Chapter 1

Intro to physical modelling

Engineering multiports

Engineering multiports are Places at which subsystems can be interconnected are where power can flow between the sbusystems, and such places are called ports. Physical subsystems with one or more ports are called multiports. The variables listed for the previous multiports are called power variables, because the product of the two variables considered as functions of time is the instantaneous power flowing between the two multiports. All power variables are called either *effort* (e) or *flow* (f).

The power flowing through a port can be expressed as the product of an effort and a flow variable:

$$P(t) = e(t)f(t)$$

Two other types of variables (*energy variables*) turn out to be important in describing dynamic systems:

- Momentum $p(t) = p_0 + \int_{t_0}^{t} e(\tau)d\tau$

- Displacement: $q(t) = q_0 + \int_{t_0}^{t} f(\tau)d\tau$

The *energy flow* is the time integral of the power flow:

$$E(t) = \int^{t} P(\tau)d\tau = \int^{t} e(\tau)f(\tau)d\tau$$

$$E(t) = \int^{t} e(\tau)dq(\tau) = \int^{t} f(\tau)dp(\tau)$$

There are cases where an effort is a function of a displacement, or a flow is a function of a momentum, which implies that

$$E(q) = \int^{q} e(\bar{q})d\bar{q} \qquad E(p) = \int^{p} f(\bar{p})d\bar{p}$$

Multiport elements can be connected to other multiport to form systems, and power can flow through the connected ports.

1.1 Bond Graphs

When two multiports are coupled so that the effort and flow variables become identical, the two multiports are said to have a common bond, in analogy to the bonds between component parts of molecules.

A bond graph consists of subsystems linked together by power bonds. When major subsystems are represented by words, the graph is called a word bond graph. The word bond graph serves to make som initial decisions about the representation of dynamic systems.

The causal stroke

In performing experiments, the notions of input and output arise. One must make a decision about what is to be done at the ports. At each port, both an effort and a flow variable exist, and one can control either one but not both of these variables simultaneously. In bond graphs, the way in which inputs and outputs are specified is by means of the causal stroke:

- the causal stroke is a short, perpendicular line made at the end of a bond

- it indicates the direction in which the effort signal is directed

- the end of a bond that does not have a causal stroke is the end toward which the flow's signal arrow points.

Pure signal flow

In the case of pure signal flow (transfer of information with negligible power flow), either an effort or a flow may be suppressed at many interconnection points. In this case, the bond degenerates to a single signal (active bond). In many cases, systems are designed so that only one of the power variables is important, i.e. a single signal is transmitted between two subsystems

- Electronic amplifier

- Ideal Ampere meter

- Control system (ideal actuator)

Control systems are useed to modify the dynamic behaviour of engineering systems. They require:

- Sensors

- Signal processors

- Actuators

It is advantageous to use composite representations in which the physical systems is represented using bond graphs and the controller is represented as a signal processor.

Chapter 2

Basic component models

2.1 Basic 1 port elements

1-port resistor

an element in which the effort and flow variables at the single port are related by a static function.

$$e = \Phi_R(f)$$

Where $\Phi_R(\cdot)$ is called *constitutive equation*. Usually, resistors dissipate energy, i.e. power flows into the resistor but never comes out of it. Pwer flows nito the port when the product of e and f is positive according to the sign convention shown, so power is always dissipated if the constitutive relation lies in the first and third quadrants of the $e - f$ plane. When a resistive element is assumed to be linear, it is conventional to indicate this on the bond graph by appending a colon (:) next to R. For passive resistors, estabilsh the power sign convention by means of a half-arrow pointing towards the resistor.
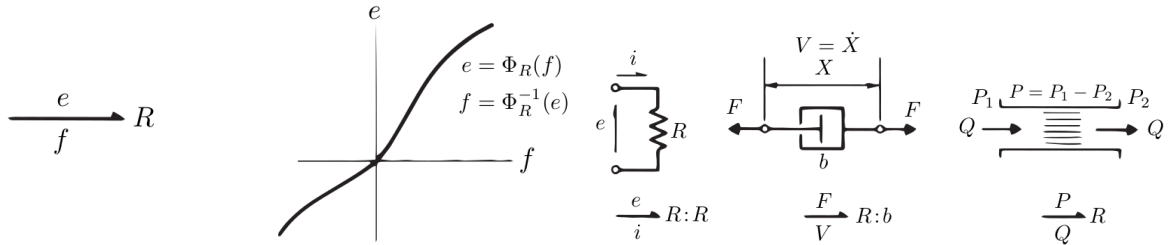
1-port capacitor - compliance

1-port device in which a static constitutive relation between an effort and a displacement.

- such a device stores and gives up energy without loss

- In bond graph terminology, an element that relates e to q is called is called a *1-port capacitor* or *compliance*

- In physical terms, a capacitor is an idealisation of devices as springs, torsion bars, electrical capacitors, gravity tanks, and hydraulic accumulators

- There are idealised linear compliance elements as well as nonlinear ones

With the same sign convention of the resistor, we have that

$$E(t) = \int_0^t e(\tau)f(\tau)d\tau + E_0$$

considering a change of variables $f\,dt = dq$ and $e = e(q)$ we get

$$E(q) = \int_{q_0}^q e(\bar{q})d\bar{q} + E_0$$

The conservation of energy for C is obvious

- The power flow into the port reverses and power flows out of the port

- During the process, no energy is lost, i.e. it is conserved

1-port inertia

Energy storing 1-port where the momentum p is related to the flow f by a static constitutive law. It is the dual of the capacitor.

- it is used to model inductance effects in electrical systems, and mass or inertia effects in mechanical or fluid systems.

As before we have

$$E(t) = \int_0^t e(\tau)f(\tau)d\tau + E_0$$

by considering a change of variables $edt = dp$ and $f = f(p)$

$$E(p) = \int_{p_0}^p f(\bar{p})d\bar{p} + E_0$$

effort source and flow source

- an effort or flow is maintained reasonably constant, or constrained to be some particular function of time

- a source maintains one of the power variables constant or a specified function of time no matter how large the other variable may be

It is possible to incorporate a modulation in the source element, thus obtaining a MSE of a MSF. Many physical devices function as amplifiers or transducers but are inadequately modelled as simple controlled sources.

2.2 Basic 2-port elements

The 2-ports to be discussed here are ideal in the sense that power is conserved ($e_1(t)f_1(t) = e_2(t)f_2(t)$). Let us consider a conservative 2-port with a through power sign convention

- Transformer (TF): $e_1 = me_2, \quad mf_1 = f_2$

- Gyrator (GY): $e_1 = rf_2, \quad rf_1 = e_2$

The scalign ration for a TF or a GY can be time-dependent. In mechanics, the MTF is particularly important and may be used to represent geometric transformations or kinematic linkages.

2.3 3-port junction elements

3-port power conserving elements that serve to interconnect other multiports into subsystems or system models

- They are the most fundamental ideas behind bond graph formalism

- they are an abstraction of the parallel and series connections of electrical circuits

- they appear in all physical domains

0-junction

$$e_1(t) = e_2(t) = e_3(t)f_1(t) + f_2(t) + f_3(t) = 0$$

1-junction

$$f_1(t) = f_2(t) = f_3(t)e_1(t) + e_2(t) + e_3(t) = 0$$