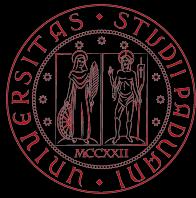
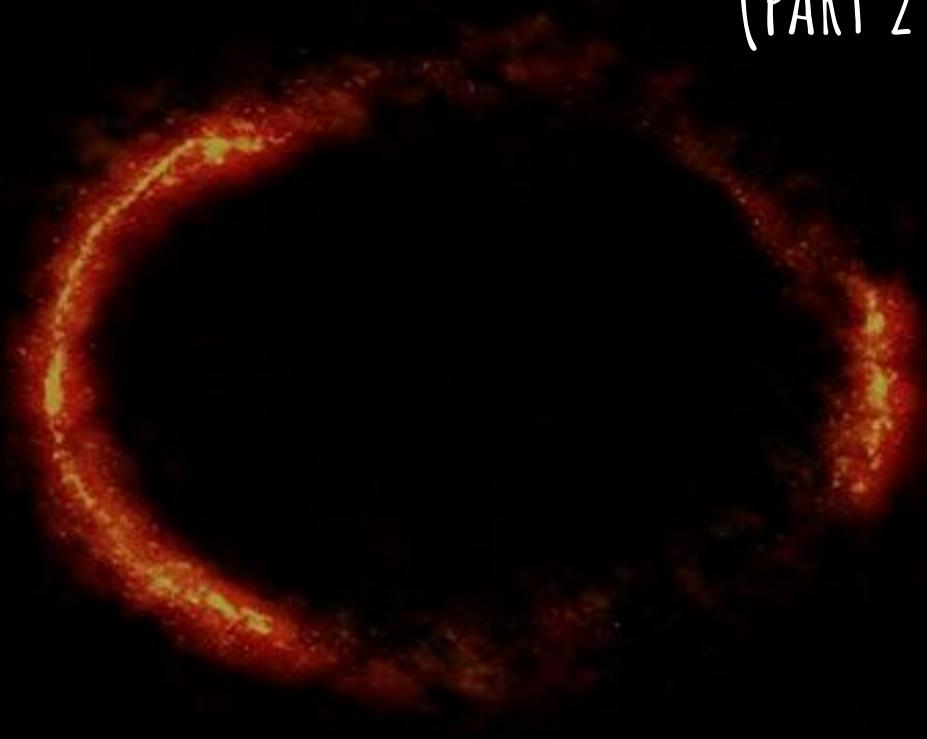


# GRAVITATIONAL LENSING (PART 2: LENSING APPLIED)



OBSERVATIONAL COSMOLOGY COURSE, MASTER'S DEGREE IN ASTROPHYSICS, A.A 2024-2025

# Theory of Gravitational Lensing

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2\vec{\xi}} \vec{e}_{\vec{\xi}} = \frac{4GM}{c^2\vec{\xi}^2} \vec{\xi}$$

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{\text{LS}}}{D_{\text{L}} D_{\text{S}}} \frac{2}{c^2} \int \Phi(D_{\text{L}} \vec{\ell}, z) dz$$

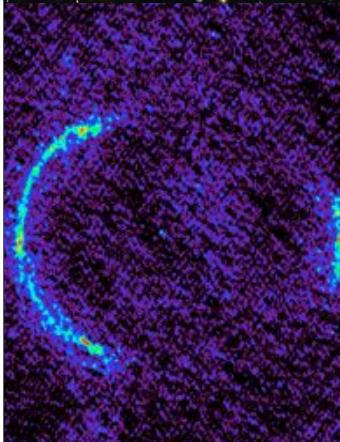
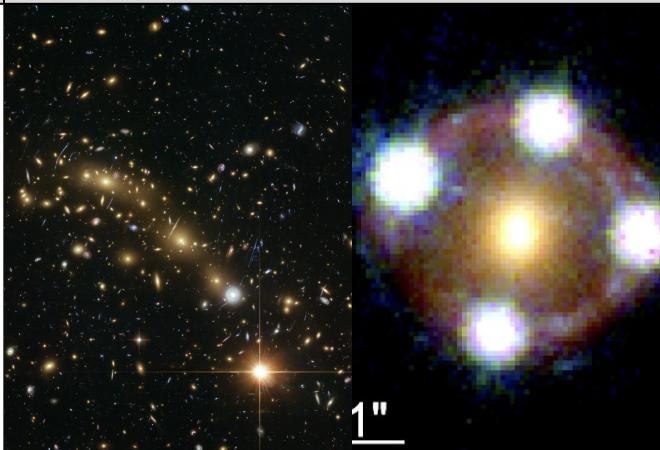
$$\hat{\vec{\alpha}}(\vec{\xi}) = \sum_i \hat{\vec{\alpha}}_i (\vec{\xi} - \vec{\xi}_i) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi}_i}{|\vec{\xi} - \vec{\xi}_i|^2}$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}') \Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2 \vec{\xi}'$$

$$\hat{\vec{\alpha}}(\vec{\theta}) = \frac{(1+z_{\text{L}})}{c} \frac{D_{\text{L}} D_{\text{S}}}{D_{\text{LS}}} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi}(\vec{\theta}) \right]$$

$$\alpha(\theta) = \frac{4GM}{c^2 D_{\text{L}} \theta} \quad \theta_E \approx (10^{-3})'' \left( \frac{M}{M_{\odot}} \right)^{1/2} \left( \frac{D}{10 \text{kpc}} \right)^{-1/2}$$

# Lensing, applied



## Mass distribution in galaxy clusters

Mass distribution in  
galaxy clusters

Galaxy clusters are the largest existing virialized objects in the Universe. Their mass distribution can be inferred through, e.g., kinematic analysis of the member galaxies (applying virial theorem), or X-ray observations of the hot ICM. Lensing analysis of the whole system is the least model-dependent method.



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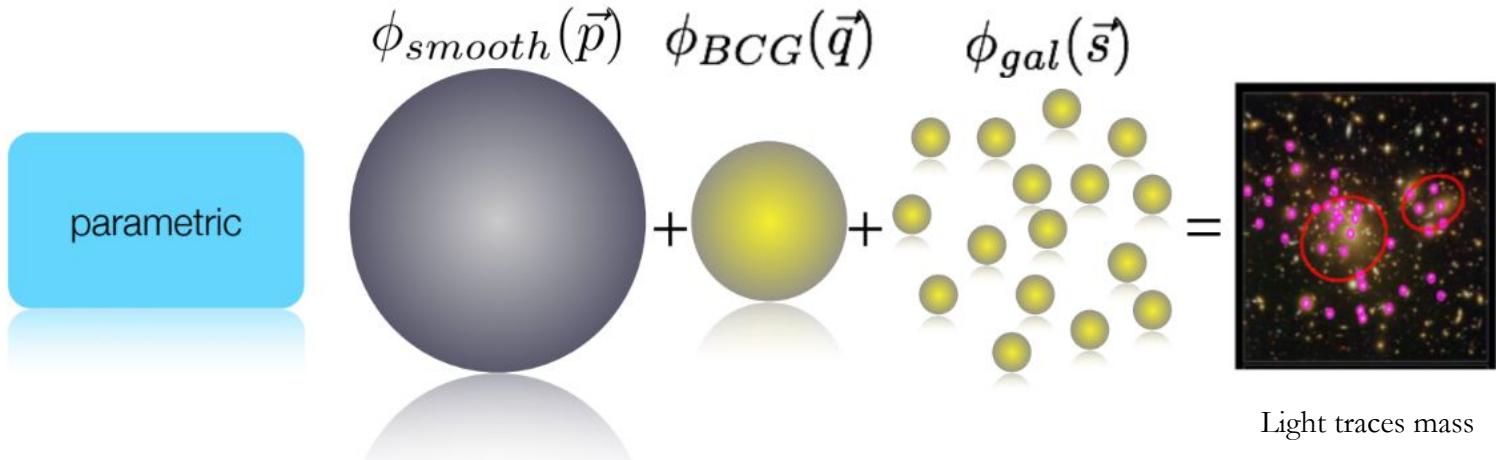
Independent  
from  $\rho$

in the case of non-relaxed / irregular clusters, gravitational lensing is the **only** robust way of measuring their mass

## Outline

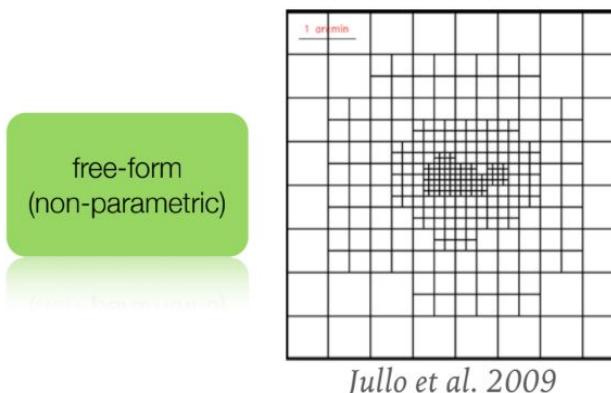
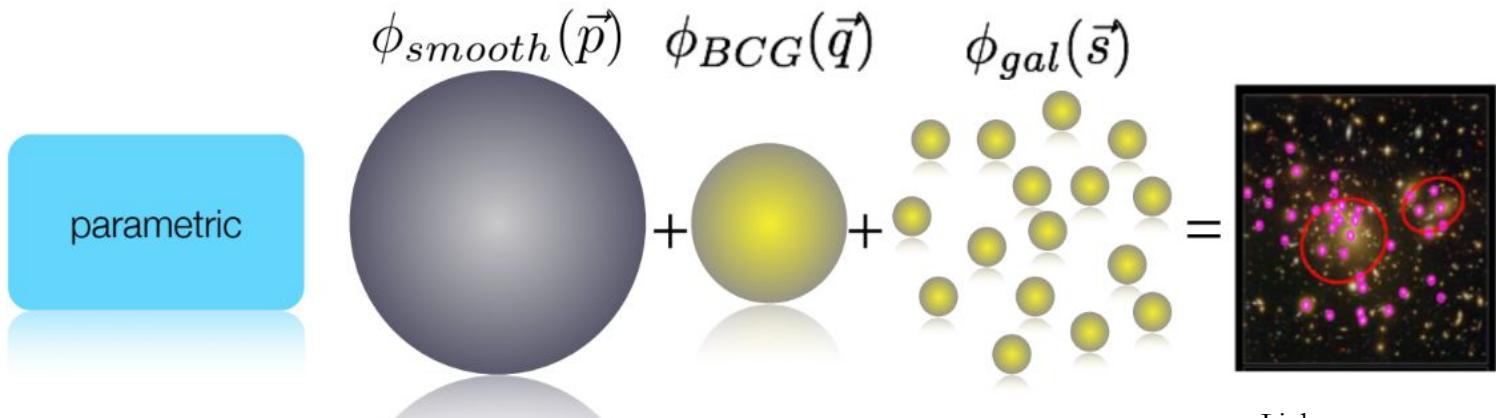
Mass distribution in  
galaxy clusters

# Mass distribution in galaxy clusters



Mass distribution in  
galaxy clusters

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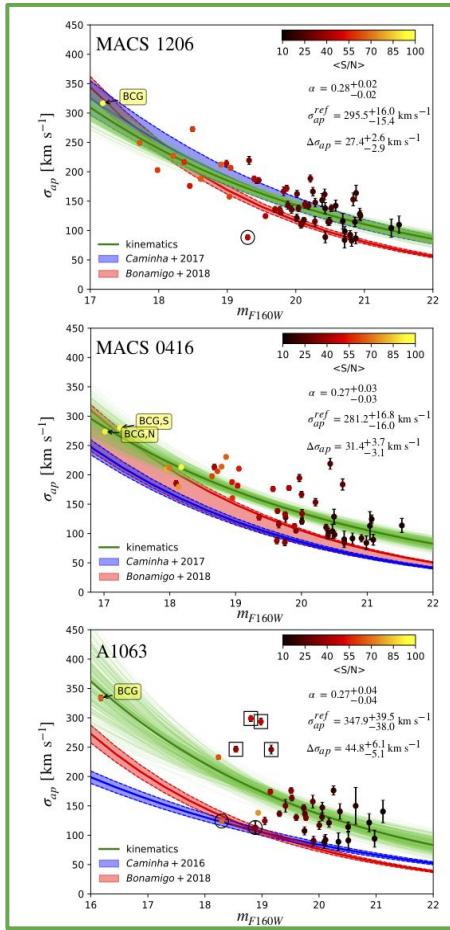


No assumption that light traces mass (in fact, it shouldn't) or on the shape of the density profiles. The cluster is decomposed into pixels (or tesserae or radial basis functions); practically, each pixel has its own mass distribution and contributes to deflecting the foreground galaxies' light.

In both cases, the best fit parameters for the parametric mass distribution / values for free-form are inferred via Bayesian statistics by maximizing a posterior distribution function between a model and the data.

## Outline

Mass distribution in galaxy clusters

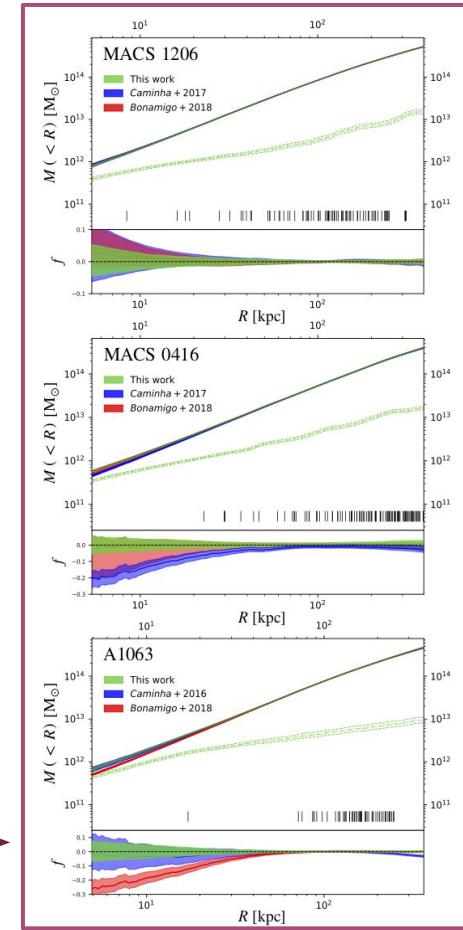


# Mass distribution in galaxy clusters

Strong lensing studies of galaxy clusters shed light on their mass distribution (total, so light + DM + gas).



Mass distribution of galaxies (in terms of velocity dispersion) as a function of their luminosity (magnitude), a.k.a. “scaling relations”



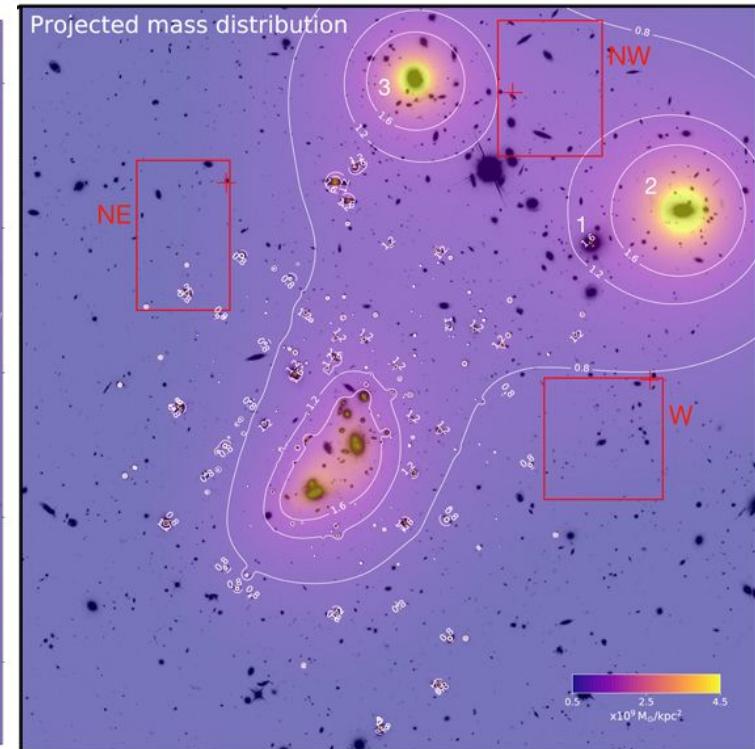
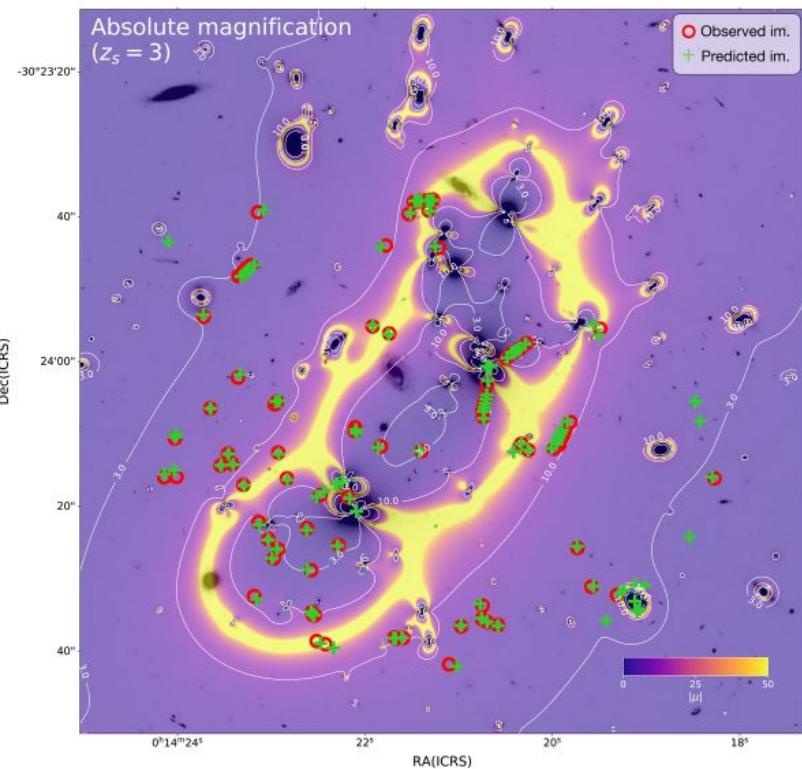
How the mass is distributed inside the cluster



