J Med Syst (2008) 32:201–206
DOI 10.1007/s10916-007-9123-7

ORIGINAL PAPER

AR Spectral Analysis Technique for Human PPG, ECG and EEG Signals

Elif Derya Übeyli · Dean Cvetkovic · Irena Cosic

Spectrums of ecg with
- FFT and welch metod
- AR estimation

COnclusion: AR is better becouse of bsence of windowiing, leading to a smooth spectrum.

AR spectrum estimation is done using YW and LS estimation. In such case the PSD is slighty different from YW one but practically is not. Has some advantages.

Received: 9 October 2007 / Accepted: 20 December 2007 / Published online: 8 January 2008

Abstract In this study, Fast Fourier transform (FFT) and autoregressive (AR) methods were selected for processing the photoplethysmogram (PPG), electrocardiogram (ECG), electroencephalogram (EEG) signals recorded in order to examine the effects of pulsed electromagnetic field (PEMF) at extremely low frequency (ELF) upon the human electrophysiological signal behavior. The parameters in the autoregressive (AR) method were found by using the least squares method. The power spectra of the PPG, ECG, and EEG signals were obtained by using these spectral analysis techniques. These power spectra were then used to compare the applied methods in terms of their frequency resolution and the effects in extraction of the features representing the PPG, ECG, and EEG signals. Some conclusions were drawn concerning the efficiency of the FFT and least squares AR methods as feature extraction methods used for representing the signals under study.

Keywords PPG · ECG · EEG · Least squares AR method

Introduction

In the present study, spectral analysis of the photoplethysmogram (PPG), electrocardiogram (ECG), electroencephalogram (EEG) signals was performed by using the Fast Fourier transform (FFT) and autoregressive (AR) methods. In the past few decades, the responses of human and animal EEG activity to non-ionising radiation of Extremely Low Frequency (ELF) have been studied [1–4]. Since that time, various studies have reported that humans and animals are particularly sensitive to ELF or ELF modulated sensory stimulation. The ELF refers to the range of electromagnetic field frequencies below 300 Hz.

Several studies have examined the effects of sinusoidal ELF magnetic fields upon the human EEG activity in the past [1]. Cvetkovic and Cosic [2] single-blind counter-balanced pilot study investigated whether the human EEG activity could be altered when stimulated by localised ELF magnetic field at the top-central human head region. The statistical results performed on the recorded EEG data did reveal a significant difference between exposure and control, found in the Alpha1 EEG band (7.5–9.5 Hz) at the vertex head position, where magnetic field stimulation was applied at the Alpha1 frequency of 8.33 Hz. It has been assumed that the effect in Alpha1 EEG findings is possibly related to 'synchronisation', 'induced rhythmic' and 'synchrony spread' theories of neuron firing rate after ELF magnetic field. Tabor et al. [3] study on 15 subjects revealed that the changes of time-domain Heart Rate Variability (HRV) parameters could be associated with the influence of 50 Hz magnetic field (20–30 μT). Tabor's time-domain HRV parameters included linear and non-linear analysis. Baldi et al. [4] study on the influence of ELF PEMF exposure on the HRV using linear analysis, revealed a HR variation in all subjects.

Feature extraction is the determination of a feature or a feature vector from a pattern vector. For pattern processing problems to be tractable requires the conversion of patterns to features, which are condensed representations of pat-

E. D. Übeyli (✉)
Faculty of Engineering, Department of Electrical and Electronics Engineering, TOBB Economics and Technology University, 06530 Söğütözü, Ankara, Turkey
e-mail: edubeyli@etu.edu.tr

D. Cvetkovic · I. Cosic
School of Electrical and Computer Engineering, RMIT University, GPO Box 2476V, Melbourne, Victoria 3001, Australia

terns, ideally containing only salient information. Feature extraction methods are subdivided into: (1) statistical characteristics and (2) syntactic descriptions. Feature selection provides a means for choosing the features which are best for classification, based on various criteria. The feature selection process performed on a set of predetermined features. Features are selected based on either (1) best representation of a given class of signals, or (2) best distinction between classes. Therefore, feature selection plays an important role in classifying systems such as neural networks [5, 6]. In the feature extraction stage, numerous different methods can be used so that several diverse features can be extracted from the same raw data.

In order to extract features representing the time-varying biomedical signals such as ECG and EEG signals, a number of spectral estimation techniques have recently been developed and have been compared to the more standard FFT method, or Welch method [7–13]. The FFT-based power spectrum estimation methods are known as classical methods and have been widely studied in the literature [7–13]. The AR method is a parametric method and the parameters of the AR method can be computed by using different algorithms such as least squares method [7–13].

