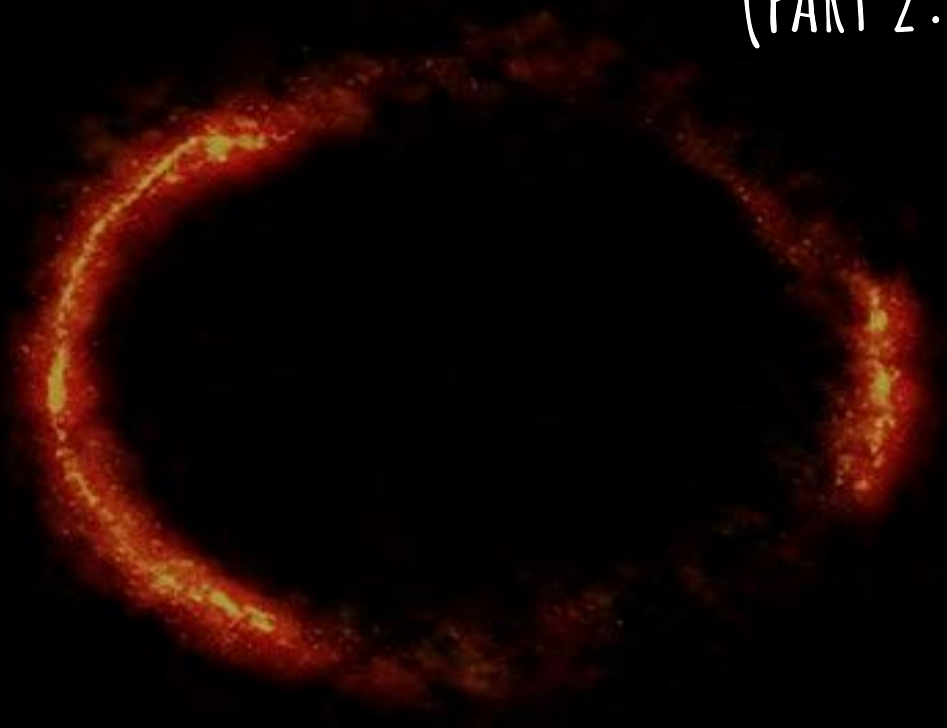
Gravitational Lensing
(part 2: lensing applied)



Observational cosmology Course, Master's Degree in Astrophysics, A.A 2023-2024

Theory of Gravitational Lensing	Lensing, applied

$$\hat{\vec{\alpha}}(\xi) = \frac{4GM}{c^2\xi}\vec{e}_\xi = \frac{4GM}{c^2\xi^2}\vec{\xi}$$

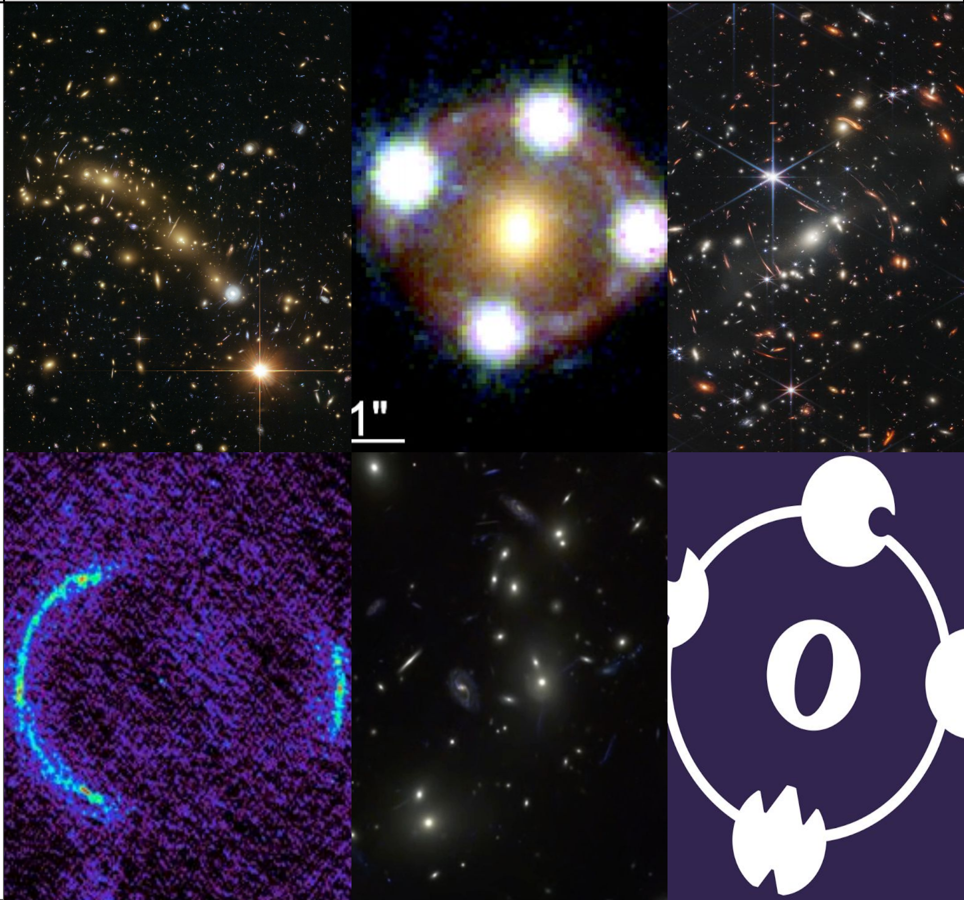
$$\hat{\Psi}(\vec{\theta}) = \frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}\frac{2}{c^2}\int \Phi(D_{\rm L}\vec{\theta}, z)\mathrm{d}z$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = \sum_i \hat{\vec{\alpha}}_i(\vec{\xi} - \vec{\xi}_i) = \frac{4G}{c^2}\sum_i M_i \frac{\vec{\xi} - \vec{\xi}_i}{|\vec{\xi} - \vec{\xi}_i|^2}$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2}\int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\,\mathrm{d}^2\xi'$$

$$(\vec{\theta}) = \frac{(1 + z_{\rm L})}{c}\frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}\left[\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]$$

$$\alpha(\theta) = \frac{4GM}{c^2 D_{\rm L}\theta} \qquad \theta_E \approx (10^{-3})''\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2}$$



Outline

Mass distribution in galaxy clusters

Mass distribution in galaxy clusters

Galaxy clusters are the largest existing virialized objects in the Universe. Their mass distribution can be inferred through e.g. kinematic analysis of the member galaxies (applying virial theorem), or X-ray observations of the hot ICM. Lensing analysis of the whole system is the least model-dependent method.

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$$\theta_E = \left(\frac{D_{DS}}{D_D D_S} \frac{4GM(\theta)}{c^2} \right)^{1/2} \sim 3 \times 10^{-6} \left(\frac{M}{M_\odot} \right)^{1/2} \frac{1}{D_{\rm Gpc}^{1/2}} \ \textbf{arcsec}$$

which is of the order of tens of arcsec for a typical galaxy cluster with 10^{15} solar masses.

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$$K_0 = \frac{kT}{4\pi G\mu} = \frac{v^2_{//}}{4\pi G}$$

and integrating ϱ(r) we obtain the projected surface mass density (with p being the impact parameter)

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so the deflection angle becomes

$$\hat{\alpha} = \frac{\pi <v_{//}^2>}{c^2} \sim 6\left[\frac{<v_{//}^2>^{1/2}}{1000\ \text{km/s}}\right]\ \text{arcsec}$$

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Independent from p

in the case of non-relaxed / irregular clusters gravitational lensing is the **only** robust way of measuring their mass

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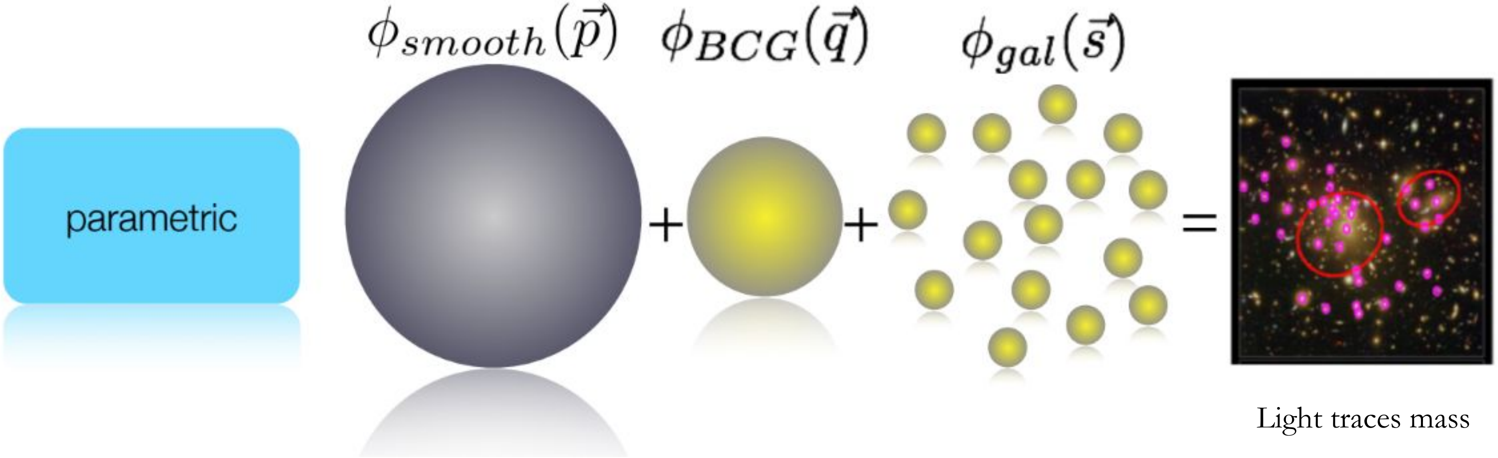
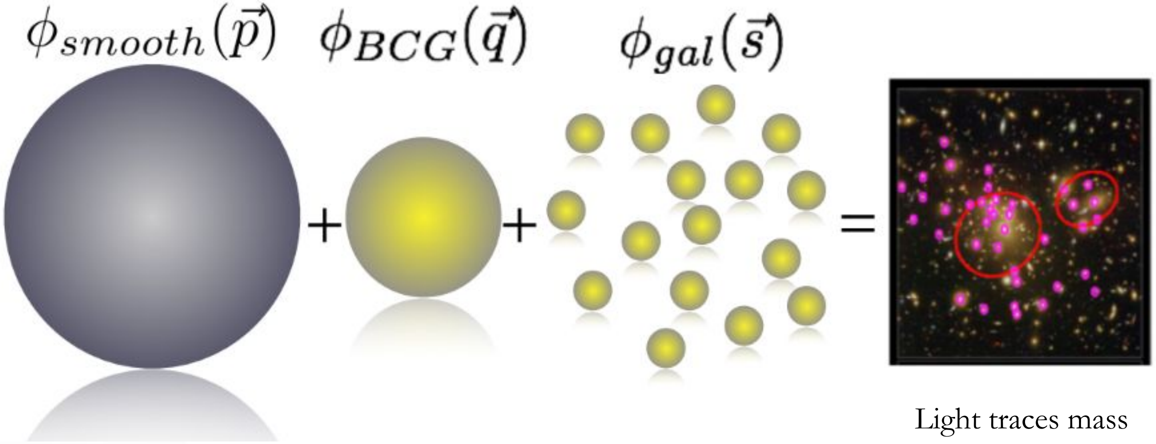


Light traces mass

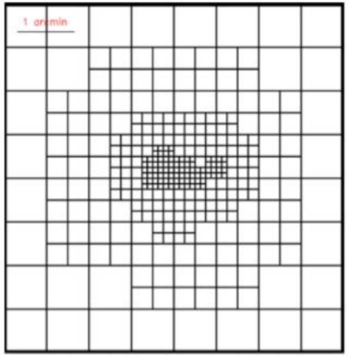
Mass distribution in galaxy clusters

parametric



Light traces mass

free-form (non-parametric)



Jullo et al. 2009

No assumption that light traces mass (in fact, it shouldn't) or on the shape of the density profiles. The cluster is decomposed into pixels (or tesserae, or radial basis functions); practically, each pixel of has its own mass distribution and contributes in deflecting the foreground galaxies' light.

In both cases, the best fit parameters for the parametric mass distribution / values for free-form are inferred via Bayesian statistics by maximizing a posterior distribution function between a model and the data.

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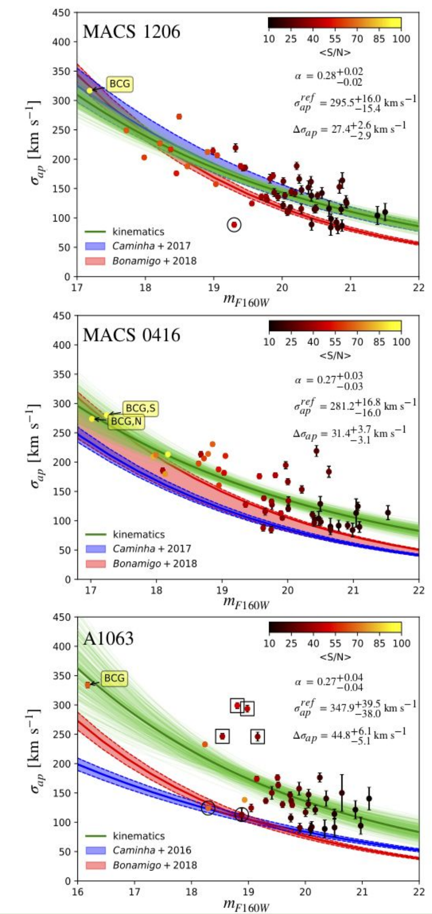
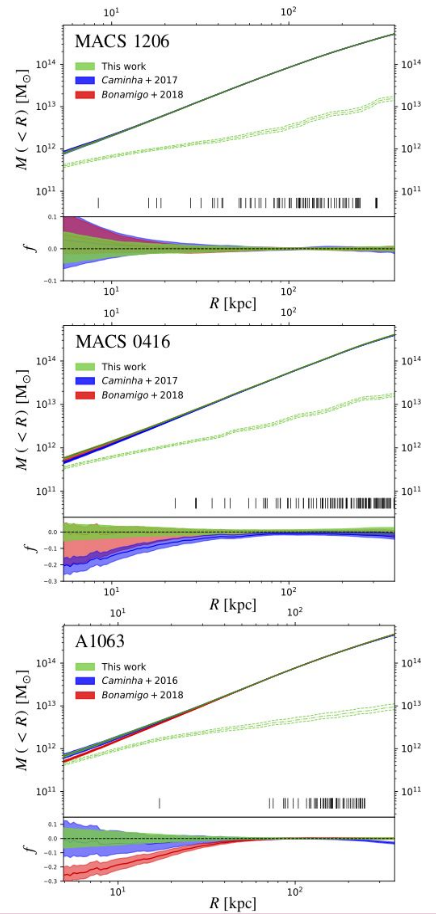


Strong lensing studies of galaxy clusters shed light on their mass distribution (total, so light + DM + gas).

