Quantum Computing

Lecture $|12\rangle$

Quantum Key Distribution

Paolo Zuliani

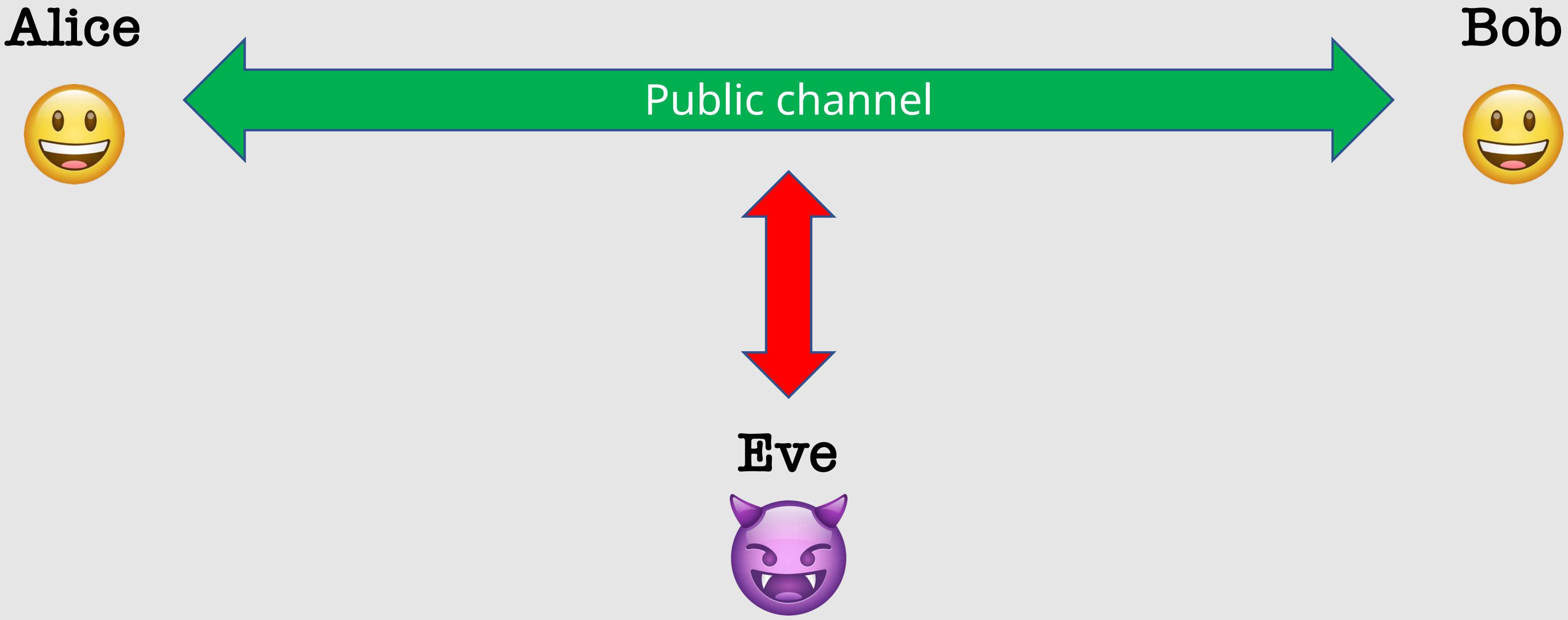
zuliani@di.uniroma1.it

Sapienza Università di Roma

Outline

- Communication context and problem
- One-time pad
- Privacy amplification
- Bennett & Brassard’s quantum key distribution (BB84)
- Post-quantum cryptography

Communication Context and Problem



Communication Context and Problem

Public-key cryptography:

- What are the prime factors of 4983749726021447820470189037379?
- Best classical algorithms for integer factoring take time **exponential** in length of number.
- No one has showed that an efficient (i.e., poly-time) algorithm cannot exist.
- Effectively: security based on unproven computational assumptions…

Communication Context and Problem

Private-key cryptography:

- Parties must share a **private** key.
- Benefit: can be **perfectly** secure!
- Effectively: security based on privacy.
- Disadvantage: how to share a private key?
  - Trusted couriers
  - Private communication lines (e.g., red phone between USA – USSR)
  - Covert operations
  - etc …

Private-key Cryptography

One-time pad

Random key (bitstring)

1 0 1 0 0 0 1 0 1 1 1 1 0 1 0

Private channel

Alice

L	O	V	E		Y	O	U	!

message

Bitwise XOR

key 1 0 1 0 0 0 1 0 1 1 1 1 0 1 0

X	E	£	$	%	R	T	Y	A

Public channel

Bob

X	E	£	$	%	R	T	Y	A

Bitwise XOR

key 1 0 1 0 0 0 1 0 1 1 1 1 0 1 0

L	O	V	E		Y	O	U	!