## Outline

# Mass distribution in galaxy clusters

Mass distribution in  
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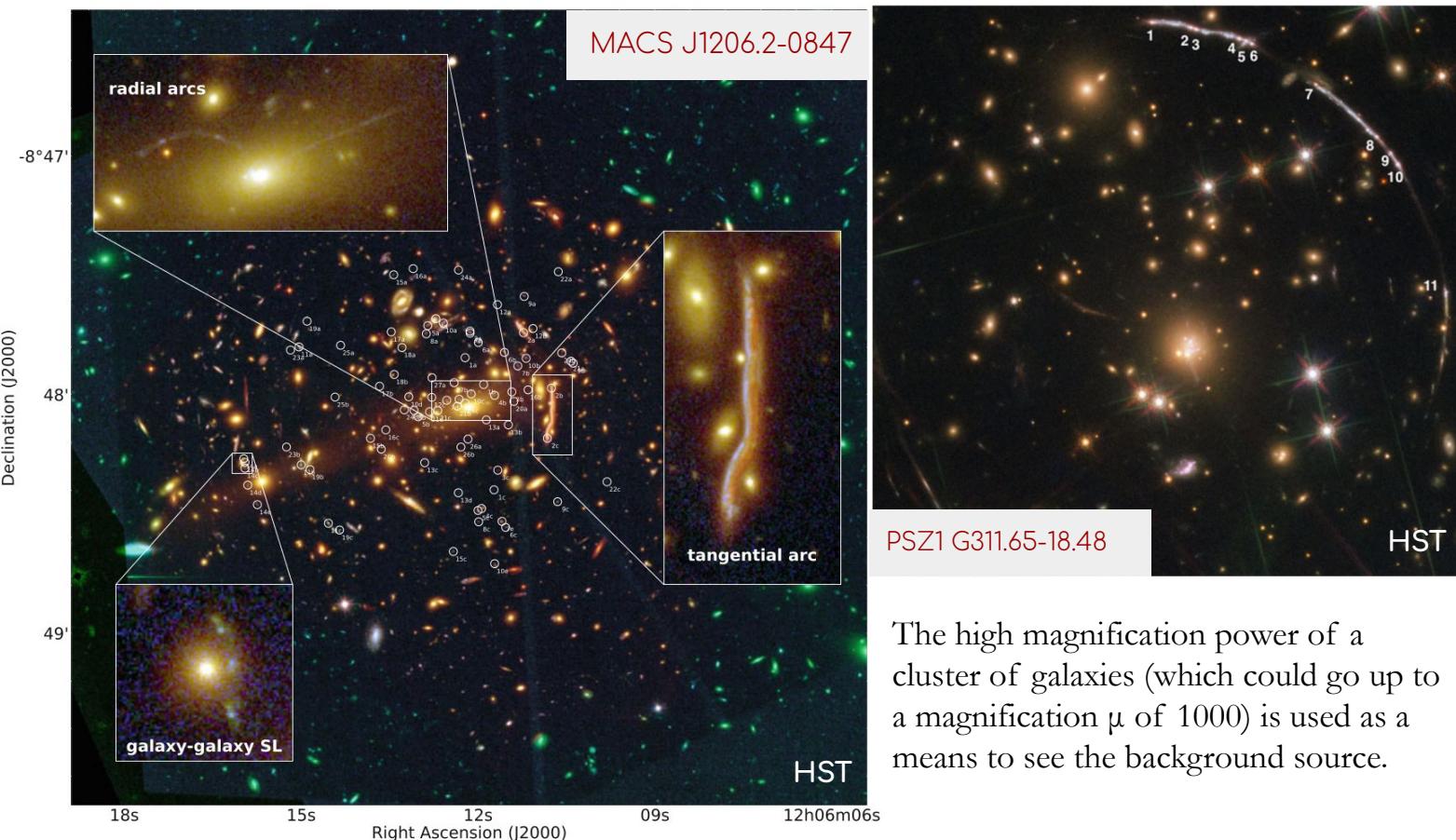


## Outline

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

# Galaxy clusters as a means to the background source



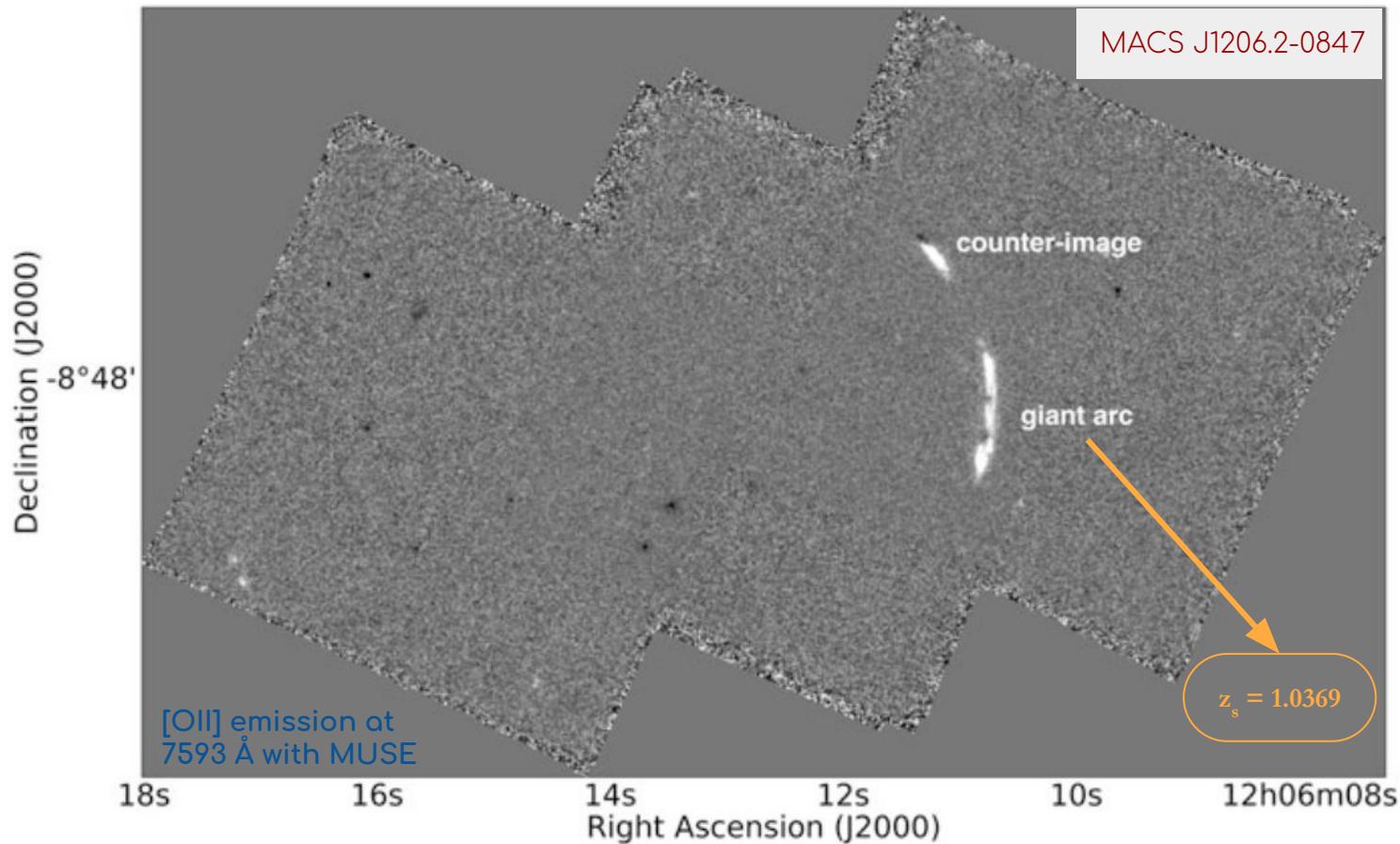
The high magnification power of a cluster of galaxies (which could go up to a magnification  $\mu$  of 1000) is used as a means to see the background source.

## Outline

# Galaxy clusters as a means to the background source

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source



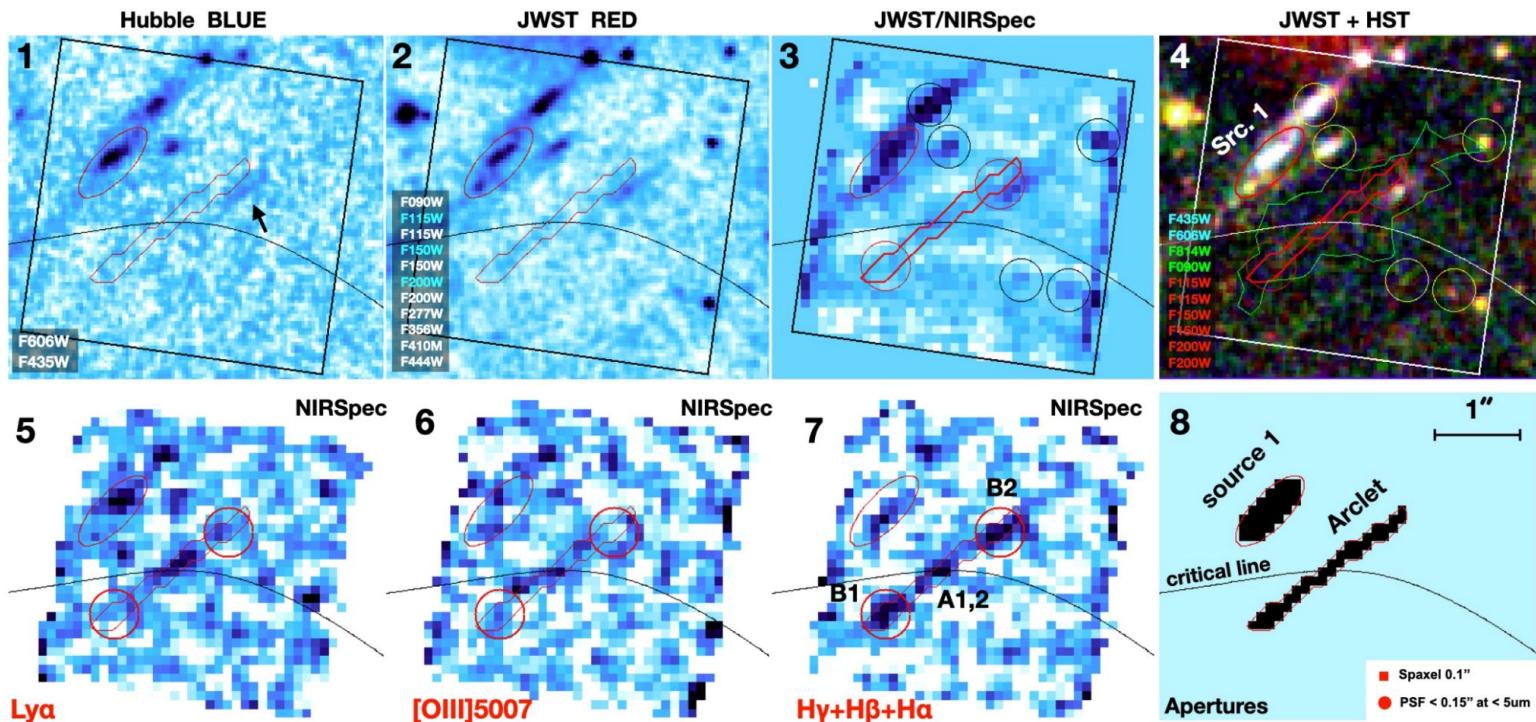
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Mass distribution in galaxy clusters

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Lensed And Pristine 1, (LAP1), a lensed ( $\mu > 100$ ) Population III candidate stellar complex behind MACS J0416, the most metal-poor star-forming region currently known in the reionization era.



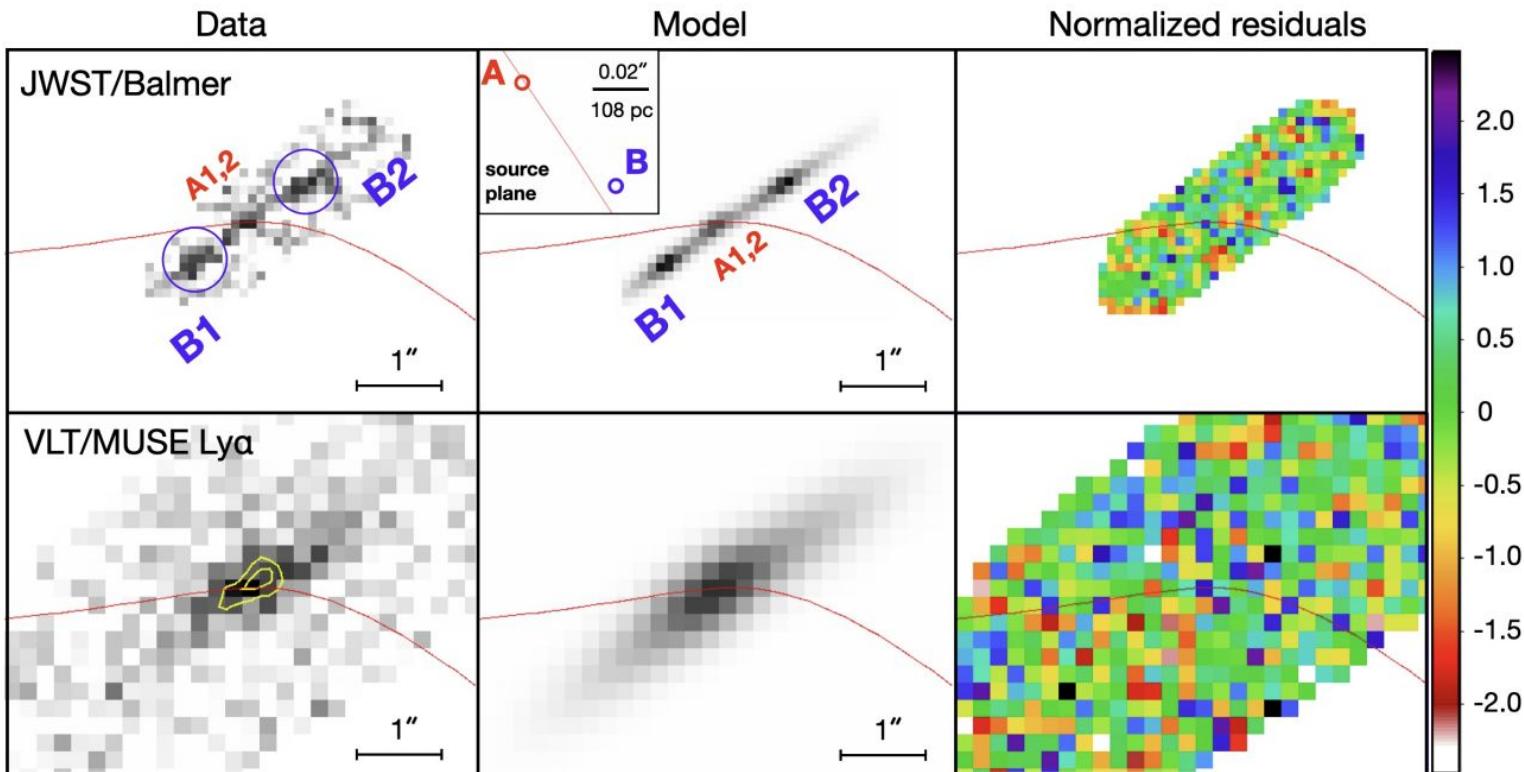
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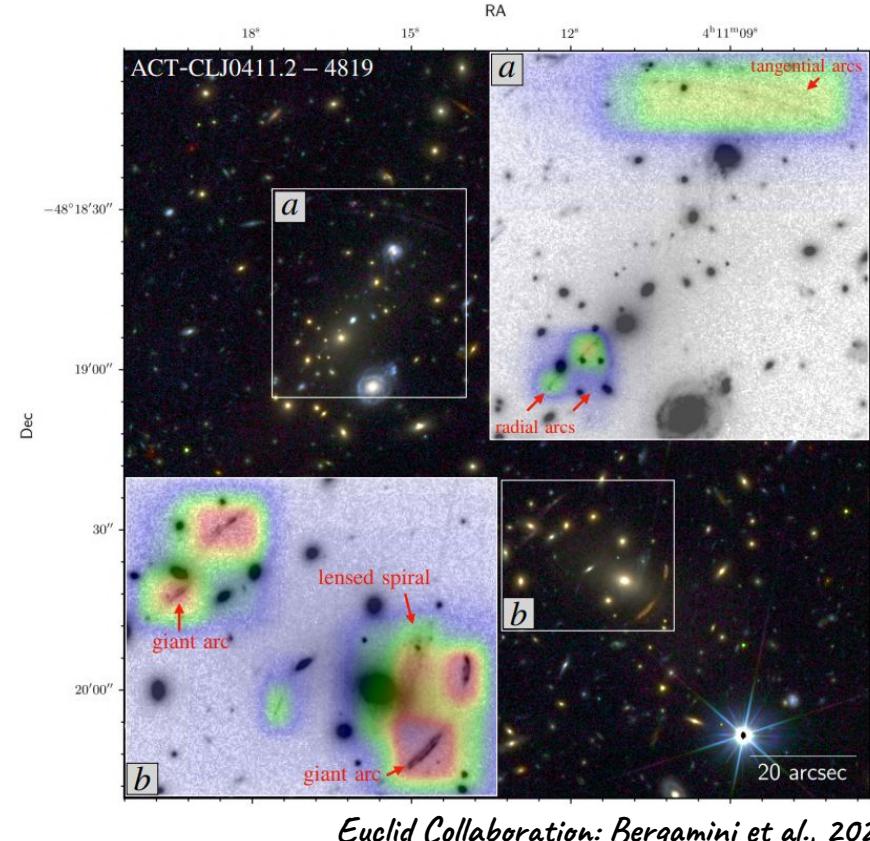
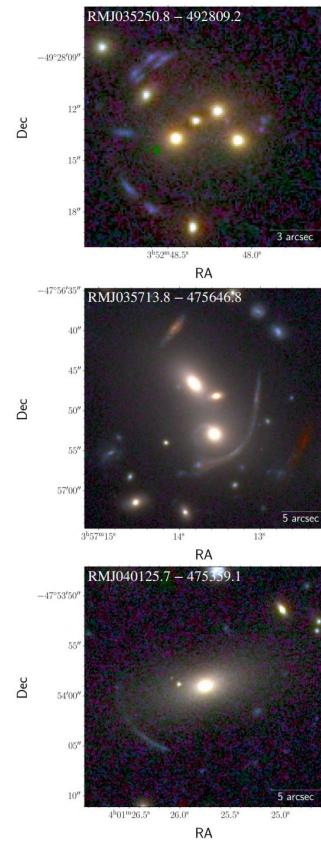
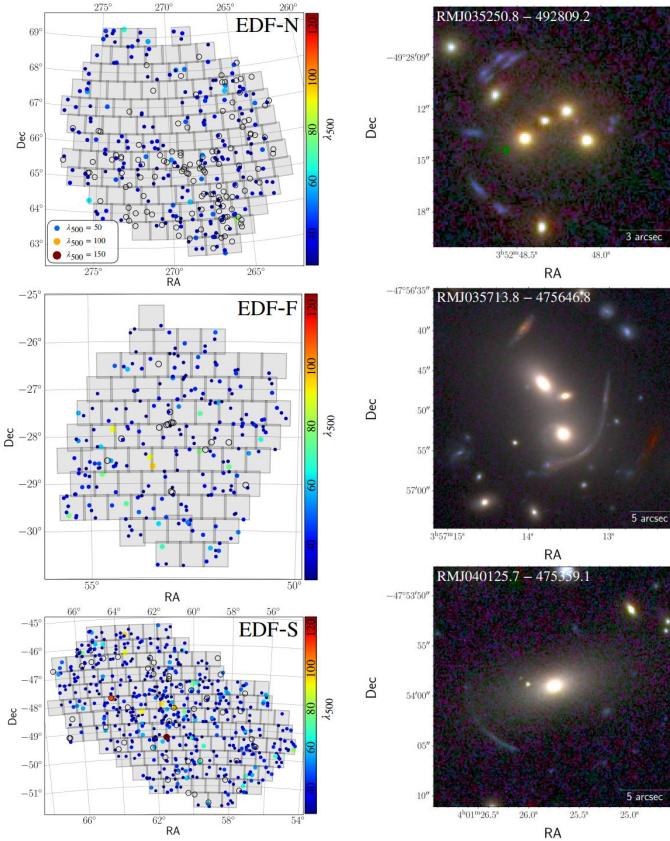
## Outline

