UNIVERSITÀ CATTOLICA DEL SACRO CUORE

INTERFACULTY ECONOMICS - BANKING, FINANCE AND INSURANCE SCIENCES

MASTER OF SCIENCE IN STATISTICAL AND ACTUARIAL SCIENCES

DATA ANALYTICS FOR BUSINESS AND ECONOMICS

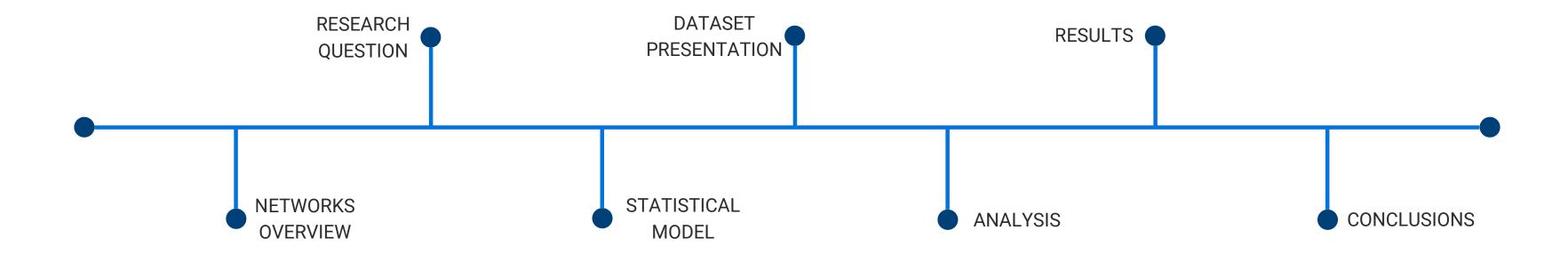


STATISTICAL MODELLING OF LONGITUDINAL NETWORK DATA: AN ANALYSIS OF THE TECH STOCK MARKET

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Definition:

• A network **N** can be defined as a set of points (or nodes) and ties between them, N = (V; E) (fig 1).

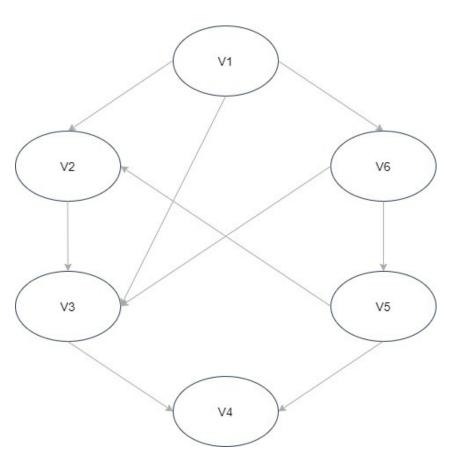


Fig.1 : Example of a network.

Possible representations:

- Adjacency *list* (fig. 2)
- Adjacency matrix (fig. 3).

| Vertices | Ties | | |
|----------|------------|--|--|
| V1 | V2, V3, V6 | | |
| V2 | V3 | | |
| V3 | V4 | | |
| V4 | None | | |
| V5 | V2, V4 | | |
| V6 | V3, V5 | | |

Fig.2 : Example of adjacency list.

| | V1 | V2 | V3 | V4 | V5 | V6 |
|----|----|----|----|----|----|----|
| V1 | 0 | 1 | 1 | 0 | 0 | 1 |
| V2 | 0 | 0 | 1 | 0 | 0 | 0 |
| V3 | 0 | 0 | 0 | 1 | 0 | 0 |
| V4 | 0 | 0 | 0 | 0 | 0 | 0 |
| V5 | 0 | 1 | 0 | 1 | 0 | 0 |
| V6 | 0 | 0 | 1 | 0 | 1 | 0 |

Fig.3: Example of adjacency matrix.

Research Object

Longitudinal Networks

Research Question

To verify whether a statistical model developed for the analysis of social networks can be extended to a different field of application and specifically used to predict the monthly correlations between the returns of 13 tech companies at a specific time point.

