1 = 0 \qquad \text{with } a_0 = \frac{\partial G}{\partial m_0} \; a_1 = \frac{\partial G}{\partial m_1}$$

- This is a system of linear equations for (a_0, a_1):
  - Two equations in 2D
  - Three equations in 3D
- Normal equation must have non-zero solution.

Gradient Correlation (cont.)

- Multiply by ∇m_0 and by ∇m_1 :

$$\begin{bmatrix} \|\nabla m_0\|^2 & \nabla^T m_0 \nabla m_1 \\ \nabla^T m_1 \nabla m_0 & \|\nabla m_1\|^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- For non-trival solution: determinant has to be zero:

$$C(m_0, m_1) = \|\nabla m_0\|^2 \|\nabla m_1\|^2 - (\nabla^T m_0 \nabla m_1)^2 = 0$$

  - Notice: always $C(m_0,m_1) \geq 0$
  - Also known as Gram determinant
- We want to minimize $C(m_0,m_1)$ in the inversion

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Cost Function

- Cost function to minimize over $\boldsymbol{m}=(m_0, m_1)$:

$$\Phi(\boldsymbol{m})=\alpha^{(1)}\underbrace{\left(\Phi_d^{(1)}(m_0)+\mu_1\Phi_r^{(1)}(m_0)\right)}_{\text{gravity}}$$

$$+\alpha^{(2)}\underbrace{\left(\Phi_d^{(2)}(m_1)+\mu_1\Phi_r^{(2)}(m_1)\right)}_{\text{MT}}+\theta\,\Phi_c\underbrace{(m_0,m_1)}_{\text{correlation}}$$

$$\Phi_c(m_1,m_2)=\frac{1}{2}\underbrace{\int_\Omega\left(\frac{1}{\|\nabla m_0\|^2}+\frac{1}{\|\nabla m_1\|^2}\right)}_{\text{balance vs. regularization}}\cdot C(m_0,m_1)\,d\boldsymbol{x}$$

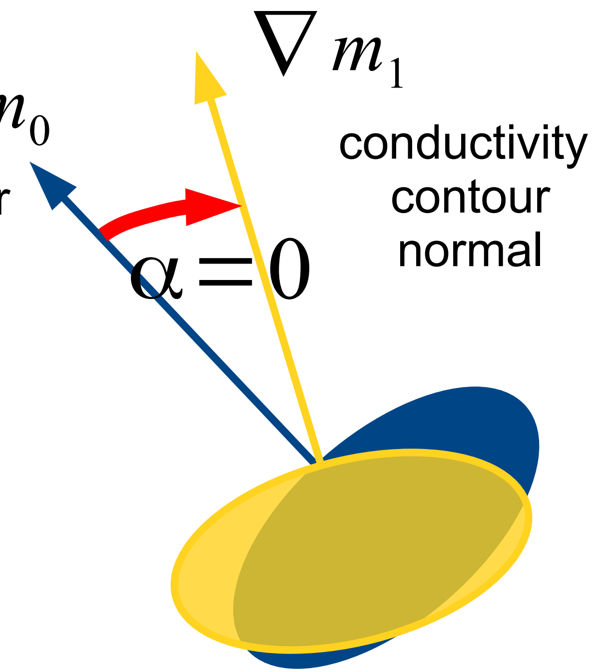
https://doi.org/10.1093/gji/ggz134

Cross Gradient Form

- Lagrange's identity:

$$C(m_0, m_1) = \|\nabla m_0\|^2 \|\nabla m_1\|^2 - (\nabla^T m_0 \nabla m_1)^2 = \|\nabla m_0 \times \nabla m_1\|^2$$

- Known as “cross-gradient” form
  - Geometrical interpretation



$$\|\nabla m_0 \times \nabla m_1\| = \|\nabla m_0\| \cdot \|\nabla m_1\| (1 - \cos^2(\alpha))$$