Mass distribution in  
galaxy clusters

Galaxy clusters as a  
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source

# Systematic search for lensing clusters

Large area surveys are systematically identifying lensing galaxy clusters. For example, 13 secure lensing clusters have been identified in Euclid Quick Data Release 1 alone ( $63 \text{ deg}^2$  of the extragalactic sky), even a merging galaxy cluster.



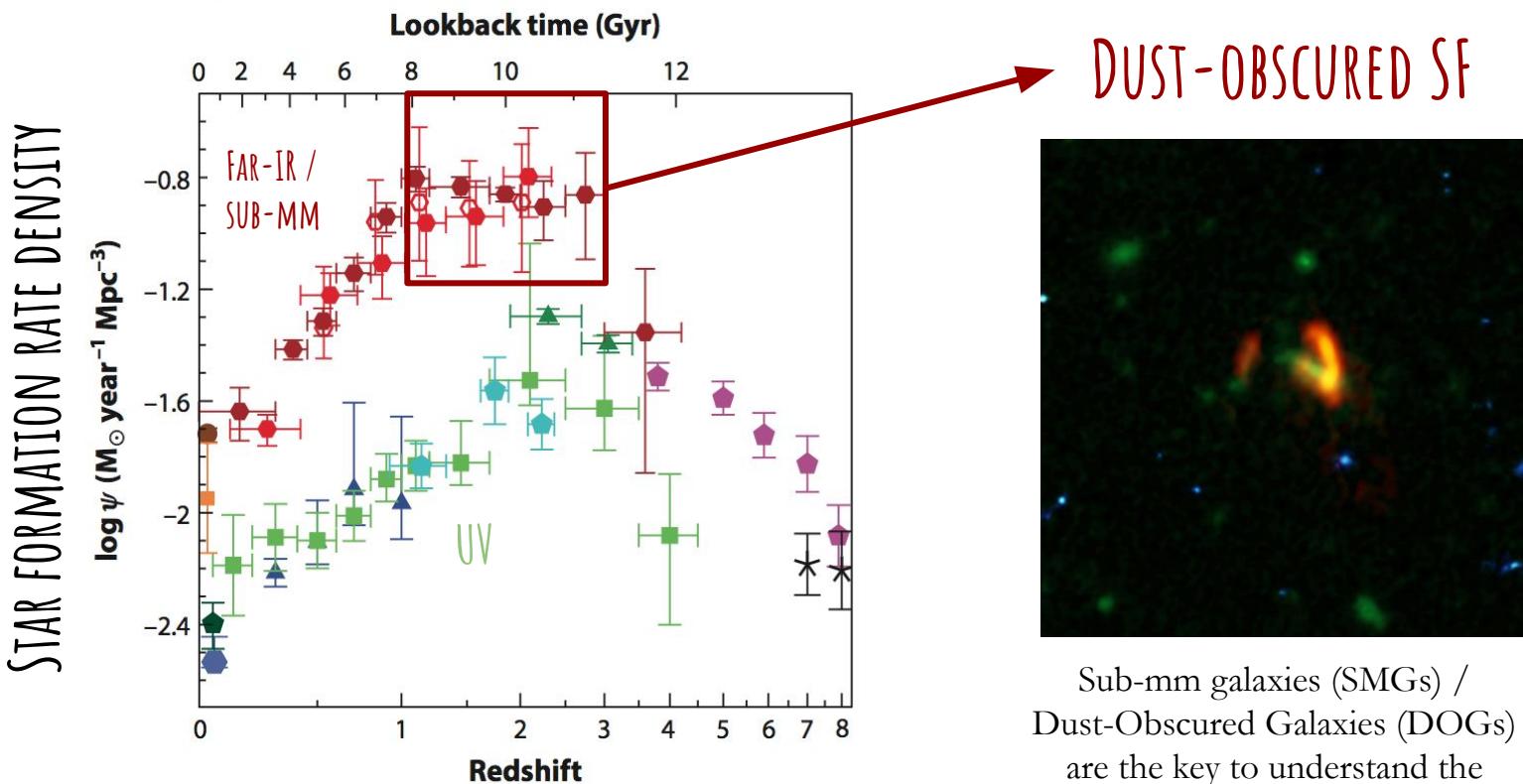
## Outline

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing

# Galaxy-galaxy strong lensing



Madav & Dickinson, ARA&A, 2014

Sub-mm galaxies (SMGs) /  
Dust-Obscured Galaxies (DOGs)  
are the key to understand the  
build-up of galaxies

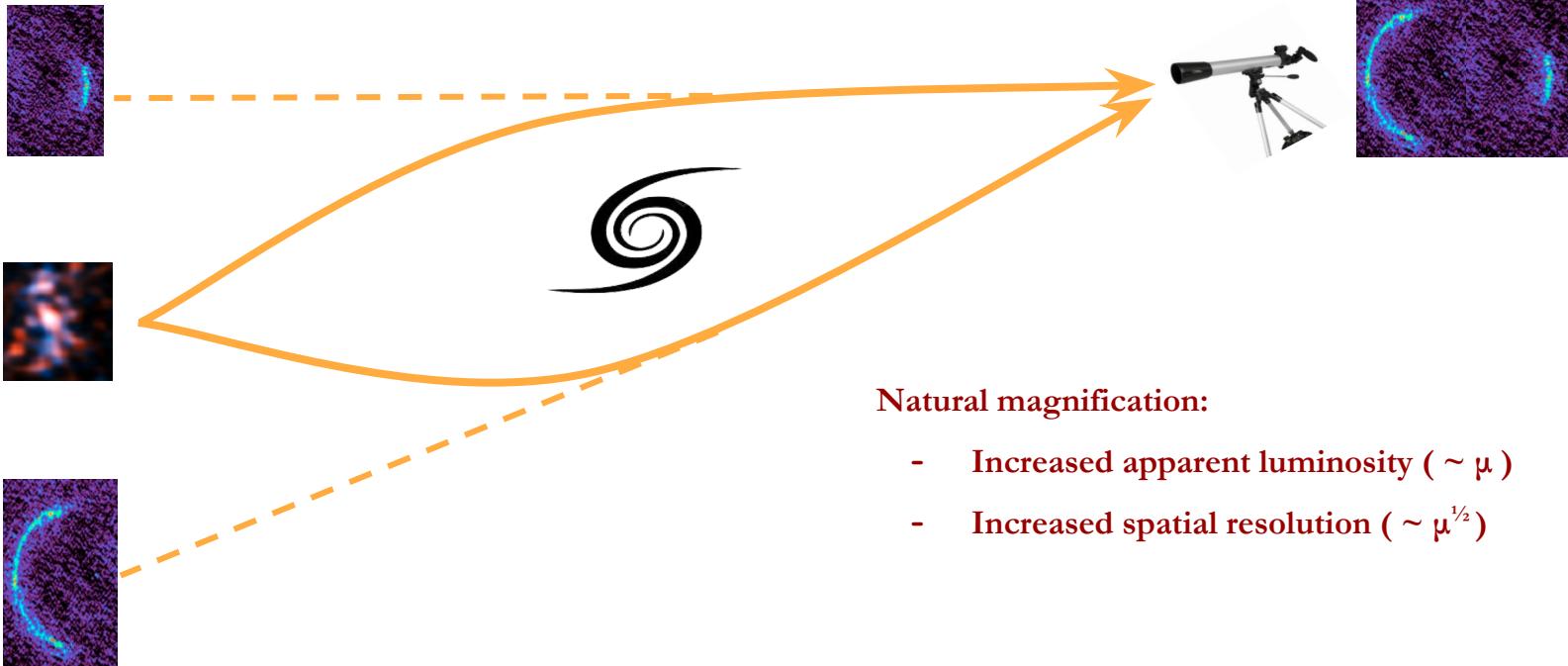
## Outline

# Galaxy-galaxy strong lensing

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing



## Natural magnification:

- Increased apparent luminosity ( $\sim \mu$ )
- Increased spatial resolution ( $\sim \mu^{1/2}$ )

## Outline

# Galaxy-dog strong lensing

Mass distribution in  
galaxy clusters

Galaxy clusters as a  
means to the background  
source

Galaxy-Galaxy strong  
lensing



## Outline

# Galaxy-dog strong lensing

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing



\* NELL'IMMAGINE NON E' STATO MALTRATTATO NELLA PRODUZIONE DI QUESTA SLIDE

## Outline

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing

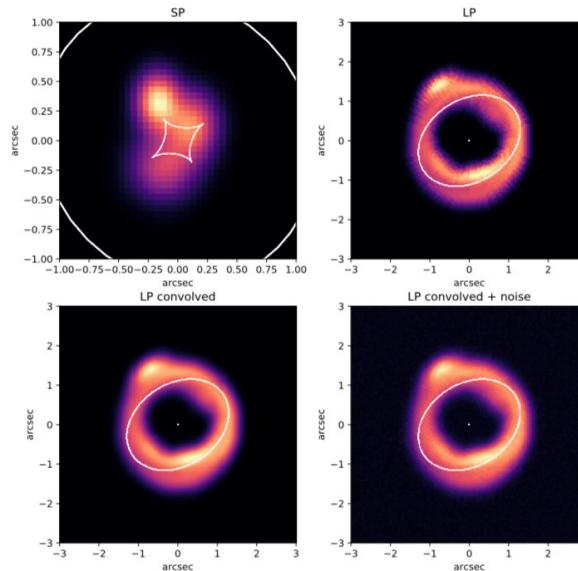
# Galaxy-galaxy strong lensing

## LENS

Find the lens mass distribution model, described by a set of parameters.

E.g.:

- Singular Isothermal Ellipsoid:
  - Velocity dispersion  $\rightarrow$  Einstein radius  $\theta_E$
  - Axis-ratio  $q$
  - Position angle  $\theta_{\text{rot}}$



## Outline

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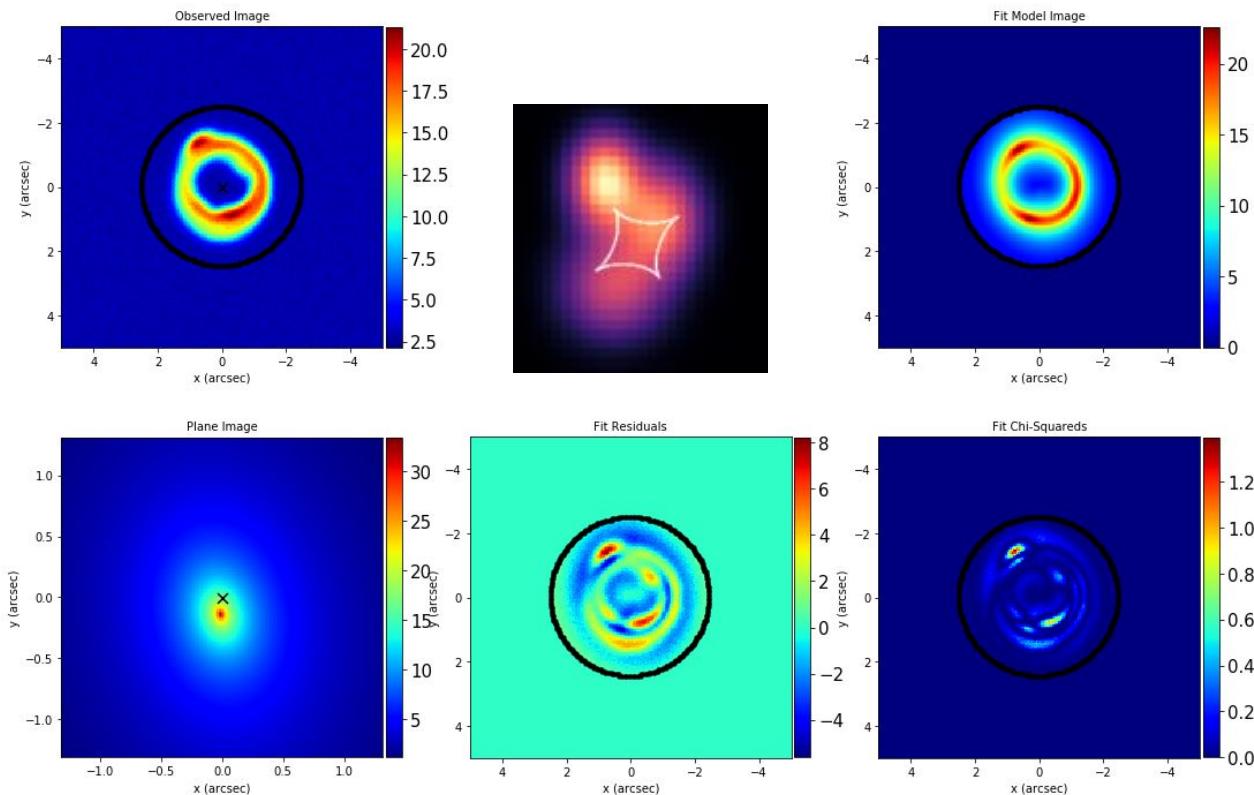
Galaxy-Galaxy strong  
lensing

# Galaxy-galaxy strong lensing

Two main approaches:

- 1) Fully-parametric: the source is also modelled with an analytical model, i.e., a Sérsic profile.