Mass distribution of galaxies (in terms of velocity dispersion) as a function of their luminosity (magnitude), a.k.a. "scaling relations"

How the mass is distributed inside the cluster



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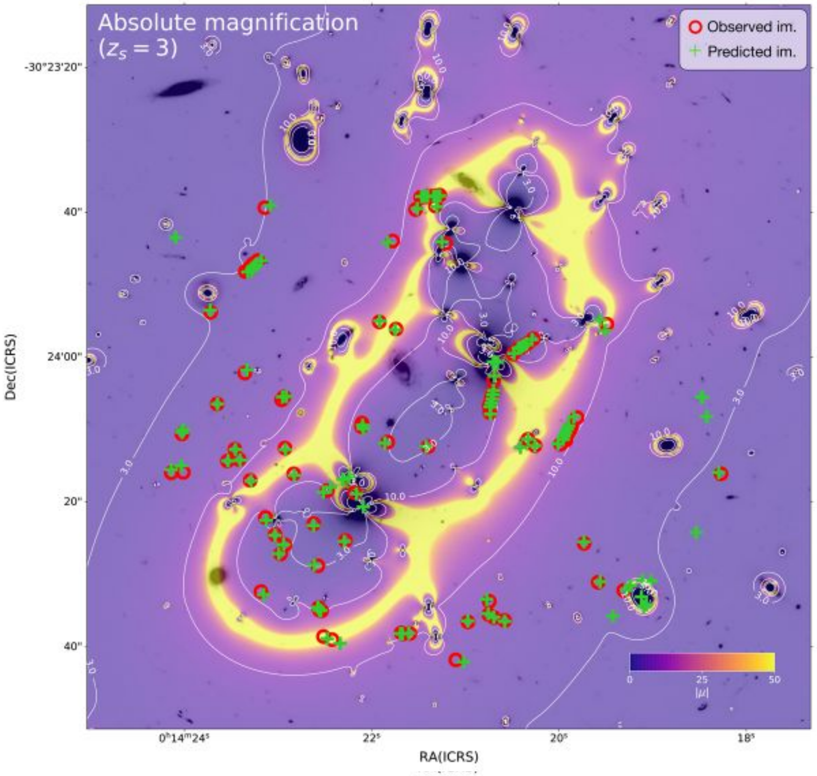

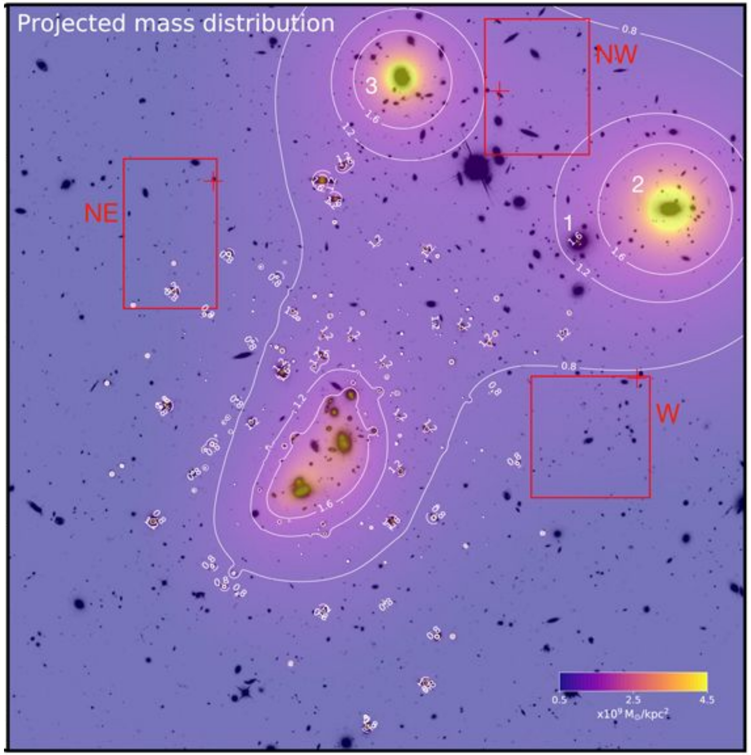

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Galaxy clusters as a means to the background source

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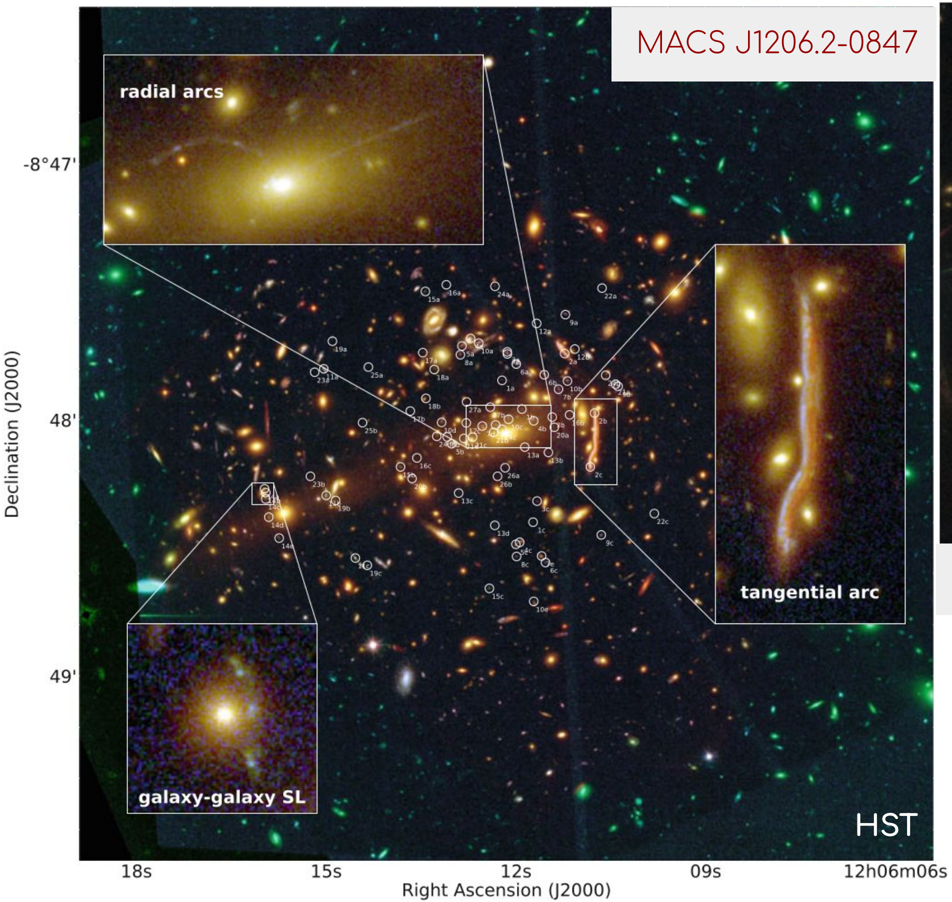

The high magnification power of a cluster of galaxies (which could go up to μ of 1000) is used as a means to the background source.

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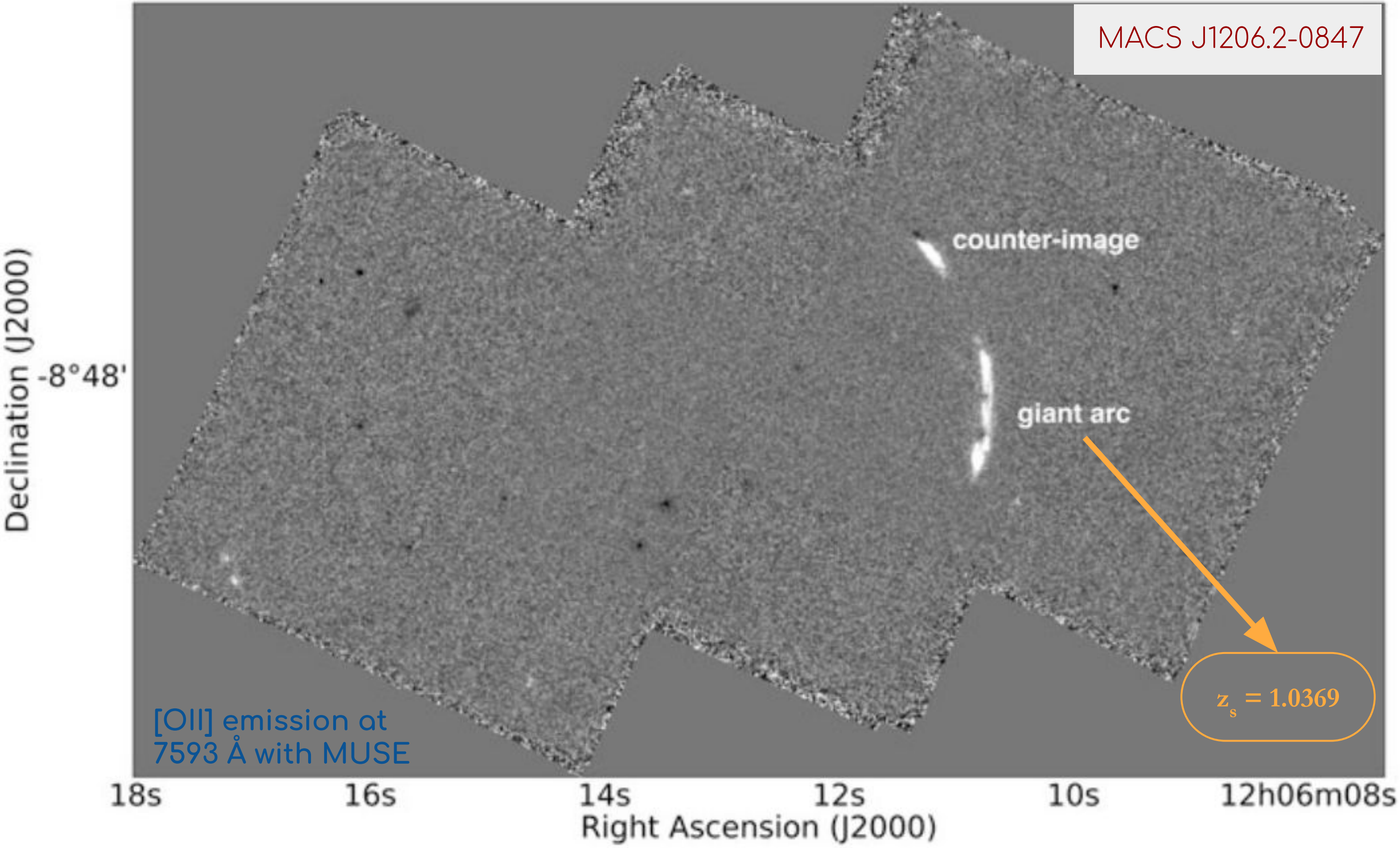

$z_s = 1.0369$

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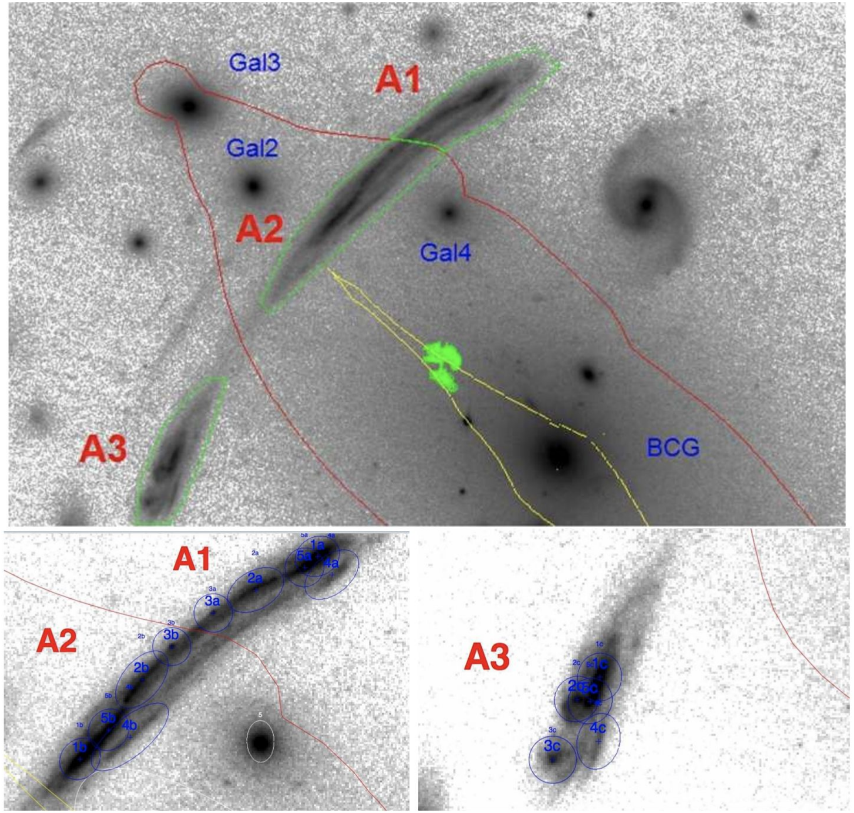
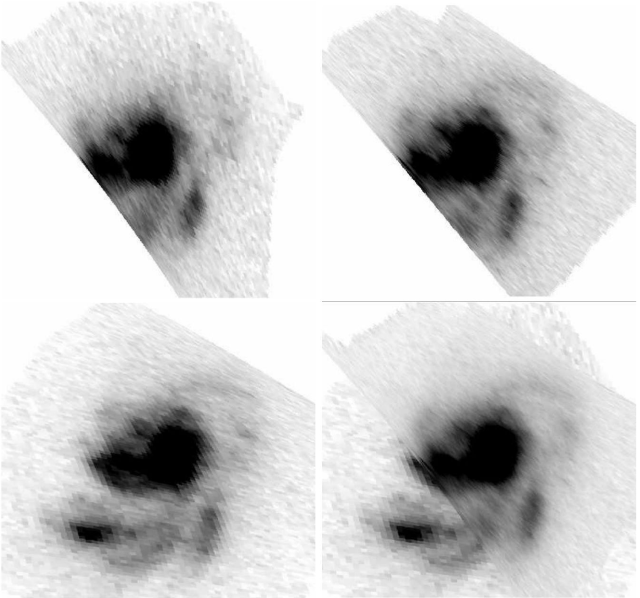


Example of source reconstruction (more details in a few minutes) for a spiral galaxy at redshift 1.0334 in Abell2667
(image plane on the left, source plane below)



Cao et al., 2015

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Lensed And Pristine 1, (LAP1), a lensed ($\mu > 100$) Population III candidate stellar complex behind MACS J0416, the most metal poor star-forming region currently known in the reionization era