  - Target: make ∇m_0 and ∇m_1 parallel

Implementation

- Solution by BFGS method
  - Gradient:

$$\langle \nabla \Phi(\boldsymbol{m}) | \delta \boldsymbol{m} \rangle = \alpha^{(1)} \langle \nabla \Phi^{(1)}(m_0) | \delta m_0 \rangle + \alpha^{(2)} \langle \nabla \Phi^{(2)}(m_1) | \delta m_1 \rangle + \theta \langle \nabla \Phi_c(\boldsymbol{m}) | \delta \boldsymbol{m} \rangle$$

$$\langle \nabla \Phi_c(\boldsymbol{m}) | \delta \boldsymbol{m} \rangle = \int_\Omega \Big((a_0 \nabla m_0 - a \nabla m_1)^T \nabla \delta m_0 + (a_1 \nabla m_1 - a \nabla m_0)^T \nabla \delta m_1 \Big) \, d\boldsymbol{x}$$

with some a_0, a_1, a depending $\nabla m_0, \nabla m_1$

https://doi.org/10.1093/gji/ggz134

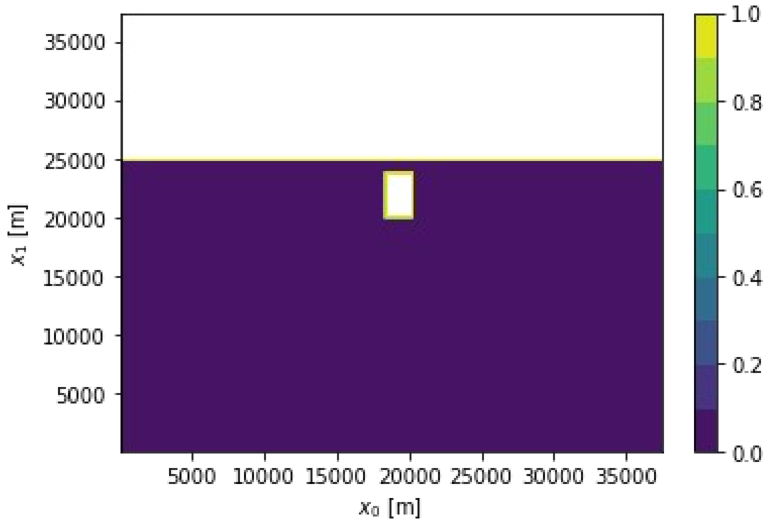
Check Correlation

- Measure of correlation:

$$R = \frac{\nabla^T m_1 \nabla m_0}{\|\nabla m_1\| \|\nabla m_0\|} \in [-1, 1]$$

  - Good correlation: $R^2 \approx 1$
  - Weak correlation: $R^2 << 1$
    - Marks regions in which G is changing with x.
    - Interpretation: interface of geological units