Methodology

- Statistical model
- Data collection
- Dataset building

Statistical Model

The TERGM has the following **formulation**:

$$\mathcal{P}\left(N^{t} \mid N^{t-1}, \boldsymbol{\theta}\right) = \frac{1}{c\left(\boldsymbol{\theta}, N^{t-1}\right)} \exp\left\{\boldsymbol{\theta}' \boldsymbol{\Psi}\left(N^{t}, N^{t-1}\right)\right\}$$

Elements of the formulation:

- θ is the vector of the model coefficients.
- Ψ is a vector of statistics computed on the network N
- ullet C is the normalizing constant.

Estimation:

- 1. Markov Chain Monte Carlo MLE
- 2. Maximum Pseudolikelihood Estimation:

MPLE replaces likelihood with the product over conditional dyadic tie probabilities:

$$\pi_{ij}(\boldsymbol{\theta}) = \Pr(N_{ij} = 1 \mid \boldsymbol{N}_{-ij}, \boldsymbol{\theta}) = 1/\left[1 + \exp\left\{-\boldsymbol{\theta}'\delta_{ij}(\boldsymbol{\Psi}(\boldsymbol{N}))\right\}\right]$$

Advantages:

- 1. Faster than MCMC-MLE.
- 2. Converges asymptotically to the MLE as the number of network samples increases.

Data collection

- 13 major tech companies.
- Daily stock returns from 2005 to 2020.
- Linear correlation of the returns grouped by month.
- Sharpe ratio and Beta of the grouped returns.



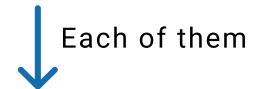
Correlations filtered according to **4 thresholds**:

- 0.6
- 0.65
- 0.7
- 0.75

If the correlation is greater than the threshold, then there is a tie between the two nodes.

Dataset building

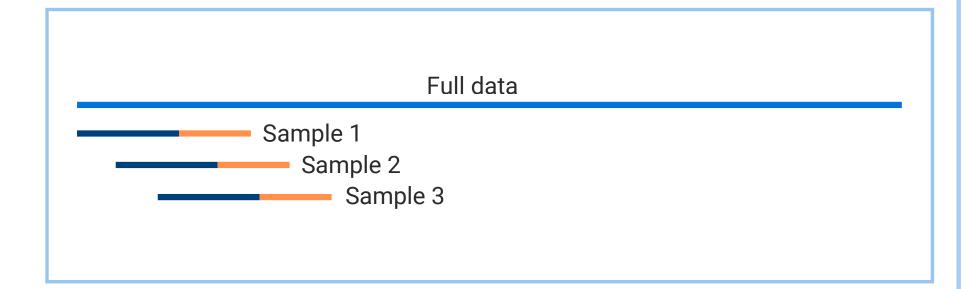
• 4 longitudinal networks built according to the thresholds



- Represented through 13x13 adjacency matrices
- 188 time points.
- 13 nodes per network.
- Two nodal attributes: SR and β .

Rolling Origin Cross Validation

Model used to predict the status of the network at certain time point, given 3 years of data.



Results evaluated through average Roc AUC for each network

Out-of-sample Prediction

Performed on the most recent data, considering one year:

- Training sample = 11 time points (July 2019 June 2020)
- Test sample = 1 time point (July 2020)

Predictive performance evaluated with Roc AUC.

Goodness of fit adressed with gof function:

- 1. Model fitted on training subset.
- 2. *n* simulations of the test subset are made.
- 3. Comparison of the simulations against the observed network.