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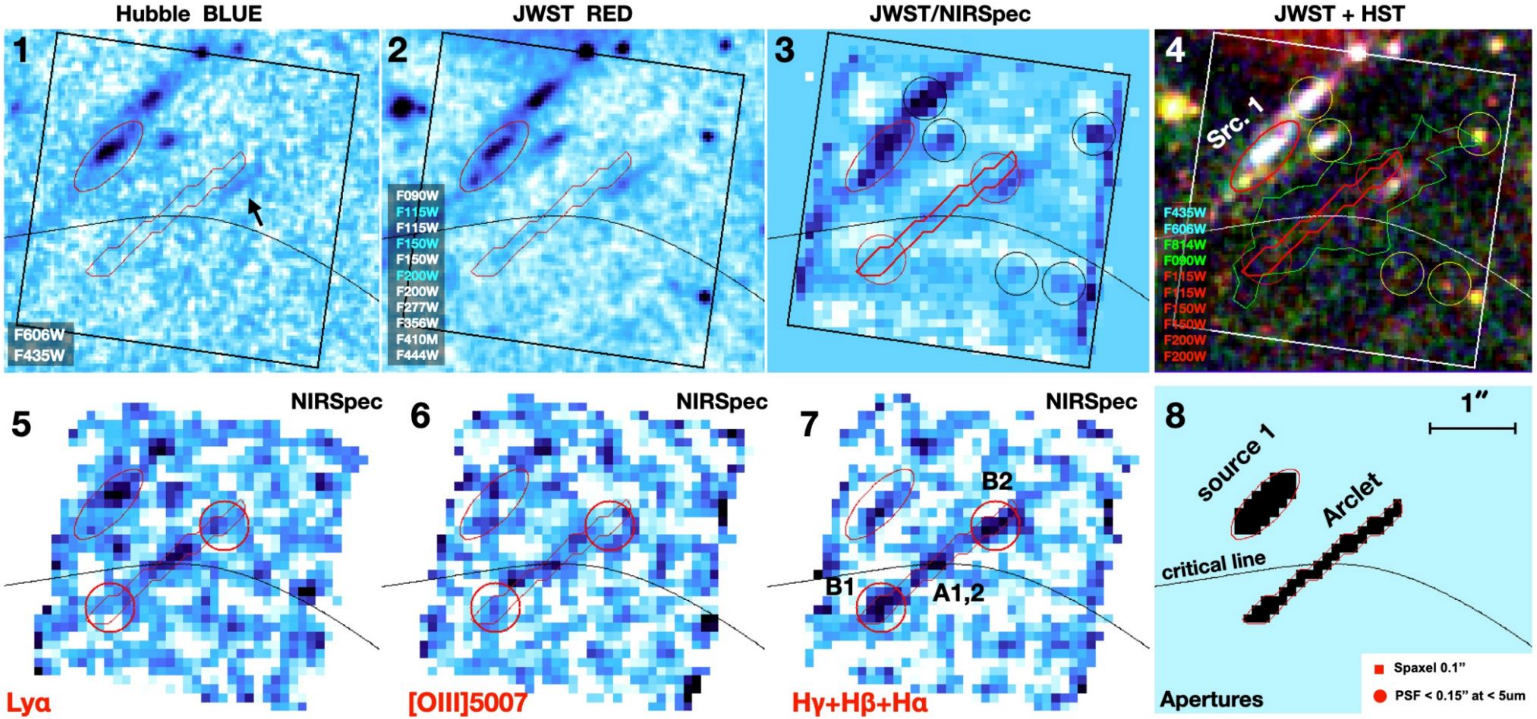

Vanzella et al., 2023

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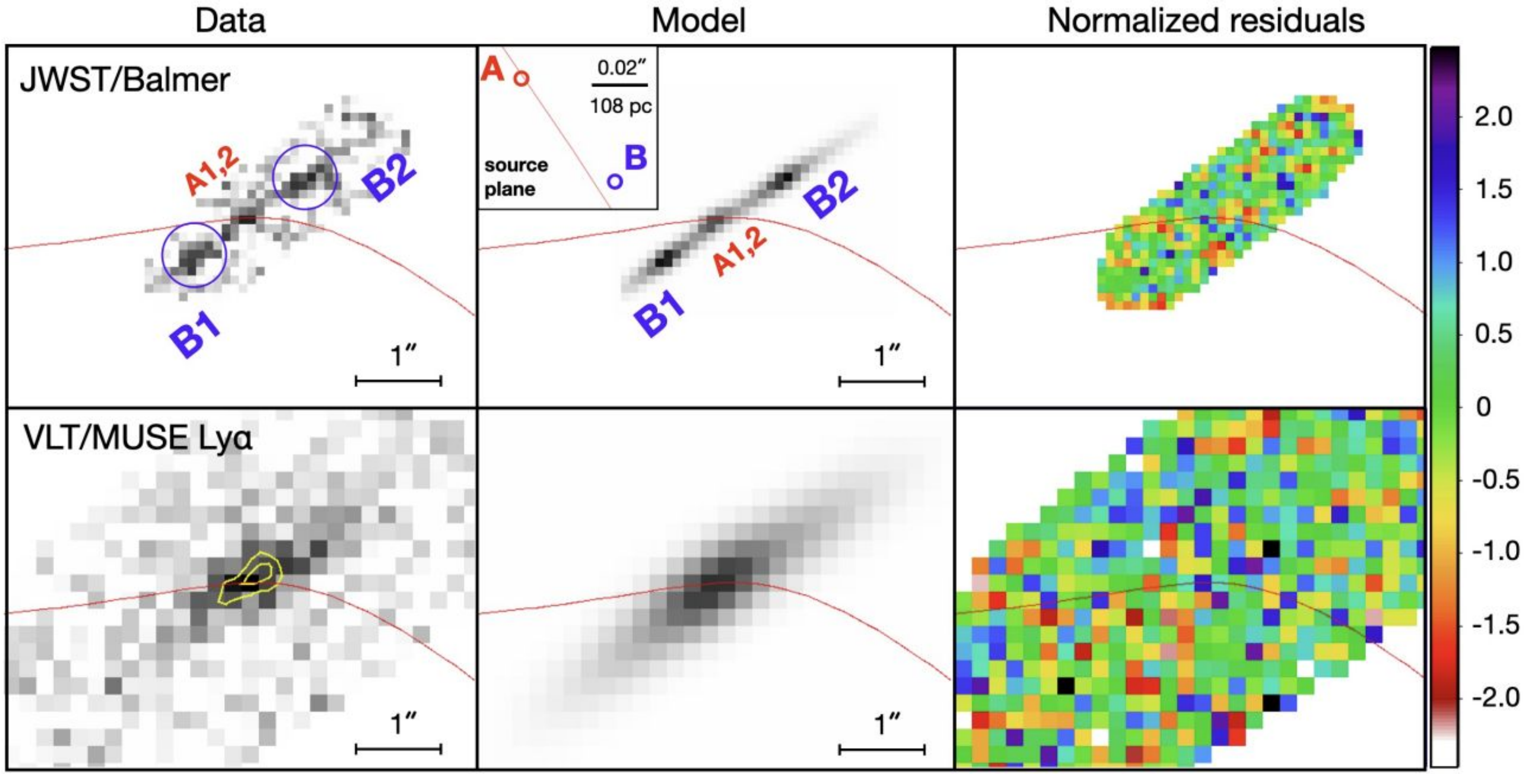

Vanzella et al., 2023

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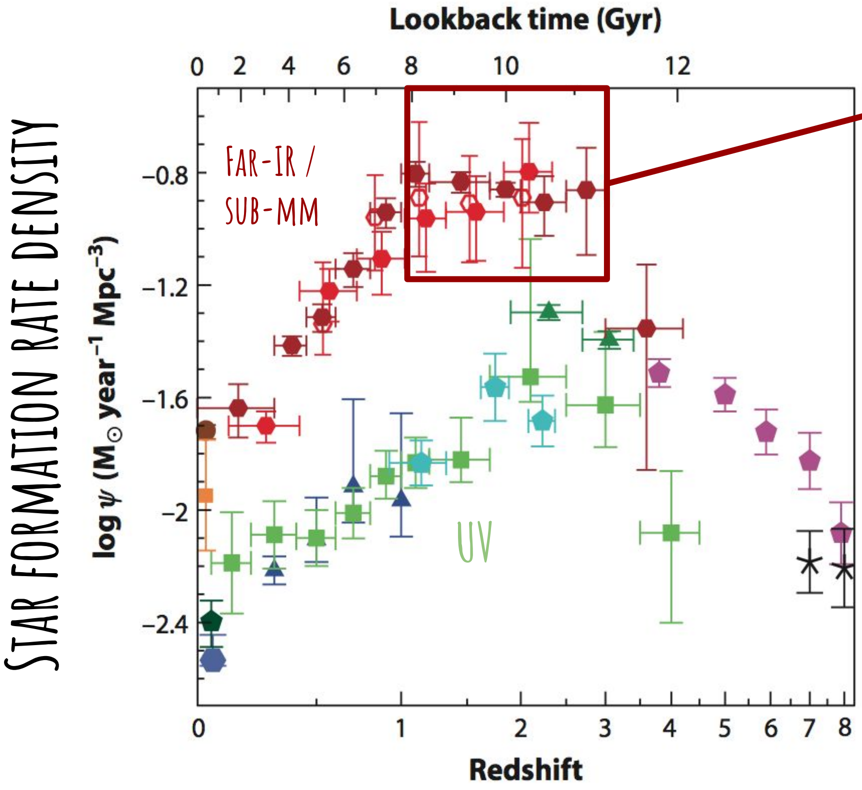
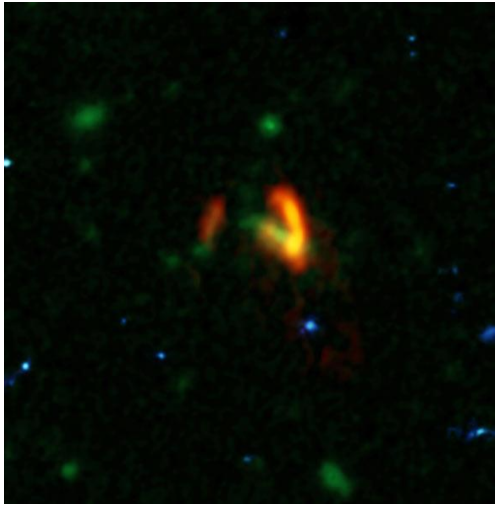
Galaxy-Galaxy strong lensing

Galaxy-galaxy strong lensing



Madau & Dickinson, ARA&A, 2014

DUST-OBSCURED SF



Sub-mm galaxies (SMGs) / Dust-Obscured Galaxies (DOGs) are the key to understand the build-up of galaxies

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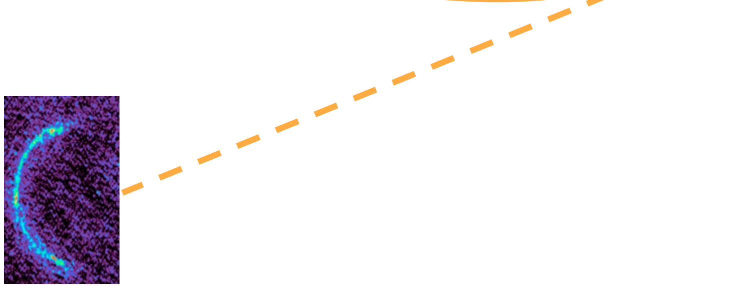

Natural magnification:

- **Increased apparent luminosity (~ μ)**
- **Increased spatial resolution (~ $\mu^{1/2}$)**

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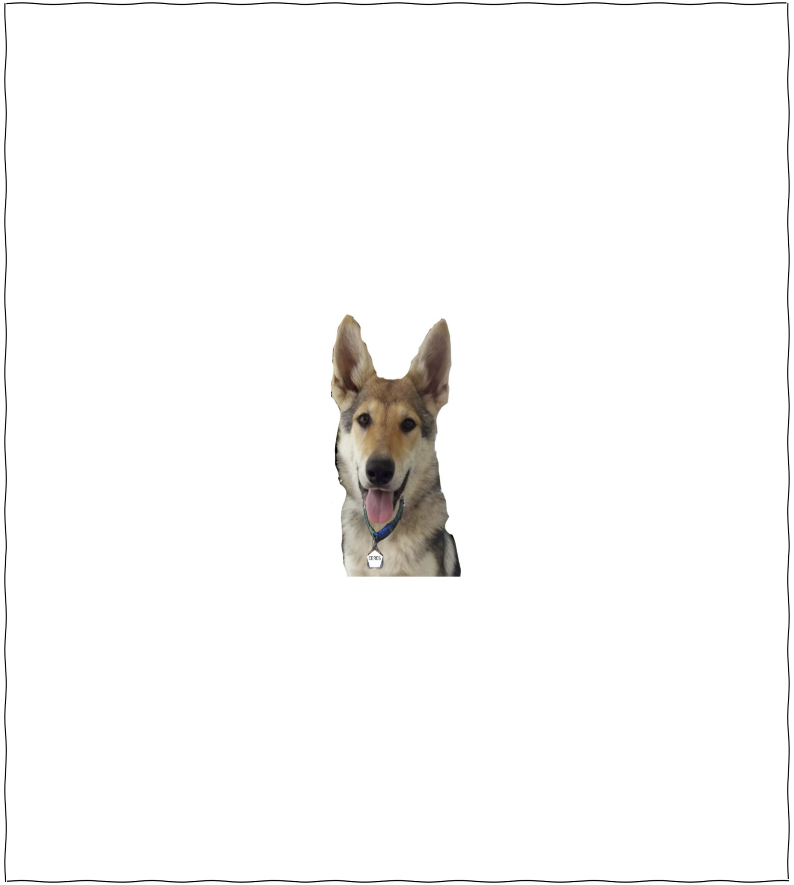
Galaxy-dog strong lensing



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* NESSUN CANE E' STATO MALTRATTATO NELLA PRODUZIONE DI QUESTA SLIDE

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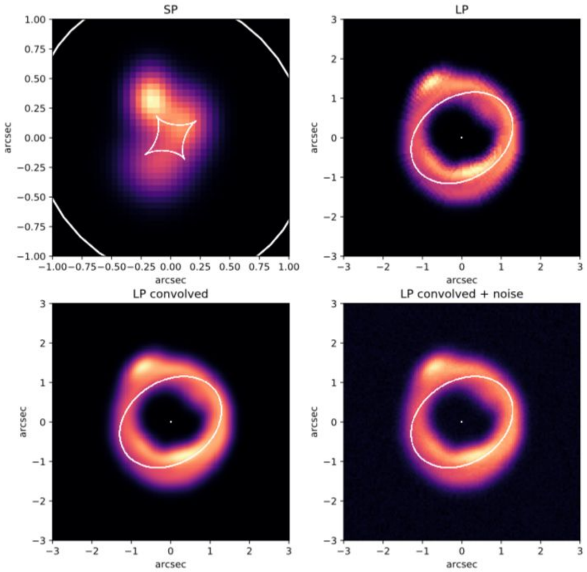
Lens

Find the lens mass distribution model, described by a set of parameters.

