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Optimal selection of wavelet basis function applied to ECG signal denoising

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Abstract

Over the years ElectroCardioGram (ECG) signal has been used to assess the cardiovascular condition of humans. In practice, real time acquisition and transmission of the ECG may contain noise signals superimposed on it. In general, the signal processing algorithms employed for denoising provide optimal performance and eliminate the high frequency noise between any two beats contained in a continuous ECG signal. Despite their optimal performance, the signal processing algorithms significantly attenuate the peaks of characteristics wave of the ECG signal. This paper presents a selection procedure of mother wavelet basis functions applied for denoising of the ECG signal in wavelet domain while retaining the signal peaks close to their full amplitude. The obtained wavelet based denoised ECG signals retain the necessary diagnostics information contained in the original ECG signal.

Keywords: ECG; DWT; Denoising; Basis function; Thresholding

1. Introduction

With astounding achievements in recent time, biomedical engineering happens to be one of the fastest growing fields. Proficiency in the interpretation of the ECG's is an essential skill for medical professionals. Errors in reading are common, and may lead to serious consequences. Over the years a systematic development took place in ECG technology incepted in late 1950s, when the professionals were able to accomplish electrical stimulated heart signals by placing electrodes in and around the heart muscle. An ECG signal is due to ionic current flow causing the cardiac fibers to contract and relax, subsequently, generating a time variant periodic signal. In an ECG system, the potential difference between two electrodes placed on the skin surface is considered as an input to the ECG plotter. Statistical data from literature reveals that there is approximately 20–50% discordance between the early ECG interpretation and final interpretation by a senior cardiologist [1]. The ECG weighs an important physiological signal for the final diagnosis and urgent treatment of the ailing patients with life-threatening cardiovascular diseases. With wide spread proliferation of the telemedicine in human health care systems the role of the signal processing community has become even more important. In the simplest form of the telemedicine ECG was interpreted through public telephone network about forty years ago [2].

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1.1. ECG wave

A single normal cycle of the ECG represents the successive arterial depolarization/repolarization and ventricular depolarization/repolarization, which occurs with every heartbeat. These can be approximately associated with the peaks and through of the ECG waveform labeled P, Q, R, S, and T as shown in Fig. 1 [3]. It has the characteristic P, T, and U waves and *PQ*, *QRS*, and *ST* segments.

The unstable recording environment, spurious signals from nearby equipment, poor electrodes, and electromagnetic pollution are few reasons for unwanted noise contamination on the ECG signal. Results from laboratory and clinical studies suggest existence of abnormal ventricular conduction during sinus rhythm in regions surrounding a myocardial infraction, which generate delayed and fractionated micro potentials (*late potentials*) recorded on the ECG signals. They are low-level, high-frequency oscillations on the terminal part of the *QRS* complex and *ST* segment, and must be recorded under a special care, from three orthogonal leads and amplitude quantized with 12 or 16 bit A/D converters. Due to very low amplitude of late potentials and the poor signal-to-noise ratio (SNR), it is difficult to remove the noise from the ECG signals by employing conventional denoising method. Therefore, reliable signal processing techniques are required for extraction of useful clinical information contained in a noisy ECG signal. The ECG signal analysis requires careful investigation and detection of the complex *QRS* segment. This may contain information related to normal and abnormal cardiac patterns. Though literature related to other time-frequency methods applied to the ECG signal processing is available, however, wavelet based technique is preferred as it has demonstrated better detection accuracy [3]. Furthermore, the noise contained in the ECG signal may be due to two main reasons. First, due to physical parameters of recording instrument and the second, due to bioelectric activity of the cells not belonging to the area of diagnostic interest (also termed as background activity) [3]. Although analog filtering is performed during acquisition, however, due to overlapping spectra classical filtering methods are not sufficient to obtain the actual ECG signals. This is due to the fact that classical filtering methods result into an overlapped random noise signal over ECG signal in time and frequency domain. Therefore, noise removal is an involved process in physiological signals. During denoising, care has to be practiced to preserve the features contained in original signal. The preserved features are often relevant and necessary for an appropriate diagnosis. In general, a person specialized in signal processing may lack the necessary expertise needed to differentiate between biologically important features of the signal and the contained noise within the signal. The biological signals may not be consistent and may be at variance, therefore, the signal processing of biological signals needs to be carried out case by case basis. This necessitates a robust, versatile, and adaptable denoising method applicable in different operative circumstances.

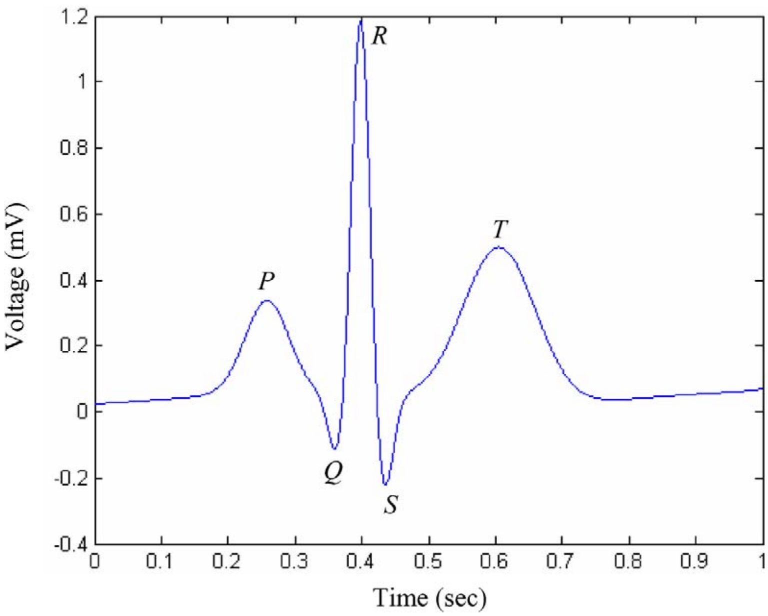


Fig. 1. Standard one beat *PQRST* complex of an ECG signal.