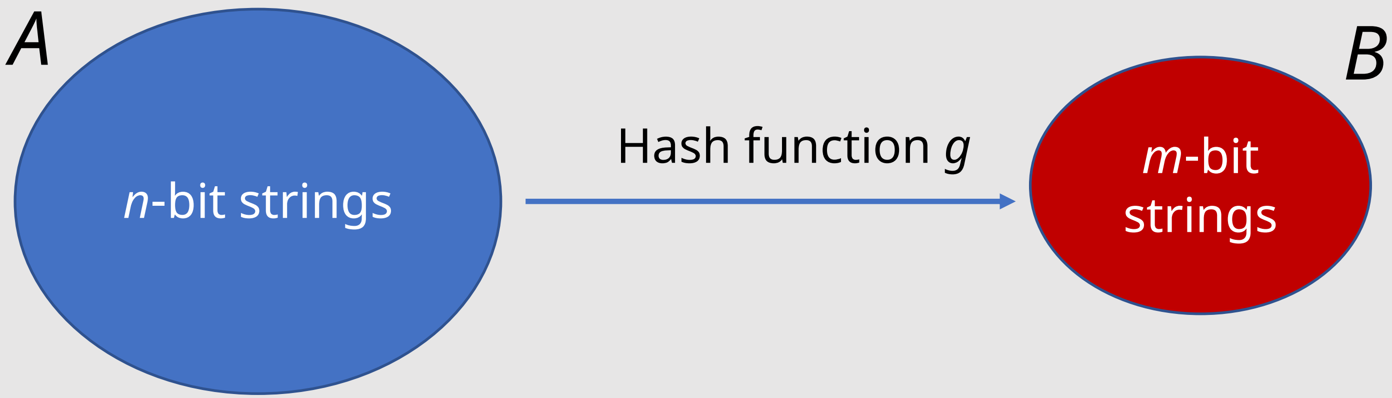
Private-key Cryptography

- Key and message must have the **same length** ⇒ the only perfectly secure cryptosystem!!
- Keys should be **guarded at all times**.
- Keys should be **destroyed after use**; else reduced privacy.
- How to distribute random keys securely? BB84!

Privacy Amplification

- Alice and Bob share a bitstring, but Eve has some knowledge of it.
- Can Alice and Bob "distil" a more secure key that reduces Eve's knowledge of the key?

Universal Hash Functions



$$\forall a_1, a_2 \in A, \quad g \text{ randomly chosen hash function} \Longrightarrow \mathrm{Prob}(g(a_1) = g(a_2)) \leq \frac{1}{|B|}$$

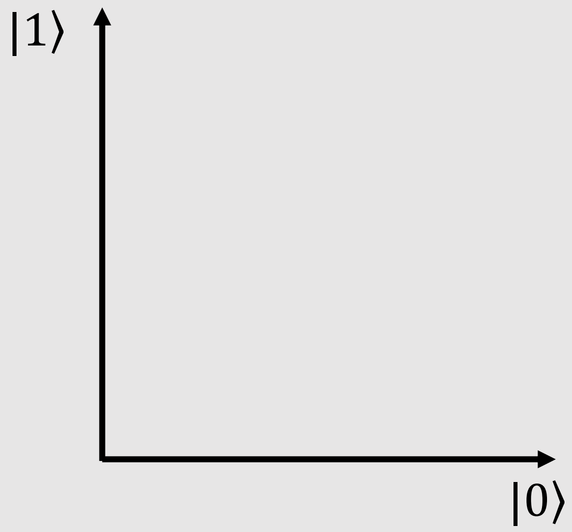
Privacy Amplification

- Alice and Bob **publicly** select a universal hash function *g*.
- They apply *g* to their shared bitstring *W* – the output *g(W)* will be their secret key *S*.
- It can be shown that if Eve's entropy (knowledge) on *W* > *d*, then

$$\text{Eve's entropy on } S \geq m - 2^{m-d}$$

The BB84 QKD Protocol

- Enables Alice and Bob to share a random bitstring (key) over **public quantum/classical channels**
- Security based on the validity of Quantum Physics only. (Philosophy mode off!)
- **<u>Main idea</u>**: trying to distinguish two **non-orthogonal** quantum states implies disturbance!