E.g.:

- Singular Isothermal Ellipsoid:
  - Velocity dispersion -> Einstein radius θ_E
  - Axis-ratio q
  - Position angle θ_{rot}

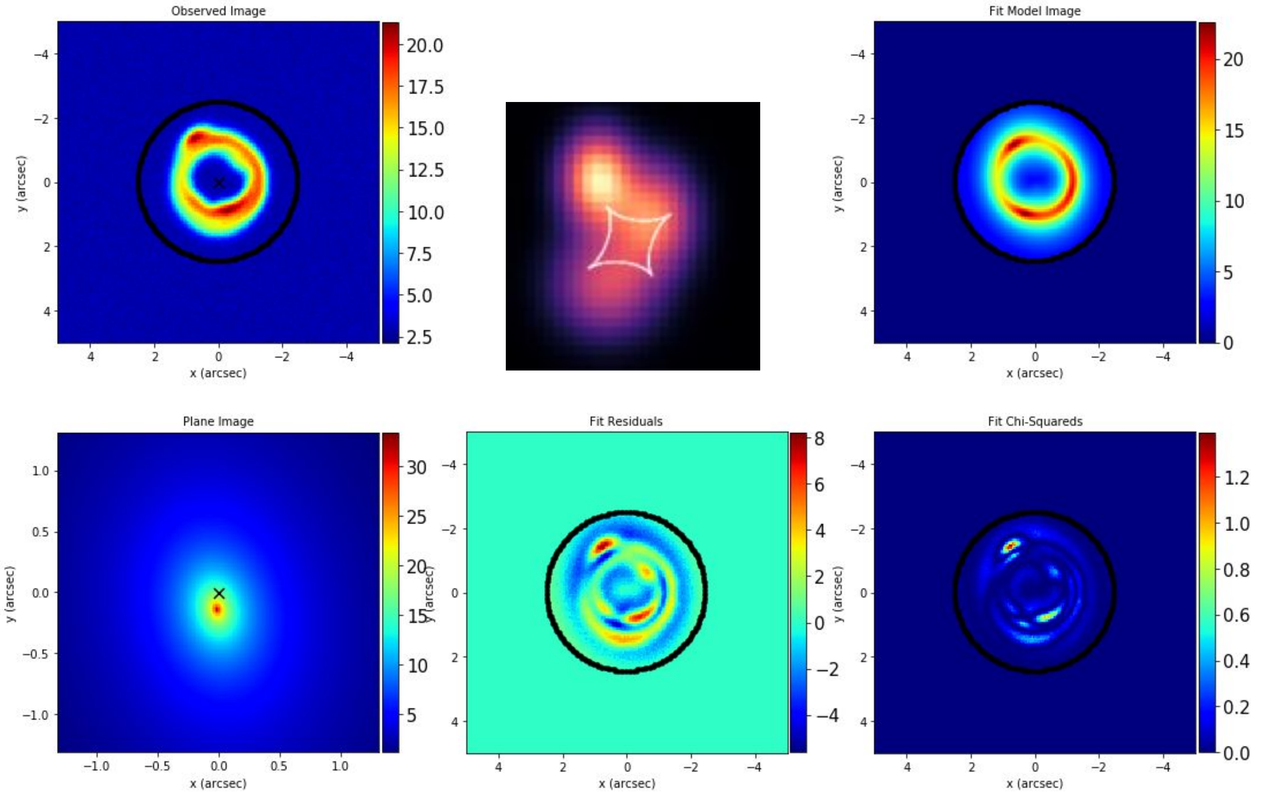


Galaxy-galaxy strong lensing

Two main approaches:

SOURCE

1) Fully-parametric: the source is also modelled with an analytical model, i.e. a Sérsic profile.



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Two main approaches:

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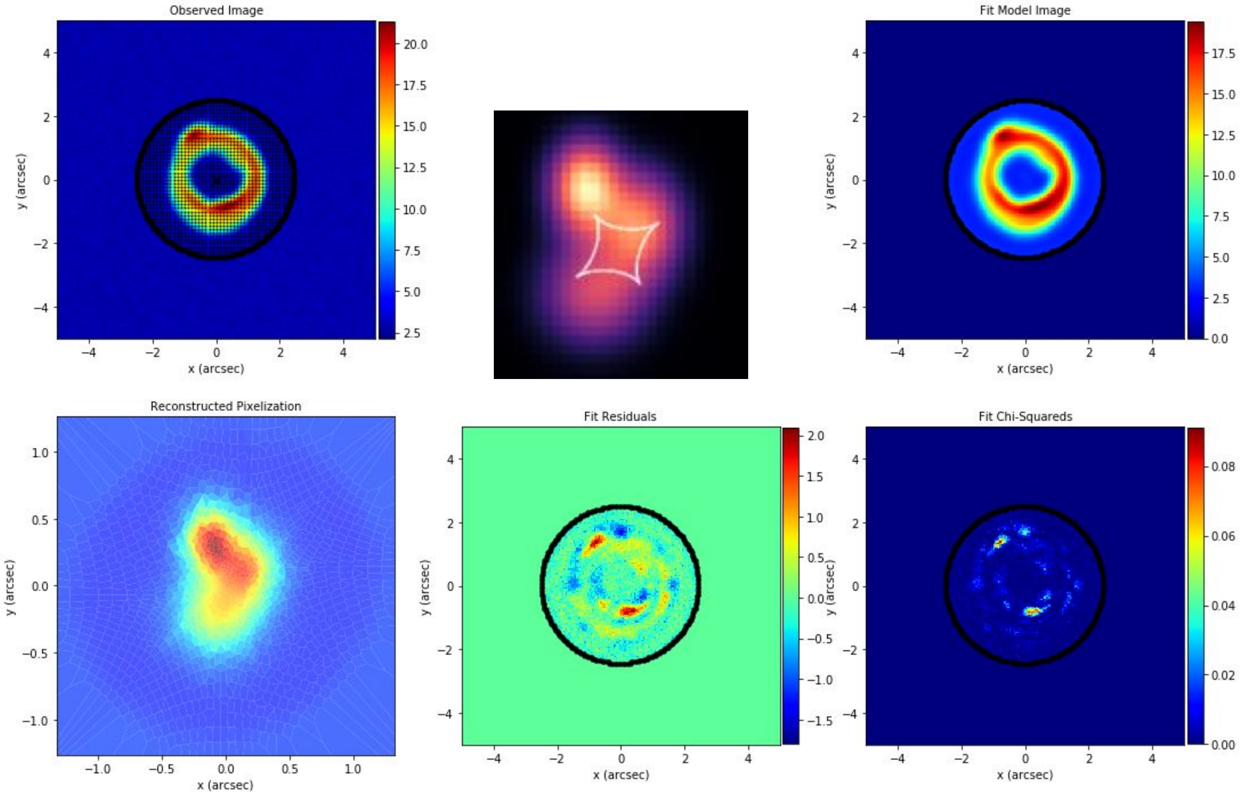
2) Semi-parametric: the source is defined with a grid of pixels/tassels, so its morphology is free to vary.



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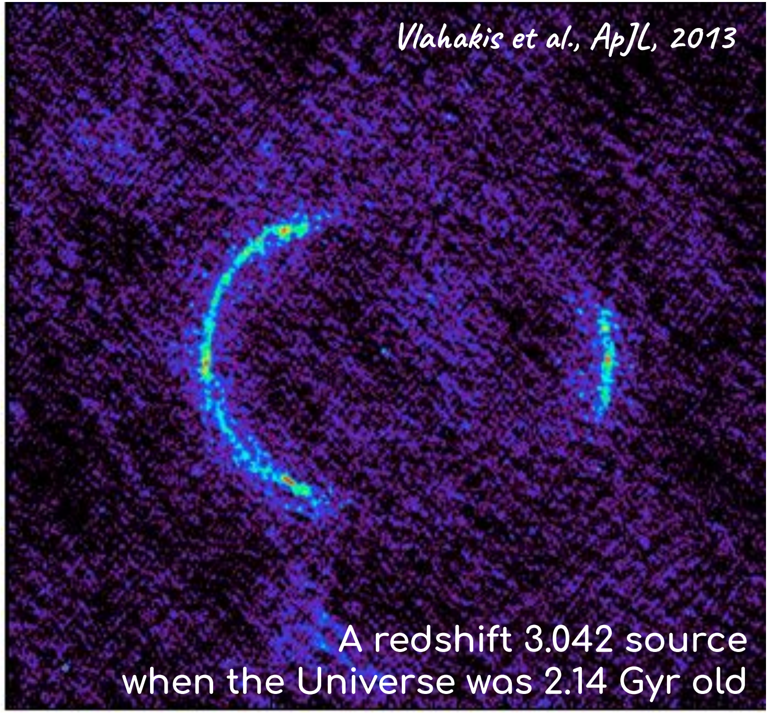
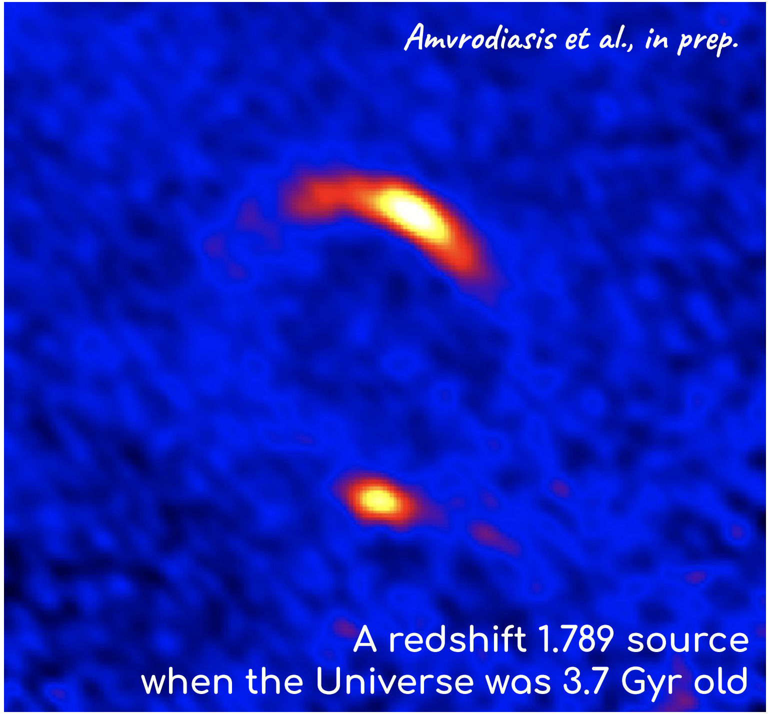
SDP.81 (ALMA, 1.3 mm)



SDP.11 (ALMA, 0.87 mm)



A strong gravitational lens reveals fine details of sources in the distant Universe
Proper lens modelling and source reconstruction is needed to reconstruct the original source morphology, and fully exploit the lens magnification

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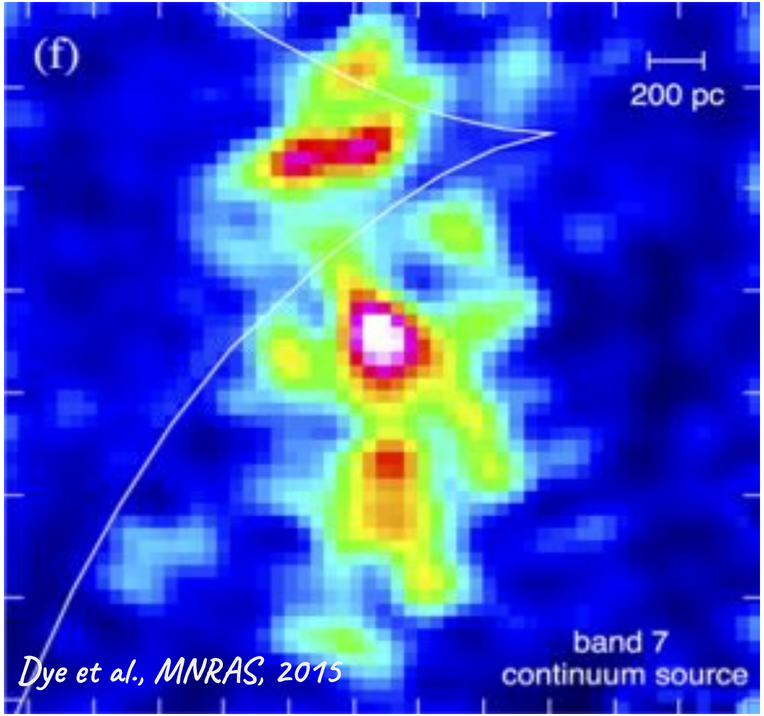

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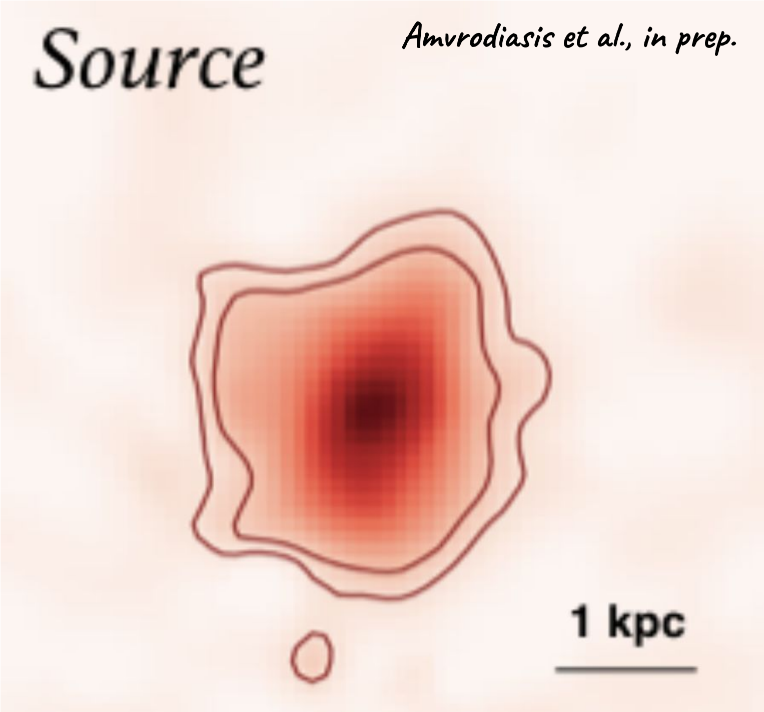

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Map the lens mass distribution

Being lensing a consequence of the presence of (lots of) matter, whatever kind of (dark, visible, invisible, solid, gaseous, living, dead), when you perform lens modelling you're not just reconstructing a source, you're, well, ***LENS*** modelling.

This means having a direct access on how matter is distributed in a galaxy (well, not the whole galaxy, within the Einstein radius).

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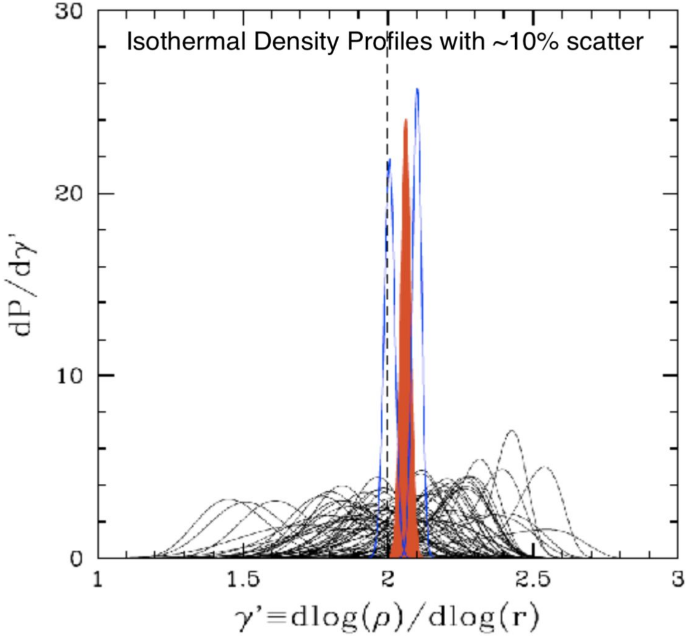
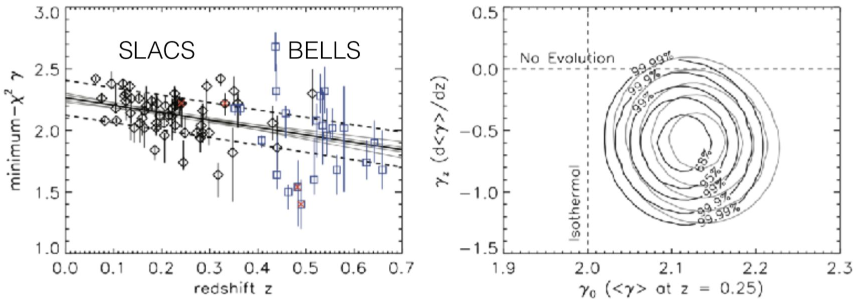
These studies found that the sum of DM + baryonic mass in elliptical galaxies is always an isothermal profile ($\varrho \propto$ r-2), despite different physical and environmental starting conditions, which is something that's not quite well understood.