SOURCE



## Outline

Mass distribution in  
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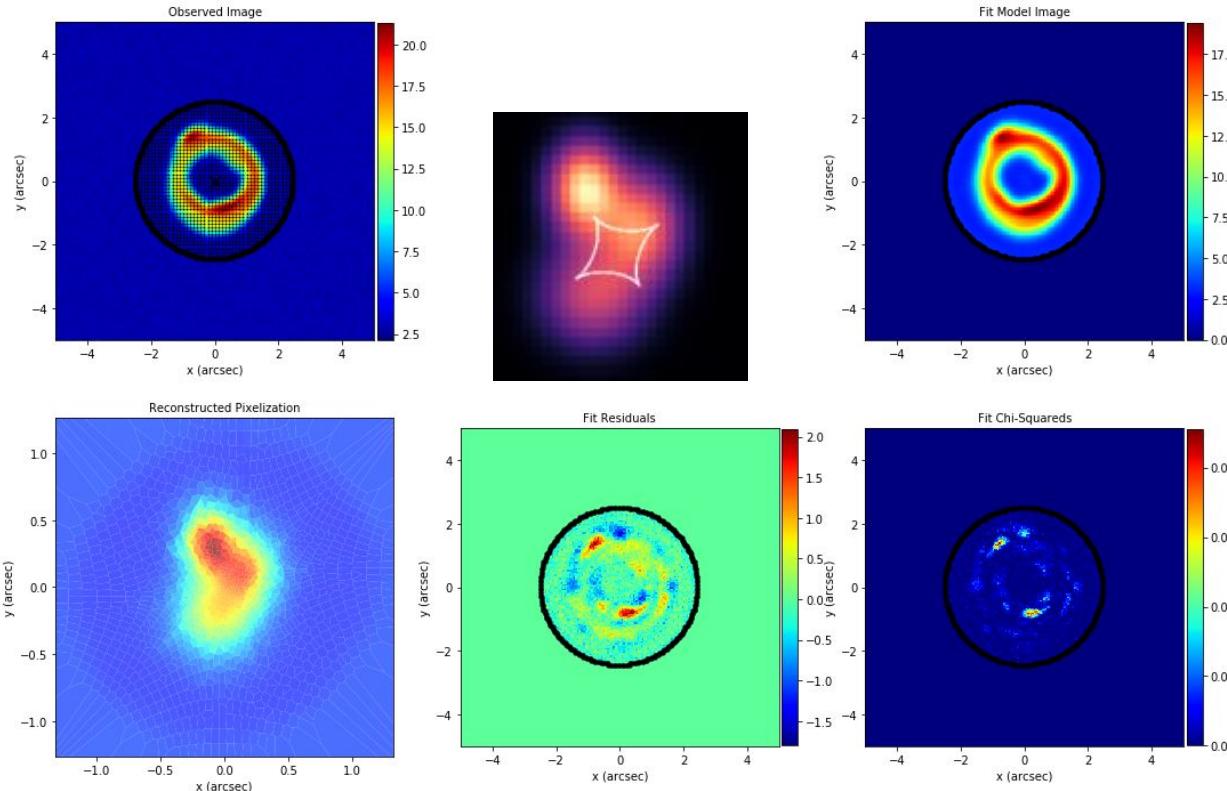
Galaxy-Galaxy strong  
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# Galaxy-galaxy strong lensing

Two main approaches:

SOURCE

- 1) Fully-parametric: the source is also modelled with an analytical model, i.e., a Sérsic profile.
- 2) Semi-parametric: the source is defined with a grid of pixels/tassels, so its morphology is free to vary.



## Outline

Mass distribution in  
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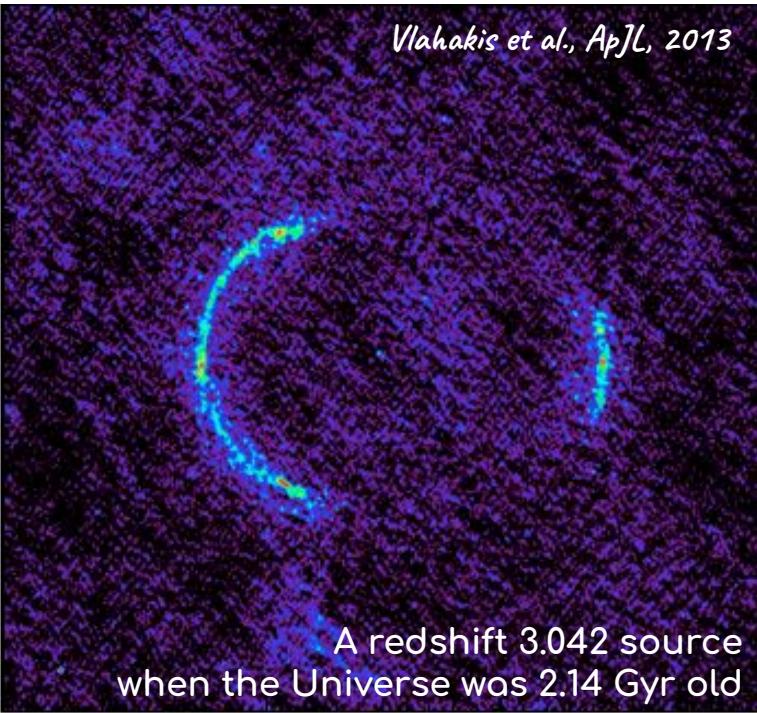
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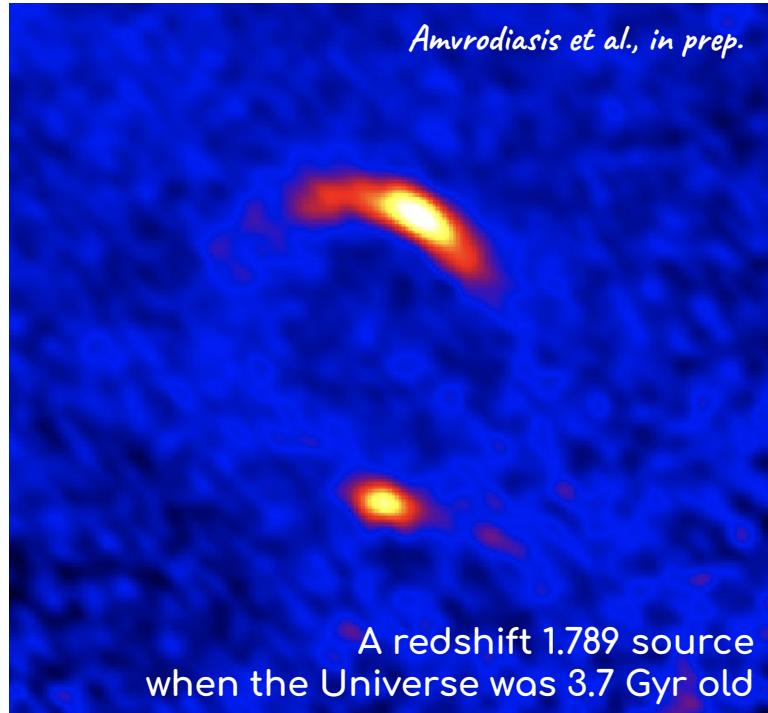
SDP.81 (ALMA, 1.3 mm)

Vlahakis et al., ApJL, 2013



SDP.11 (ALMA, 0.87 mm)

Amvrodiasis et al., in prep.



A strong gravitational lens reveals fine details of sources in the distant Universe  
Proper lens modelling and source reconstruction is needed to reconstruct the original source morphology,  
and fully exploit the lens magnification

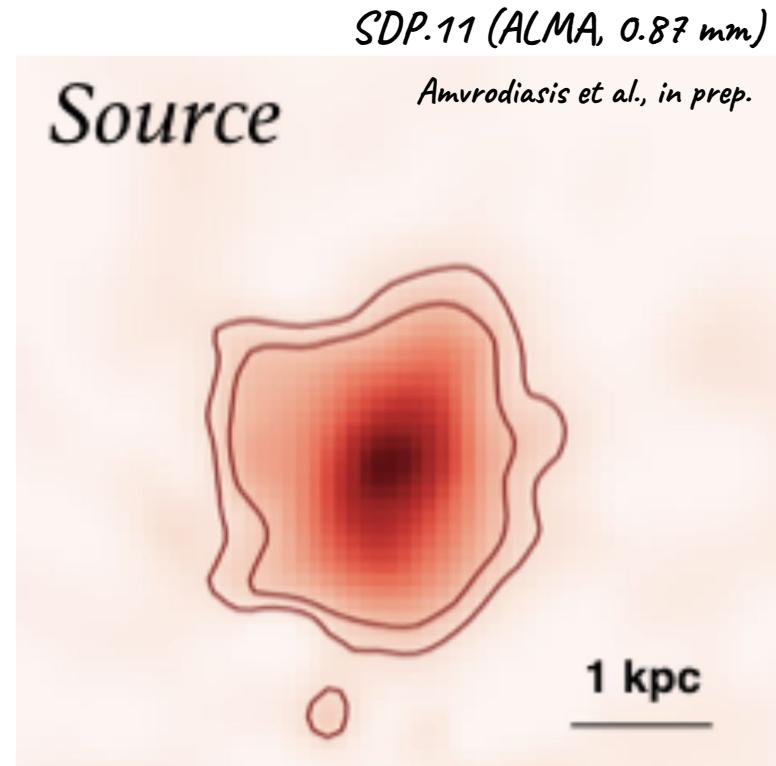
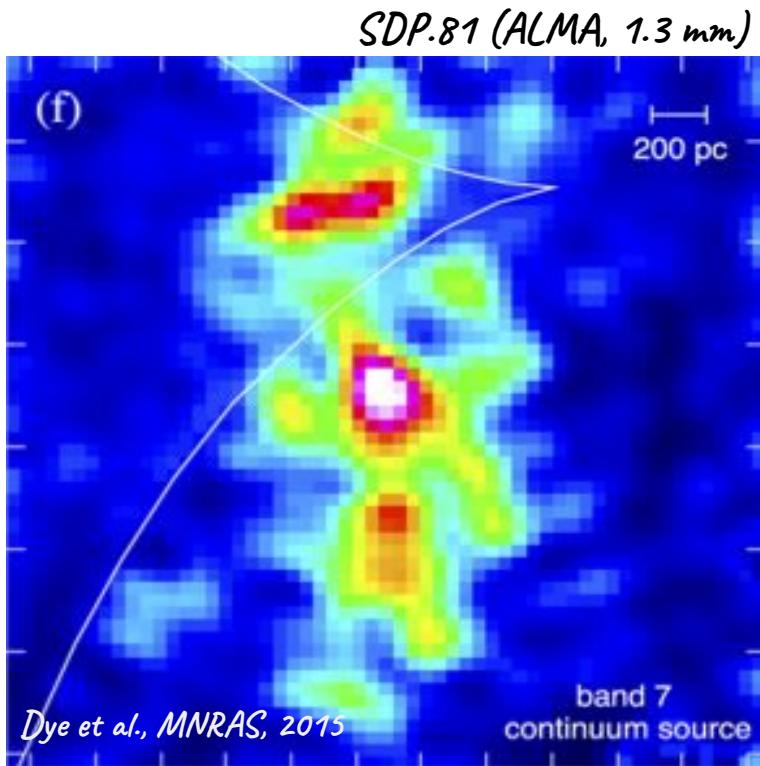
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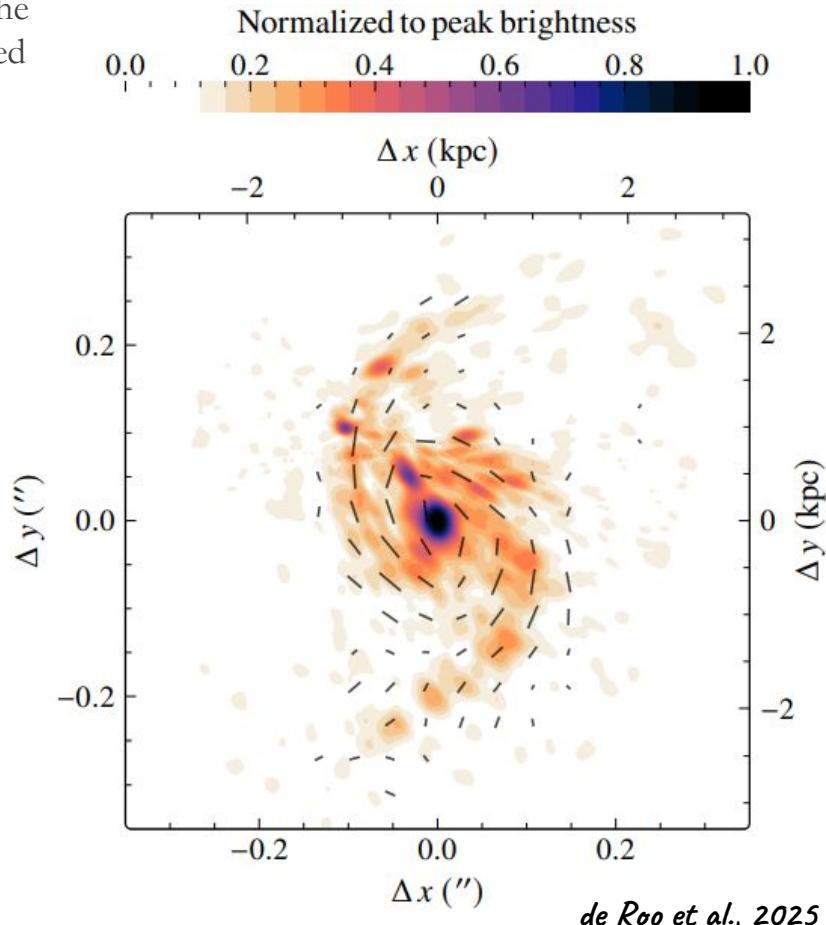
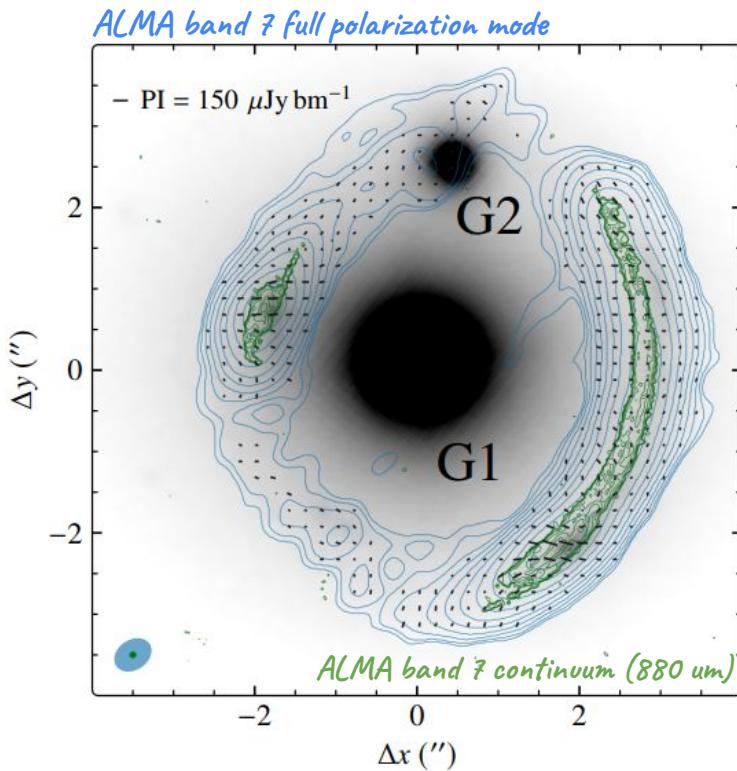
Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing

# Galaxy-galaxy strong lensing

A grand-design spiral at redshift 2.6 – when the Universe was only 2.6 Gyr old – with an ordered magnetic field, traced with ALMA observations.



## Outline

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# Galaxy-galaxy strong lensing



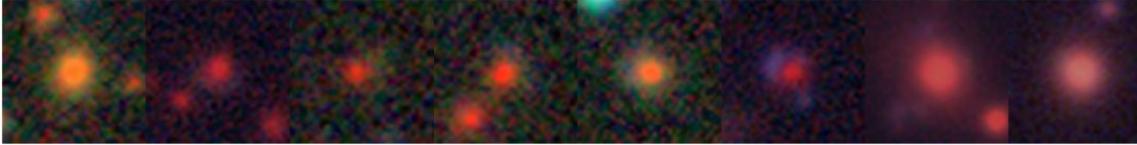
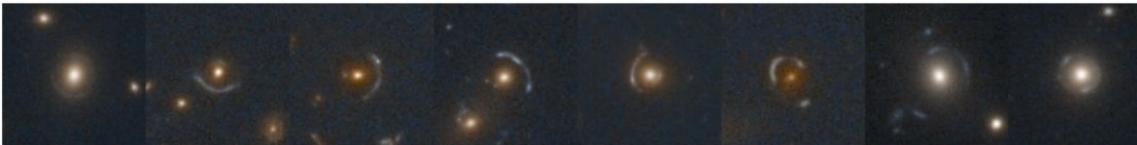
(a) Double source plane (left) and quadruple image (right) lens candidates



(b) Top-ranked edge-on lens candidates



(c) Top-ranked Einstein ring lens candidates



(d) Top-ranked lens candidates (excluding those above) shown with *Euclid* (upper row) vs. Legacy Survey (lower row) images

In the same Euclid Quick Data Release 1, we identified 497 strong lensing candidates, comparable to the most successful extragalactic campaigns while looking only at  $63 \text{ deg}^2$  of the extragalactic sky in approximately six weeks

In the end, Euclid will cover about  $13.000 \text{ deg}^2$  of the extragalactic sky, raising the total number of galaxy-galaxy strong lens by about three orders of magnitude

## Outline

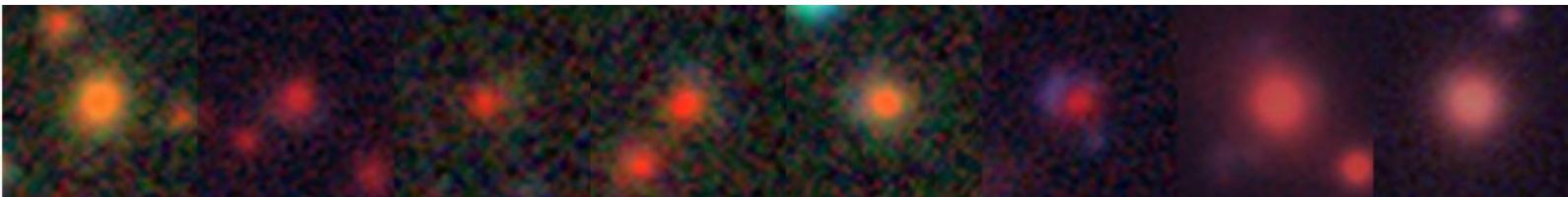
# Galaxy-galaxy strong lensing

Mass distribution in  
galaxy clusters

Galaxy clusters as a  
means to the background  
source

Galaxy-Galaxy strong  
lensing

Before...



