Additional exercises

Exercise 1: let $M = \{m_1, m_2, m_3\}$ be a set of messages such that:

$$P(m_1) = \frac{1}{2}, P(m_2) = \frac{1}{4}, P(m_3) = \frac{1}{4}$$

Can we design a prefix-free binary encoding f such that the average length of f is 1.2 and the compression is lossless? Motivate your answer

Exercise 2:
Consider the following set of messages $M = \{m_1, m_1, m_{3,} m_{4,} m_5\}$:
$P(m_1) = \frac{2}{5}, P(m_2) = \frac{1}{5}, P(m_3) = \frac{1}{5}, P(m_4) = \frac{1}{10}, P(m_5) = \frac{1}{10}$
Which of the following encodings achieves the highest compression?

1. $f(m_1) = 1, f(m_2) = 01, f(m_3) = 000, f(m_4) = 0011, f(m_5) = 0010$
2. $f(m_1) = 00, f(m_2) = 01, f(m_3) = 10, f(m_4) = 110, f(m_5) = 111$

Motivate your answer

Exercise 3:
Consider the following set of messages

$M = \{m_1, m_{2,} m_3, m_{4,} m_5, m_6\}$

$P(m_1) = P(m_2) = \frac{1}{24}$

$P(m_3) = P(m_4) = \frac{1}{12}$

$P(m_5) = \frac{1}{3}$

$P(m_6) = \frac{5}{12}$

Find a maximally compressed prefix-free binary encoding

Exercise 4:
Consider a channel with noise matrix

$$W(Y|X) = \begin{array}{c c} & \begin{array}{cc} 0 & \;\;1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 1 & 0 \\ 0.2 & 0.8 \end{pmatrix} \end{array}.$$

For what of the following choices of P(x) is the mutual information between X and Y maximized?

1.$P(X = 0) = 1, P(X = 1) = 0$
2.$P(X = 0) = 0.6, P(X = 1) = 0.4$
3.$P(X = 0) = 0.5, P(X = 1) = 0.5$