Koopmans et al., 2009

Also, there's some evidence for a mild evolution of the mass-density slope in ETGs with redshift.

Anyway, lensing is a powerful key for galaxy evolution and cosmic structure formation.



Bolton et al., 2012

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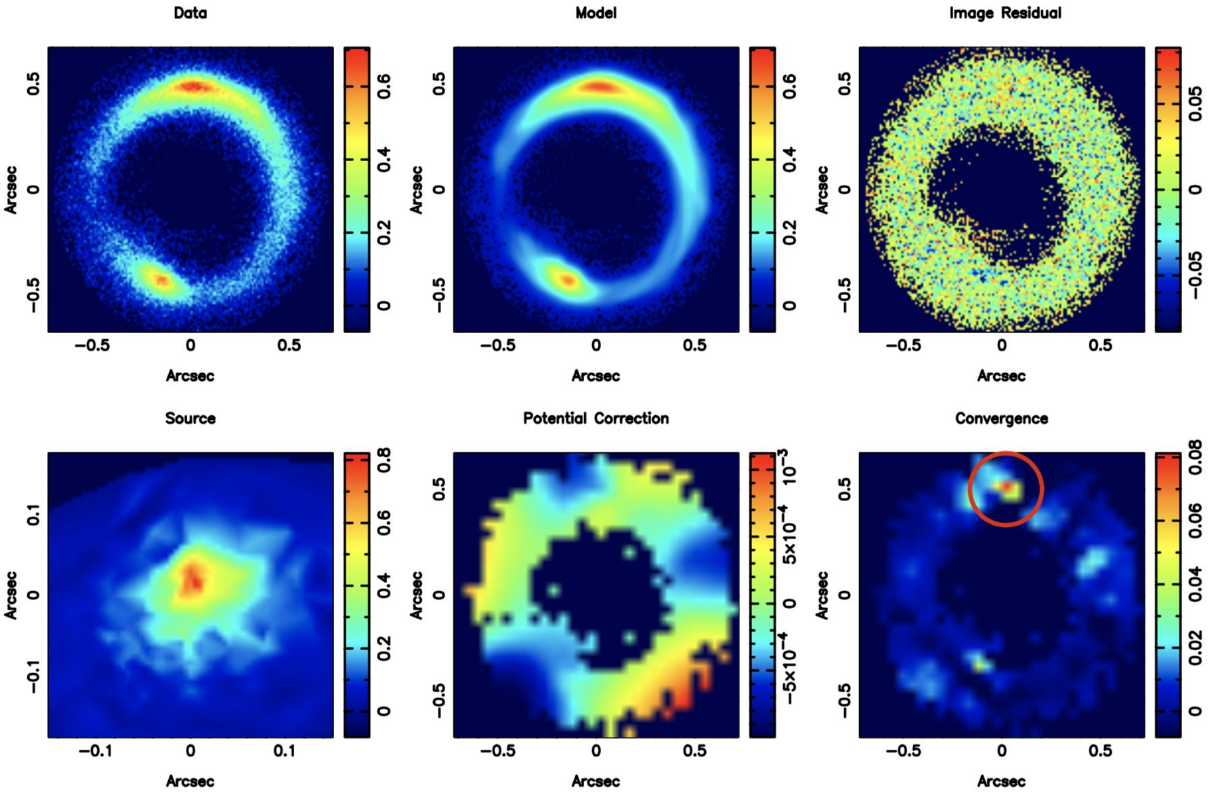
Mapping the lens mass distribution

Map the lens mass distribution

Being lensing a consequence of the presence of (lots of) matter, whatever kind of (dark, visible, invisible, solid, gaseous, living, dead), when you perform lens modelling you're not just reconstructing a source, you're, well, ***LENS*** modelling.

This means having a direct access on how matter is distributed in a galaxy (well, not the whole galaxy, within the Einstein radius).



Also, strong lensing is able to answer (or at least give it a try) to the famous "missing satellite problem". The presence of DM substructures is the smoking gun for cosmological models such as ΛCDM, and characterizing their distribution (or absence of) can wipe out lots and lots of theoretical models.

Lagattuta et al., 2012; Vegetti et al, Nature, 2012

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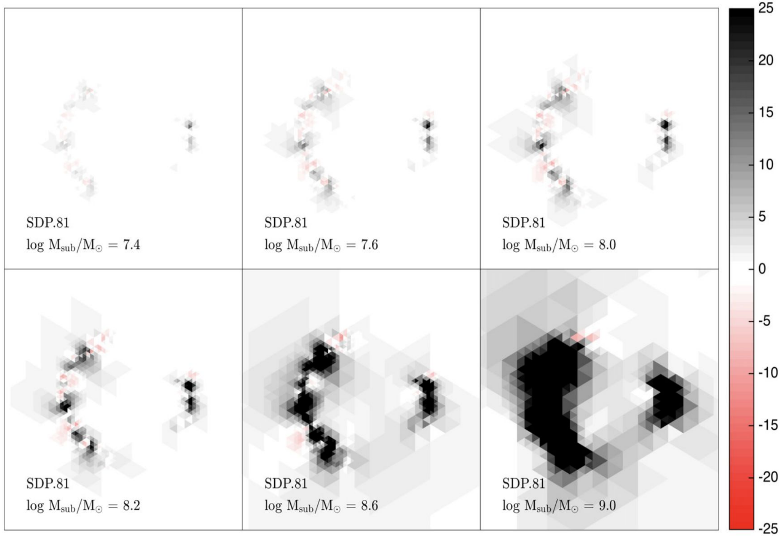

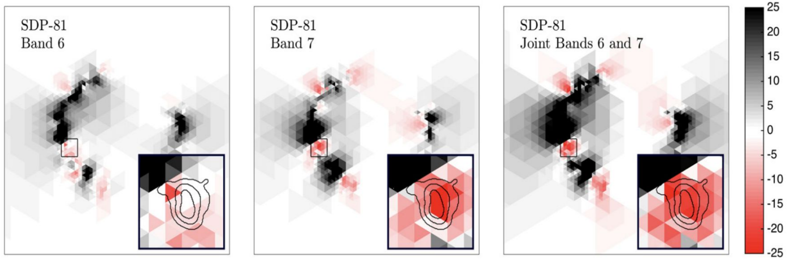

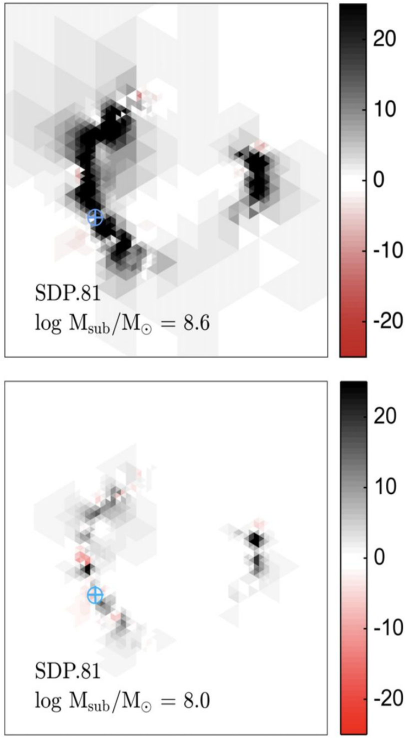

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Hezaveh et al., 2015

Outline

The fraction of strongly lensed objects at high-z

What is the probability to obtain a strong lensing event along a generic line of sight?
The answer is given by the integral of the number density of lenses n(z) times the strong lensing cross section:

$$P(z) = \int_0^z n(z')\sigma(z')dr_{\rm prop}(z')$$

also viewed in terms of **lensing optical depth** (the fraction of sky covered with Einstein rings)

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Let's take the cross section corresponding to the Einstein radius ϑ_E

$$\sigma = \pi R^2 = \pi(\theta_E D_D)^2 = \frac{4\pi GM}{c^2}\frac{D_D D_{DS}}{D_S}$$

this distance here peaks when $D_D = D_{DS}$
→ lens halfway between the source and the observer

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In the end:

$$\tau(z) = P(z) \sim \frac{3\Omega_D}{2}\int_0^{z_s}\frac{d_D d_{DS}}{d_S}\frac{1+z}{\sqrt{1+\Omega z}}dz$$

where $d_D \equiv H_0 D_D / c$ and similarly for d_S and d_{DS}

For $\Omega = 1$ and $z_s = 3$, considering that $d_D d_{DS}/d_S \sim 1$ → $\tau(z_s = 3) \sim 0.5\Omega_D$

The most effective lensing structures are galaxy's cores (especially early-type, massive elliptical ones)

$$\Omega_D \sim 10^{-1}\Omega_{\text{gal}} \sim 10^{-3}$$ → roughly one out of 10^3-10^4 high-z quasars are strongly lensed

The fraction of strongly lensed objects at high-z

A deeper quantitative analysis require a more precise evaluation of the strong lensing probability as a function of the angular distance between the source and the lens. i.e. for point mass lenses

(obtained from the μ definition)

$$\theta_\pm = \frac{1}{2}(\beta \pm \sqrt{\beta^2 - 4\theta_E^2}) \quad \text{and magnification} \quad \mu(x) = \frac{x^2+2}{x\sqrt{x^2+4}} \quad \text{with} \quad x \equiv \beta/\theta_E$$

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Now let's compute the lensing cross-section as a function of the flux amplification

= the proper area around a given lens through which the un-deflected light ray would need to pass to cause amplifications greater than μ

$$\sigma(>\mu) = \pi[D_D\beta(\mu)]^2$$

$\beta(\mu)$ being the source undeflected angle on sky within which the amplification is larger than μ

$$\beta^2 = 2\theta_E^2\frac{\mu-\sqrt{\mu^2-1}}{\sqrt{\mu^2-1}} \Longrightarrow \sigma(>\mu) = \frac{8\pi GM}{c^2}\frac{D_D D_{DS}}{D_S}\frac{1}{\mu^2+1+\mu\sqrt{\mu^2-1}} \propto \mu^{-2}$$

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For an isothermal sphere (Peacock, 1999):

$$\sigma(>\mu) = \left[\frac{4\pi G<v_{//}^2>}{c^2}\right]^2\left(\frac{D_D D_{DS}}{D_S}\right)^2\frac{4\pi}{\mu^2} \quad \text{if } \mu > 2$$

$$\sigma(>\mu) = \left[\frac{4\pi G<v_{//}^2>}{c^2}\right]^2\left(\frac{D_D D_{DS}}{D_S}\right)^2\frac{\pi}{(\mu-1)^2} \quad \text{if } \mu < 2$$

And the probability to obtain a strong lensing event along an arbitrary line of sight with amplification > μ is:

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This of course depends also on other cosmological / population parameters, in particular Ω_m and Ω_Λ.

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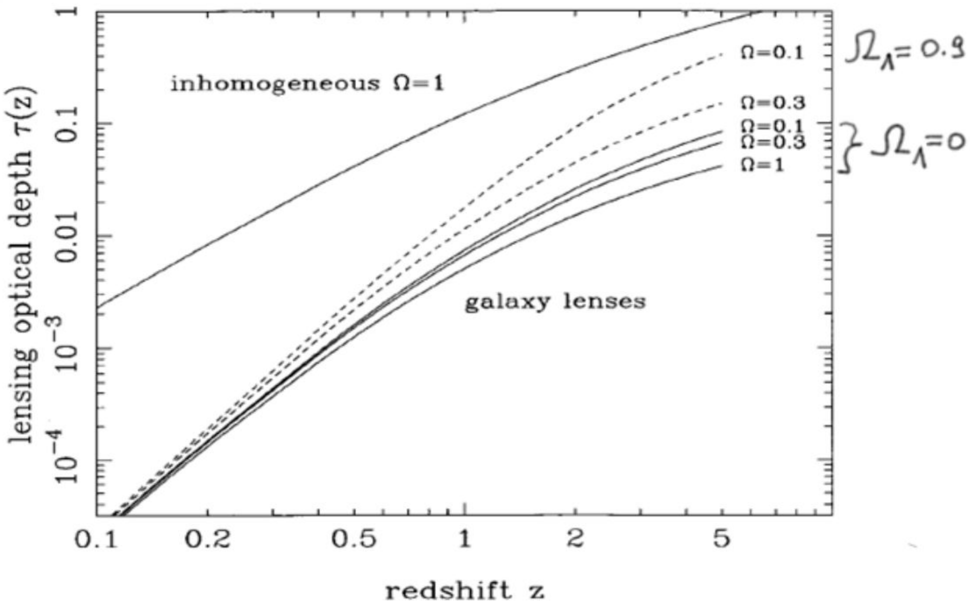
The fraction of strongly lensed objects at high-z

Cosmic Lens All Sky Survey (CLASS), a radio survey with the Very Large Array (VLA), the Very Long Baseline Array (VLBA) and MERLIN.
They found a ratio of lensed:unlensed radio sources of one per 690 ± 190 targets (Mitchell et al., 2005), consistent with

$$\Omega_\Lambda + \Omega_m = 1$$

and leading to:

$$\Omega_\Lambda = 0.69^{+0.31}_{-0.24}$$



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Time delays

The deflection of light always causes a delay in the travel-time of light between the source and the observer. This time delay has two components:

$$\Delta t = \boxed{\Delta t_{\rm grav}} + \boxed{\Delta t_{\rm geom}}$$

Shapiro time delay

Geometric delay

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Remember: light travelling in a different metric travels with a speed c' = c/n (in the observer reference frame)

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Flying Madrid - Oslo, Oslo - Malta, Malta - Palermo is slower than directly flying Madrid - Palermo

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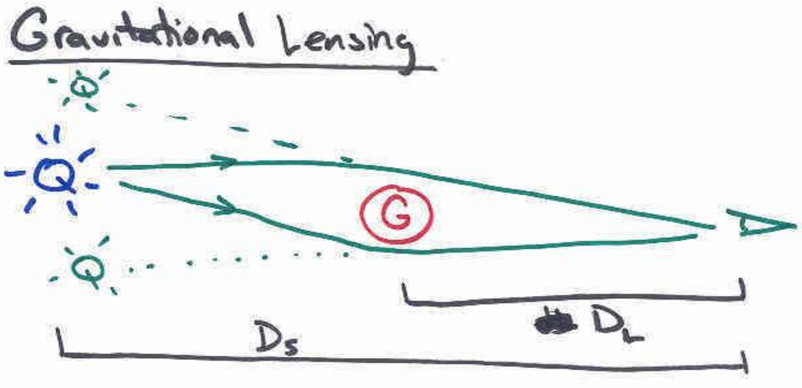
As you can see, both depends on lens specifics (potential, deflection angle), therefore the different multiply lensed images of the same source will be delayed in time: one arrives first, the others follow, and the actual delay depends on lens + cosmology.