## Outline

# Galaxy-galaxy strong lensing

Mass distribution in  
galaxy clusters

Galaxy clusters as a  
means to the background  
source

Galaxy-Galaxy strong  
lensing

... and after Euclid.



## Outline

Mass distribution in galaxy clusters

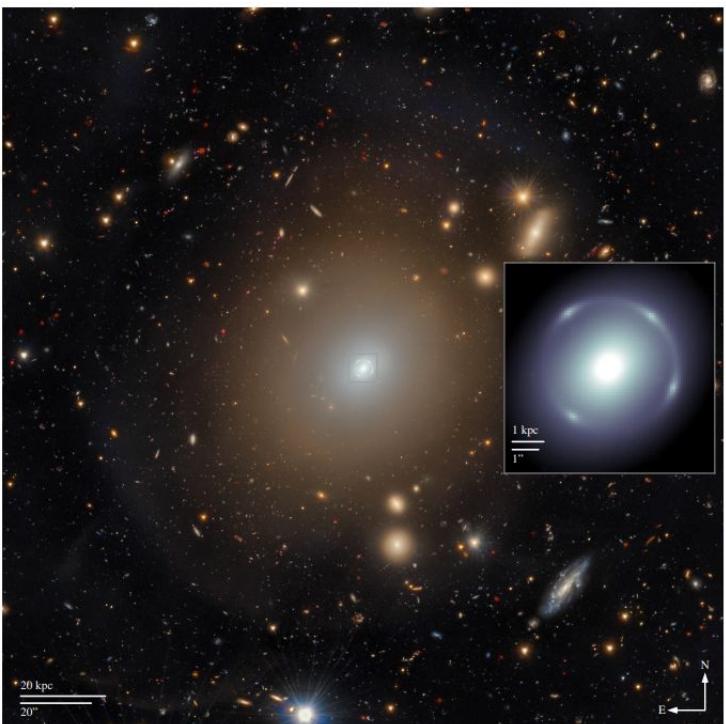
Galaxy clusters as a means to the background source

Galaxy-Galaxy strong lensing

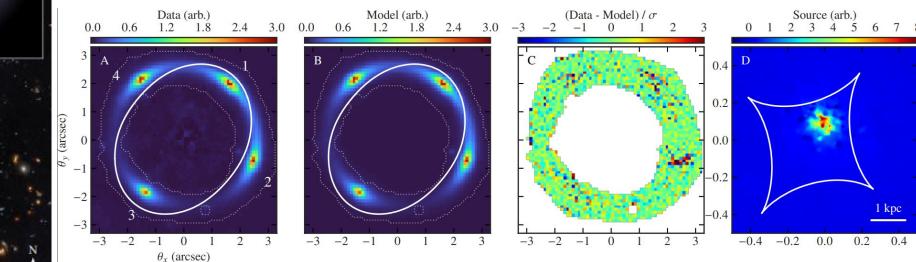
# Galaxy-galaxy strong lensing

“Altieri’s Lens”: a complete Einstein ring around the elliptical galaxy NGC 6505, at  $z = 0.042$ .

This is the *first* strong gravitational lens in an NGC object from any survey. The combination of the low redshift of the lens galaxy, the brightness of the source galaxy ( $I_E = 18.1$  lensed,  $I_E = 21.3$  unlensed), and the completeness of the ring make this an exceptionally rare strong lens.



By combining the model of the Einstein ring (see below) and the distribution of stars of the galaxy, it has been possible to determine that the mass fraction composed of dark matter within the Einstein ring of the lens (2.5 arcsec) is only  $f_{DM} = 11.1^{+5.4}_{-3.5}\%$ .



## Outline

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Mapping the lens mass distribution

## Map the lens mass distribution

Being lensing a consequence of the presence of (lots of) matter, whatever kind of (dark, visible, invisible, solid, gaseous, living, or dead), when you perform lens modelling, you're not just reconstructing a source; you're, well, **LENS** modelling.

This means having direct access to how matter is distributed in a galaxy (well, not the whole galaxy, but within the Einstein radius).

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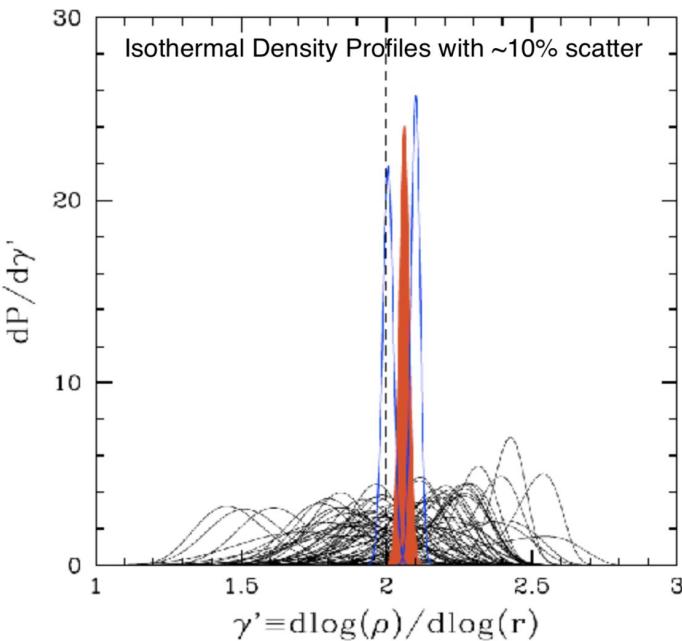
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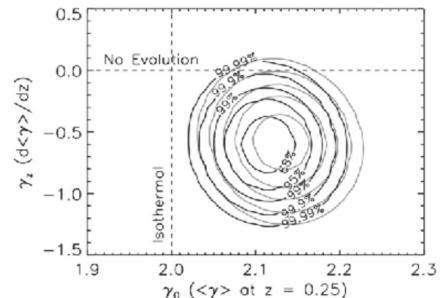
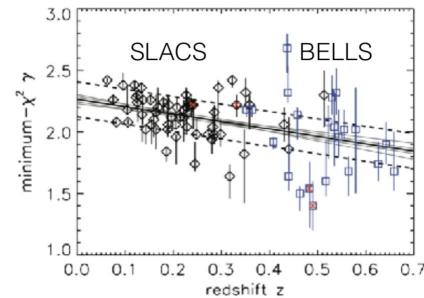
These studies found that the sum of DM + baryonic mass in elliptical galaxies is always an isothermal profile ( $\rho \propto r^{-2}$ ), despite different physical and environmental starting conditions, which is something that's not quite well understood.



Koopmans et al., 2009

Also, there's some evidence for a mild evolution of the mass-density slope in ETGs with redshift.

Anyway, lensing is a powerful key for galaxy evolution and cosmic structure formation.



Bolton et al., 2012

## Outline

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

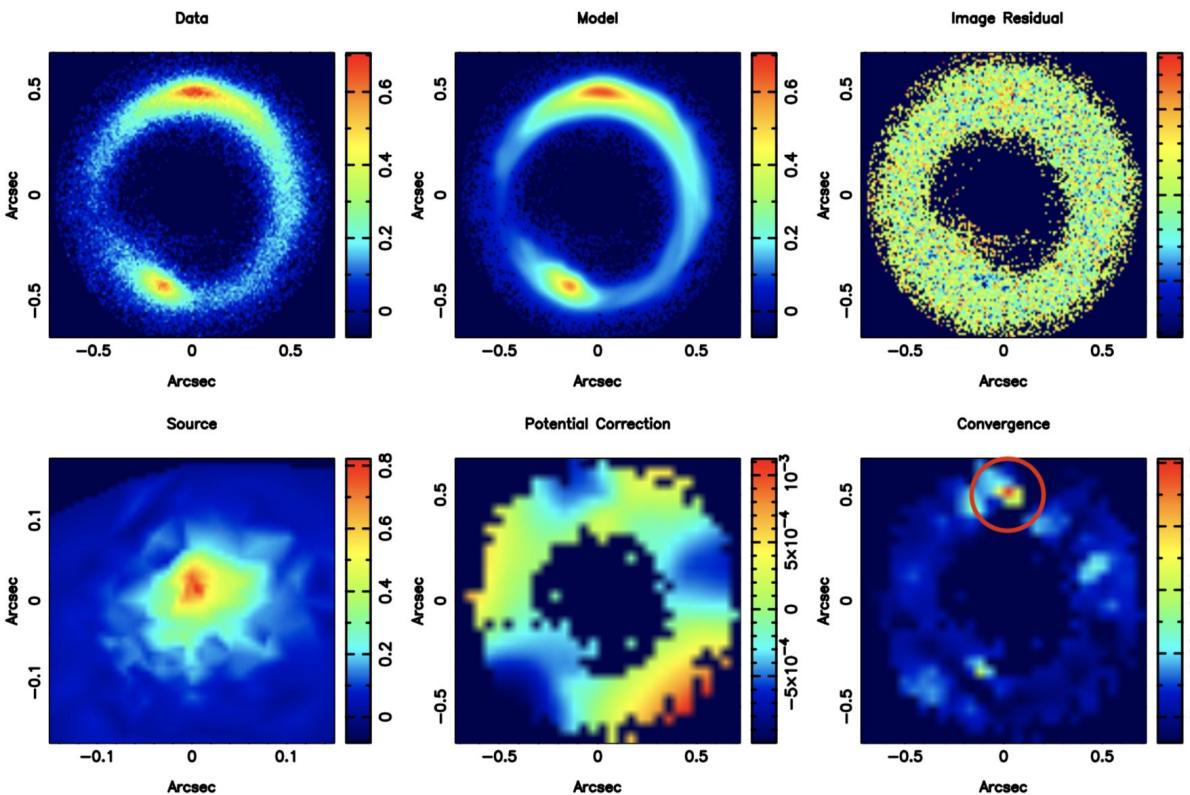
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Also, strong lensing is able to answer (or at least give it a try) to the famous “missing satellite problem”. The presence of DM substructures is the smoking gun for cosmological models such as  $\Lambda$ CDM, and characterizing their distribution (or absence of) can wipe out lots and lots of theoretical models.

## Outline

Mass distribution in galaxy clusters

Galaxy clusters as a means to the background source

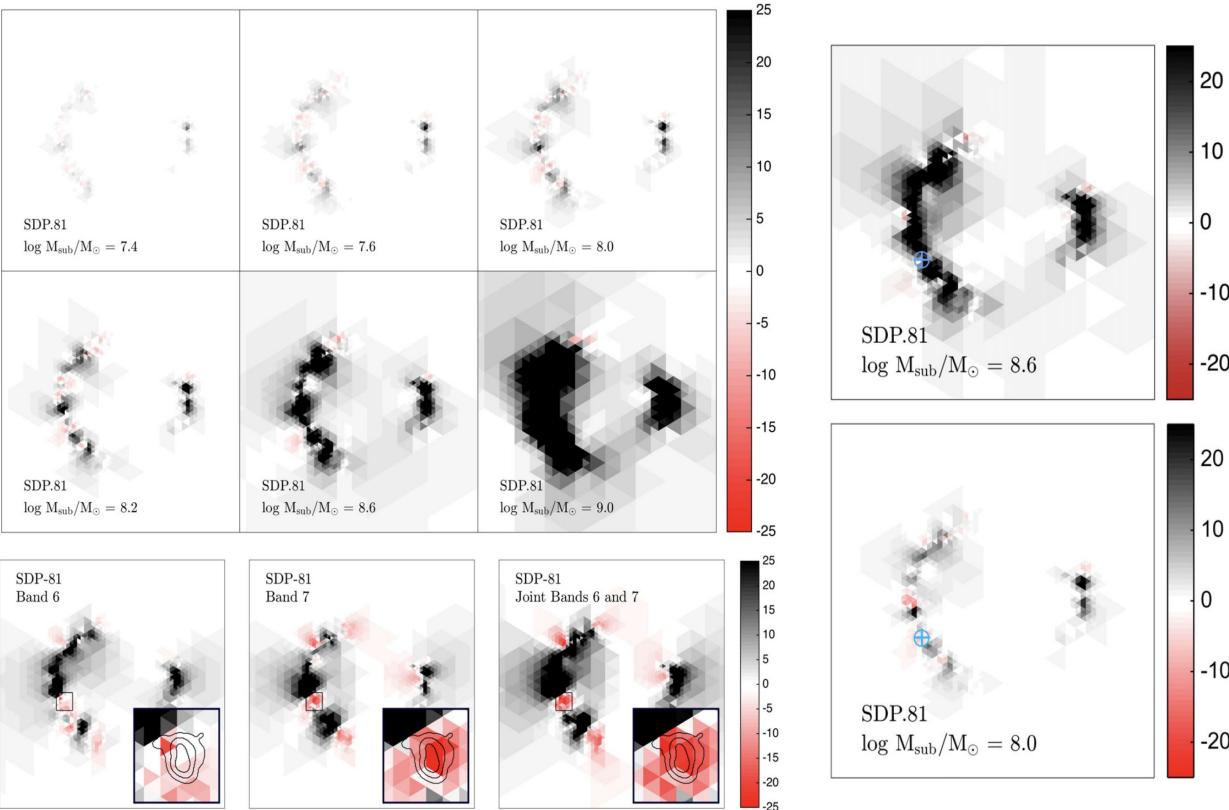
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The fraction of strongly lensed objects

# The fraction of strongly lensed objects at high-z

What is the probability to obtain a strong lensing event along a generic line of sight?

The answer is given by the integral of the number density of lenses  $n(z)$  times the strong lensing cross section:

$$P(z) = \int_0^z n(z')\sigma(z')dr_{\text{prop}}(z')$$

also viewed in terms of **lensing optical depth**  
(the fraction of sky covered with Einstein rings)

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Let's take the cross section corresponding to the Einstein radius  $\theta_E$

$$\sigma = \pi R^2 = \pi(\theta_E D_D)^2 = \frac{4\pi GM}{c^2} \frac{D_D D_{DS}}{D_S}$$

this distance here peaks when  $D_D = D_{DS}$   
→ lens halfway between the source and the observer

**The maximal lensing probability happens for lenses falling between  $z \approx 0.3 - 1$  and sources at  $z \approx 1 - 3$**

# The fraction of strongly lensed objects at high-z

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In the end:

$$\tau(z) = P(z) \sim \frac{3\Omega_D}{2} \int_0^{z_s} \frac{d_D d_{DS}}{d_S} \frac{1+z}{\sqrt{1+\Omega z}} dz \quad \text{where } d_D \equiv H_0 D_D / c \quad \text{and similarly for } d_S \text{ and } d_{DS}$$

For  $\Omega = 1$  and  $z_s = 3$ , considering that  $d_D d_{DS} / d_S \sim 1 \rightarrow \tau(z_s = 3) \sim 0.5\Omega_D$

The most effective lensing structures are galaxy's cores (especially early-type, massive elliptical ones)

$$\Omega_D \sim 10^{-1} \Omega_{\text{gal}} \sim 10^{-3} \rightarrow \text{roughly one out of } 10^3 - 10^4 \text{ high-z quasars are strongly lensed}$$

## Outline

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A deeper quantitative analysis require a more precise evaluation of the strong lensing probability as a function of the angular distance between the source and the lens.  
i.e. for point mass lenses

$$\theta_{\pm} = \frac{1}{2}(\beta \pm \sqrt{\beta^2 - 4\theta_E^2}) \quad \text{and magnification} \quad \mu(x) = \frac{x^2 + 2}{x\sqrt{x^2 + 4}} \quad \text{with} \quad x \equiv \beta/\theta_E$$

(obtained from the  $\mu$  definition)

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Now let's compute the lensing cross-section as a function of the flux amplification

= the proper area around a given lens through which the un-deflected light ray would need to pass to cause amplifications greater than  $\mu$

$$\sigma(>\mu) = \pi[D_D \beta(\mu)]^2 \quad \beta(\mu) \text{ being the source undeflected angle on sky within which the amplification is larger than } \mu$$

$$\beta^2 = 2\theta_E^2 \frac{\mu - \sqrt{\mu^2 - 1}}{\sqrt{\mu^2 - 1}} \implies \sigma(>\mu) = \frac{8\pi GM}{c^2} \frac{D_D D_{DS}}{D_S} \frac{1}{\mu^2 + 1 + \mu\sqrt{\mu^2 - 1}} \propto \mu^{-2}$$

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For an isothermal sphere (Peacock, 1999):

$$\sigma(>\mu) = \left[ \frac{4\pi G <v_{||}^2>}{c^2} \right]^2 \left( \frac{D_D D_{DS}}{D_S} \right)^2 \frac{4\pi}{\mu^2} \quad \text{if } \mu > 2$$

$$\sigma(>\mu) = \left[ \frac{4\pi G <v_{||}^2>}{c^2} \right]^2 \left( \frac{D_D D_{DS}}{D_S} \right)^2 \frac{\pi}{(\mu-1)^2} \quad \text{if } \mu < 2$$

And the probability to obtain a strong lensing event along an arbitrary line of sight with amplification  $> \mu$  is:

$$P(z) = \int_0^z n(z') \sigma(>\mu, z') dr_{\text{prop}}(z') \propto \mu^{-2}$$

This, of course, depends also on other cosmological / population parameters, in particular  $\Omega_m$  and  $\Omega_\Lambda$ .