1.2. Literature review on ECG signal denoising

Wavelet theory has already proven its ability in splitting signal and noise in wavelet domain. Recently, researchers from biomedical signal processing community have applied wavelet theory in signal compression, feature extraction, and to small extent in denoising [4–6]. Wavelet based signal processing over conventional techniques offers definite advantages such as coefficient compaction characteristics of wavelet, dilution of noise in wavelet domain, and the removal of redundancy. These inherent potentials of wavelet transform have been harnessed in a large scale for medical data compression and retrieval. The data compression capability of the wavelet becomes very useful in the ECG, this is due to the reason that the EGC data requires enormous memory to store. For example, an ECG signal sampled at the rate of 1 kHz requires on average 100 MB digital storage per day. Thus real time telemedicine application necessitates data compression of the ECG signal before a successful and efficient transmission of the ECG signal can be carried out to a remote diagnosis facility. In an ECG beat the most important part is brief QRS complex [7–9]. An inverse of the RR interval (time between successive R wave peaks) of the ECG signal is an important pathological parameter as it provides instantaneous heart rate. Also, the information pertaining to detection of R peak, QT interval, nature of S, and T waves are considered as important diagnostics parameters. Presently, experts are using Physionet [10] data base to evaluate performance of biomedical signal processing algorithms. Although the database helps the user but in practice there may be possibility of variations in results due to overlapped noise with the ECG signals.

Khamene et al. [11] have developed a wavelet transform based method for extracting fetal ECG from composite abdominal signals. This method detects singularities from the composite abdominal signal using modulus maxima in the wavelet domain. They proposed a DWT based method to detect QRS complex, which is robust to noise. They designed a spline wavelet that is suitable to QRS detection [12].

1.3. Statement of the problem

The extraction of the actual ECG signal from the noisy recording is formulated as the problem of signal denoising. Our main objective is to investigate applicability of wavelet based thresholding scheme for the ECG signal denoising. The reason for selection of wavelet domain processing of the ECG signal is due to non-stationary characteristics of signal and temporal or spreaded contamination of noise, which restricts application of conventional linear filtering scheme. The main issue is selection of best wavelet basis function (within an infinite continuous family of candidates). The methodology adopted here is experimental: to find out the influence of the type of wavelet transform, numerous test experiments have been performed. This necessitates definition of an objective performance measure that can be used to rank the various transforms or to optimize the structure parameter.

The scope of this paper is to investigate whether the properties of decomposition filters play an important role in ECG signal denoising. This will enable the researcher to select an optimal filter bank for their signal processing applications. This is important for application like biomedical signal processing where signal features and noise both are of narrowband in nature. We carried out denoising experimentation on ECG signal in presence of additive Gaussian noise. Finally, we conclude the most relevant decomposition filter bank in wavelet based ECG signal noise removal. The paper is organized as follows. Section 2 summarizes the basics of wavelet transforms and their properties. Basics of wavelet thresholding applied to signal denoising in general and applied to ECG in particular have been presented in Section 3. Section 4 presents results and discussion pertaining to an optimal selection of the wavelet basis function suitable for the ECG denoising. Finally, in Section 5 we conclude our findings.

2. Wavelet transform revisited

Wavelet basis function plays a key role in multiresolunal analysis. Discrete wavelet transform serves a link between wavelets in mathematics on one hand and applications of wavelets on the other hand, because in real-time we usually deal with discrete data sets in place of functions. Representation of a function in Fourier domain is not economical as there is no localization in time—each coefficient depends on values of the function across the whole interval $(0, 2\pi)$. Consequently, discontinuity in underlying function affects all the coefficients and necessitates a large number of terms to approximate discontinuous function accurately.

In wavelet domain signal processing and processed output signal is based on a function ψ, called as mother wavelet. For each $m \geqslant 0$ the wavelet function as defined by Meyer [13,14] is as follows.

Definition. A function $\psi : R \to R$ is called a mother wavelet of order m if the following five properties are satisfied:

1. If $m > 1$ then ψ is $(m-1)$-times differentiable.
2. $\psi \in L^\infty(R)$. If $m > 1$, for each $j \in \{1, \dots, m-1\}$ $\psi^{(j)} \in L^\infty(R)$.
3. ψ and all its derivatives up to order $(m-1)$ decay rapidly: For each $r > 0$ there is a $\gamma > 0$ such that
$$\left|\psi^{(j)}(t)\right| < \frac{1}{t}, \quad j \in \{0, 1, \dots, m-1\} \text{ for each } |t| > \gamma.$$
4. For each $j \in \{0, 1, \dots, m\}$ we have $\int t^j \psi(t)\,dt = 0$. This property of mother wavelet also called property of vanishing moment is very useful as it leads to economical representations of functions under study.
5. The set $\{\psi_{j,k}\}_{j,k\in Z}$ is an orthonormal basis of $L^2(R)$ where $\psi_{j,k}$ are derived from the mother wavelet by relationship $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$.
   Thus expression for wavelet coefficients is given as

$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$

$$f_{j,k} = \int_{-\infty}^{\infty} f(t)\psi_{j,k}(t)\,dt. \tag{1}$$

Base => rappresenta tutto lo spazio scala traslo
=> mi bastano j,k.

2.1. *Discrete wavelet transform (DWT)*

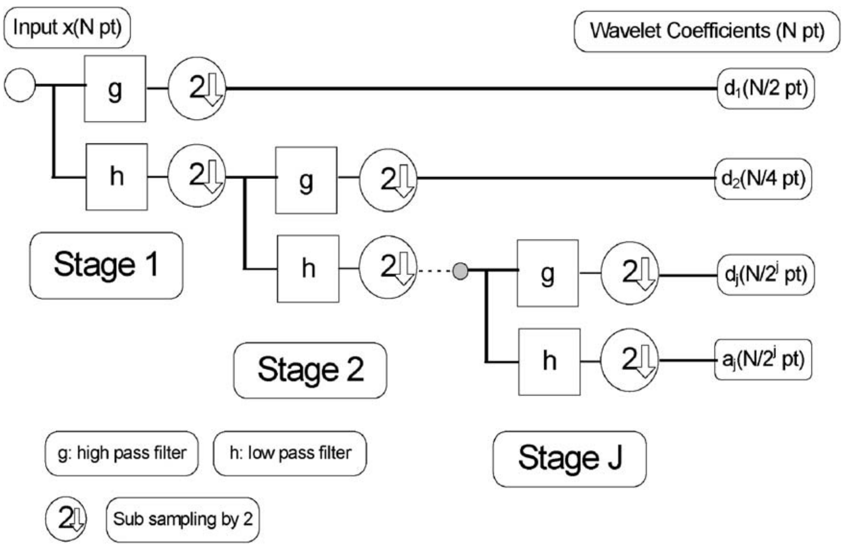
Computation of the wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. Selection of a subset of scales and positions based on powers of two (dyadic scales and positions) results in a more efficient and accurate analysis. Mallat [14,15] has introduced repetitive application of *high pass* and *low pass* filters to calculate the wavelet expansion of a given sequence of discrete numbers as depicted in Fig. 2.

