Chapter 1

Mobile Robot Control

1.1 Configuration Space

- It has dimensions equal to the number of parameters needed to uniquely describe the configutation of a mobile robot
- Heavily dependent on the structure of the considered robot
- Equivalent to the joint space for manipulators

1.2 Constraints

Definition 1.2.1 (Constraint)**.** A constraint is any condition imposed to a material system that prevents it from assuming a generic position and/or act of motion

Definition 1.2.2 (holonomic constraints)**.** A material system is subject to *holonomic constraints* if finite relations between the coordinates of the system are present (position costraints) or if differentiable/integrable relations between the coordinates of the system are present

Definition 1.2.3 (non holonomic constraints)**.** A constraint is said to be *non-holonomic* if the differential relation between the coordinates is not reducible to finite form.

Some insight on non-holonomic constraints:

- They cannot be fully integrated
- They cannot be written in the configuration space
- They do not restrict the space of configurations but the instant robot mobility

1.2.1 Constraints in Pfaffian form

- Constraint vector equation: $a(q)\dot{q} = 0$ (1 wheel)
- Constraint matrix equation $A(q)\dot{q} = 0$ (N wheels)

Definition 1.2.4 (Pfaffian constraint)**.** A constraint that can be written in *Pfaffian Form* (i.e. $A(q)\dot{q} = 0$), is called a Pfaffian Constraint

Remark. Admissible speeds may be generated by a matrix $G(q)$ such that:

$$\mathrm{Im}(G(q)) = \ker(A(q)), \forall q \in \mathbb{C}$$

where $\mathbb{C}$ is the configuration space

1.2.2 Types of vehicles

Definition 1.2.5 (Unicycle model)**.** A unicycle is a vehicle with a single adjustable wheel.

Configuration	Constraints	Pfaffian form
$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$	$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$	$A(q) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 \end{bmatrix}$

Definition 1.2.6 (Bycicle kinematic model)**.** A bycicle is a vehicle haveing a castor and a fixed wheel with their rotation axes perpendicular to the longitudinal plane

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Configuration	Constraints	Pfaffian form
$q = \begin{bmatrix} x \\ y \\ \theta \\ \gamma \end{bmatrix}$	$\begin{cases} \dot{x}_f \sin(\theta+\gamma) - \dot{y}_f \cos(\theta+\gamma) = 0 \\ \dot{x}_r \sin(\theta) - \dot{y}_r \cos(\theta) = 0 \end{cases}$	$A(q) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 \\ \sin(\theta+\gamma) & -\cos(\theta+\gamma) & -L\cos\gamma & 0 \end{bmatrix}$

1.3 Motion Control

1.3.1 Trajectory Following

Input-Output State Feedback Linearization

We define a point B outside the wheel's axle so that we can control the point b which *pulls* the vehicle

$$\begin{cases} x_b = x_r + b\cos\theta_r \\ y_b = y_r + bsin\theta_r \end{cases} \qquad b \neq 0$$

The point B is not constrained in any way, therefore it can instantly move in any direction. We can define two inputs $(v_{x,b}, v_{y,b})$ to control the system:

$$\begin{cases} \dot{x}_b = v_{x,b} \\ \dot{y}_b = v_{y,b} \end{cases} \qquad b \neq 0$$

which are related to the unicycle's configuration as follows:

$$\begin{cases} \dot{x}_b = \dot{x}_r - b\omega\sin\theta_r = v\cos\theta_r - b\omega\sin\theta_r \\ \dot{y}_b = \dot{y}_r - b\omega\cos\theta_r = v\cos\theta_r - b\omega\cos\theta_r \end{cases} \qquad b \neq 0$$