The PPG, ECG, and EEG signals were examined by taking into consideration of their power spectral density (PSD) estimates. The FFT and the least squares AR methods were used for spectral analysis of the PPG, ECG, and EEG signals. Using these spectral analysis techniques, the time-dependent spectral distributions were visualized and detailed documentations of the PPG, ECG, and EEG signals were obtained. These methods were compared in terms of their frequency resolution and the effects in extraction of the features representing the PPG, ECG, and EEG signals.

In comparison to previous studies, the pilot study described in this paper consists of measuring the PPG, ECG, and EEG signal responses to ELF PEMF exposures over a 5 day period. A significant contribution of the present work was the computation of power levels of the PSD estimations obtained by the FFT and the least squares AR methods which were used to extract the representative features of the signals under study in order to obtain the accurate classification models.

Data description

The pilot experiment consisted of one healthy subject, recruited to participate for five consecutive days (excluding weekends). The experiment was a double-blinding and counter-balanced experimental design, where neither the subject nor the investigator was aware of the EMF exposure applied to the subject. At each sessions (day) the experimental protocol was designed to record the biosignals before (baseline) and after (post) EMF exposure during: control (magnetic field turned OFF) and exposure (magnetic field turned ON). The experiment was conducted between 1 and 3 pm for the 5 days. The ELF PEMF stimulations were undertaken with MEDEC Bioresonance Therapy System. The 10 min ELF PEMF exposure were generated by the magnetic flux density at the top (2.33 μT), middle (5.24 μT) and bottom (6.45 μT) of the applicator mattress at the operating frequency of 16 Hz.

All the biosignals were recorded for 60 s after the ELF PEMF exposure. The subject was connected to MINDSET EEG machine to record the EEG activity of the brain using standard Neuroscan 19-electrode EEG cap. The cap was placed on subject's head according to 10/20 International System. The Referential Montage of 16 channels was used throughout this investigation. The left brain hemisphere electrodes: Fp1, F7, F3, T7, C3, P7, P3 and O1 were all referenced to M1 or A1 (left masteroid). While the right brain hemisphere electrodes: Fp2, F8, F4, T8, C4, P8, P4 and O2 were referenced to right masteroid (M2 or A2). The EEG signals were sampled at the rate of 256 Hz. Two other signals were recorded using Data Acquisition device BIOPAC Inc., which consisted of MP100A system with ECG100C Electrocardiogram amplifier (ECG) and PPG100C Photo-plethysmogram amplifier (PPG). The 50 Hz notch filter was activated to shield the subject and electrodes from the electric fields in the laboratory. The signals were digitised at a rate of 100 Hz and transmitted to a PC's Acknowledge 3.7 software via USB cable. Recording took place in a dim room, inside a Faraday cage constructed of mesh wire and steel frames. Inside the cage, subjects were laid in a comfortable semi-reclining METRON chair.

Spectral analysis techniques

Fast Fourier transform method for spectral analysis

The FFT methods such as the Welch method are defined as classical (nonparametric) methods. The Welch spectral estimator is one of the FFT methods and relies on the definition of the periodogram method. If available information on the signal consists of the samples $\{x(n)\}_{n=1}^{N}$, the periodogram spectral estimator is defined as

$$\hat{P}_{\mathrm{PER}}(f) = \frac{1}{N}\left|\sum_{n=1}^{N} x(n)\exp(-j2\pi fn)\right|^2 \tag{1}$$

where $\hat{P}_{\mathrm{PER}}(f)$ is the estimation of periodogram.

In the Welch method, signals are divided into overlapping segments, each data segment is windowed, the periodograms are calculated and then average of the periodograms is found. $\{x_l(n)\}$, l=1,…,S are data segments and each segment's length equals to M. In this method, generally overlapping ratio is taken as 50% (M/2). The Welch spectral estimator is defined as

$$\hat{P}_l(f) = \frac{1}{M}\frac{1}{P}\left|\sum_{n=1}^{M} v(n)x_l(n)\exp(-j2\pi fn)\right|^2 \text{ and}$$
$$\hat{P}_W(f) = \frac{1}{S}\sum_{l=1}^{S}\hat{P}_l(f) \tag{2}$$

where $\hat{P}_l(f)$ is the periodogram estimate of each signal interval, $v(n)$ is the data window, P is the average of $v(n)$ given as $P = \frac{1}{M}\sum_{n=1}^{M}|v(n)|^2$, $\hat{P}_W(f)$ is the Welch PSD estimate, M is the length of each signal interval and S is the number of signal intervals.