Lens potential

$$\Delta t \propto D_{\Delta t} \times \phi_{\rm lens} \rightarrow D_{\Delta t} \propto 1/\mathbf{H_0}$$

Time-delay

Time-delay distance

$$D_{\Delta t} = (1+z_D)\frac{D_S D_D}{D_{DS}}$$

Hubble Constant

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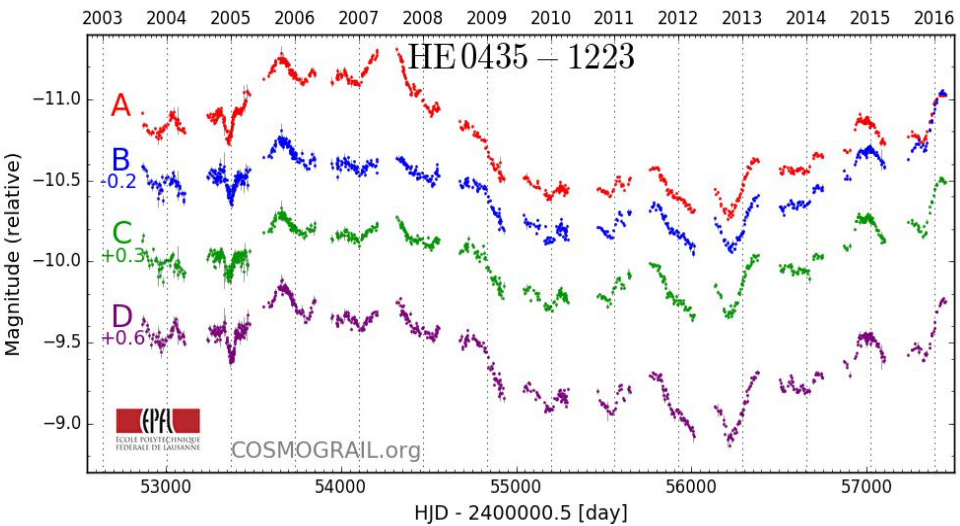
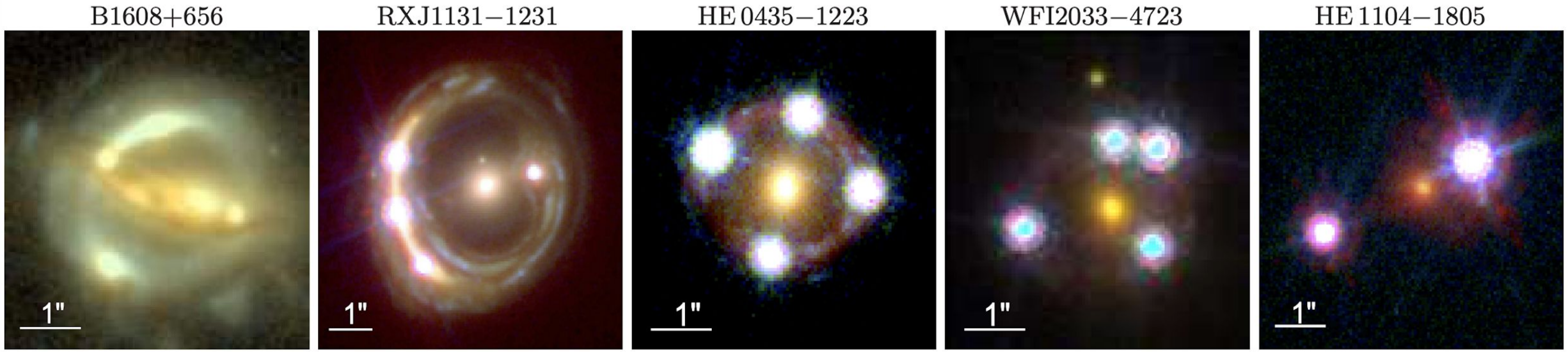


https://shsuyu.github.io/H0LiCOW/site/



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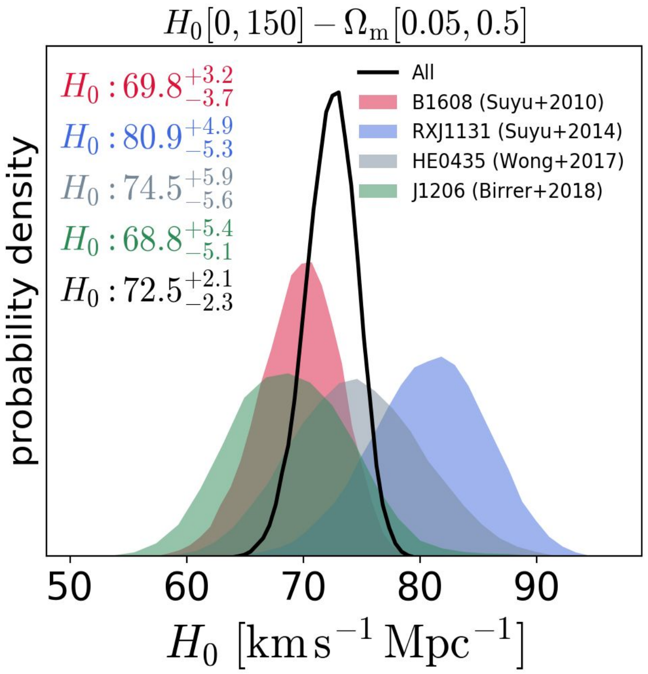

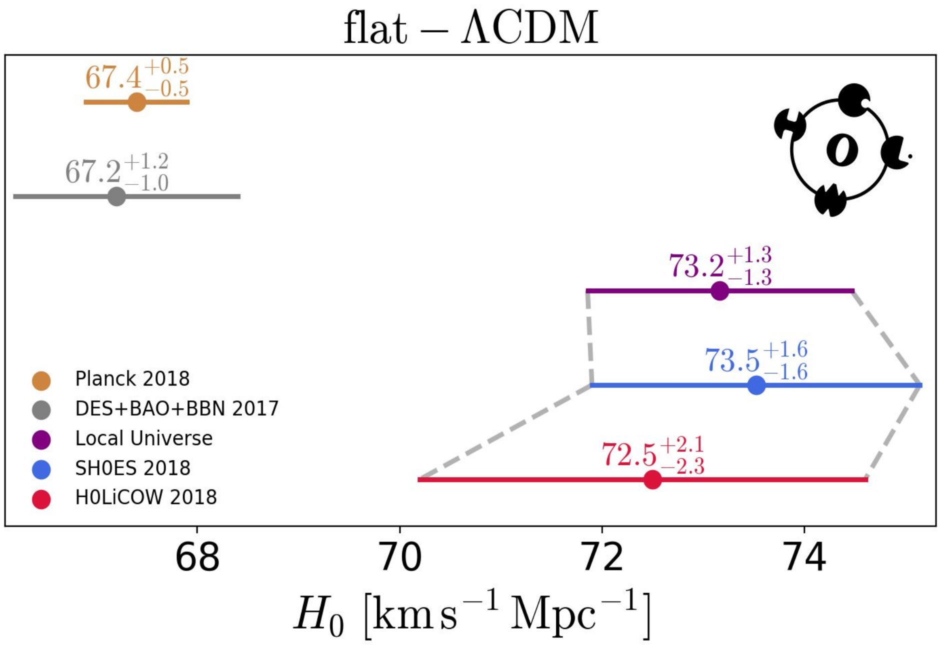

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Hubble constant H0 measurement from blind analysis of 4 multiply-imaged quasar systems through strong gravitational lensing: $H_0 = 72.5^{+2.1}_{-2.3}$ km/s/Mpc, at 3% precision, in the standard flat ΛCDM model.

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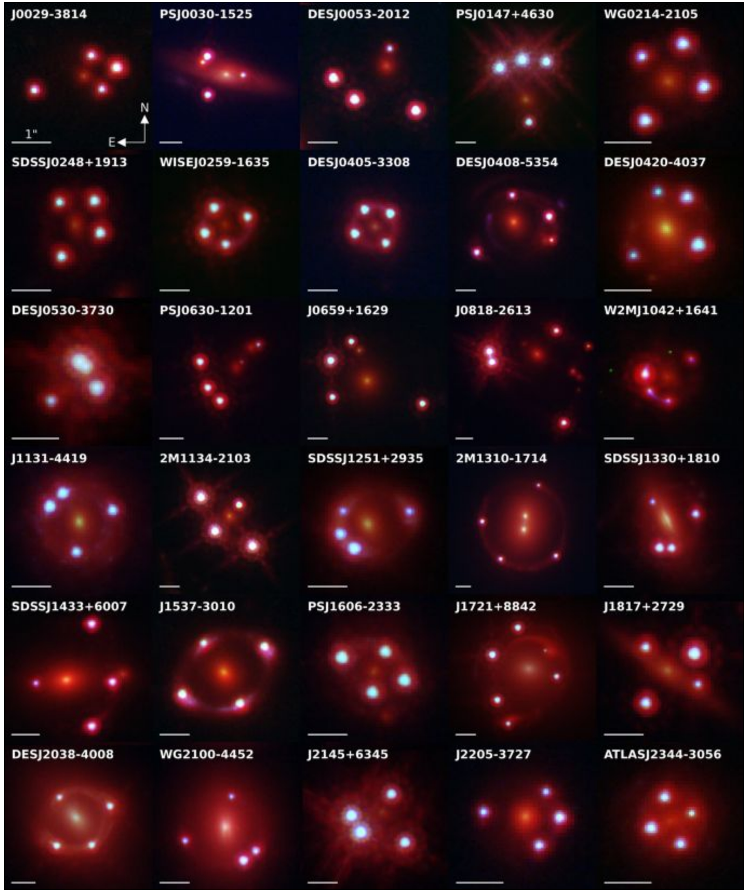
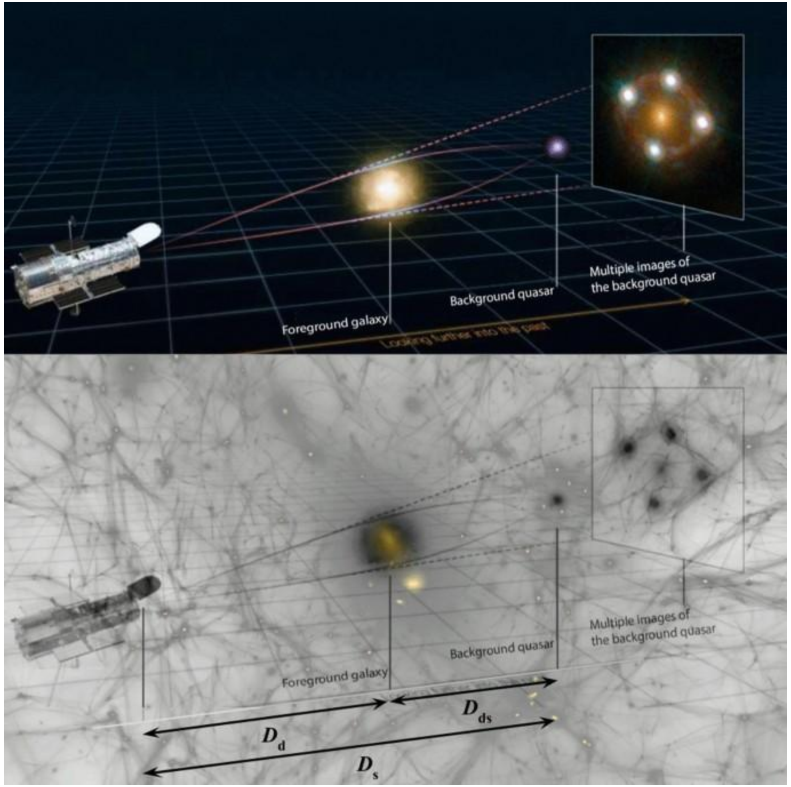


Schmidt et al., 2022



Treu et al., 2022

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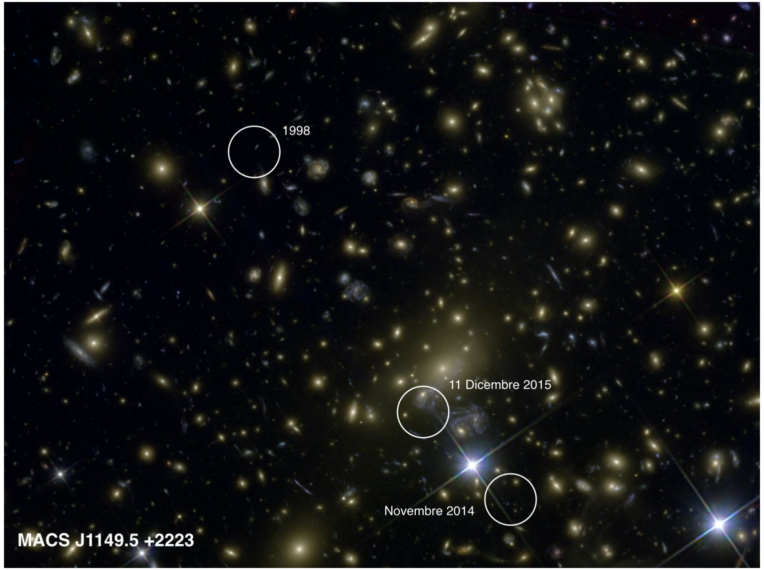
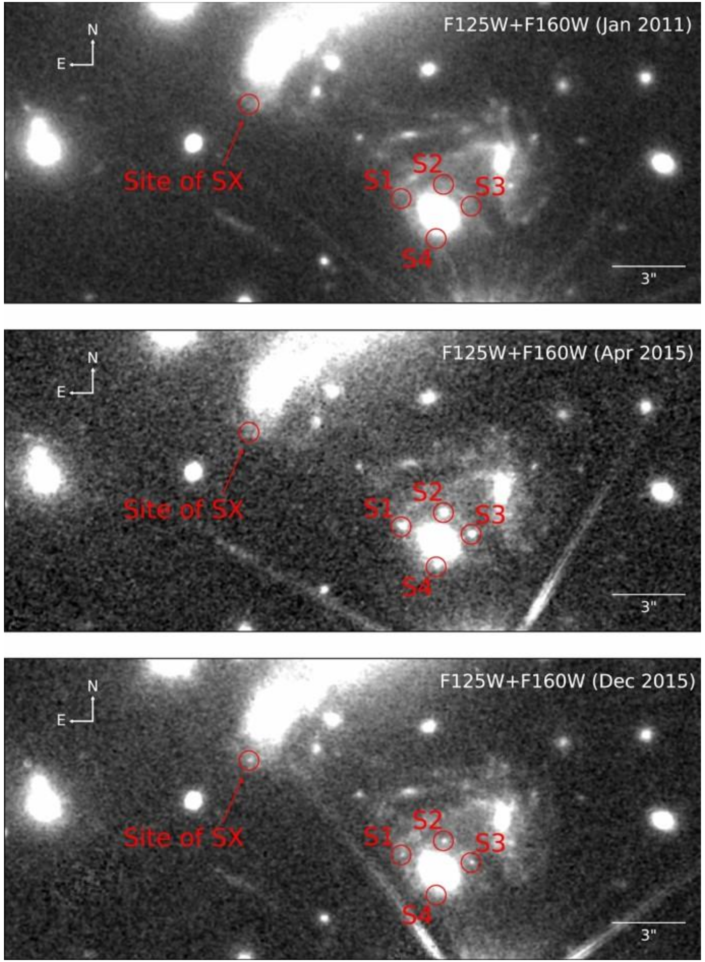
Time delays and the Refsdal SNa

SN Refsdal

There is so much mass in a galaxy cluster, and therefore lots of multiple images, that you can see the same event happen multiple times, like a SN explosion in a redshift 1.49 distant galaxy.
The timing of the apparitions strongly depends on cosmological parameters.