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# The fraction of strongly lensed objects at high-z

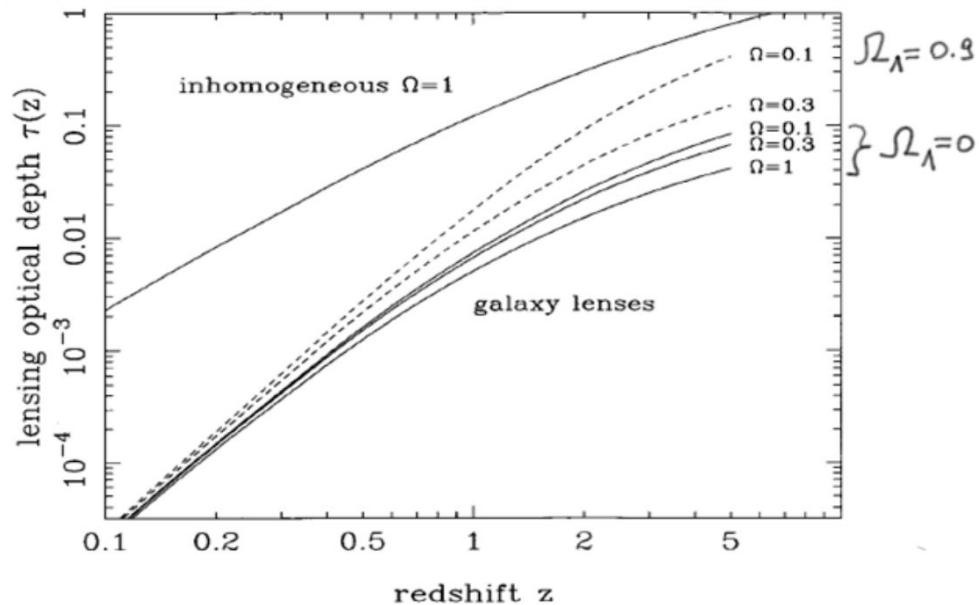
Cosmic Lens All Sky Survey (CLASS), a radio survey with the Very Large Array (VLA), the Very Long Baseline Array (VLBA) and MERLIN.

They found a ratio of lensed:unlensed radio sources of one per  $690 \pm 190$  targets (Mitchell et al., 2005), consistent with

$$\Omega_\Lambda + \Omega_m = 1$$

and leading to

$$\Omega_\Lambda = 0.69^{+0.31}_{-0.24}$$



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## Outline

Mass distribution in galaxy clusters

The deflection of light always causes a delay in the travel-time of light between the source and the observer. This time delay has two components:

$$\Delta t = \boxed{\Delta t_{\text{grav}}} + \boxed{\Delta t_{\text{geom}}}$$

*Shapiro time delay* ← → *Geometric delay*

The diagram illustrates the formula for time delay. It shows the equation  $\Delta t = \Delta t_{\text{grav}} + \Delta t_{\text{geom}}$ . The term  $\Delta t_{\text{grav}}$  is enclosed in a blue box and labeled "Shapiro time delay" with a blue arrow pointing to it. The term  $\Delta t_{\text{geom}}$  is enclosed in an orange box and labeled "Geometric delay" with an orange arrow pointing to it.

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Remember: light travelling in a different metric travels with a speed  $c' = c/n$  (in the observer reference frame)

$$c\Delta t_{\text{grav}} = - \int (1 + z_D) \frac{2\phi}{2} dl$$

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Flying Madrid - Oslo, Oslo - Malta, Malta - Palermo is slower than directly flying Madrid - Palermo

$$c\Delta t_{\text{geom}} = (1 + z_D) \frac{D_D D_S}{D_{DS}} \frac{\alpha^2}{2}$$

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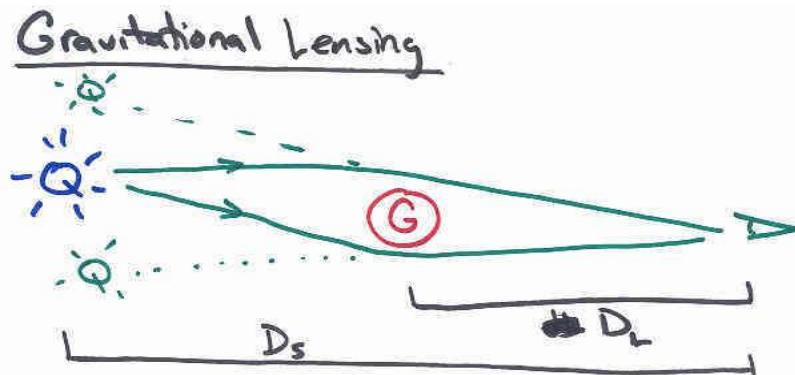
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Both depend on lens specifics (potential and deflection angle); therefore, **the different multiply lensed images of the same source will be delayed in time**: one arrives first, the others follow, and the actual delay depends on the lens and cosmology.



Lens potential

$$\Delta t \propto D_{\Delta t} \times \phi_{\text{lens}} \rightarrow D_{\Delta t} \propto 1/H_0$$

Time-delay

Time-delay distance

$$D_{\Delta t} = (1 + z_D) \frac{D_S D_D}{D_{DS}}$$

Hubble Constant

## Outline

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Galaxy clusters as a means to the background source

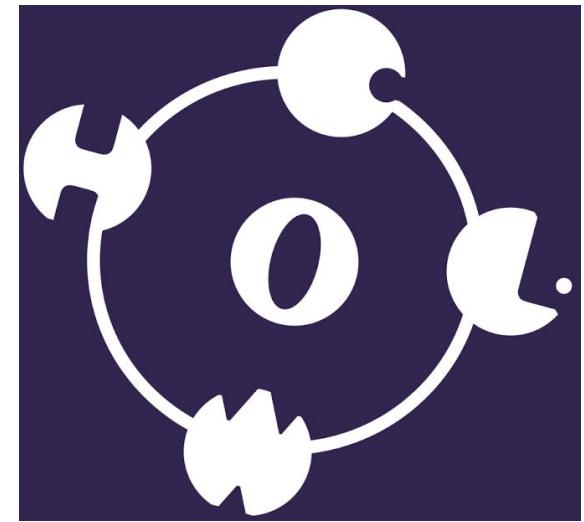
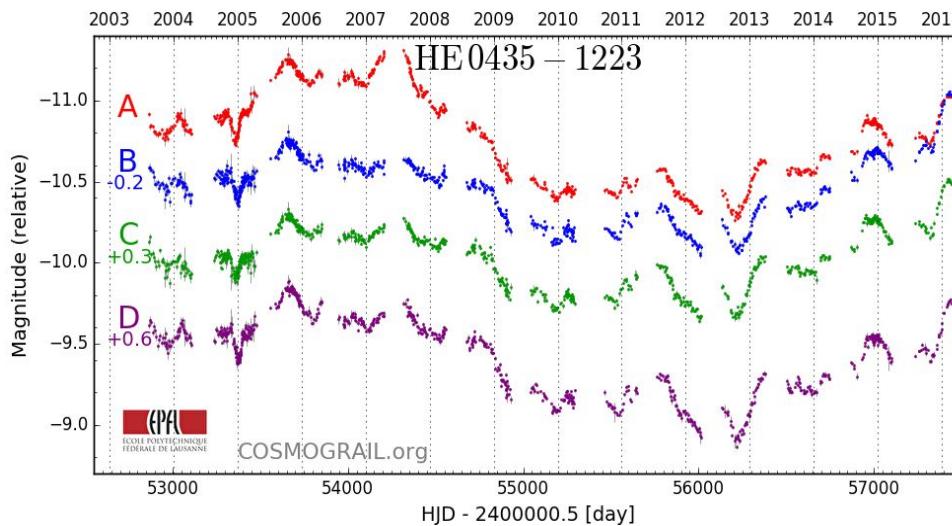
Galaxy-Galaxy strong lensing

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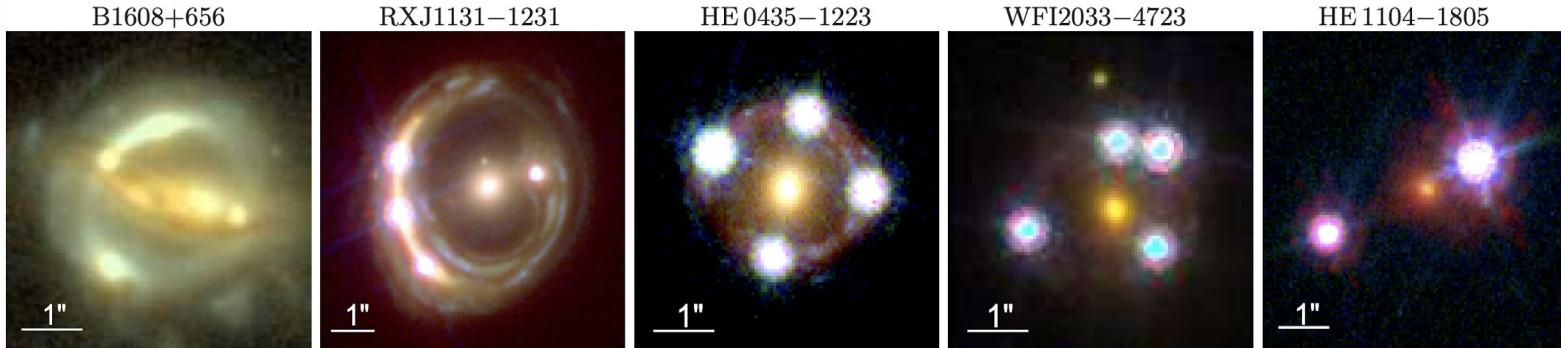
The fraction of strongly lensed objects

Time delays and lensing cosmography

# Time delay cosmography



<https://shsuyu.github.io/HOLiCOW/site/>



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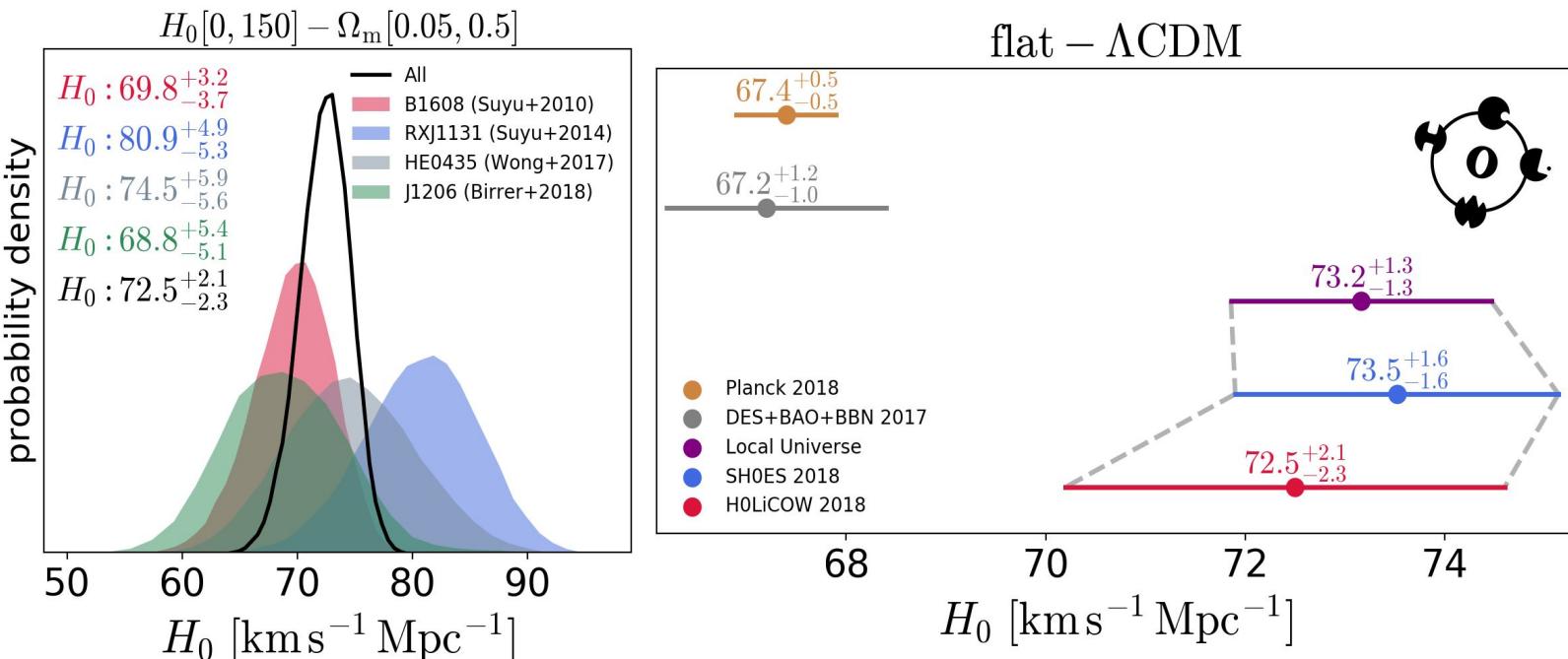
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<https://shsuyu.github.io/HoLiCOW/site/>

Hubble constant  $H_0$  measurement from blind analysis of four multiply-imaged quasar systems through strong gravitational lensing:  $H_0 = 72.5^{+2.1}_{-2.3}$  km/s/Mpc, at 3% precision, in the standard flat  $\Lambda$ CDM model.

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Galaxy clusters as a means to the background source

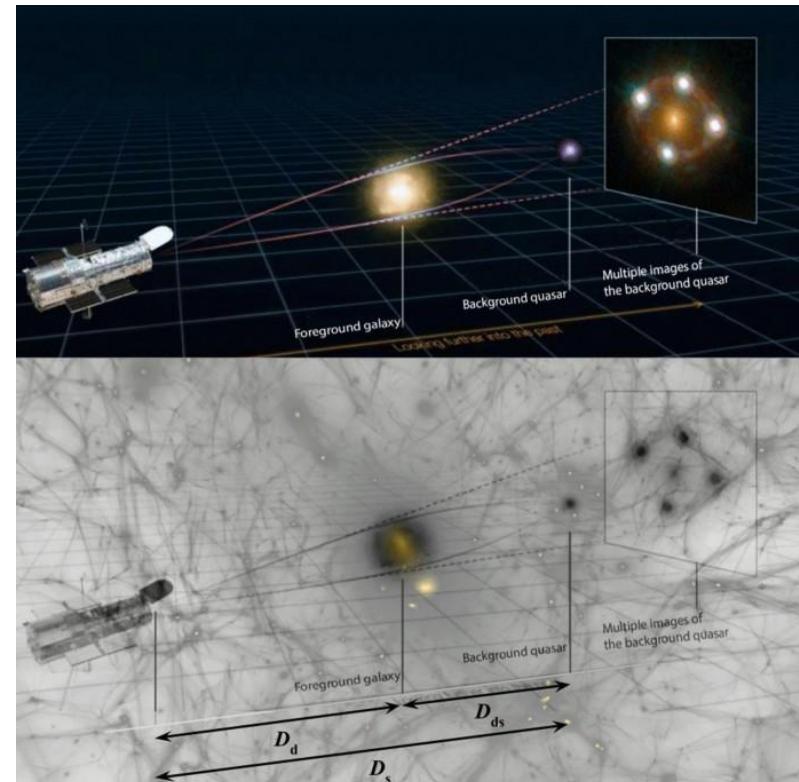
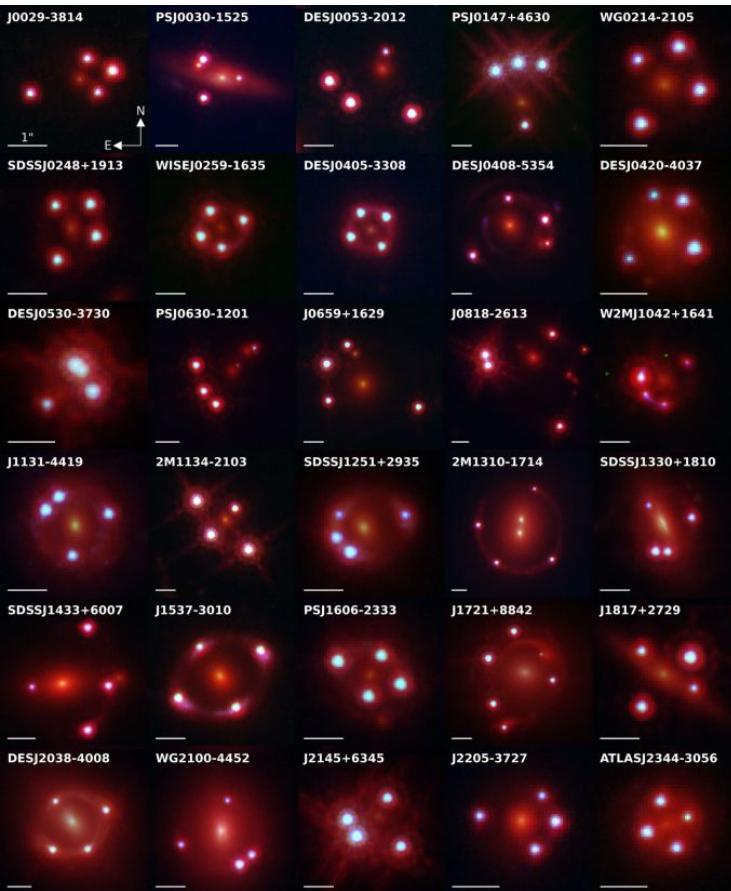
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Schmidt et al., 2022