Then, evaluation of $\hat{P}_W(f)$ at the frequency samples basically reduces to the computation of the following discrete Fourier transform (DFT):

$$X(k) = \sum_{n=1}^{N} x(n)\exp\left(-j\frac{2\pi}{N}\right)^{nk}, \; k = 0, \ldots, N-1 \tag{3}$$

where $X(k)$ is expressed as the discrete Fourier coefficient, N is the length of available data and $x(n)$ is the input signal on the time domain. The procedure that computes Eq. 3 is called as FFT algorithm. The estimated Welch PSD can be computed by use of the DFT, which in turn is efficiently computed by the FFT algorithm. Variance of an estimator is one of the measures often used to characterize its performance. For 50% overlap and triangular window, variance of the Welch method is defined as

$$\text{var}\left(\hat{P}_w(f)\right) = \frac{9}{8S}\text{var}\left(\hat{P}_l(f)\right) \tag{4}$$

where $\hat{P}_W(f)$ is the Welch PSD estimate and $\hat{P}_l(f)$ is the periodogram estimate of each signal interval.

Since $\lim_{S\to\infty} \text{var}\left(\hat{P}_W(f)\right) = 0$, the Welch method is asymptotically consistent estimator.

The Welch spectral estimator can be efficiently computed via the FFT and is one of the most frequently used PSD estimation methods. The FFT method is a poor spectral estimator because its variance is high and frequency resolution of the FFT method is limited by the available data record duration, independent of the characteristics of the data. In addition, a smeared spectral estimate is a consequence of the windowing. Owing to these limitations of the FFT method, the model-based (parametric) spectral estimation methods, like the AR methods, are extremely valuable for data analysis [7–13].

AR method for spectral analysis

The model-based methods are based on modeling the data sequence $x(n)$ as the output of a linear system characterized by a rational system. In the model-based methods, the spectrum estimation procedure consists of two steps. Given the data sequence $x(n)$, $0 \le n \le N-1$, the parameters of the method are estimated. Then from these estimates, the PSD estimate is computed. The AR method is the most frequently used model-based method because estimation of the AR parameters can be done easily by solving linear equations. In the AR method, data can be modeled as output of a causal, all-pole, discrete filter whose input is white noise. The AR method of order p is expressed as the following equation:

$$x(n) = -\sum_{k=1}^{p} a(k)x(n-k) + w(n) \tag{5}$$

where $a(k)$ are the AR coefficients and $w(n)$ is white noise of variance equal to σ^2. AR (p) model can be characterized by the AR parameters $\{a[1], a[2], \ldots, a[p], \sigma^2\}$. The PSD is

$$P_{AR}(f) = \frac{\sigma^2}{|A(f)|^2} \tag{6}$$

where $A(f) = 1 + a_1e^{-j2\pi f} + \ldots + a_pe^{-j2\pi fp}$ [7–13].

Least squares method for AR parameter estimation

Linear prediction of AR method is to predict the unobserved data sample $x(n)$ based on the observed data samples $\{x(n-1), x(n-2), \ldots, x(n-p)\}$,

$$\hat{x}(n) = -\sum_{k=1}^{p}\alpha_k x(n-k) \tag{7}$$

the prediction coefficients $\{\alpha_1, \alpha_2, \ldots, \alpha_p\}$ are chosen to minimize the power of the prediction error $e(n)$:

$$\rho = E\left\{|e(n)|^2\right\} = E\left\{\left|x(n) - \hat{x}(n)\right|^2\right\} \tag{8}$$

For minimizing ρ the orthogonality principle is used,

$$r(k) = -\sum_{l=1}^{p}\alpha_l r(k-l), \; k = 1, 2\ldots, p \tag{9}$$

$$\rho_{\min} = r(0) + \sum_{k=1}^{p}\alpha_k r(-k) \tag{10}$$

where α_k=[k] for k=1,2…, p and $\rho_{\min}=\sigma^2$.

la forma ECG è ragionevole? Abbiamo, ragionevolmente, nei singoli battiti → non si vedono le forme d'onda caratteristiche confronto sui tuoi dati tra Welch e PSD.

→ non sono sicuro del metodo usato per la stima...