Vetterli [16] has presented the approximation properties of filter banks and their relation to wavelets in his paper. An orthonormal compactly supported wavelet basis of $L^2(R)$ is formed by the dilation and translation of a single real-valued function $\psi(t)$, called the *mother wavelet*. The introduction to wavelet analysis will be simplified by defining an auxiliary function $\varphi(t)$, a companion function of the *wavelet* function, which is called *scaling function*, used to define the approximations and forms a set of orthonormal bases of $L^2(R)$ as given below:

$$\varphi_{j,k}(t) = 2^{-j/2}\varphi\left(\frac{t - k2^j}{2^j}\right), \quad j,k \in Z. \tag{2}$$

≠ mother wavelet

dividi il segnale andando ad estrarre coefficienti di approssimazione e dettaglio. E lo fai sottocampionando il segnale stesso.
Dettagli: HF
Approx: LF
Sottocampionando diventa tutto più grezzo.



detail

approximation

Fig. 2. Mallat's cascaded filter MRA: Tree structure DWT.

The *scaling function* $\varphi(t)$ satisfies

$$\int_{-\infty}^{+\infty} \varphi(t)\,dt = 1 \tag{3}$$

and two *scale difference equation*,

$$\varphi(t) = \sqrt{2}\sum_{k=0}^{L-1} h_k \varphi(2t-k), \quad j,k \in Z, \tag{4}$$

h_k permette di ricostruire $\varphi(t)$ da una sua versione sotto-campionata.

where Z is the set of integers.

The *wavelet* function is given by

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-k2^j}{2^j}\right), \quad j,k \in Z. \tag{5}$$

→ momenti nulli

In Eq. (2), the function ψ has M vanishing moments [13] up to order $(m-1)$ and it satisfies the following *two scale difference equation*,

$$\psi(t) = \sqrt{2}\sum_{k=0}^{L-1} g_k \varphi(2t-k). \tag{6}$$

g_k permette la ricostruzione di $\psi(t)$

The wavelet transform computation requires a pair of filters. One filter in the pair calculates wavelet coefficients, whereas, the other applies the scaling function. This scaling function, implemented with filter coefficients $\{h_k\}$, provides an approximation of the signal via the following equation [14]:

$$W_L(n,j) = \sum_m W_L(m,j-1)h(m-2n). \tag{7}$$

Low Pass n coeff. j livell:

The wavelet function gives us the detail signals, which are also called high, pass output as given in [14].

$$W_H(n,j) = \sum_m W_L(m,j-1)g(m-2n), \tag{8}$$

High Pass

where $W_L(p,q)$ is the pth scaling coefficient at the qth stage, $W_H(p,q)$ is the pth wavelet coefficient at the qth stage, and $h(n), g(n)$ are the filter coefficients corresponding to the scaling (*low pass* filter) and wavelet (*high pass* filter) functions, respectively [16]. These two filters are related by

$$g_k = (-1)^k h_{L-k}, \quad k = 0,\ldots,L-1 \tag{9}$$

are called as *quadrature mirror filters* (QMF) [16]. Table 1 lists the popular wavelet basis functions and their properties used for experimentation in present investigation [17].

Table 1
General characteristics of popular wavelet families

Family	Daubechies	Symmlet	Coiflet
Short name	Db	Sym	Coif
Order N	N strictly positive integer	$N = 2, 3, \ldots$	$N = 1, 2, \ldots, 5$
Examples	Db1 or haar, Db4, Db15	Sym2, Sym8	Coif2, Coif4
Orthogonal	Yes	Yes	Yes
Biorthogonal	Yes	Yes	Yes
Compact support	Yes	Yes	Yes
DWT	Possible	Possible	Possible
CWT	Possible	Possible	Possible
Support width	$2N-1$	$2N-1$	$6N-1$
Filters length	$2N$	$2N$	$6N$
Regularity	About $0.2N$ for large N		
Symmetry	Far from	Near from	Near from
Number of vanishing moments for ψ	N	N	$2N$

During literature review and our ground study we have observed that as compared to odd length an even length bio-orthogonal filters exhibit superior performance. This can be explained that symmetric even length filters have significantly less shift variance than odd length filters. Filters with lower shift variance and with a reasonable number of vanishing moments represent an optimal selection for the wavelet based signal denoising.

2.2. Filter bank and regularity

The low pass filter $H(z)$ is also called refinement filter must be factorisable as $H(z) = 2^{-\gamma}(1 + z^{-1})^{\gamma} Q(z)$ which is an expression that involves some number of regularity factors $(1 + z^{-1})$ as well as a stable residual term $Q(z)$ satisfying the low pass constraint $Q(1) = 1$.

The presence of regularity term is essential for theoretical reasons. It is responsible for a number of key wavelet properties such as an order of approximation, vanishing moments, reproduction of polynomials, and smoothness of the basis functions. The importance of these properties of wavelet lies in its ability to provide explanation about its suitability for approximating piecewise-smooth signals and in characterizing singularities. It is important to identify which mother wavelet is best suited for the detection of given pattern.

The different mother wavelets (basis functions) vary according to following criteria. The important of them are support of ψ, and its speed of convergence to 0. The number of vanishing moments is useful for compression purposes. Whereas, regularity is useful for getting nice features, like smoothness of the reconstructed signal or image, and for the estimated function in nonlinear regression analysis.

