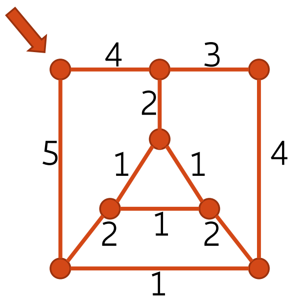
HEURISTICS (1)

In [Wieselthier, Nguyen, Ephremides, 00]: three heuristics all based on the greedy technique:

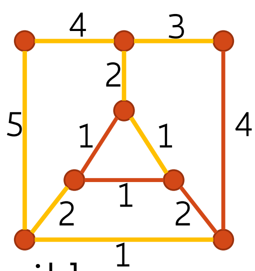
- SPT (spanning path tree): it runs Dijkstra algorithm to get the minimum path tree, then it directs the edges of the tree from the root to the leaves.
- BAIP (Broadcast Average Incremental Power): it is a modification of the Dijkstra algorithm based on the nodes (i.e. a new node is added to the tree on the basis of its minimum average cost).
- MST (min spanning tree): it runs Prim algorithm to get a MST, then it directs the edges of the tree from the root to the leaves.

A PARENTHESIS

SPT (spanning path tree) and MST (min spanning tree) can be different:



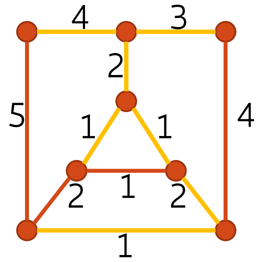
Dijkstra algorithm (SPT)



Possibly many trees, not all of the same weight

(e.g., add the upper edge of weight 1)

Prim algorithm (MST)



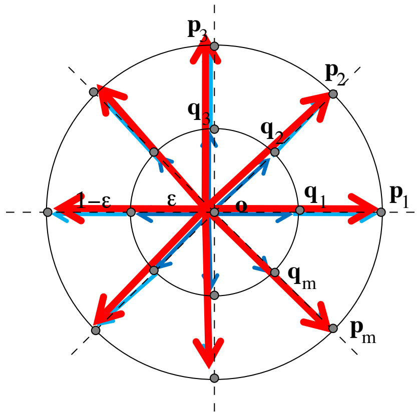
Possibly many trees, all of the same (minimum) weight

HEURISTICS (2)

GREEDY IS NOT ALWAYS GOOD

Greedy is not always good [Wan, Calinescu, Li, Frieder '02]:

- SPT: it runs Dijkstra algorithm to get the minimum path tree, then it directs the edges of the tree from the root to the leaves



(let α=2)

- SPT outputs a tree with total energy: $\varepsilon^2+n/2(1-\varepsilon)^2$
- If the root transmits with radius 1 the energy is 1
- When $\varepsilon \rightarrow 0$ SPT is far n/2 from the optimal solution.

HEURISTICS (3)

GREEDY IS NOT ALWAYS GOOD

BAIP (Broadcast Average Incremental Power): it is a modification of the Dijkstra algorithm based on the nodes: a new node is added to the tree on the basis of:

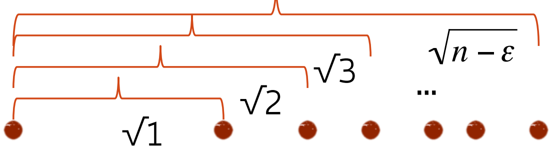
min avg cost = energy increasing / # of added nodes.

If it is <1, it is considered profitable.

It has been designed to solve the problems of SPT.

HEURISTICS (4)

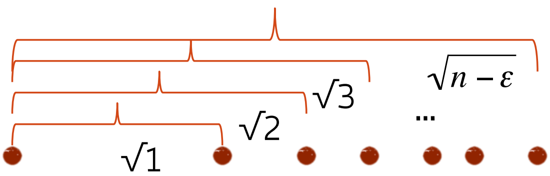
GREEDY IS NOT ALWAYS GOOD

(let α=2): 

- The min transmission power of the source to reach k receiving nodes is $\sqrt{k^2} = k$ and thus the average power efficiency is k / k = 1
- The min transmission power of the source to reach all receiving nodes is $(\sqrt{n-\varepsilon})^2 = n-\varepsilon$ and thus the average power efficiency is
  $(n-\varepsilon)/n = 1 - \varepsilon/n$...

HEURISTICS (5)

GREEDY IS NOT ALWAYS GOOD



BAIP will let the source to transmit at power $\sqrt{n-\varepsilon}$ to reach all nodes in a single step with power n-ε. However, the opt. routing is a path consisting of all nodes from left to right. Its min power is:

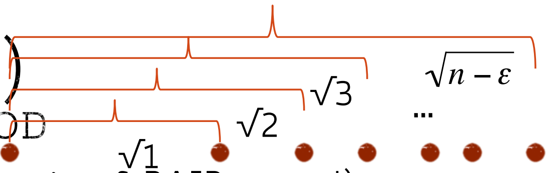
$$\sum_{i=1}^{n-1}(\sqrt{i}-\sqrt{i-1})^2+(\sqrt{n-\varepsilon}-\sqrt{n-1})^2 < \sum_{i=1}^{n}(\sqrt{i}-\sqrt{i-1})^2 =$$

$$\sum_{i=1}^{n}(\sqrt{i}-\sqrt{i-1})^2\frac{(\sqrt{i}+\sqrt{i-1})^2}{(\sqrt{i}+\sqrt{i-1})^2} = \sum_{i=1}^{n}\frac{((\sqrt{i}-\sqrt{i-1})(\sqrt{i}+\sqrt{i-1}))^2}{(\sqrt{i}+\sqrt{i-1})^2} =$$

$$= \sum_{i=1}^{n}\frac{(i-(i-1))^2}{(\sqrt{i}+\sqrt{i-1})^2} = \sum_{i=1}^{n}\frac{1}{(\sqrt{i}+\sqrt{i-1})^2} = 1+\sum_{i=2}^{n}\frac{1}{(\sqrt{i}+\sqrt{i-1})^2} \leq$$