Treu et al., 2022

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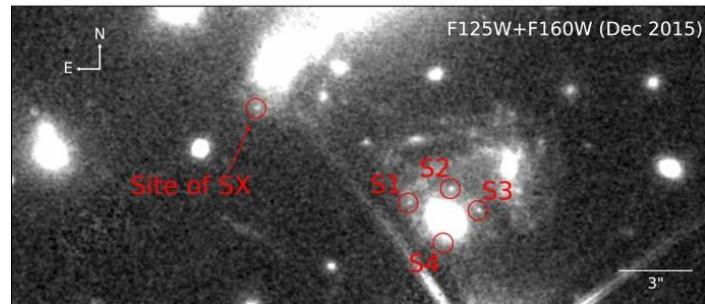
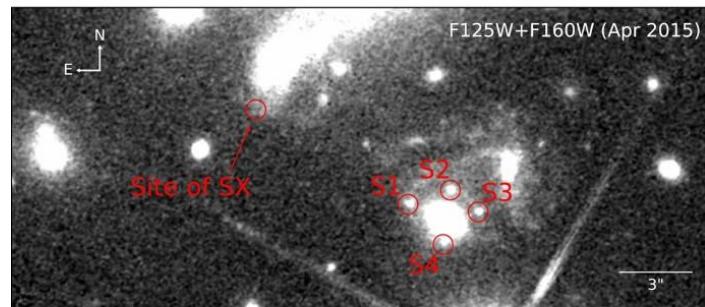
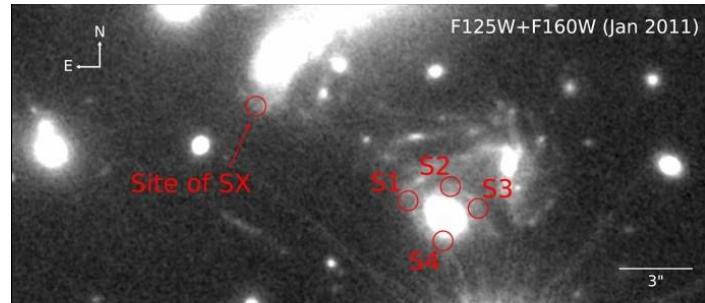
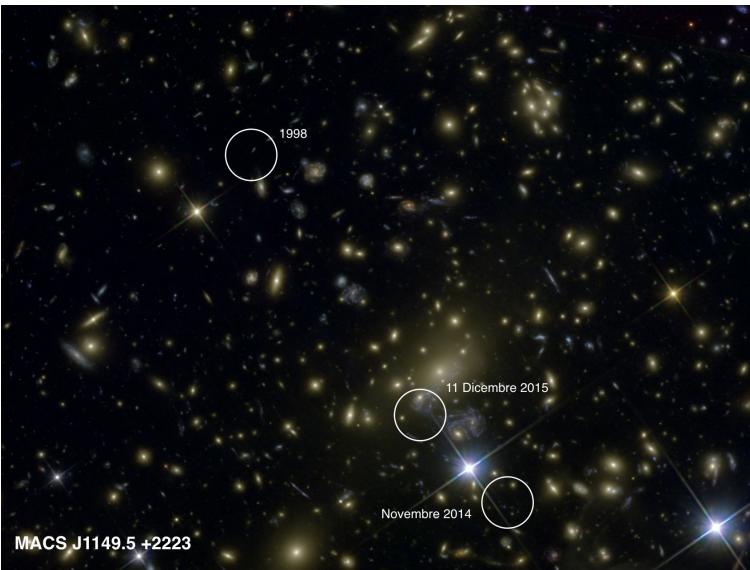
Time delays and lensing cosmography

# Time delays and the Refsdal SNa

## SN Refsdal

There is so much mass in a galaxy cluster, and therefore lots of multiple images, that you can see the same event happen multiple times, like an SN explosion in a redshift 1.49 distant galaxy.

The timing of the apparitions strongly depends on cosmological parameters.



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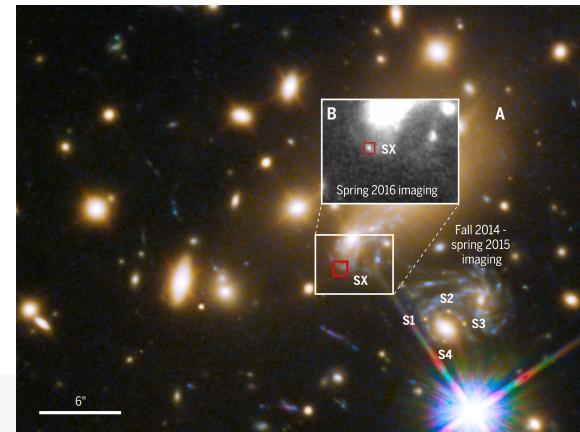
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### CONCLUSION

We infer a value of  $H_0$  of  $64.8^{+4.4}_{-4.3} \text{ km s}^{-1} \text{ Mpc}^{-1}$  using the full set of eight pre-reappearance models and of  $66.6^{+4.1}_{-3.3} \text{ km s}^{-1} \text{ Mpc}^{-1}$  from the two preferred models. Our results are most consistent with the  $H_0$  value measured from the CMB but do not exclude the higher value from nearby SNe.

We used a simulation of a galaxy cluster lens to verify that the uncertainty on our measurement of  $H_0$  is consistent with expectations. The ability of the lens models to reproduce the positions of the SN images also implies an expected uncertainty on  $H_0$ , which we find agrees with our constraints. The best agreement between lens models and observations that are independent of  $H_0$  is achieved by the models that were constructed by assigning dark-matter halos to both the cluster and to individual galaxies in the cluster.



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Microlensing

## Microlensing

We talk about microlensing when the spatial resolution is not enough to resolve the lensing features (i.e. Einstein rings, multiple images at the order of *milliarcsec* or *microarcsec*), but a lensing event is still happening: flux magnification, with a unique light-curve shape.

$$\theta_E = \left( \frac{D_{DS}}{D_D D_S} \frac{4GM(\theta)}{c^2} \right)^{1/2} \sim 0.3 \times 10^{-3} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D'}{10 \text{ kpc}} \right)^{1/2} \text{ arcsec}$$

Broad range of masses: planets, stars, star clusters, compact objects in MW or other galaxies (but mainly the LMC/SMC). Historically, the first method to actually probe if dark matter is made of MAssive Compact Halo ObjectS (MACHOs).

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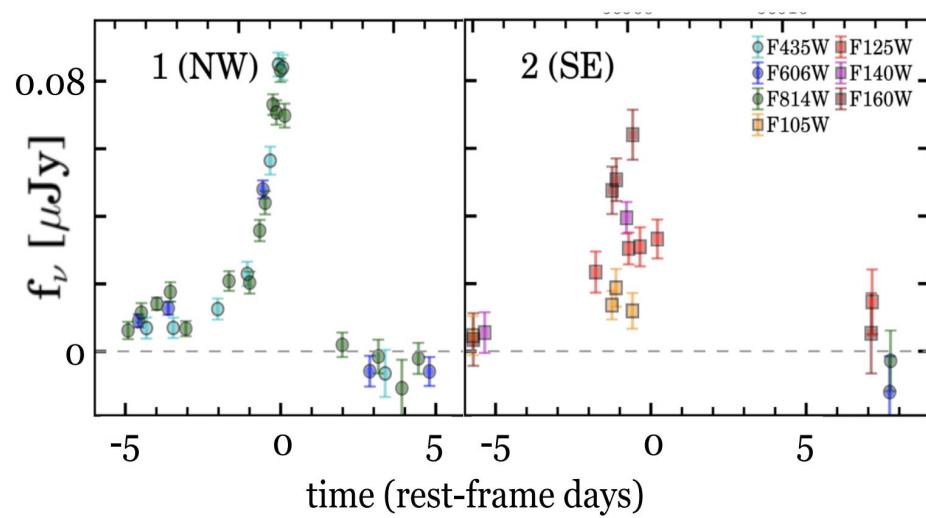
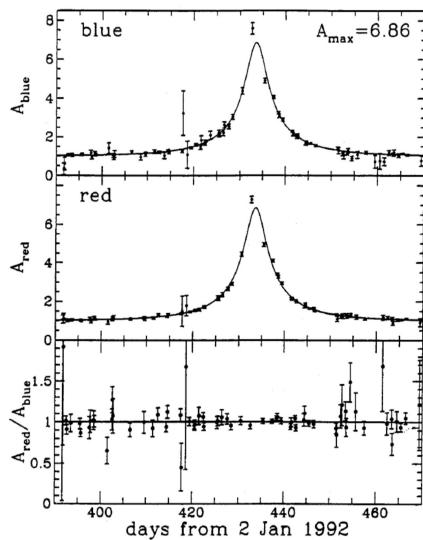
Microlensing

## Microlensing

We talk about microlensing when the spatial resolution is not enough to resolve the lensing features (i.e. Einstein rings, multiple images at the order of *milliarcsec* or *microarcsec*), but a lensing event is still happening: flux magnification, with a unique light-curve shape.

$$\theta_E = \left( \frac{D_{DS}}{D_D D_S} \frac{4GM(\theta)}{c^2} \right)^{1/2} \sim 0.3 \times 10^{-3} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D'}{10 \text{ kpc}} \right)^{1/2} \text{ arcsec}$$

Broad range of masses: planets, stars, star clusters, compact objects in MW or other galaxies (but mainly the LMC/SMC). Historically, the first method to actually probe if dark matter is made of MAssive Compact Halo ObjectS (MACHOs).



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# Microlensing

The characteristic time-scale for a microlensing event is:

$$\dot{\theta} = \frac{v}{D_D} = 4 \times 10^{-3} \left( \frac{v}{200 \text{ Km/sec}} \right) \left( \frac{D_D}{10 \text{ kpc}} \right) \text{ arcsec yr}^{-1}$$

and the typical time-scales of the flux variations due to the lensing event is:

$$t_E = \frac{\theta_E}{\dot{\theta}} = 0.2 \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{v}{200 \text{ Km/sec}} \right)^{-1} \left( \frac{D_D}{10 \text{ kpc}} \right)^{1/2} \left( 1 - \frac{D_D}{D_S} \right)^{1/2} \text{ yr}$$

which is a couple of months for typical parameter values. Finally, the microlensing cross-section is directly related to the Einstein ring as:

$$\sigma_{\text{micro}} = \pi \theta_E^2$$

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Great, so microlensing is measurable in human timescales. The problem is that there are billions of reasons why a star could suddenly change its flux, all linked with their intrinsic variabilities / environmental things all unrelated to gravitational lensing.

Solutions:

- lensing is achromatic, so if the same flux variability is detected in multiple bands its a clear indication that microlensing is happening there
- smoking gun: **microlensing light light curve:**
  - 1) Unique shape
  - 2) Symmetric with respect to the maximum
  - 3) Achromatic

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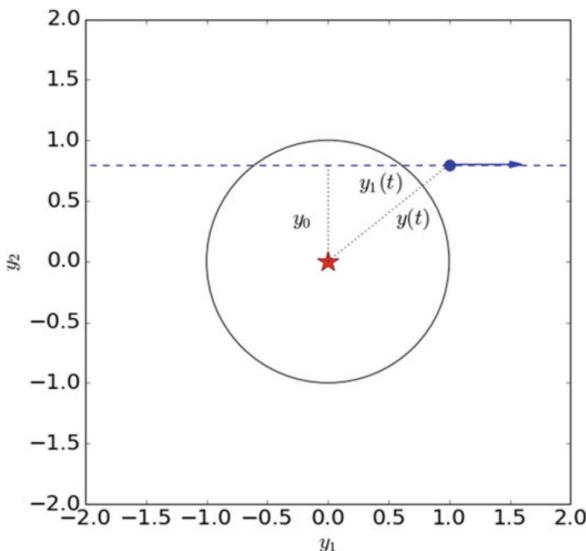
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## Microlensing



We have a foreground **lens** (center) and a moving background **source** moving along a straight line (which is a good approximation of a real microlensing application).

$y_0$  is the normalized impact parameter (in units of  $\theta_E$ ), which is the closest lens-source distance at time  $t_0$ .

The source trajectory is:

$$\begin{aligned}y(t) &= (y_0^2 + y_1^2(t))^{1/2} = \left( y_0^2 + \frac{\dot{\theta}^2(t-t_0)^2}{\theta_e^2} \right)^{1/2} \\&= \sqrt{y_0^2 + \left( \frac{t-t_0}{t_E} \right)^2}\end{aligned}$$

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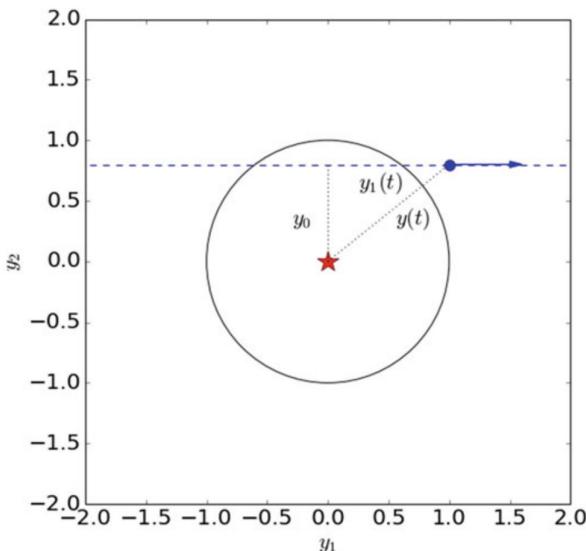
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The magnification  $\mu$  for a point mass lens is:

$$\mu(y) = \frac{y^2+2}{y\sqrt{y^2+4}}$$

Therefore the source flux changes in time as:

$$S(t) = S_0 \times \mu[y(t)] = S_0 \times \frac{y^2+2}{y\sqrt{y^2+4}}$$

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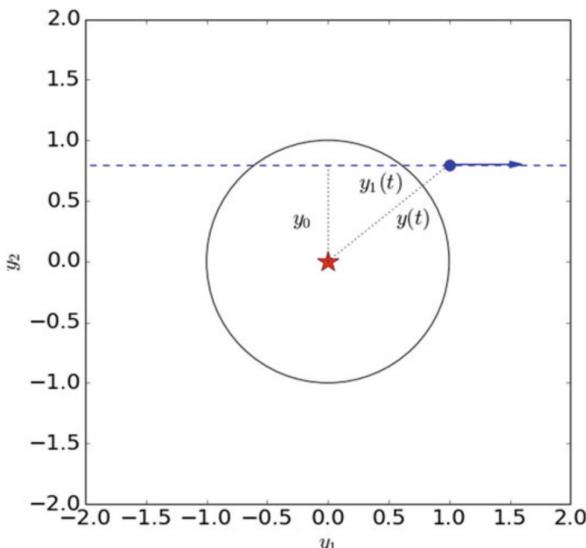
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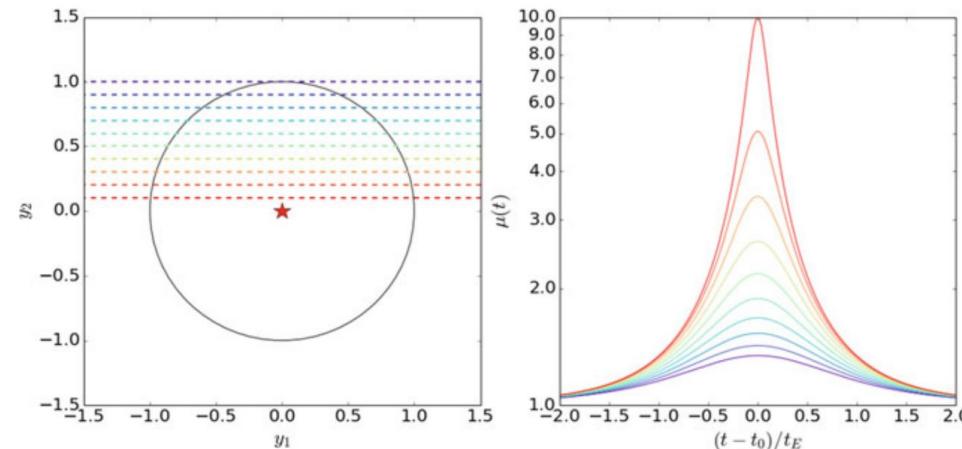
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## Microlensing and star masses

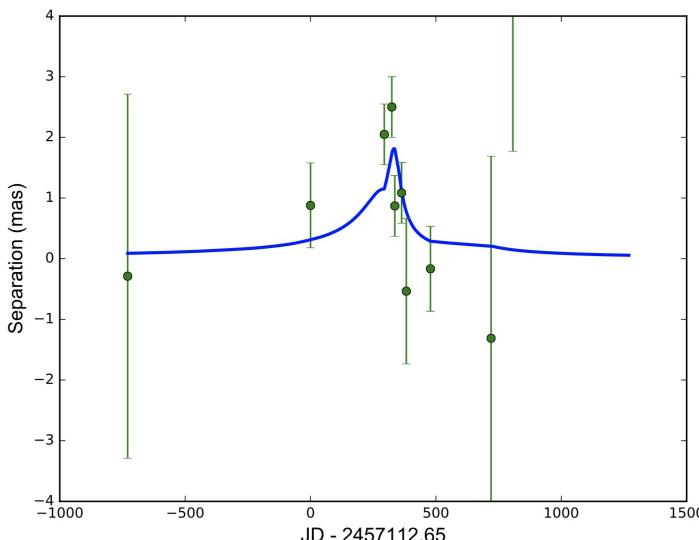
$$S(t) = S_0 \times \mu[y(t)] = S_0 \times \frac{y^2 + 2}{y\sqrt{y^2 + 4}}$$

- Unlensed flux  $S_0$  ✓
- Time of max  $t_0$  ✓
- Smallest distance  $y_0$  ✓
- Typical time scale  $t_E$  ✓

$t_E \propto \sqrt{MD_D}/v$  ✗  
Cannot disentangle these three quantities

*Microlensing degeneracy:* one cannot infer the distances, the velocity, and the lens mass uniquely from the microlensing light curve.