3. Optimal wavelet selection

Selection of a suitable basis mother wavelet filter is necessary for the ECG signal processing in wavelet domain. For the ECG signal under investigation, an optimal wavelet will lead maximization of coefficient values in wavelet domain. This will produce highest local maxima of the ECG signal in wavelet domain. The possibility of best characterization of frequency content of the ECG signals is possible with optimally selected wavelet filter bank. Figure 3a plots ECG signal (with 512 pt), Matlab generated wavelet coefficients (decomposition level 10) and ordered wavelet coefficients. It is clear from the plot that wavelet domain contains few significant coefficients, which after reconstruction will generate the ECG signal of interest. Figure 3b plots 50 most significant wavelet coefficients of underlying ECG signal decomposed with Haar wavelet and Daubechies wavelet (Db8). Plot reveals better localization property of wavelet coefficients with Db8 in comparison to Haar wavelet. Figure 3b also plots the reconstructed ECG signal with these two wavelet basis functions. It is clear from Fig. 3b that Db8 wavelet results into better-reconstructed ECG signal as opposed to Harr wavelet. This signifies need of an optimal wavelet basis function for ECG signal processing, resulting into better-reconstructed ECG signal in wavelet domain.

Thus following steps will lead determination of optimal wavelet applied to the ECG signal:

LAGO

1. Select the basis wavelet filter, low pass, decomposition [18] from wavelet filter bank library.
2. Compute the cross correlation coefficient [17] between ECG signal and selected wavelet filter.
3. Select the optimum wavelet filter which maximizes the cross correlation coefficient.

Figure 4 plots Matlab generated cross correlation coefficient [17] of the single bit ECG signal with various wavelet filters (available in Matlab library). Results strengthens the suitability of Daubechies wavelet filter of order 8 is an optimal one for the ECG signal processing. The experimental results reported in Section 4 will further strengthen our findings with respect to selection of optimal wavelet.

3.1. Wavelet thresholding

The recovery of signal from its noisy version in wavelet domain is carried out with an assumption that smooth functions have economical wavelet representations thereby most of the coefficients are set to zero without introducing larger error [18]. Due to its orthogonal properties a DWT transforms white noise to white noise with same variance. Thus wavelet thresholding leads setting of small wavelet coefficient to zero and retaining or shrinking the coefficients corresponding to desired signal. Classical thresholding assumes that the wavelet transform of smooth functions have economical representations, so that most of the coefficients are nearly zero, and white noise is transformed to white

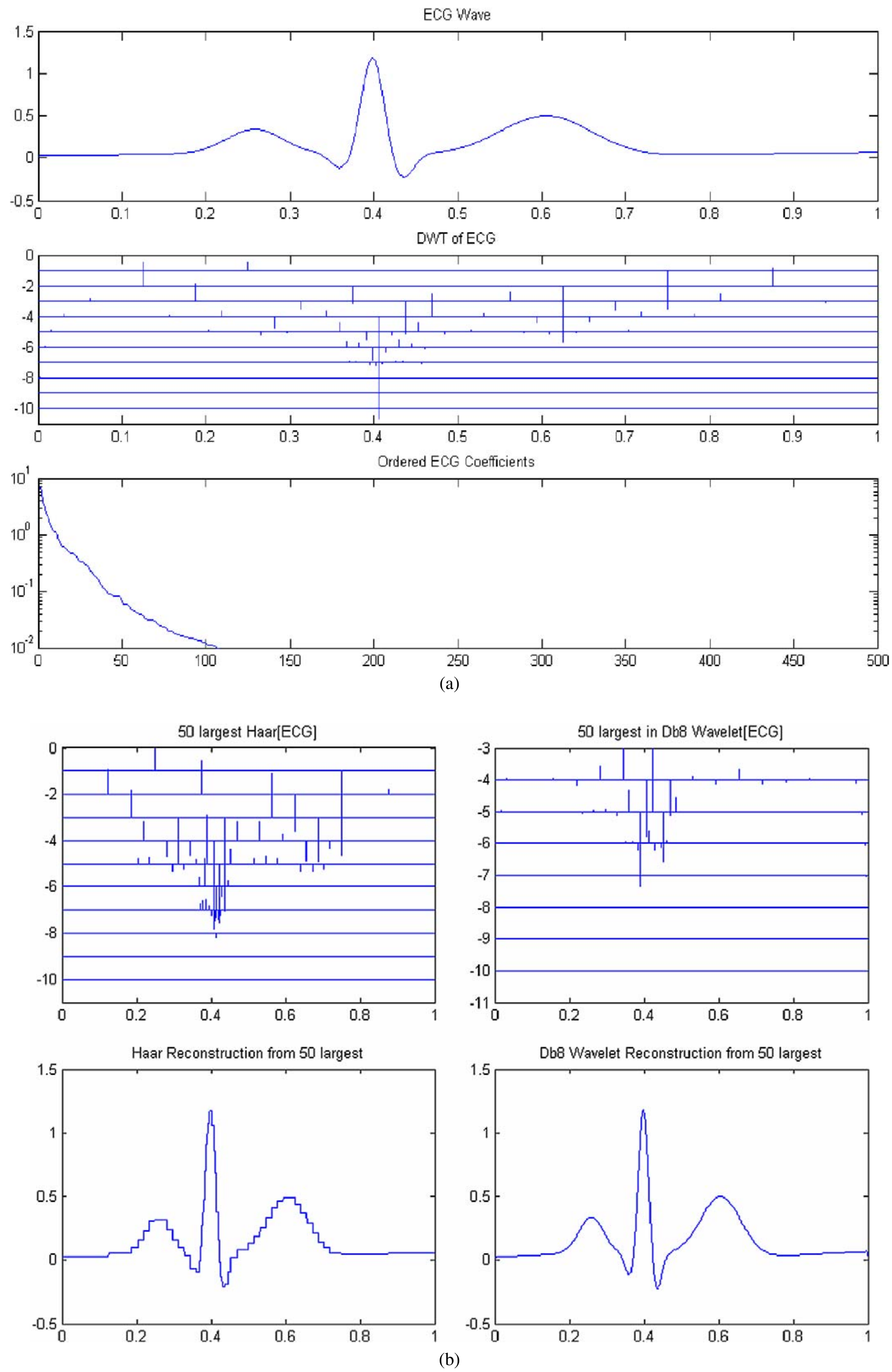


Fig. 3. (a) Matlab generated plot of one bit ECG signal (512 pt) and decomposed wavelet coefficients. (b) Matlab generated ECG signal decomposition and reconstruction using Haar and Daubechies (order 8) filter.