Test Case

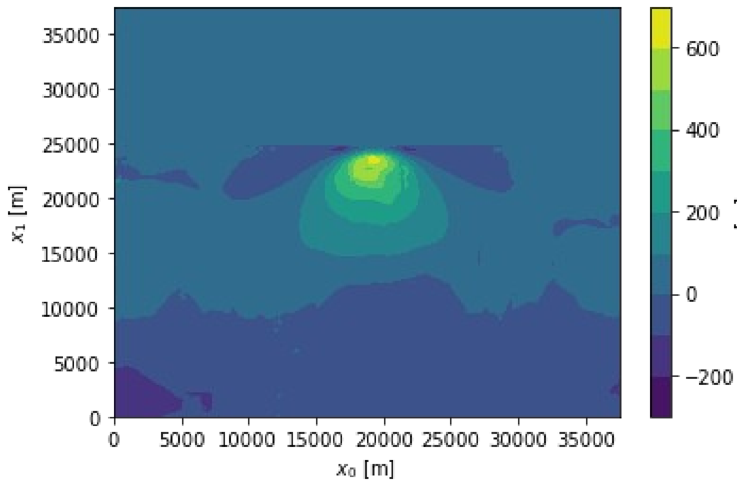


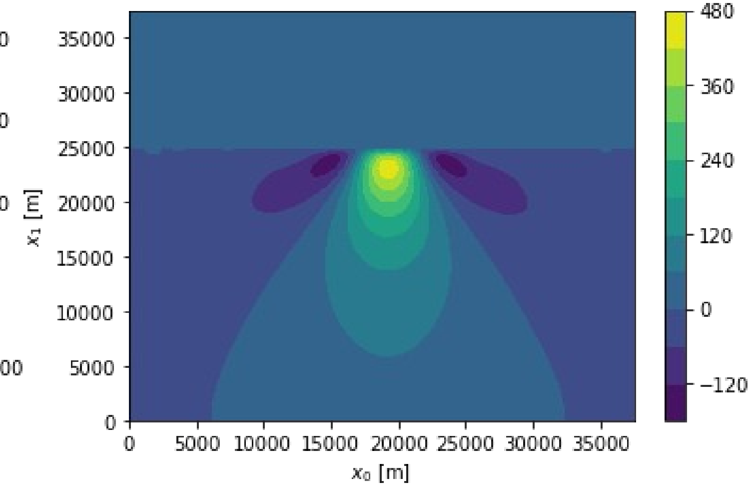
- f=5Hz, σ_1=2, ρ_1=1500
- σ_{bg}=0.01, ρ'=500
- α^g=1e-4, α^{mt}=0.0
- μ^g, μ^{mt}=1e6
- θ=1e8
- 62500 cells.