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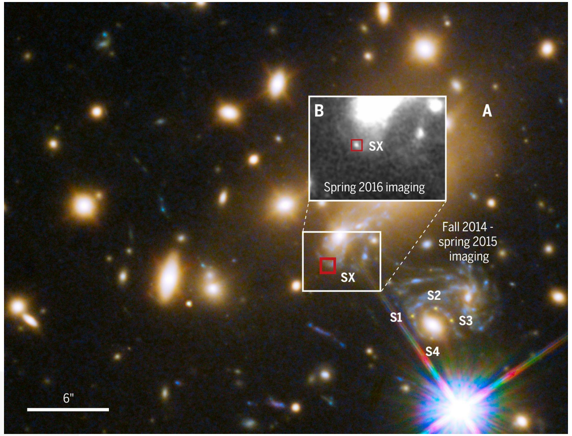

CONCLUSION

We infer a value of H_0 of $64.8^{+4.4}_{-4.3}$ km s^{-1} Mpc^{-1} using the full set of eight pre-reappearance models and of $66.6^{+4.1}_{-3.3}$ km s^{-1} Mpc^{-1} from the two preferred models. Our results are most consistent with the H_0 value measured from the CMB but do not exclude the higher value from nearby SNe.

We used a simulation of a galaxy cluster lens to verify that the uncertainty on our measurement of H_0 is consistent with expectations. The ability of the lens models to reproduce the positions of the SN images also implies an expected uncertainty on H_0, which we find agrees with our constraints. The best agreement between lens models and observations that are independent of H_0 is achieved by the models that were constructed by assigning dark-matter halos to both the cluster and to individual galaxies in the cluster.

Kelly et al., 2023

Microlensing

We talk about microlensing when the spatial resolution is not enough to resolve the lensing features (i.e. Einstein rings, multiple images at the order of *milli*arcsec or *micro*arcsec), but a lensing event is still happening: flux magnification, with a unique light-curve shape.

$$\theta_E = \left(\frac{D_{DS}}{D_D D_S} \frac{4GM(\theta)}{c^2} \right)^{1/2} \sim 0.3 \times 10^{-3} \left(\frac{M}{M_\odot} \right)^{1/2} \left(\frac{D'}{10 \text{ kpc}} \right)^{1/2} \text{ arcsec}$$

Broad range of masses: planets, stars, star clusters, compact objects in MW or other galaxies (but mainly the LMC/SMC). Historically, the first method to actually probe if dark matter is made of MAssive Compact Halo ObjectS (MACHOs).

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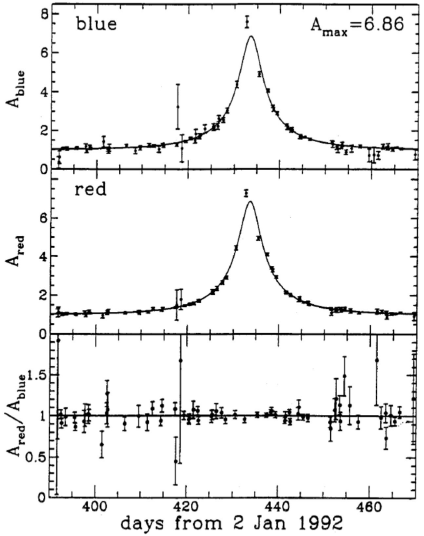

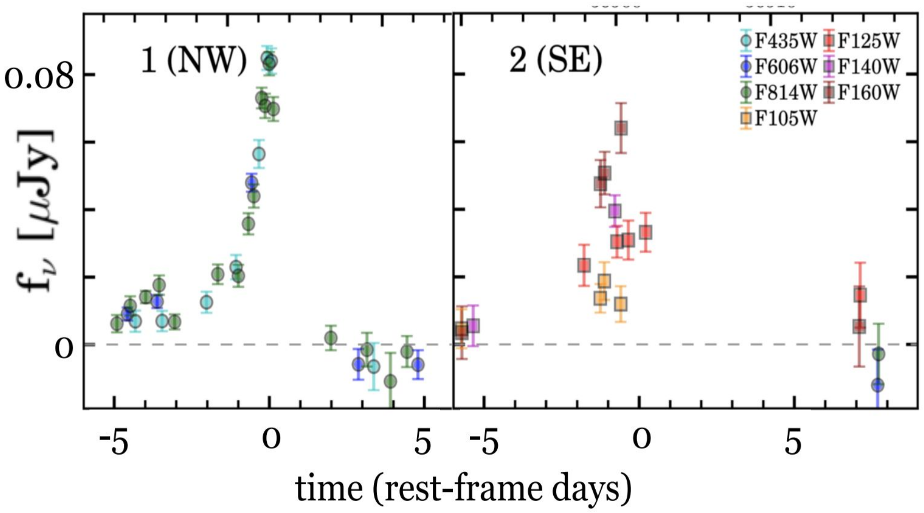

Microlensing

The characteristic time-scale for a microlensing event is:

$$\dot{\theta} = \frac{v}{D_D} = 4 \times 10^{-3} \left(\frac{v}{200\ \mathrm{Km/sec}}\right) \left(\frac{D_D}{10\ \mathrm{kpc}}\right)\ \mathrm{arcsec\ yr}^{-1}$$

and the typical time-scales of the flux variations due to the lensing event is:

$$t_E = \frac{\theta_E}{\dot{\theta}} = 0.2 \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{v}{200\ \mathrm{Km/sec}}\right)^{-1} \left(\frac{D_D}{10\ \mathrm{kpc}}\right)^{1/2} \left(1 - \frac{D_D}{D_S}\right)^{1/2}\ \mathrm{yr}$$

which is a couple of months for typical parameter values. Finally, the microlensing cross-section is directly related to the Einstein ring as:

$$\sigma_{\mathrm{micro}} = \pi \theta_E^2$$

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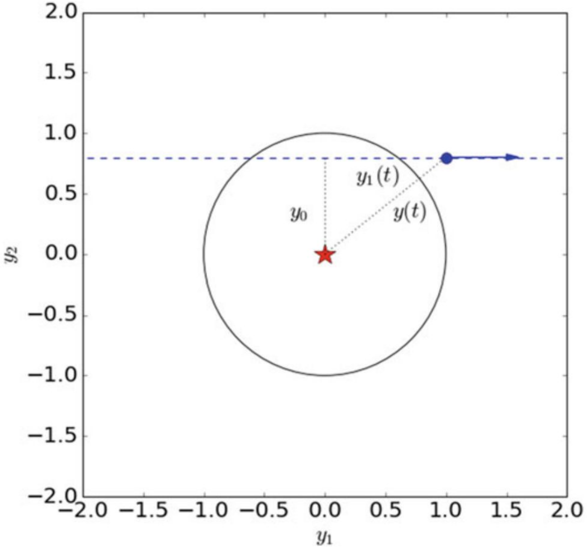
Great, so microlensing is measurable in human timescales. The problem is that there are billions of reasons why a star could suddenly change its flux, all linked with their intrinsic variabilities / environmental things all unrelated to gravitational lensing.

Solutions:

- lensing is achromatic, so if the same flux variability is detected in multiple bands its a clear indication that microlensing is happening there
- smoking gun: microlensing light light curve:
    1) Unique shape
    2) Symmetric with respect to the maximum
    3) Achromatic

Microlensing



We have a foreground lens (center) and a moving background source moving along a straight line (which is a good approximation of a real microlensing application).

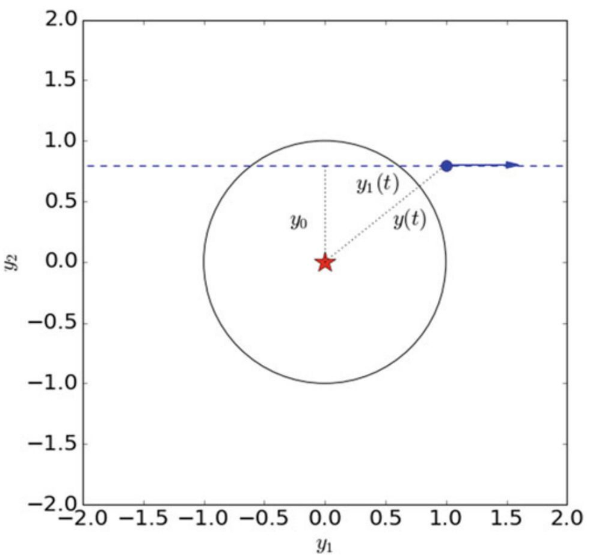
y_0 is the normalized impact parameter (in units of ϑ_E), which is the closest lens-source distance at time t_0.

The source trajectory is:

$$y(t) = \left(y_0^2 + y_1^2(t)\right)^{1/2} = \left(y_0^2 + \frac{\dot{\theta}^2 (t-t_0)^2}{\theta_e^2}\right)^{1/2}$$

$$= \sqrt{y_0^2 + \left(\frac{t-t_0}{t_E}\right)^2}$$

Microlensing



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The magnification μ for a point mass lens is:

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Therefore the source flux changes in time as:

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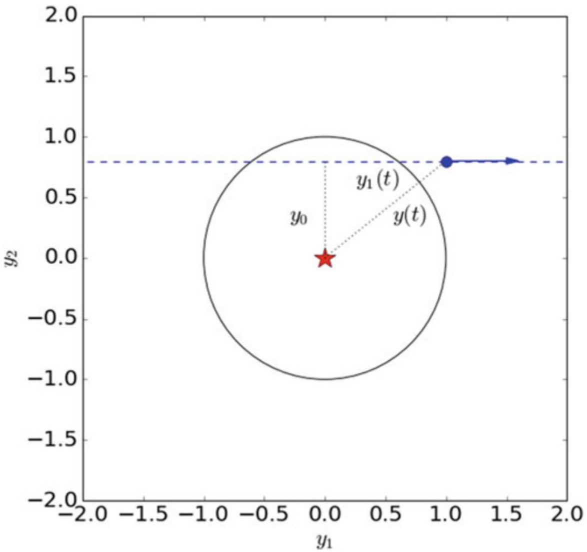

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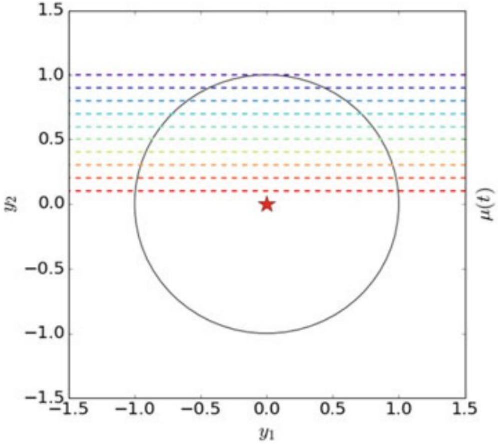
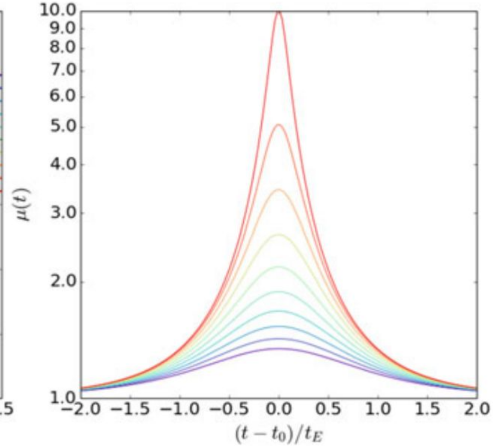




Microlensing and star masses

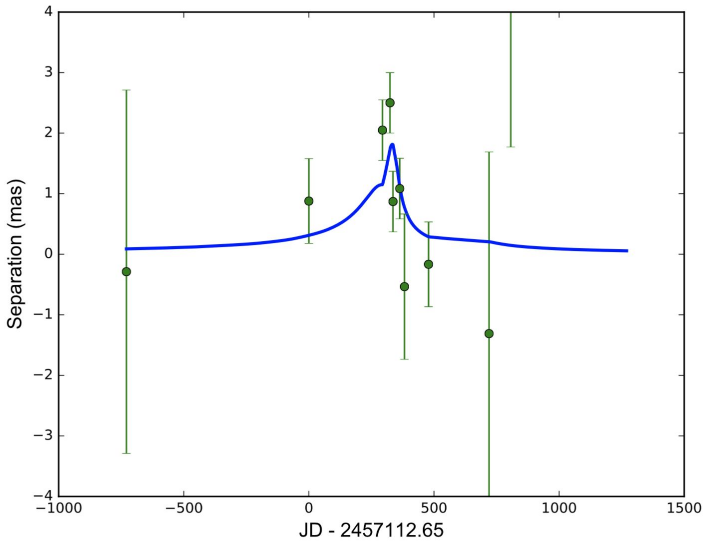
$$S(t) = S_0 \times \mu[y(t)] = S_0 \times \frac{y^2+2}{y\sqrt{y^2+4}}$$

- Unlensed flux S_0 ✓
- Time of max t_0 ✓
- Smallest distance y_0 ✓
- Typical time scale t_E ✓

$t_E \propto \sqrt{MD_D}/v$ ✖

Cannot disentangle these three quantities

Microlensing degeneracy: one cannot infer the distances, the velocity and the lens mass uniquely from the microlensing light curve.
Of course, if you know two of them from other independent measurement than a microlensing event can give you a way to measure the third one.



For example, you can measure the mass of a foreground star acting as a lens **if** background source distance and lens-star relative velocity are known independently.

Independent measure of Proxima Centauri mass (0.15 Mo) in agreement with literature, when it passed in front of two foreground stars in 2014 and 2016.

Zurlo et al., 2018

Outline

Microlensing and exoplanets

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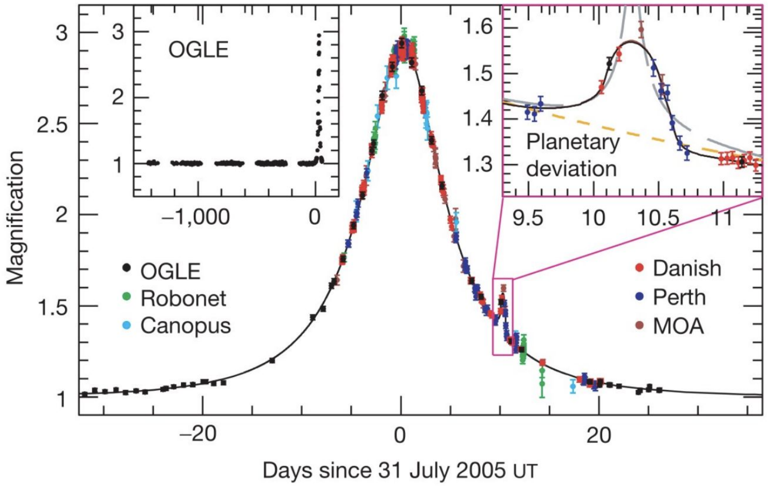

Microlensing is also a nice technique to detect planets you'll never actually see again, but still carrying useful information from a statistical standpoint.

