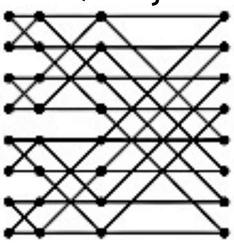
OPTIMAL AREA LAYOUT OF THE BUTTERFLY NETWORK

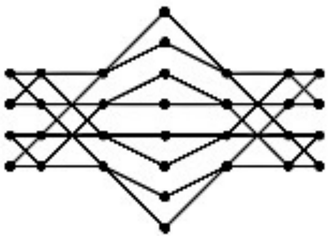
OPTIMAL AREA LAYOUT – IDEA (1)

The two papers that provide an optimal area layout base their results on the following lemma:

Lemma: For any non-negative integers j, k, $0 \le j \le j+k \le n$, the subgraph of the n-dim. Butterfly induced by the nodes of levels j, j+1, …, j+k is the disjoint union of 2^{n-k} copies of k-dimensional butterflies.

In particular, if j=0 and k=n-1:

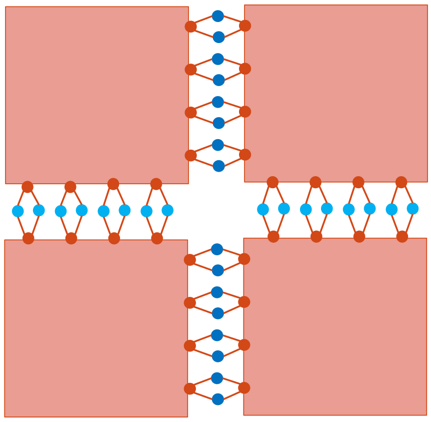




OPTIMAL AREA LAYOUT - IDEA (2)

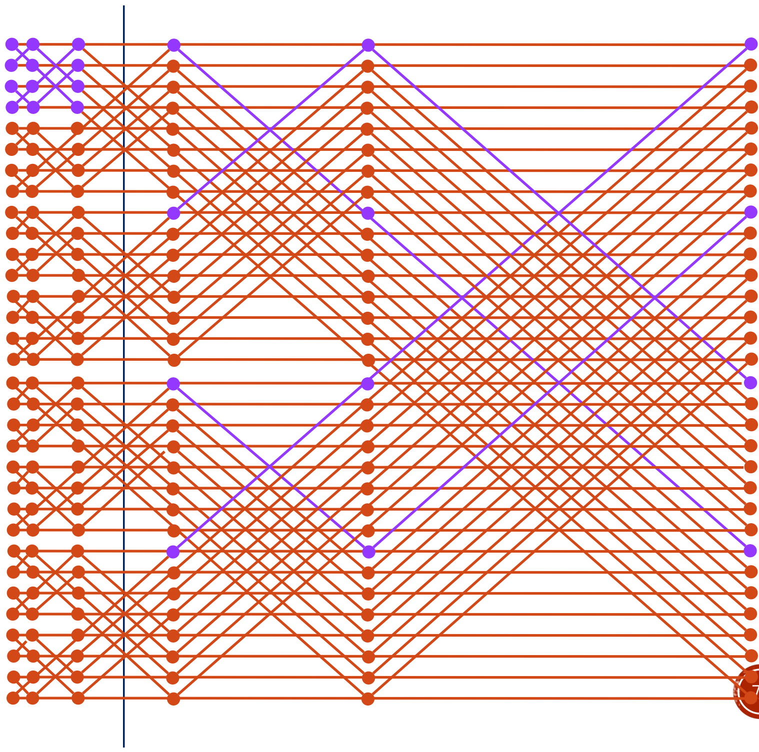
Hence, an (n-1)-dimensional butterfly can be built as a pair of (n-2)-dim. butterflies connected by one node layer and one edge layer.

If we cut out the input and output nodes from an n-dim. Butterfly, we get:



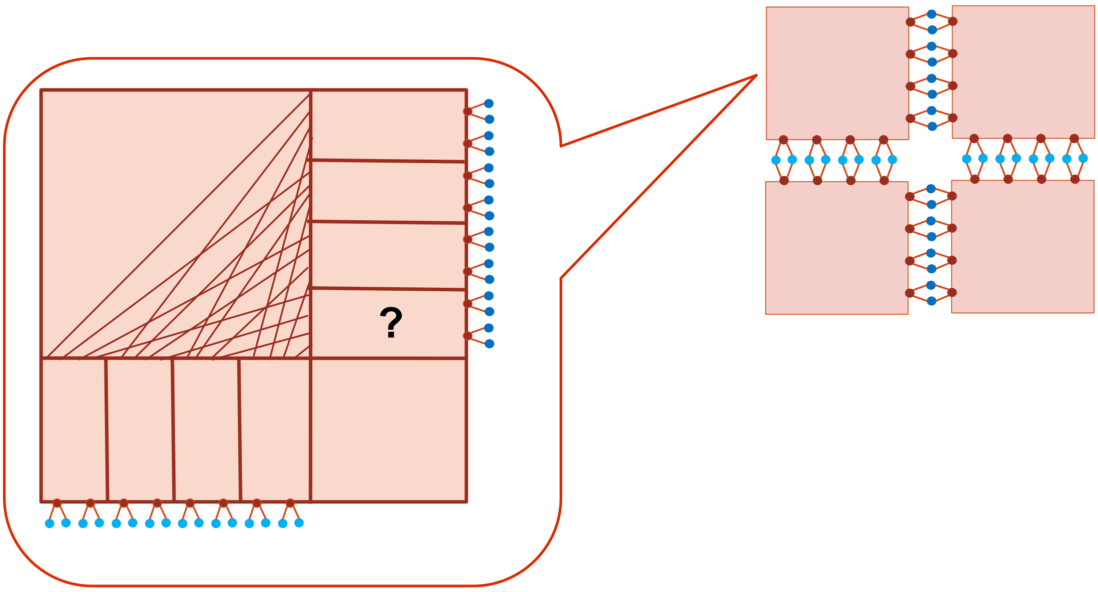
OPTIMAL AREA LAYOUT - IDEA (3)