Given a finite set of data samples $\{x(n)\}_{n=1}^{N}$ minimum of $E\left\{|e(n)|^2\right\}$ is calculated with respect to α_k (k=1,2,…, p).

$$\begin{aligned} f(\alpha) &= E\left\{|e(n)|^2\right\} = \sum_{n=N_1}^{N_2} |e(n)|^2 \\ &= \sum_{n=N_1}^{N_2} \left| x(n) + \sum_{k=1}^{p} \alpha[k] x(n-k) \right|^2, \ k = 1,2,\ldots,p \\ &= \left\| \begin{bmatrix} x(N_1) \\ x(N_1+1) \\ \vdots \\ x(N_2) \end{bmatrix} + \begin{bmatrix} x(N_1-1) & \cdots & \cdots & x(N_1-p) \\ x(N_1) & \cdots & \cdots & x(N_1+1-p) \\ \vdots & & & \vdots \\ x(N_2-1) & \cdots & \cdots & x(N_2-p) \end{bmatrix} \alpha \right\|^2 \\ &= \|x + X\alpha\|^2 \end{aligned} \tag{11}$$

The vector α that minimizes $f(\alpha)$ is given by

$$\hat{\alpha} = -\left(X^* X\right)^{-1}\left(X^* x\right) \tag{12}$$

where, as seen from Eq. 11, the definitions of X and x depend on the choice of (N_1, N_2) considered. By substitution autocorrelation function estimates $\left\{\hat{r}(k)\right\}_{k=0}^{p}$ and $\hat{\alpha}$ in Eq. 10 $\hat{\rho}_{\min}$ is obtained,

$$\hat{\rho}_{\min} = \hat{r}(0) + \sum_{k=1}^{p} \hat{\alpha}\hat{r}(-k) \tag{13}$$

From the estimates of AR parameters, PSD estimation is formed as [7–13]:

$$\hat{P}_{LS}(f) = \frac{\hat{\rho}_{\min}}{\left|1 + \sum_{k=1}^{p} \hat{a}_p(k) e^{-j2\pi fk}\right|^2} \tag{14}$$

Least squares method's performance characteristics have been found to be superior to the other AR estimation methods, in the sense that the least squares method does not exhibit the same sensitivity to such problems as line splitting, frequency bias and spurious peaks. With the least squares method there is no guarantee that the estimated AR parameters yield a stable AR model. However, in spectrum estimation, this is not considered to be a problem. The least squares method has sharper response for narrowband processes than other AR estimates.

ampiezze comparabili...

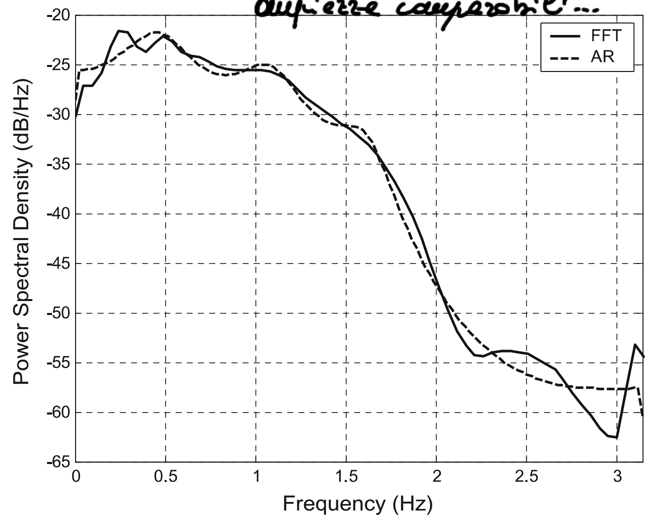

Fig. 2 PSDs of ECG obtained by FFT and least squares AR

Results and discussion

In this study, the PSDs of the PPG, ECG, and EEG signals recorded under the effects of PEMF at ELF were obtained by using the FFT and the least squares AR methods. The power spectra describe the distribution of power with frequency. The sample PSDs of the PPG, ECG, and EEG signals are presented in Figs. 1, 2 and 3. When the PSDs of