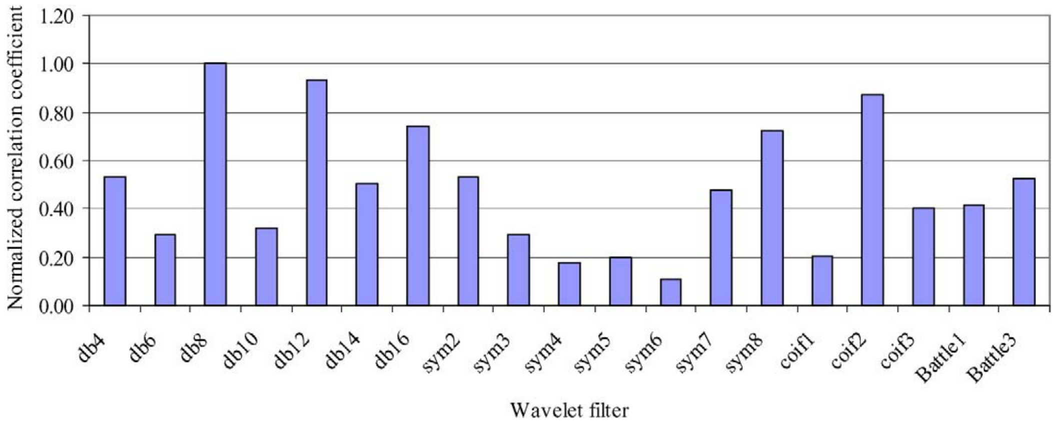


Fig. 4. Comparative plot of correlation coefficients with selected mother wavelet filter for ECG signal under test.

noise. Therefore, it is reasonable to assume that small coefficients are due to noise and can be set to zero, while the signal is stored in a few large coefficients, which should be retained. In fact, pre-processing steps are necessary for removing noise from the ECG signal before extracting the morphological parameters.

There have been vast investigations into removal of noise in signals and images using wavelet transform. The principal work is that of Donoho and Johnstone [19–22] based on thresholding of wavelet coefficients and reconstructing it. The method relies on the fact that in wavelet domain noise tends to be represented by wavelet coefficients at finer scales [23]. As the coefficients at such scales are also the primary carriers of edge information, it requires developing a methodology for removal of these coefficients. The removal must be in order that one looses minimum of primary signal components at the same time removing the noise to a maximum.

Considering following model of a discrete noisy signal:

$$y_i = f(t_i) + \varepsilon_i, \quad i = 1, 2, \ldots, N \tag{10}$$

or in vector notation,

$$y = f + \varepsilon. \tag{11}$$

The vector $\boldsymbol{y}$ represents input signal, $t_i = i/N$ and f is an unknown deterministic signal. It is assumed that the noise is a stationary stochastic signal, i.e., all its values are identically distributed with zero mean and variance σ^2. Thus

$$E\varepsilon_i = 0 \quad \text{and} \quad E\varepsilon_i^2 = \sigma^2, \quad \forall i = 1, 2, \ldots N, \tag{12}$$

where noise is uncorrelated (white) noise. This means that $E\varepsilon_i\varepsilon_j = \delta_{ij}\sigma^2$.

Recovery of original function in wavelet domain is possible by setting a threshold value, which sets the coefficients corresponding to noise to zero. Hence, the question arises:

(i) How to distinguish between the coefficients that are mainly due to signal and those mainly due to noise?
(ii) How should the thresholds be adjusted to obtain a noise free signal?

3.2. *Threshold selection method*

The universal threshold selection by Donoho and Johnstone [19–22] explicitly proposes a threshold value $\sigma\sqrt{2\log M}$ proportional to the amount of noise σ (assumed to be known or estimated from data) and M the number of samples. Donoho and Johnstone [22] proposed the minimax threshold, which applies the optimal threshold in terms of L^2 risk. This optimal (minimax) threshold depends on the sample size M, and is derived to minimize the constant term in an upper bound of the risk involved in estimating a function.

Other well-known threshold selection procedure is based on Stein's unbiased risk estimator (SURE) [24]. The *SureShrink* threshold have serious drawback in situations of extreme sparsity of the of the wavelet coefficients. To overcome this drawback Donoho et al. proposed a hybrid scheme of the *SureShrink* using a hybrid of Universal threshold and the *Sure* threshold and has been shown to perform well [25]. The *HybridSureShrink* is considered as main method for comparative study of wavelet basis functions applied for thresholding. Further, performance of optimally selected basis functions has been compared with a number of thresholding schemes available in literature. A popular alternative (to analytic methods) in selection of basis functions is based on resampling, i.e., leave-one-out cross-validation (CV). Under this approach, prediction risk is estimated via cross-validation, and the model providing lowest estimated risk is chosen. Based on the work of Nason [26] and Janesen [27], in this work the minimizer of the generalized cross validation (GCV) function for threshold selection has been used.

4. Results and discussion

4.1. Description of simulation

In present investigation, the ECG signal was taken up from biological signal processing (BSP) demonstration data sheet (400 s @ 500 Hz) [28]. The data (x_i, y_i) were generated from a model of the form $y_i = f(x_i) + \varepsilon_i$, $\{\varepsilon_i\}$ i.i.d. $N(0, \sigma^2)$, where $\{x_i\}$ are equispaced in [0, 1], $x_0 = 0$ and $x_n = 1$. The factors are:

(a) ECG signal sample sizes n.
(b) Values of σ^2.
(c) The thresholding methods.
(d) Type of Mother Wavelet basis function [29]:
  Daubechies filter (Db) of order 4, 6, 8, 10, 12;
  Symmlet filter (Sym) of order 4, 5, 6, 7, 8;
  Coiflet filter (C) of order 1, 2, 3, 4, 5;
  Battle–Lemarie (Bt) filter of order 1, 3, 5;
  Beylkin filter (Bl);
  Vaidyanathan filter (Vd).

For each combination of these factors, a simulation run was repeated 50 times keeping all factor levels constant, except the $\{\varepsilon_i\}$ that were regenerated. In order to compare the behavior of the estimation methods performance criteria viz. root mean square error (RMSE), root means square bias (RMSB), and $L1$ norm [18,29] are employed. The computational platform selected is Intel Pentium (III), 500 MHz processor, 128 MB RAM. All simulations are performed in MATLAB [16].

4.2. Discussion

Figure 5 is a plot of RMSE of the resulted denoised ECG signals by different thresholding schemes [18]. The result pertains to signals with sample length 256, σ as 1.0 and Daubechies 8 wavelet basis function. The result provides a comparative study of effectiveness of HybridSure method over other thresholding rules used in denoising of the ECG signal.