The principle is simple: the star lensing generates the typical lightcurve, and the planet lensing causes a small (though measurable) deviation (on the left, from a object with 5 Earth masses).

Beaulieu et al., 2006

Microlensing and DM: MACHOs

MACHO stands for MAssive Compact Halo Object. The first observational campaigns started in the '80s. The idea is that the Milky Way halo should be full of compact objects (i.e. isolated black holes, neutron stars, very low-mass stars), undetectable with the instrumentation available at that time.
But hey, we are talking about massive invisible object… maybe baryonic candidates for dark matter?
The concept is: if the MW halo is full of MACHOs, those should generate microlensing events in background objects such as the Small/Large Magellanic Cloud. The number density of MACHOs would be proportional to the number of microlensing events, and their characteristic mass proportional to the timescale t_E.
In the early '90s collaborations like MACHO, EROS (*Experience de Recherche d'Objets Sombres*) and OGLE (*Optical Gravitational Lensing Experiment*) repeatedly looked at the Magellanic Clouds and the galactic bulge looking for these kind of events.

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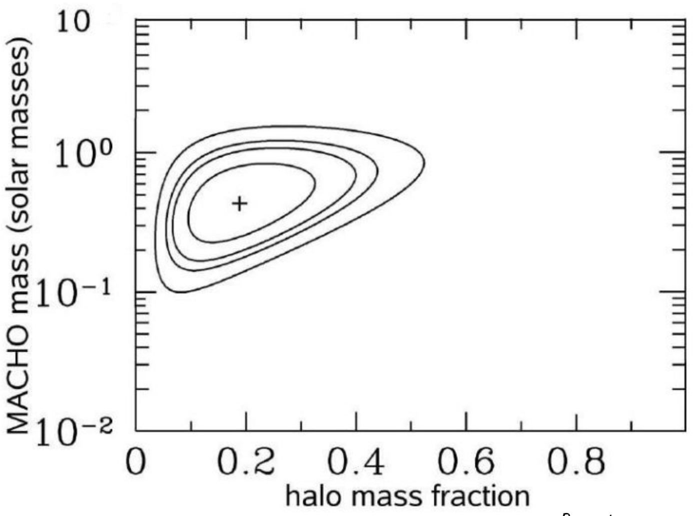
Some results:

- relatively high rate of detection → MW is barred
- looking at MCs, no short events (< 20 days) → strong limits on the presence of Jupiters-like objects in the halo: these objects contribute less than 10% of the dark matter around our Galaxy
- most microlensing events toward the bulge are most likely caused by known stellar populations. BH can contribute to 2% of the total mass of the halo
- hints at the presence of free-floating planets in the MW disk (Sumi et al. 2011)



Bartelmann, 2005

Weak lensing and the Large Scale Structure

The zero order definition is:

strong lensing is when multiple images are generated, ***weak*** lensing is when these are not

Weak lensing and the Large Scale Structure

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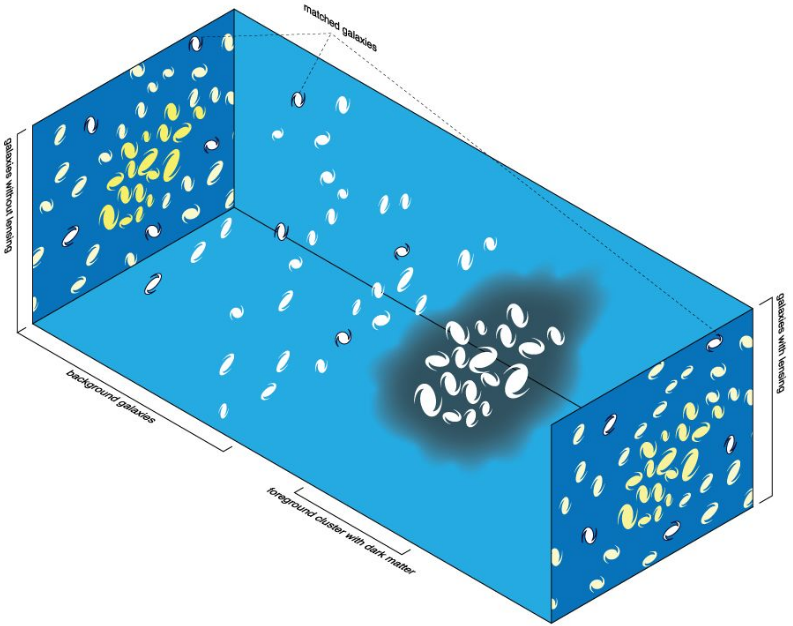
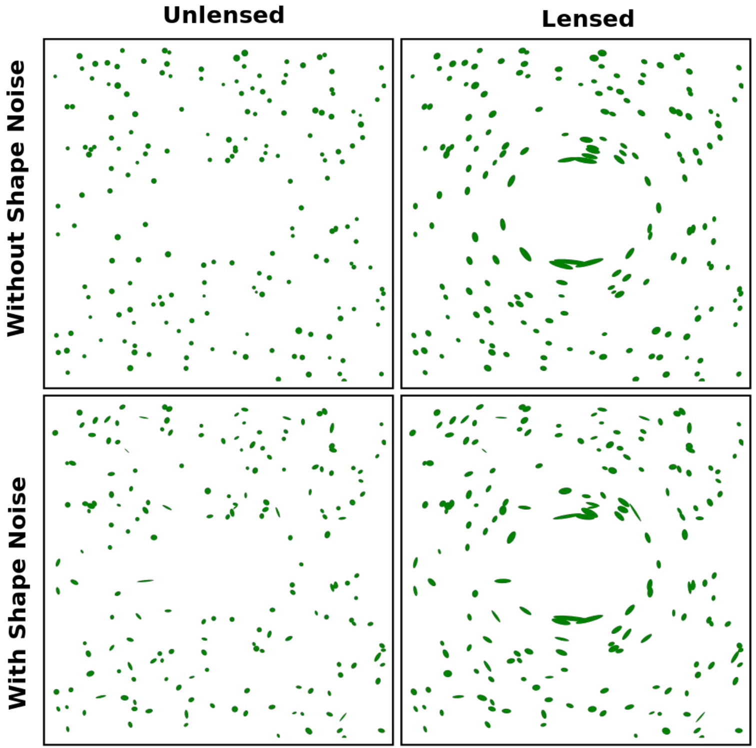
The lens, however, is still there, and imprints its signature on the *shapes* and *orientation* of the background galaxies



Brutal Wikipedia



Weak lensing and the Large Scale Structure

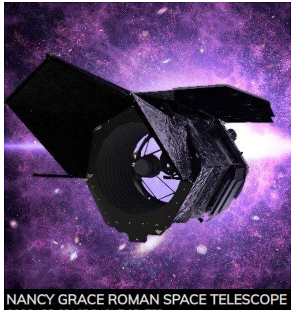
I will spare you lots and lots of math, and go directly with the convergence power spectrum:

$$P_\kappa(l) = \frac{9H_0^4\Omega_{m,0}^2}{4c^4}\int_0^{w_H} \frac{W^2(w)}{a^2(w)} P_\delta\left(\frac{l}{f_K(w)}, w\right) \mathrm{d}w$$

This depends on cosmology in several ways: it is sensitive to the growth of structures within the Universe, to the square of the matter density Ωm, to the geometry of the Universe in the factor f_K (panel A). This is a measurable quantity, assuming you are able to accurately map slices of the Universe in the whole sky up to high redshift (weak lensing *tomography*).

Euclid consortium

July 2023



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VERA C. RUBIN OBSERVATORY

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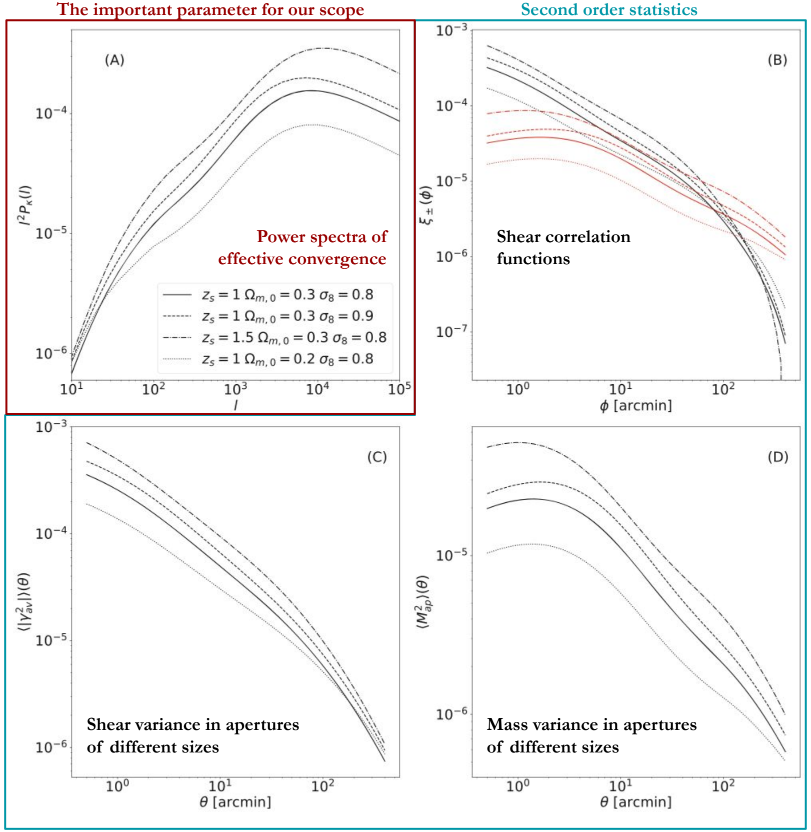

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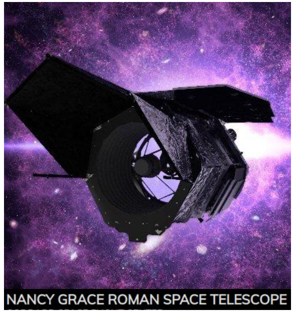

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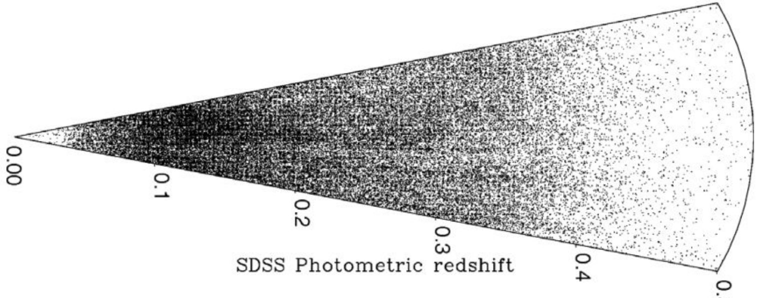
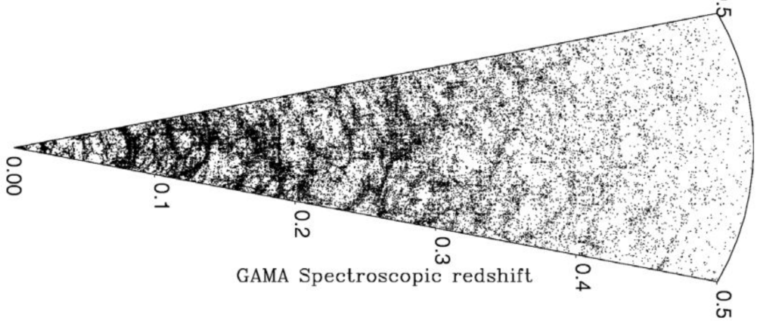
Of course, there are LOTS of complication in doing so:

- signal amplitude is tiny (order of 0.01)
- degenerate cosmological parameters
- systematics (intrinsic shapes/alignments), to accurately model
- **high** accuracy in measurements is crucial for the theoretical measurement (photo-z $\sigma_{NMAD} < 0.01$)





Outline

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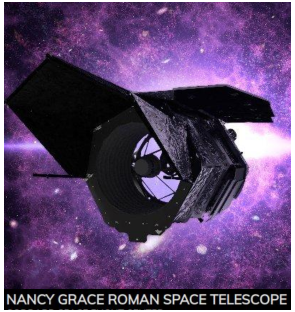

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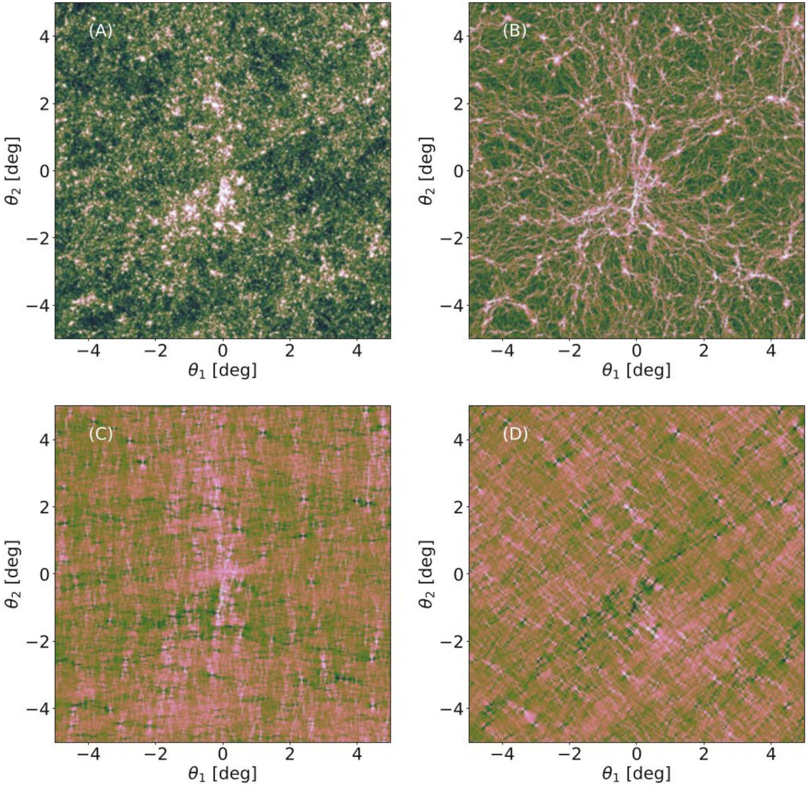

Example on a cosmological simulation.
Panel (A) is the effective convergence, panel (B) the shear modulus γ panels (C) and (D) the shear components

Outline

Thanks for your attention, good luck with the exam.

* nessun SOBA e' stato maltrattato nella produzione di questa slide

for anything, contact me at **andrea.enia@unibo.it**