Rolling Origin Cross Validation

The average Roc AUC for each network is:

- Network 1: 0.63
- Network 2: 0.63
- Network 3: 0.63
- Network 4: 0.65

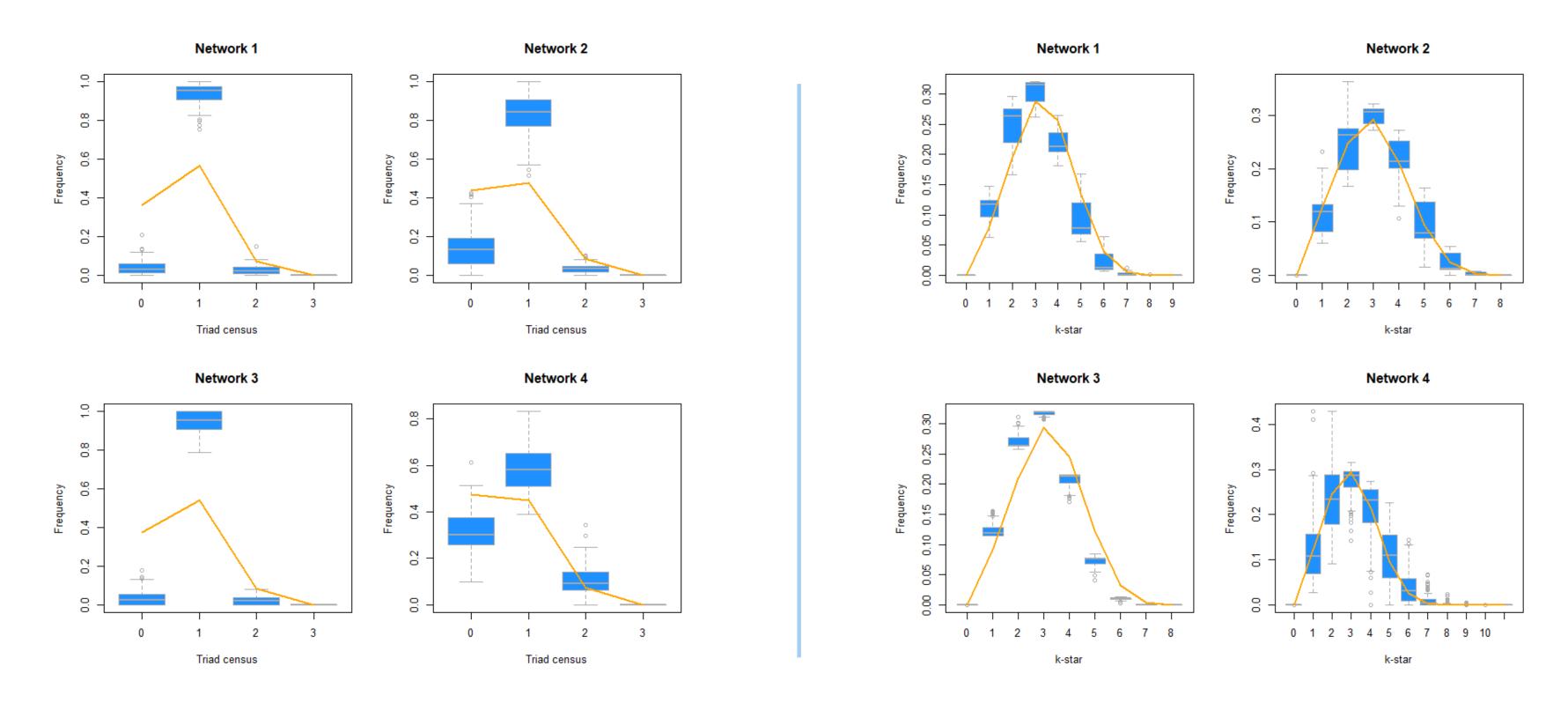
The model performs well in specific time windows but it is not very consistent, given the ROCV.