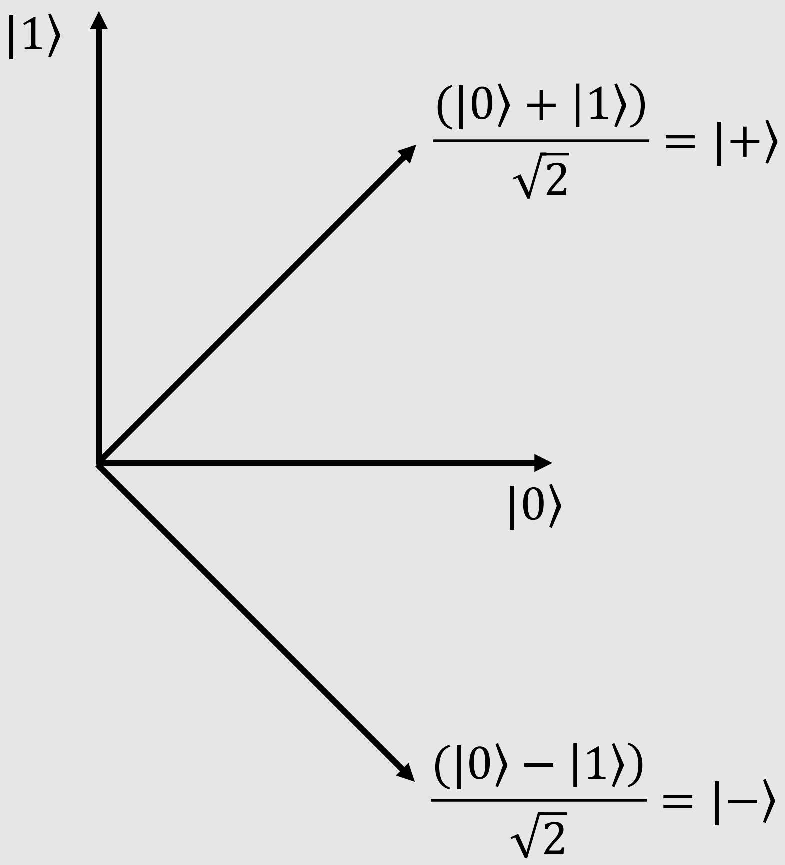


Suppose you know a qubit is either in state $|0\rangle$ or $|1\rangle$.

(These two states are orthogonal.)

By measuring it you can learn **precisely** its state.