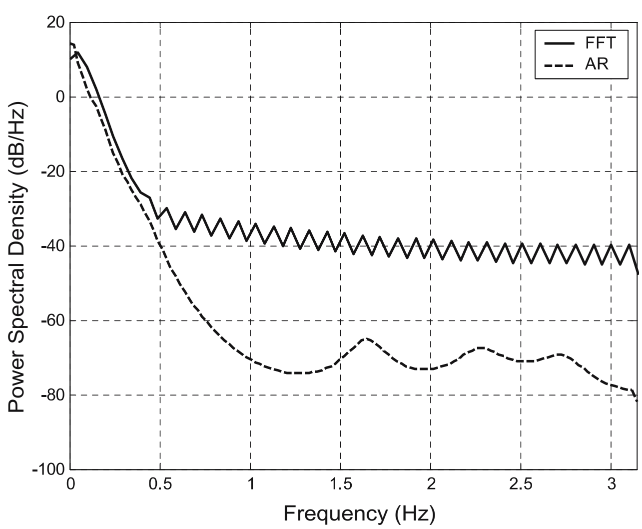

Fig. 1 PSDs of PPG obtained by FFT and least squares AR

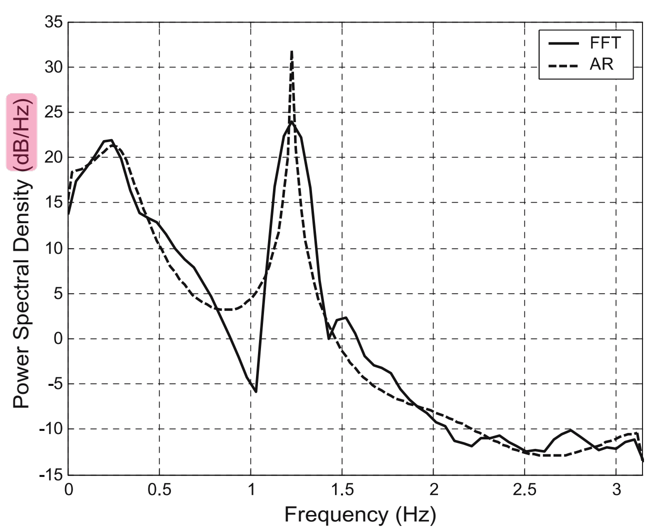

Fig. 3 PSDs of EEG obtained by FFT and least squares AR

the sample PPG, ECG, and EEG signals were examined, it is seen that the classical method (FFT) had large variance (Figs. 1, 2 and 3). The FFT method is based on a finite record of data, the frequency resolution of these methods equal to the spectral width of the window length N, which is approximately $1/N$. The principal effect of windowing that occurs when processing with the FFT is to smear or smooth the estimated spectrum. Owing to smearing the FFT method cannot resolve details in the studied spectrum that are separated by less than $1/N$ in cycles per sampling interval. For this reason, $1/N$ is called the spectral resolution limit of the FFT method. Furthermore, this method suffers from spectral leakage effects, due to windowing that are inherent in finite-length data records. Often, the spectral leakage masks weak signals that are present in the data. Smearing and spectral leakage are particularly critical for spectra with large amplitude ranges, such as peaky spectra.

In the model-based methods, a model for the signal generation can be constructed with a number of parameters that can be estimated from the observed data. From the model and the estimated parameters, the PSD can be computed. Since the modeling approach eliminates the need for window functions, the model-based methods do not suffer from smearing and spectral leakage effects. As it is seen from Figs. 1, 2 and 3, the least squares AR PSD estimation method avoided the problem of leakage and provided better frequency resolution than did the FFT method. In addition, the estimation of parameters in the AR signal models is a well-established topic; the estimates are found by solving linear equations of the system. The least squares method is usually employed for good estimates of the AR parameters. Spectra with both sharp peaks can be modeled by the least squares AR method. When the PSDs of the PPG, ECG, and EEG signals were examined, performance characteristics of the least squares AR method were found to be superior to the FFT method.

The selection of the model order in the AR spectral estimator is an important subject. If the selected order is low, there will be no definite peaks in the spectrum. So, the frequency details of the signal cannot be identified. If the selected order is very high, spurious peaks may be seen in the frequency spectrum which is not related to the original signal. In the case of the dimension of autocorrelation matrix is inappropriate and the model orders chosen incorrect, poor spectral estimates are obtained by the AR spectral estimator. Heavy biases and/or large variabilities may be exhibited. In this study, Akaike Information Criteria [14] was taken as the base for choosing the model order. According to the criteria, model order p was taken as 10 for the AR method.

The peak frequencies and the power levels of the sample PPG, ECG, and EEG signals are given in Tables 1, 2 and 3, respectively. From Tables 1, 2 and 3, it is seen that the spectra of the FFT method had spurious peaks and did not produce accurate spectral estimates due to limits on resolution. On the other hand, the least squares AR method produced accurate frequency estimates. The peak frequencies and the power levels of the least squares AR PSDs can be used as the features representing the PPG, ECG, and EEG signals.