Of course, if you know two of them from other independent measurements, then a microlensing event can give you a way to measure the third one.



For example, you can measure the mass of a foreground star acting as a lens **if** background source distance and lens-star relative velocity are known independently.

Independent measure of Proxima Centauri mass (0.15  $M_\odot$ ) in agreement with literature, when it passed in front of two foreground stars in 2014 and 2016.

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# Microlensing and exoplanets

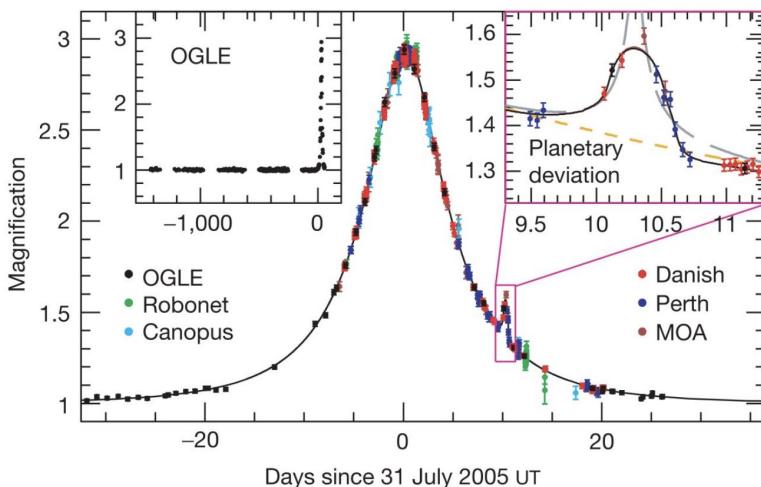
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Of course, if you know two of them from other independent measurements, then a microlensing event can give you a way to measure the third one.



Microlensing is also a nice technique to detect planets you'll never actually see again, but still carrying useful information from a statistical standpoint.

The principle is simple: the star lensing generates the typical lightcurve, and the planet lensing causes a small (though measurable) deviation (on the left, from a object with 5 Earth masses).

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# Microlensing and DM: MACHOs

MACHO stands for MAssive Compact Halo Object. The first observational campaigns started in the '80s. The idea is that the Milky Way halo should be full of compact objects (i.e., isolated black holes, neutron stars, very low-mass stars), undetectable with the instrumentation available at that time.

But hey, we are talking about massive invisible object... maybe baryonic candidates for dark matter?

The concept is: if the MW halo is full of MACHOs, those should generate microlensing events in background objects such as the Small/Large Magellanic Cloud. The number density of MACHOs would be proportional to the number of microlensing events, and their characteristic mass proportional to the timescale  $t_E$ .

In the early '90s collaborations like MACHO, EROS (*Experience de Recherche d'Objets Sombres*) and OGLE (*Optical Gravitational Lensing Experiment*) repeatedly looked at the Magellanic Clouds and the galactic bulge looking for these kinds of events.

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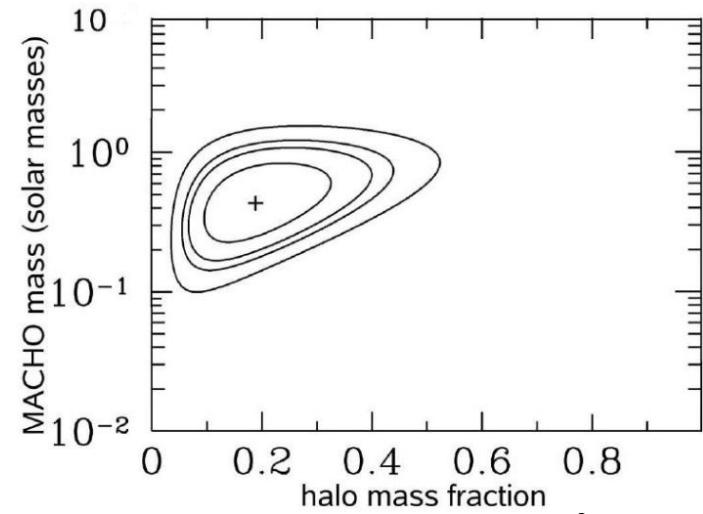
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Some results:

- relatively high rate of detection → MW is barred
- looking at MCs, no short events (< 20 days) → strong limits on the presence of Jupiters-like objects in the halo: these objects contribute less than 10% of the dark matter around our Galaxy
- most microlensing events toward the bulge are most likely caused by known stellar populations. BH can contribute to 2% of the total mass of the halo
- hints at the presence of free-floating planets in the MW disk (Sumi et al. 2011)



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The zero-order definition is:

**strong** lensing is when multiple images are generated; **weak** lensing is when these are not

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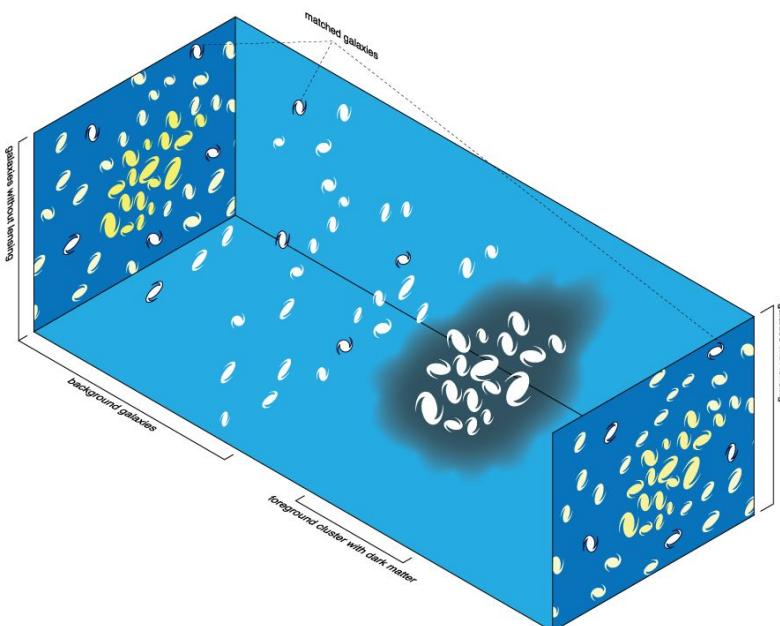
Weak Lensing and the Large Scale Structure

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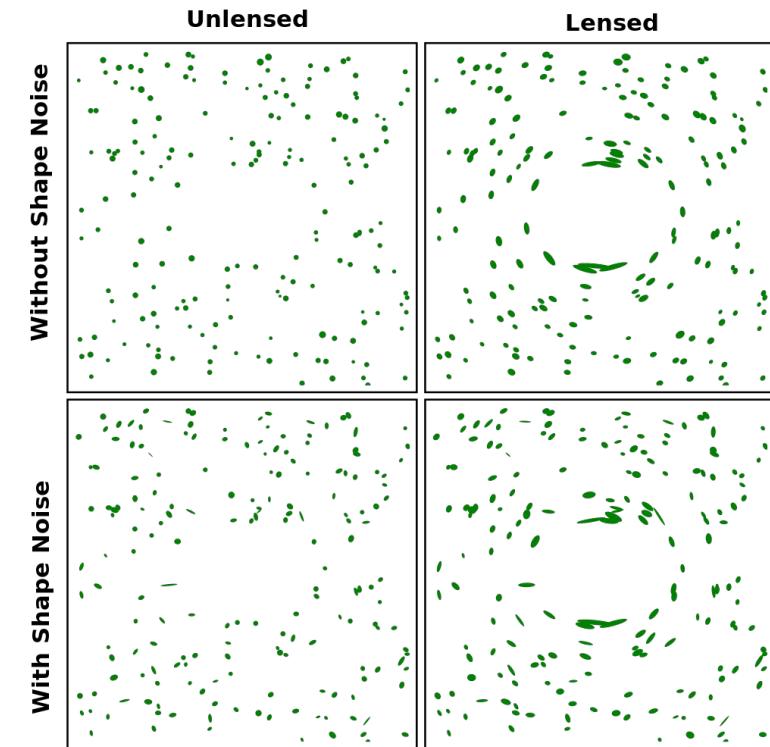
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The lens, however, is still there, and imprints its signature on the *shapes* and *orientation* of the background galaxies



Brutal Wikipedia



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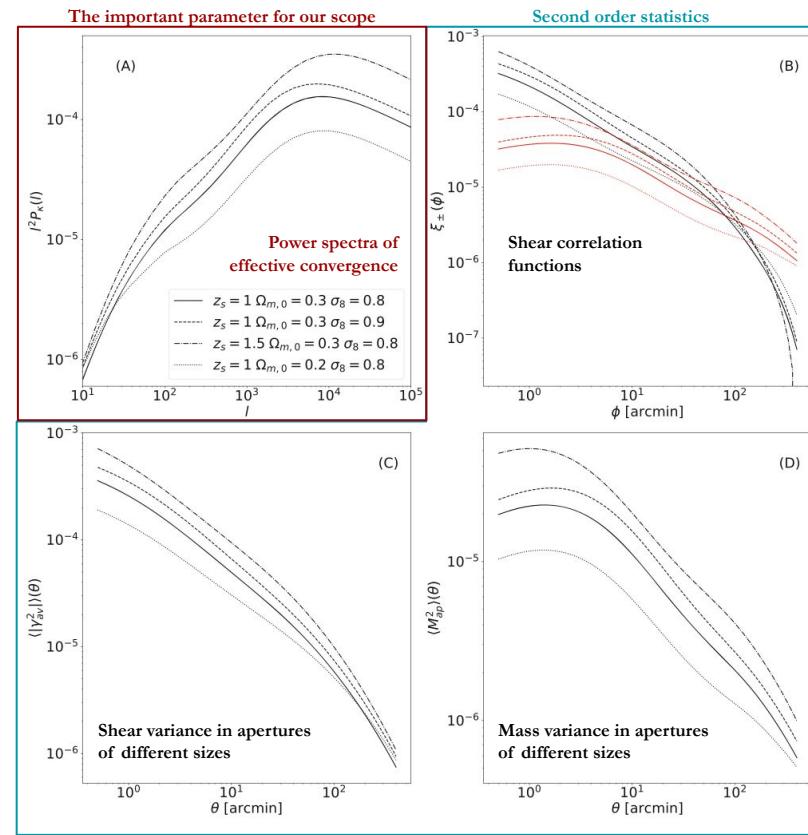
I will spare you lots and lots of math and go directly with the convergence power spectrum:

$$P_\kappa(l) = \frac{9H_0^4\Omega_{m,0}^2}{4c^4} \int_0^{w_H} \frac{W^2(w)}{a^2(w)} P_\delta \left( \frac{l}{f_K(w)}, w \right) dw$$

This depends on cosmology in several ways: it is sensitive to the growth of structures within the Universe, to the square of the matter density  $\Omega_m$ , to the geometry of the Universe in the factor  $f_K$  (panel A). This is a measurable quantity, assuming you are able to accurately map slices of the Universe in the whole sky up to high redshift (weak lensing *tomography*).



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2024



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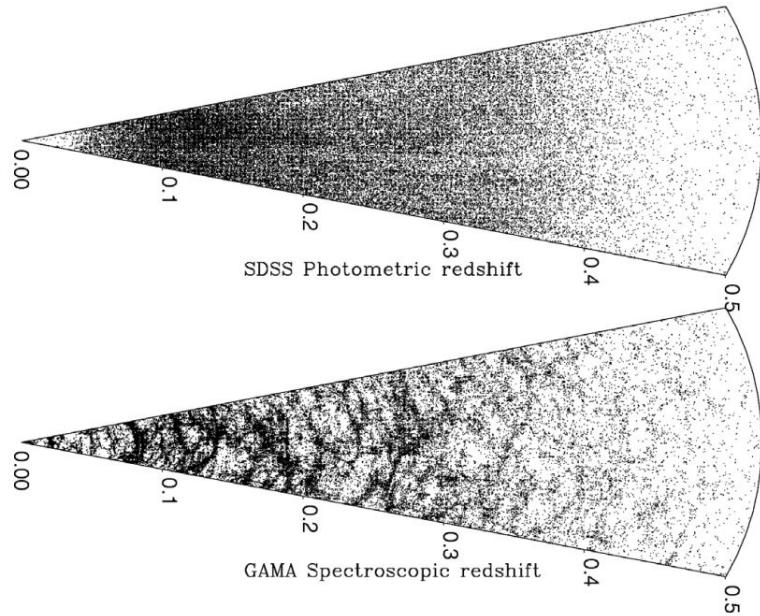
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2027

Of course, there are LOTS of complications in doing so:

- signal amplitude is tiny (order of 0.01)
- degenerate cosmological parameters
- systematics (intrinsic shapes/alignments), to accurately model
- **high** accuracy in measurements is crucial for the theoretical measurement ( $\text{photo-z } \sigma_{\text{NMAD}} < 0.01$ )



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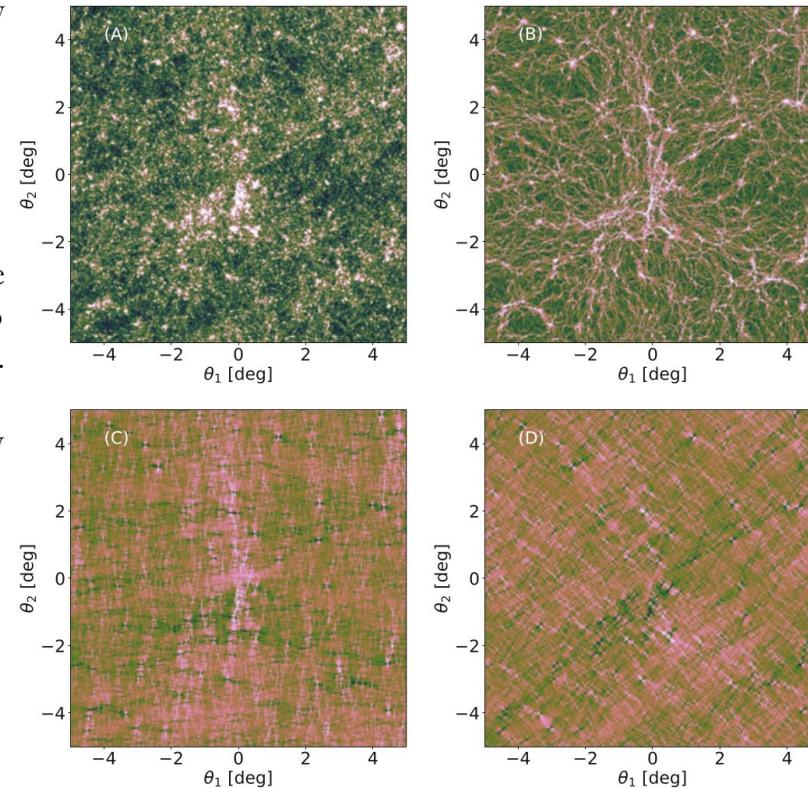
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Example on a cosmological simulation.

Panel (A) is the effective convergence; panel (B), the shear modulus  $\gamma$ ; panels (C) and (D), the shear components

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Good luck with the exam!

Thanks for your attention, good luck with the exam.



\* nessun SOBA e' stato maltrattato nella produzione di questa slide

for anything, contact me at [andrea.enia@unibo.it](mailto:andrea.enia@unibo.it)