Out-of-sample Prediction

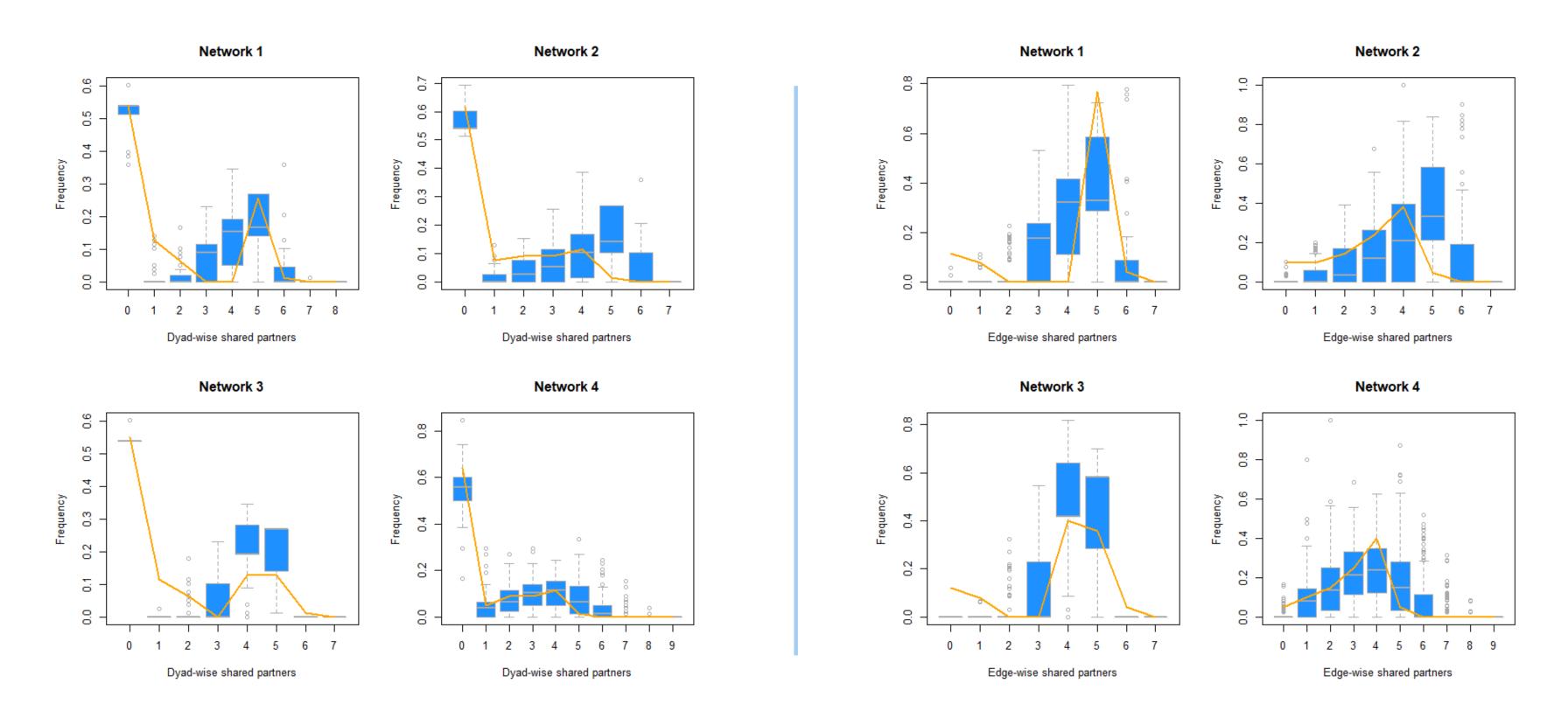
The Roc AUC for each network is:

- Network 1: 0.68
- Network 2: 0.75
- Network 3: 0.80
- Network 4: 0.78

Goodness of fit of the "Out-of-sample Prediction" model.



Goodness of fit of the "Out-of-sample Prediction" model.



- Although the results of the **Rolling Origin** aren't consistent for each sample of the data, the performance on specific time frames is promising, as shown by the goodness of fit.
- Indeed, the results of the gof of the "Out-of-sample Prediction" model show that it is able to represent the original structure of the network, especially regarding specific statistics. Moreover it has obtained a good overall predictive performance.