Each one of these (n-2)-dim. Butterflies can be, in turn, cut into many smaller butterflies:



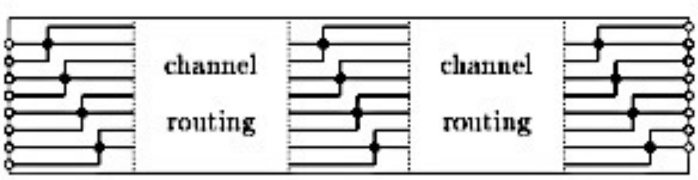
OPTIMAL AREA LAYOUT - IDEA (4)

The previous layout can be better specified as follows:



OPTIMAL AREA LAYOUT - IDEA (5)

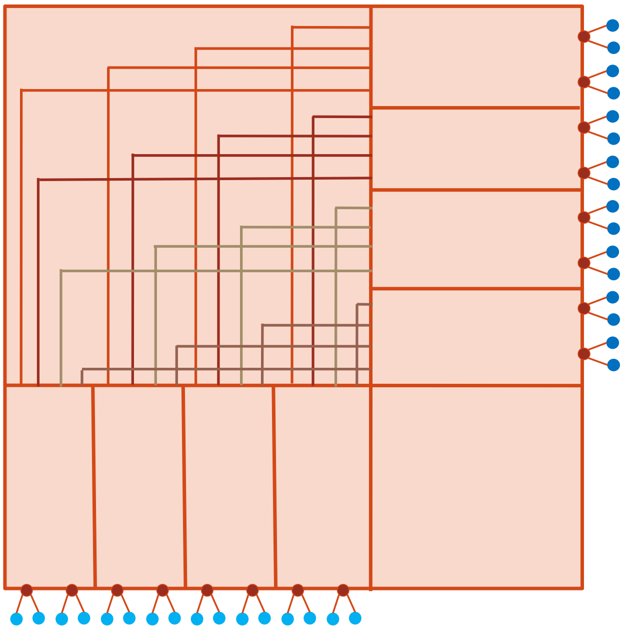
Each rectangle contains a Butterfly that can be represented, either horizontally or vertically, layer by layer as follows:



Obs.: this layout is far from being optimal; nevertheless it allows to produce a final optimal layout.

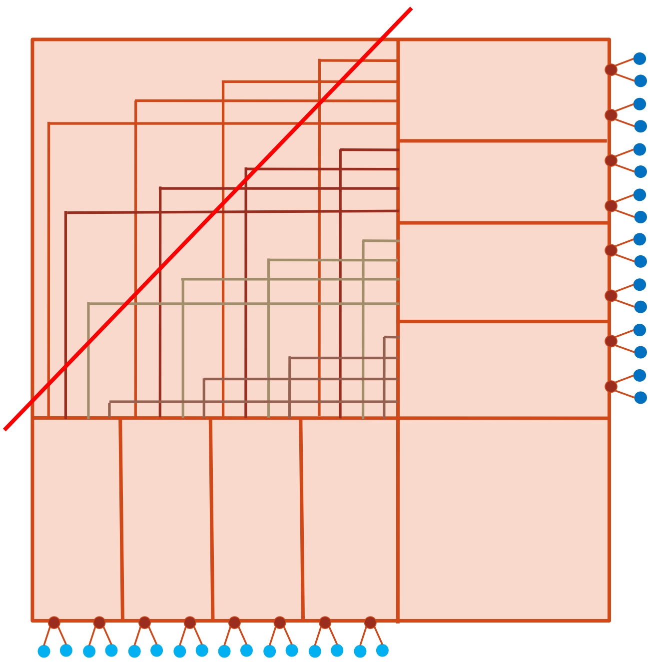
OPTIMAL AREA LAYOUT - IDEA (6)

It remains to connect the small rectangular butterflies:



OPTIMAL AREA LAYOUT - IDEA (7)

In the case of slanted layout, it can be bent along the line:



OPTIMAL AREA LAYOUT – IDEA (8)

It is possible to prove tight lower and upper bounds on the layout area for both the models (usual and slanted).

The interested students can look at:

- A. Avior, T.C., S. Even, A. Litman, A.L. Rosenberg: A Tight Layout of the Butterfly Network. Theory of Computing Systems 31, 1998.
- Y. Dinitz, S. Even, M. Zapolotsky: A Compact Layout of the Butterfly. J. of Interconnection Networks 4, 2003.

LAYOUT OF THE HYPERCUBE NETWORK