Conclusions

The PPG, ECG, and EEG signals recorded under the effects of PEMF at ELF were processed by using the FFT and the least squares AR methods. Interpretation and performance of these methods were compared in terms of their frequency resolution and the effects in feature extraction from the PPG, ECG, EEG signals. Since the FFT method had low spectral resolution, it was not found appropriate for evaluating the PSDs of the PPG, ECG, and EEG signals. The results of this study clearly demonstrated the power of the least squares AR method in resolution of frequency. In conclusion, it should be emphasized that the performance characteristics of the least squares AR method were found extremely valuable for extraction of the features representing the PPG, ECG, and EEG signals.

Table 1 Peak frequencies and power levels of PPG

Method	P_1/f_1	P_2/f_2	P_3/f_3	P_4/f_4
FFT	Spurious peaks			
Least squares AR	13.8625/ 0.0245	−64.9827/ 1.6444	−67.5245/ 2.2826	−69.2789/ 2.6998

Table 2 Peak frequencies and power levels of ECG

Method	P_1/f_1	P_2/f_2	P_3/f_3	P_4/f_4
FFT	Not specific peaks			
Least squares AR	−25.4095/ 0.0982	−21.8245/ 0.4663	−25.1002/ 1.0554	−57.6178/ 3.1170

Table 3 Peak frequencies and power levels of EEG

Method	P_1/f_1	P_2/f_2	P_3/f_3
FFT	Spurious peaks		
Least squares AR	21.3376/ 0.2454	31.9129/ 1.2272	−10.4352/ 3.1170

P_i denotes the power level of the ith peak in decibel per hertz, and f_i denotes the frequency of the ith peak in hertz

References

1. Bell, G. B., Marino, A. A., and Chesson, A., Alterations in brain electrical activity caused by magnetic fields: detecting the detection process. *Electroencephalogr. Clin. Neurophysiol.* 83:389–397, 1991.
2. Cvetkovic, D., and Cosic, I., Automated ELF magnetic field stimulation of the human EEG activity. *Integr. Comput.-Aided Eng.* 13:4313–328, 2006.
3. Tabor, Z., Michalski, J., and Rokita, E., Influence of 50 Hz magnetic field on human heart rate variability: Linear and nonlinear analysis. *Bioelectromagnetics* 25:474–480, 2004.
4. Baldi, E., Baldi, C., and Lithgow, B. J., A pilot investigation of the effets of extremely low frequency pulsed electromagnetic fields on human's heart rate variability. *Bioelectromagnetics* 28:64–68, 2007.
5. Kwak, N., and Choi, C.-H., Input feature selection for classification problems. *IEEE Trans. Neural Netw.* 13:1143–159, 2002.
6. Übeyli, E. D., and Güler, İ., Feature extraction from Doppler ultrasound signals for automated diagnostic systems. *Comput. Biol. Med.* 35:9735–764, 2005.
7. Kay, S. M., and Marple, S. L., Spectrum analysis—A modern perspective. *Proc. I. E. E. E.* 69:1380–1419, 1981.
8. Übeyli, E. D., and Güler, İ., Selection of optimal AR spectral estimation method for internal carotid arterial Doppler signals using Cramer-Rao bound. *Comput. Electr. Eng.* 30:7491–508, 2004.
9. Kay, S. M., *Modern spectral estimation: Theory and application*. Prentice Hall, New Jersey, 1988.
10. Übeyli, E. D., and Güler, İ., Determination of stenosis and occlusion in arteries with the application of FFT, AR and ARMA methods. *J. Med. Syst.* 27:2105–120, 2003.
11. Übeyli, E. D., Measuring saliency of features extracted by model-based methods from internal carotid arterial Doppler signals using signal-to-noise ratios. *Digit. Signal Process* 18:12–14, 2008.
12. Übeyli, E. D., Feature extraction by AR spectral analysis using maximum likelihood estimation: Internal carotid arterial Doppler signals. *Expert Syst.* 2007 (in press).
13. Übeyli, E. D., and Güler, İ., Spectral analysis of internal carotid arterial Doppler signals using FFT, AR, MA, and ARMA methods. *Comput. Biol. Med.* 34:4293–306, 2004.
14. Akaike, H., A new look at the statistical model identification. *IEEE Trans. Automat. Contr* AC-19:716–723, 1974.