Thus, the Temporal Exponential Random Graph model **can be extended** beyond its original field of application. However it is not consistent enough to perform well over long time span but it has to be fine tuned **to specific time frames**; in such applications the model can be exploited to predict the correlations between the nodes at specific time points.

- The financial world is by nature subject to changes depending both on endogenous and exogenous factors, which make it hard to have long-term predictions. The **networks** have been modelled using **mainly internal data**, the daily returns, and only partly using external data regarding the computing of the beta.
- This represents indeed a limit but also a **possible developement of the research**.

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It is necessary to remark that the tech sector **evolves at a fast pace** which once more make it hard to have reliable long-term predictions. In the following plots are represented two time steps of network 4: July 2019 and July 2020. It is evident the change in the network structure:

- In 2019 there was a triad composed by the japanese companies which isn't present at one years distance.
- The position of Microsoft is peculiar since it isn't highly correlated with any company both in 2019 and in 2020.
- Google is at the center of a group in both the time steps.
- Also the position outside of the main group of Amazon in 2020 is rather interesting.

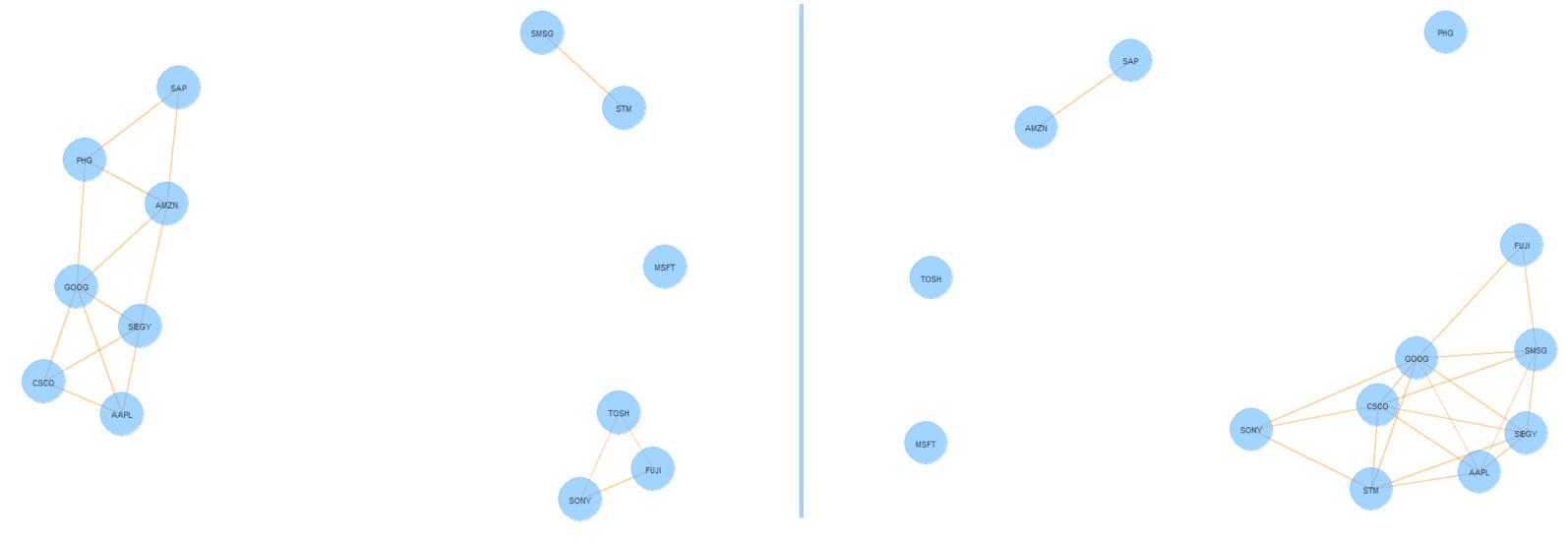


Fig.2: Network 4 at July 2020

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THANK YOU FOR YOUR ATTENTION.