HEURISTICS (6)

GREEDY IS NOT ALWAYS GOOD



(computation of the performance ratio of BAIP – cntd)

$$\leq 1+\sum_{i=2}^{n}\frac{1}{i+(i-1)+2\sqrt{i}\sqrt{i-1}} \leq 1+\sum_{i=2}^{n}\frac{1}{2i-1+2(i-1)} \leq$$

$$\leq 1+\sum_{i=2}^{n}\frac{1}{2i-1+2(i-1)} = 1+\sum_{i=2}^{n}\frac{1}{4i-3} \leq 1+\sum_{i=2}^{n}\frac{1}{4(i-1)} \leq$$

Substituting i=j+1:

$$\leq 1+\sum_{j=1}^{n-1}\frac{1}{4j} \leq 1+\frac{1}{4}\sum_{j=1}^{n-1}\frac{1}{j} \leq 1+\frac{1}{4}(\ln(n-1)+1) = \frac{\ln(n-1)+5}{4}$$

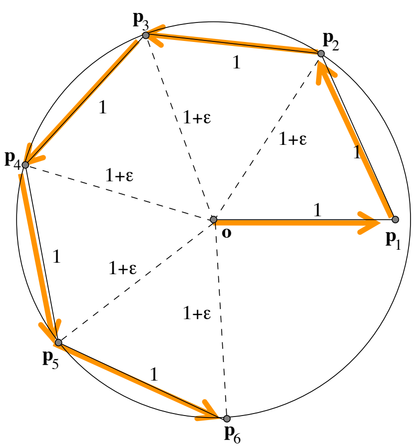
Thus the approx ratio of BAIP is at least:

$$\frac{n-\varepsilon}{\frac{\ln(n-1)+5}{4}} \rightarrow (\varepsilon \rightarrow 0) \frac{4n}{\ln(n-1)+5} = \frac{4n}{\ln n}+o(1)$$

HEURISTICS (7)

GREEDY IS NOT ALWAYS GOOD

MST: it runs Prim algorithm to get a MST, then it directs the edges of the tree from the root to the leaves



- Path $op_1 \ldots p_6$ is the unique MST, and its total energy is 6.
- On the other hand, the opt. routing is the star centered at o, whose energy is $(1+\varepsilon)^{\alpha}$.
- The approx. ratio converges to 6, as ε goes to 0.

HEURISTICS (8)

- We have just shown a lower bound on the approximation ratio of MST.
- This ratio is a constant and an upper bound is 12.
- The proof involves complicated geometric arguments...

HEURISTICS (9)

Obs. The proof in [Wan, Calinescu, Li, Frieder '02] contains a flaw that can be solved, arriving to an approximation ratio of 12,15 [Klasing, Navarra, Papadopoulos, Perennes '04]

Indipendently, an approximation ratio of 20 has been stated in [Clementi, Crescenzi, Penna, Rossi, Vocca '01]

Approx. ratio improved to 7,6 [Flammini, Klasing, Navarra, Perennes '04]

Approx. ratio improved to 6,33 [Navarra '05]

Optimal bound 6 [Ambüehl '05]

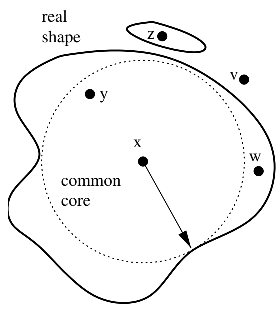
For realistic instances, experiments suggest that the tight approximation ratio is not 6 but 4 [Flammini, Navarra, Perennes '06]

HEURISTICS (10)

The 3-dimensional space better models practical environments:
in real life scenarios, radio stations are distributed over a 3-dimensional Euclidean space.
The extension to the 3-dimensional case of the assumption that transmissions are propagated uniformly in a spherical shape naturally comes from the 2- dimensional case…

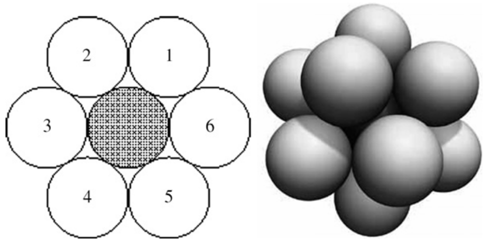
HEURISTICS (11)

…although it is not realistic: in general, in real world scenarios, the propagation is not uniform but a common core (not necessarily connected) covering a sphere.



HEURISTICS (12)

- the approximation ratio of 6 for the MST heuristic in the 2- dimensional Euclidean space for a=2 coincides with the 2-dimensional kissing number.
- the d-dimensional kissing number is the maximum number of d-spheres of a given radius r that can simultaneously touch a d-sphere of the same radius r in the d-dimensional Euclidean space



HEURISTICS (13)

- In general, the d-dimensional kissing number was proven to be a lower bound for the approximation ratio of the MST heuristic for any dimension d > 1 and power a ≥d
- The 3-dimensional kissing number is 12, but the best known approximation ratio of the MST heuristic so far is 18,8 [Navarra '08]
  - ☞ student lesson