Properties

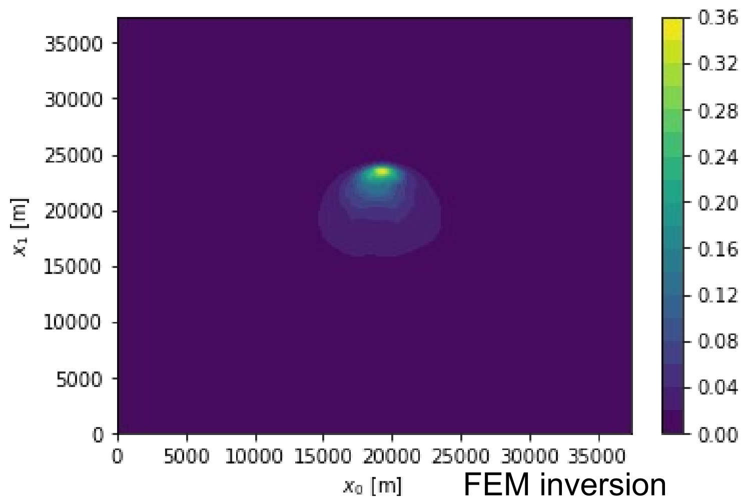
Joint inversion | Inversion $\theta=0$

Density

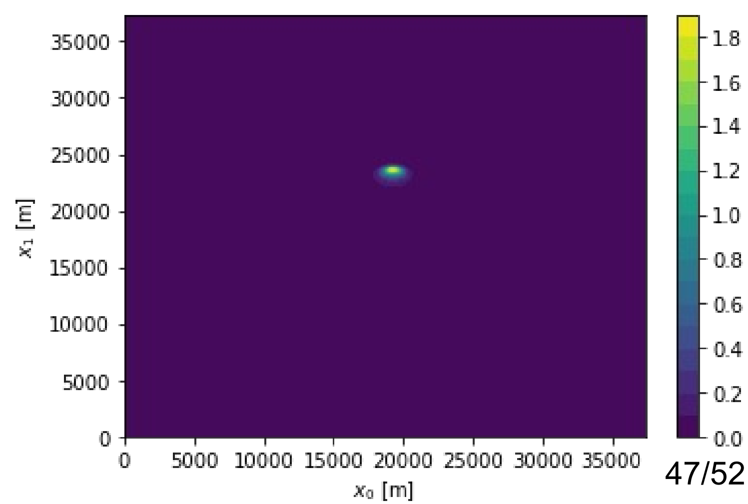




Conductivity

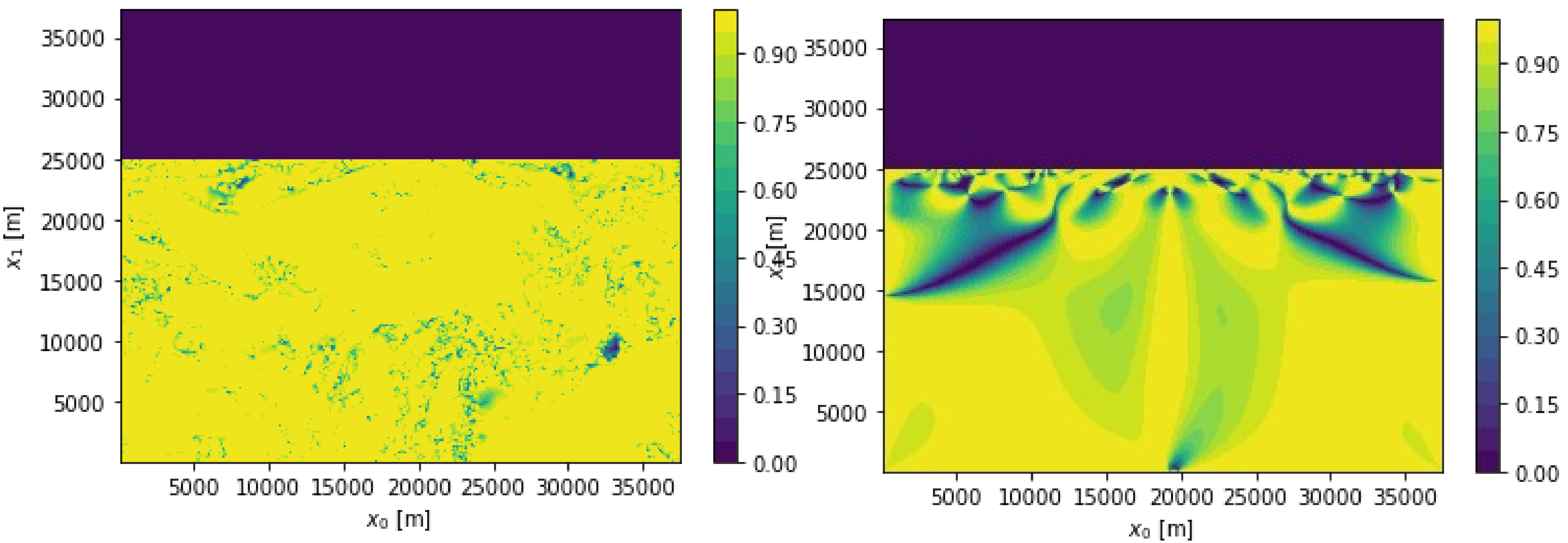


FEM inversion



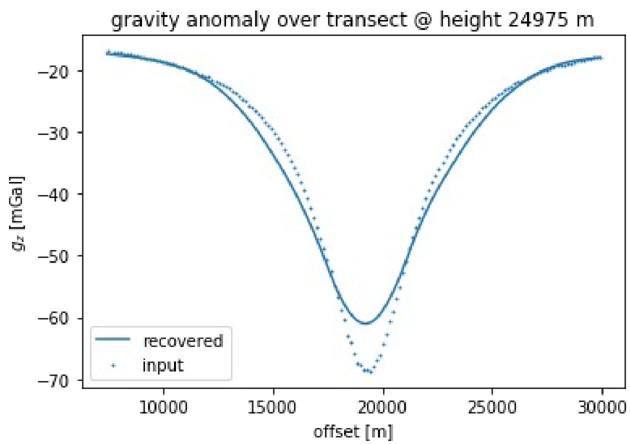
Correlation R^2

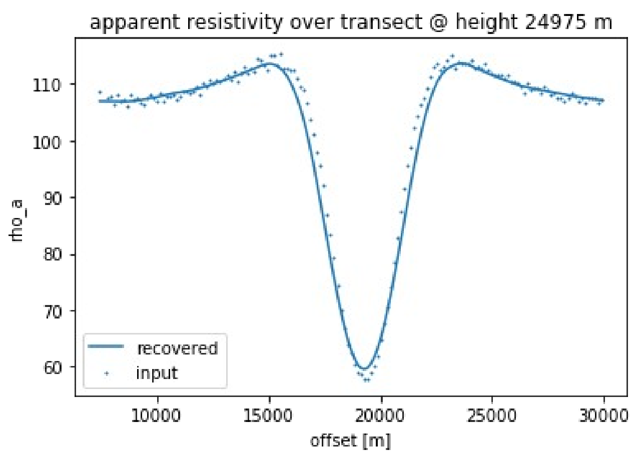