Figure 6 plots RMSE of resulting denoised ECG signals with noise variance 1 and 2 using different mother wavelet basis functions. It is observed that performance of Daubechies wavelet of order 8 is best in comparison to other wavelet basis functions under test. The plots further strengthen need for an optimal selection methodology of mother wavelet basis functions for denoising due to wide variations in results with change in mother wavelet basis functions. Furthermore, it is observed that in the same family of mother wavelet basis functions (viz. Daubechies, Symmlet, Coiflet) change in order of basis function has influence on results. The results further strengthened that the statement that wavelet filters with lower shift variance and with a reasonable number of vanishing moments represent an optimal selection for the wavelet based signal denoising. Figures 7 and 8 plot RMSB and $L - 1$ norm of the resulted denoised ECG signal using different mother wavelet basis functions. This strengthens the selection of Daubechies filter of order 8 as an appropriate wavelet basis function.

Figure 9 plots original ECG signal (blue line) and denoised version (red line) of an ECG signal (Fig. 1) with different wavelet basis functions. The selected thresholding scheme is based on *HybridSureShrink* method. While processing the ECG signal it is important to retain its characteristics wave features.

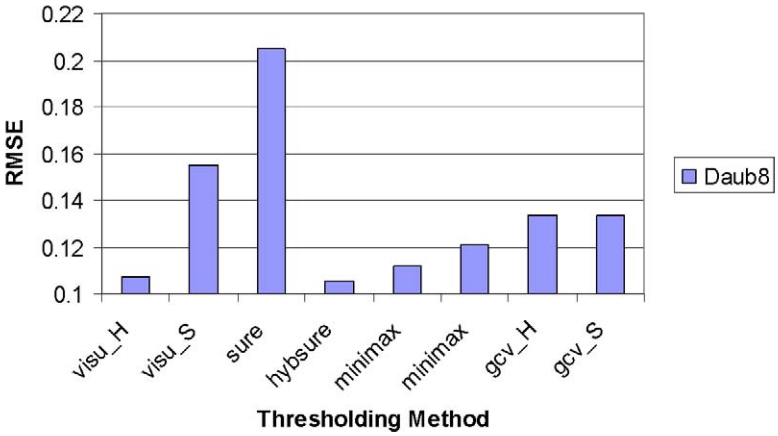


Fig. 5. Comparative plot of various thresholding schemes applied to signal denoising.

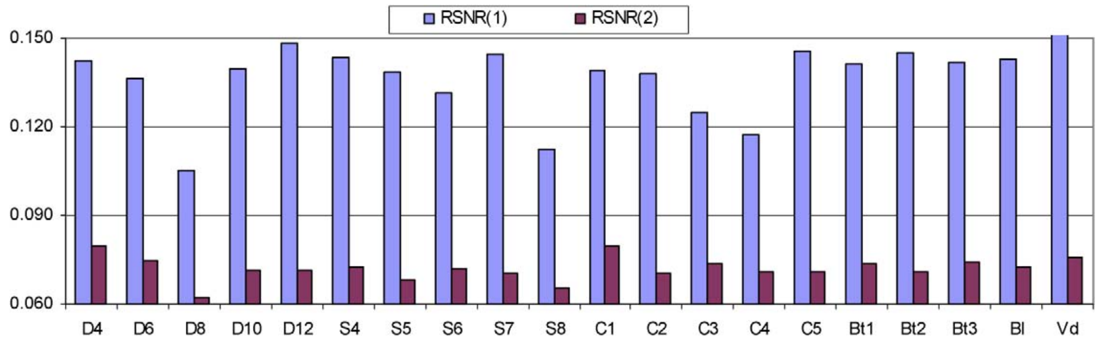


Fig. 6. Comparative plot of RMSE in denoised ECG with varied wavelet basis function.

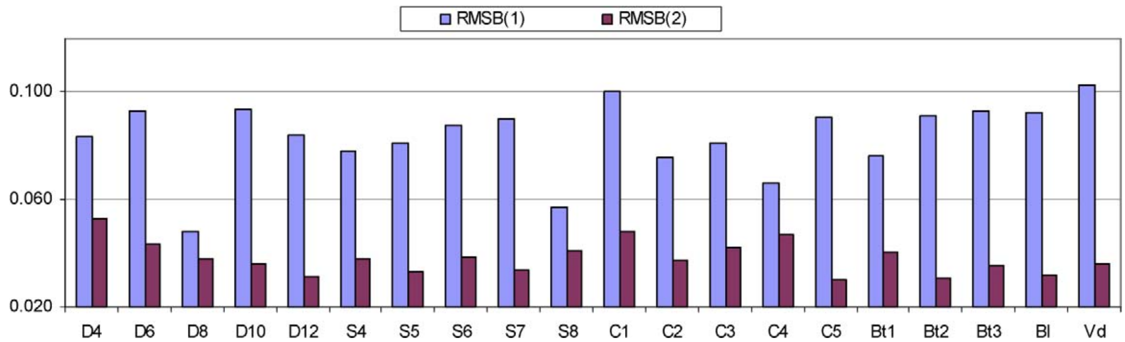


Fig. 7. Comparative plot of RMSB in denoised ECG with varied wavelet basis function.

The plots shown in Fig. 10 reveal that under same thresholding scheme (*HybridSure*), the different wavelet basis functions perform differently with respect to retaining the characteristics wave features of the ECG. We further conducted experiments on ECG signal under test, its noisy version and denoised signal in terms of detection of following characteristic peaks containing important physiological information [28]:

B peak count: percusion peak (P1) index,

A peak count: minima preceding the P1 index,

C peak count: dicrotic notch (DN) index,

D peak count: dicrotic peak (P2) index.