The BB84 QKD Protocol

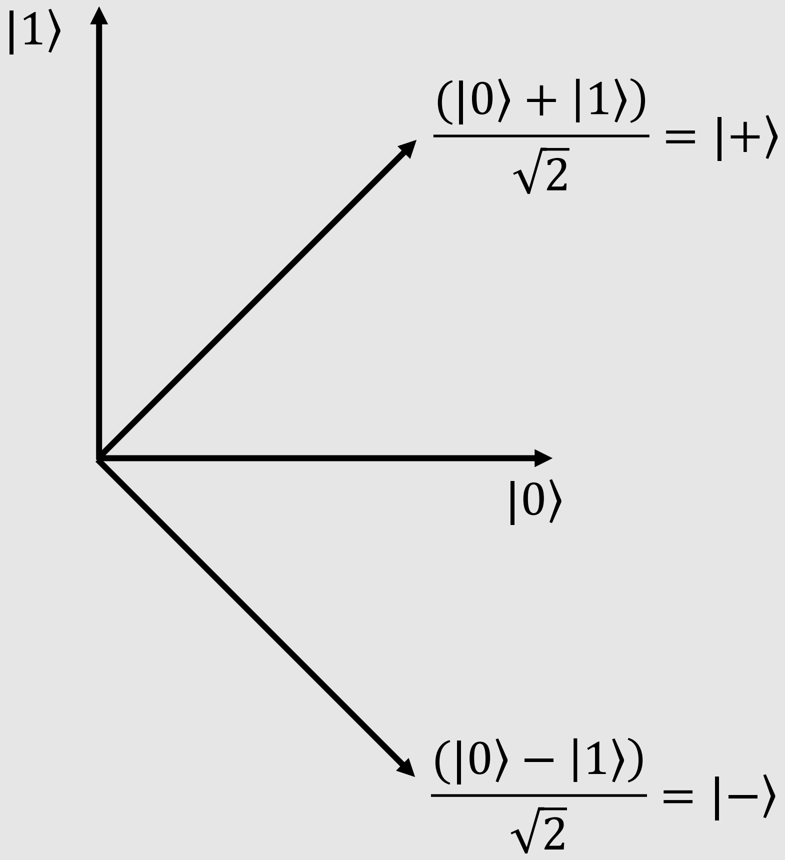


Suppose your qubit is either in state $|+\rangle$ or $|-\rangle$.

(These two states are again orthogonal.)

By rotating it and then measuring it you can learn **precisely** its state.

The BB84 QKD Protocol



Suppose your qubit is either $|0\rangle, |1\rangle, |+\rangle$ or $|-\rangle$.

These states are no longer orthogonal!

Distinguishing is now trickier …

One measurement is not sufficient! (It will destroy the state.)

The BB84 QKD Protocol

- Distinguishing non-orthogonal states implies **disturbance** (of said states)

$|\psi\rangle, |\varphi\rangle$ non-orthogonal (hence $\langle\psi|\varphi\rangle \neq 0$). Cloning is impossible, so Eve must apply some unitary U on an ancilla state $|x\rangle$:

$$|\psi \otimes x\rangle \rightarrow |\psi \otimes y\rangle$$

$$|\varphi \otimes x\rangle \rightarrow |\varphi \otimes y'\rangle$$

$|y\rangle, |y'\rangle$ should be different so that Eve can distinguish $|\psi\rangle$ from $|\varphi\rangle$.

However, U is unitary and therefore:

$\langle\psi \otimes x|\varphi \otimes x\rangle = \langle\psi \otimes y|\varphi \otimes y'\rangle \implies \langle\psi|\varphi\rangle\langle x|x\rangle = \langle\psi|\varphi\rangle\langle y|y'\rangle$

$\implies 1 = \langle x|x\rangle = \langle y|y'\rangle \implies y = y'$ so Eve does not gain anything

The BB84 QKD Protocol

Alice

Bob

time

Choose random bitstrings *a*, *b* of length *N* ‘big enough’

Produce *N* qubits $Q = \otimes_{i=1}^{N} |q_{a_i b_i}\rangle$

$$|q_{00}\rangle = |0\rangle$$
$$|q_{10}\rangle = |1\rangle$$
$$|q_{01}\rangle = |+\rangle$$
$$|q_{11}\rangle = |-\rangle$$

Send Q to Bob

Public quantum channel

$Q' = \otimes_{i=1}^{N} |q_{a_i b_i}\rangle$

The BB84 QKD Protocol

Alice

Bob

time

Choose random bitstring b' of length N ‘big enough’

Measure each qubit in Q' with basis $|0\rangle, |1\rangle$ or $|+\rangle, |-\rangle$ depending on b'

Store measurement results in bitstring a'

Publish bitstring b

Public classical channel

Publish bitstring b'

The BB84 QKD Protocol

Alice

Bob

time

Store in *W* the bits of *a* for which *b*=*b'*

Select a random substring of *W* and compare with Bob

Public classical channel

Store in *W'* the bits of *a'* for which *b*=*b'*

Select a random substring of *W'* and compare with Alice

Check fails

OK

Abort

Check fails

OK

Privacy amplification on remaining *W* bits

Highly secure key

Privacy amplification on remaining *W'* bits

Highly secure key

Post-Quantum Cryptography

- Quantum computers can (in principle) break factoring-based cryptosystems.
- Large-scale quantum computers are probably still 10-15 years away but getting too close for comfort.
- Stored data (might?) need re-encoding with longer keys.

Post-Quantum Cryptography

- Cryptosystems that are quantum-safe: lattice based?
  - A putative quantum-safe cryptosystem was recently broken on a laptop
  - April 2024: mistake found in quantum algo for breaking lattice-based cryptosystem …
- Currently, no provably quantum-secure cryptosystem is available.
- Links:
  - https://csrc.nist.gov/projects/post-quantum-cryptography
  - https://ianix.com/pqcrypto/pqcrypto-deployment.html