Joint inversion   Inversion $\theta=0$



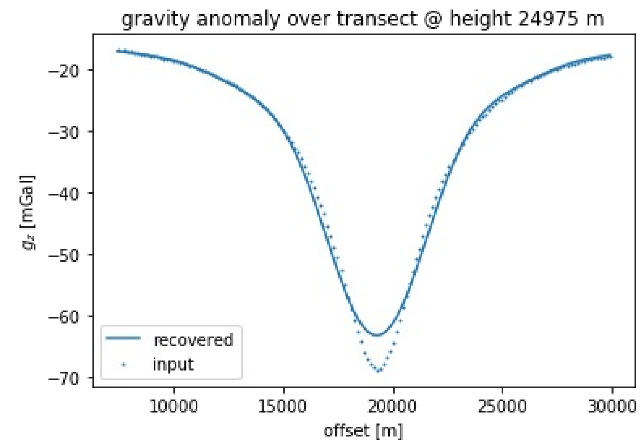
Data Recovery

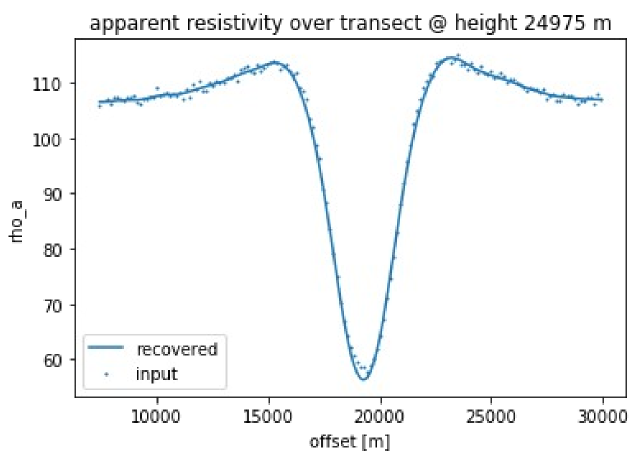
Joint inversion





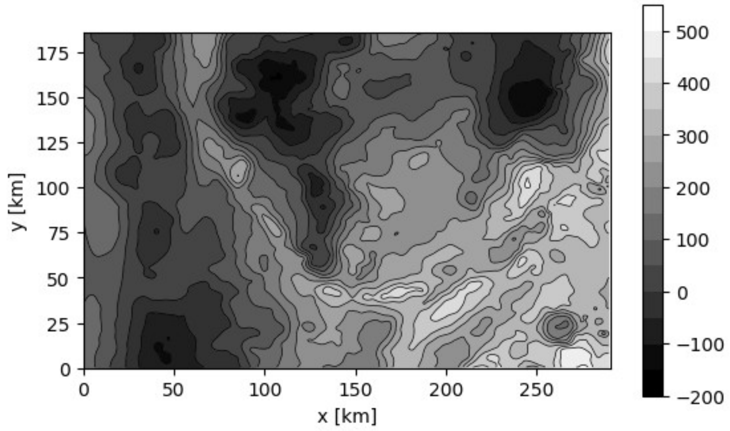
Inversion $\theta = 0$



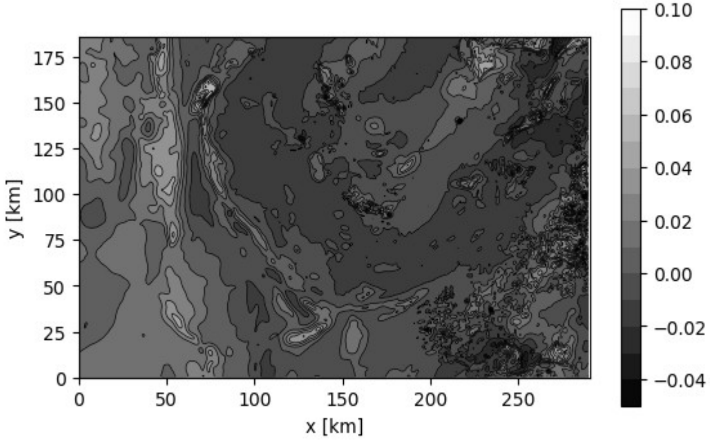


Field Data set from Central QLD

Gravity Anomalies QLD:



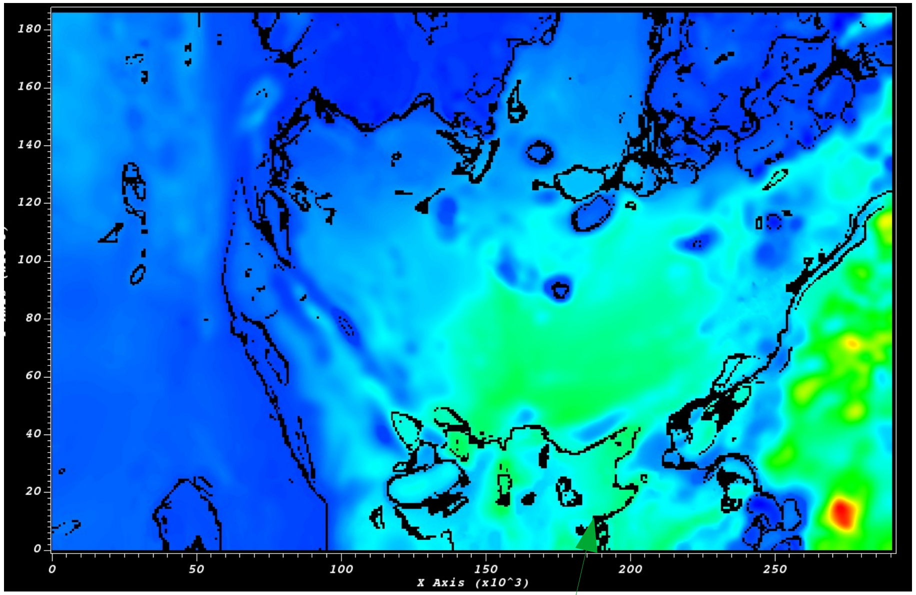
Magnetic field intensity QLD



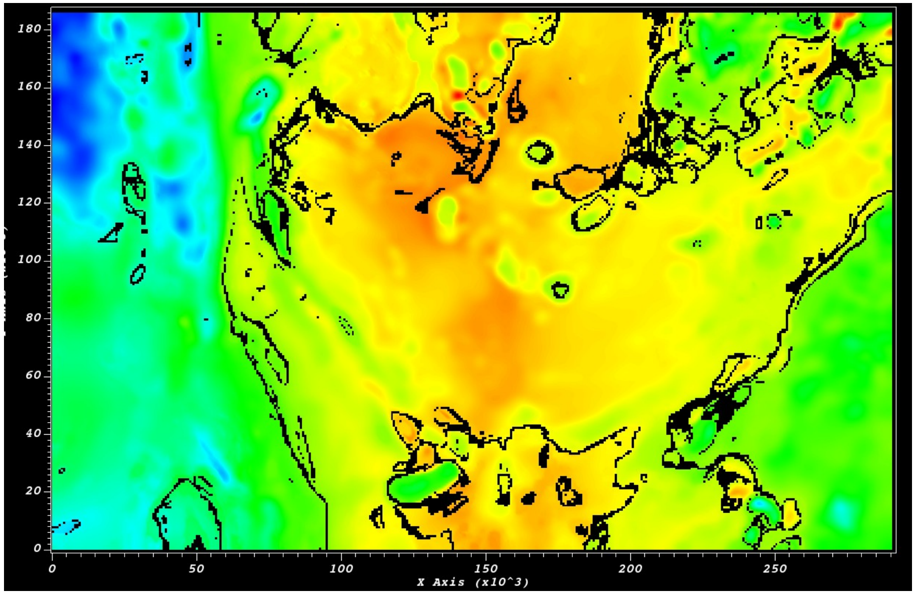
Joint Inversion with 30 Million cells on ~200 cores
Implementation in python3 + esys-escript

Some Results

Density at depth 500m



Log of susceptibility at depth 500m



Locations with R^2<0.97
= boundary of geological units

Thanks!

Questions, Suggestions, Share programs:
Please get in contact on e-mail:
l.gross@uq.edu.au
Or WeChat