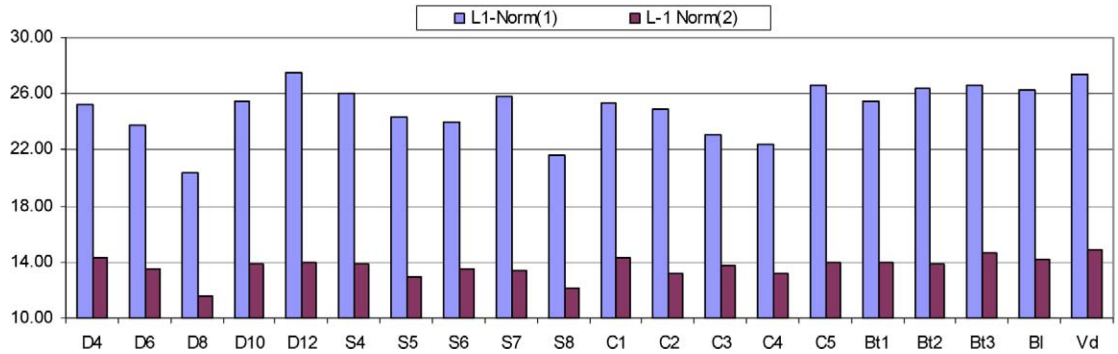


Fig. 8. Comparative plot of $L-1$ norm in denoised ECG with varied wavelet basis function.

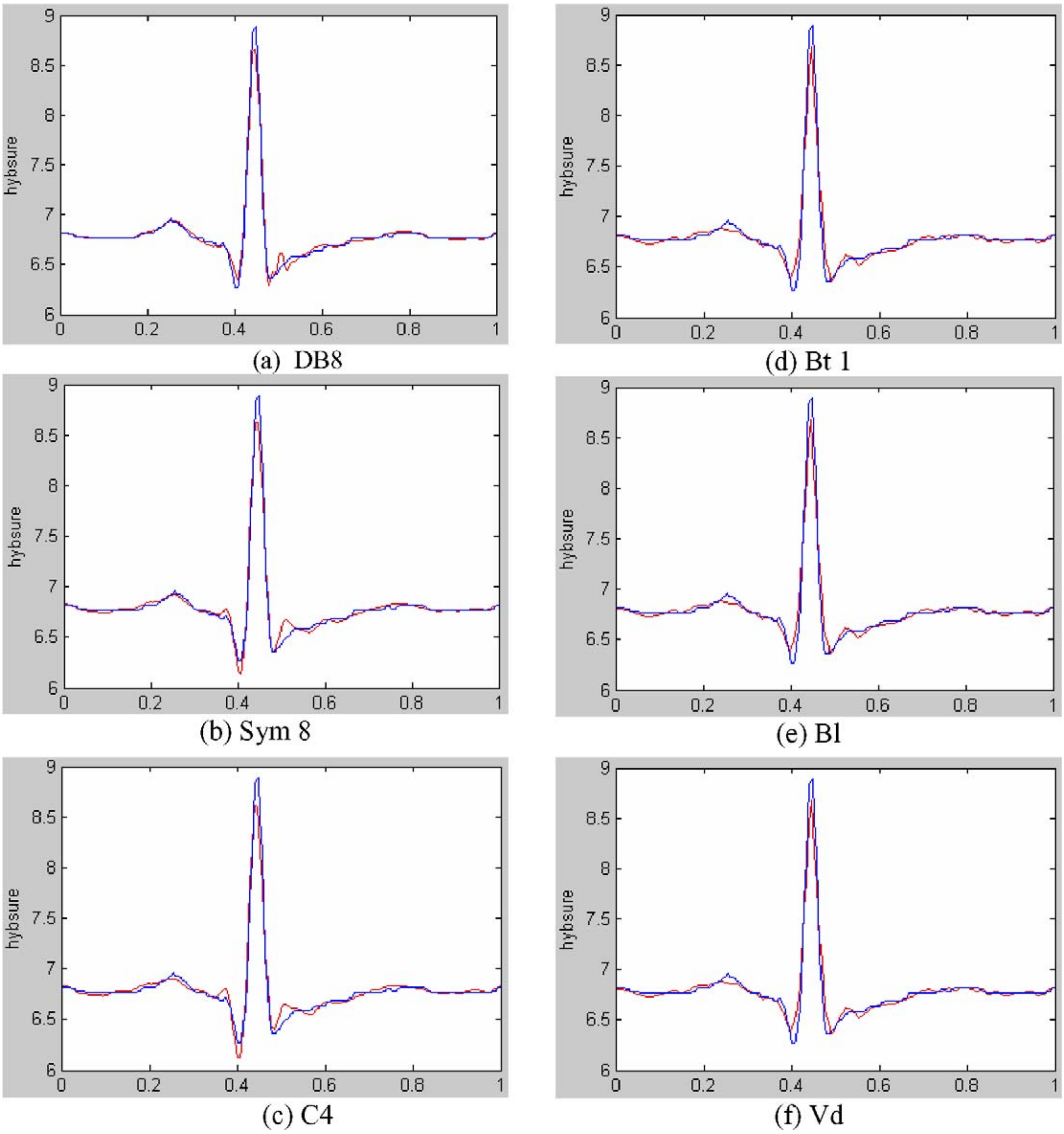


Fig. 9. Plots of original ECG signal (blue line) and denoised version (red line) with different wavelet basis function under *HybridSureShrink* thresholding scheme.

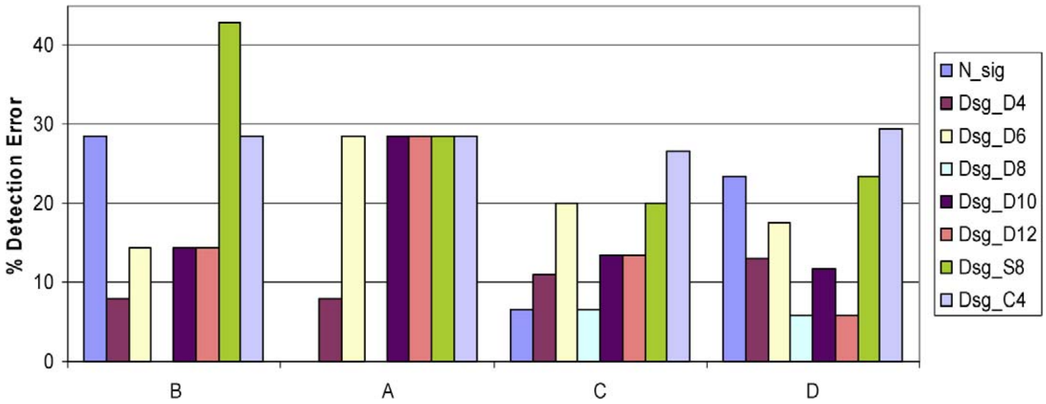


Fig. 10. Comparative plot of detection error in characteristics waves in denoised ECG signal with varied wavelet basis function.

In the investigation we performed suitable experiments for obtaining these counts in both noisy and the denoised ECG signal and compared the results from original ECG signal. Figure 10 plots the percent error in counts in the denoised ECG signal with varied wavelet basis functions. The results are compared with counts of the noisy ECG signals. It is once again observed that performance of Daubechies filter of order 8 is most appropriate for the ECG signal denoising.

5. Conclusions

In this paper selection of an optimal wavelet basis function applied to denoising of an ECG signal has been carried out. The experimental results have revealed suitability of Daubechies mother wavelet of order 8 to be the most appropriate wavelet basis function for the denoising application. The selected basis function has been found to be optimal not only in terms of root mean square errors (RMSE), but also it preserves the peaks of the ECG signal, which contains valuable physiological information for diagnostic purpose. We expect that the analysis carried out in this paper can be useful for paramedics to accurately diagnose cardiovascular ailments in patients.

References

[1] W.G. Morrison, I.J. Swann, Electrocardiograph interpretation by junior doctors, Arch. Emerg. Med. 7 (1990) 108–110.
[2] J.C. Lin, Current developments in telemedicine, IEEE Eng. Med. Biol. 18 (4) (1999) 27.
[3] P.E. McSharry, G.D. Clifford, L. Terassenko, L.A. Smith, A dynamical model for generating synthetic electrocardiogram signals, IEEE Trans. Biomed. Eng. 50 (3) (2003) 289–294.
[4] M. Unser, A. Aldroubi, A review of wavelets in biomedical applications, Proc. IEEE 84 (4) (1996) 626–638.
[5] J.B. Weaver, X. Yansum, D.M. Healy Jr., L.D. Cromwell, Filtering noise from images with wavelet transforms, Magnet. Resonance Med. 21 (1991) 288–295.
[6] A. Khamene, S. Negahdaripour, A new method for the extraction of fetal ECG from the composite abdominal signal, IEEE Eng. Med. Biol. Mag. 47 (4) (2000) 507–516.
[7] S.G. Miaou, H.L. Yen, C.L. Lin, Wavelet-based ECG compression using dynamic vector quantization with tree code vectors in single codebook, IEEE Trans. Biomed. Eng. 49 (7) (2002) 671–680.
[8] R.H. Istepanian, A.A. Petrocian, Optimal zonal wavelet-based ECG data compression for a mobile telecardiology system, IEEE Trans. Inform. Technol. Biomed. 4 (3) (2000) 200–211.
[9] W.J. Hwang, C.F. Chine, K.J. Li, Scalable medical data compression and transmission using wavelet transform for telemedicine application, IEEE Trans. Inform. Technol. Biomed. 7 (1) (2003) 54–63.
[10] A.L. Goldberger, L. Amaral, L. Glass, J.M. Hausdorff, P.Ch. Ivanov, R.G. Mark, J.E. Mietus, G.B. Moody, C.K. Peng, H.E. Stanley, PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals, Circulation 101 (23) (2000) e215–e220.
[11] A. Khamene, S. Negahdaripour, A new method for the extraction of fetal ECG from the composite abdominal signal, IEEE Eng. Med. Biol. Mag. 47 (4) (2000) 507–516.
[12] S. Kadambe, R. Murray, G.F. Boudreaux-Bartels, Wavelet transform-based QRS complex detector, IEEE Trans. Biomed. Eng. 46 (7) (1999) 838–848.
[13] I. Daubechies, The wavelet transform, time/frequency location and signal analysis, IEEE Trans. Inform. Theory 36 (5) (1990) 961–1005.

[14] S.G. Mallat, A theory of multiresolution signal decomposition: The wavelet representation, IEEE Trans. Pattern Anal. Machine Intell. 1 (7) (1989) 674–693.
[15] O. Rioul, M. Vetterli, Wavelets and signal processing, IEEE Signal Process. Mag. 8 (4) (1991) 14–38.
[16] M. Vettereli, Wavelets, approximation and compression, IEEE Signal Process. Mag. 18 (5) (2001) 59–73.
[17] MATLAB: The Language of Technical Computing, The Math Works Inc., 2003, http://www.mathworks.com/products/matlab.
[18] A.K. Tiwari, New implementation issues of DWT and its application to signal denoising and classification, Ph.D. thesis, Banaras Hindu University, Varanasi, India, 2002.
[19] D.L. Donoho, I.M. Johnstone, Ideal denoising in an orthonormal basis chosen from a library of bases, Technical report, vol. 461, Department of Statistics, Stanford University, 1994.
[20] D.L. Donoho, Denoising by soft thresholding, IEEE Trans. Inform. Theory 41 (3) (1995) 613–627.
[21] D.L. Donoho, M. Vetterli, R.A. Devore, I. Daubechies, Data compression and harmonic analysis, IEEE Trans. Inform. Theory 44 (6) (1998) 2435–2476.
[22] D.L. Donoho, I.M. Johnstone, Minimax estimation via wavelet shrinkage, Ann. Statist. 26 (1998) 879–921.
[23] R.M. Rao, A.S. Bopardikar, Wavelet Transforms: Introduction to Theory and Applications, Addison–Wesley, Reading, MA, 1998.
[24] C.M. Stein, Estimation of the mean of a multivariate normal distribution, Ann. Statist. 9 (6) (1981) 1135–1151.
[25] S.G. Chang, B. Yu, M. Vettreli, Adaptive wavelet thresholding for image denoising and compression, IEEE Trans. Image Process. 9 (9) (2000) 1532–1546.
[26] G.P. Nason, Wavelet regression by cross-validation, Department of Mathematics, University of Bristol, UK, 1994, preprint.
[27] M. Jansen, M. Malfait, A. Bultheel, Generalized cross validation for wavelet thresholding, Signal Process. 56 (1) (1997) 33–44.
[28] M. Aboy, C. Crespo, J. McNames, Biomedical signal processing toolbox, http://bsp.pdx.edu.
[29] J.B. Buckheit, S. Chen, D.L. Donoho, I.M. Johnstone, J. Scargle, About wavelab technical report, Department of Statistics, Stanford University, USA, 1995, http://www.stat.stanford.edu/~wavelab.

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