# DICHIARAZIONI

$D ::= \text{nil} \mid \underbrace{\text{const } x : \overset{\tau}{\cancel{\text{int}}} = E \mid \text{var } x : \overset{\tau}{\cancel{\text{int}}} = E}_{\text{Dichiarazioni elementari}} \mid \underbrace{D_1 ; D_0 \mid D_0 \text{ in } D_1}_{\text{Dichiarazione Composte}} \mid \underbrace{D_0 \text{ and } D_1}_{\text{Dichiarazione Composte}}$

Le costanti non vanno nell'ambiente $\rho$ perché non vengono mai modificate

$\rho : \text{IDE} \rightarrow \text{DVal} \cup \{?, \perp\}$

$\text{DVal} = \text{loc} \mid K$

$\rho(x) = K \Rightarrow x$ è costate

Essendo che è stato modificato l'ambiente dinamico devo cambiare l'ambiente statico:

$\Delta : \{(x, \tau) \ldots\}$ : non si riesce a capire se è una costante.

Adottiamo una notazione : $\forall \tau : Typ \Rightarrow \underline{\tau loc}$ → locazione di tipo $\tau$

$\Rightarrow$ $\text{const } x : \tau = E \rightarrow (x, \tau)$

$\text{var } x : \tau = E \rightarrow (x, \tau loc)$

# SEMANTICA STATICA DICHIARAZIONI

$FI(nil) = \emptyset$

$FI(const\ x : \tau = E) = FI(E)$

$FI(var\ x : \tau = E) = FI(E)$

$FI(D_0 ; D_1) = FI(D_0) \cup (FI(D_1) \setminus BI(D_0))$

$FI(D_0\ in\ D_1) = FI(D_0) \cup (FI(D_1) \setminus BI(D_0))$

$FI(D_0\ and\ D_1) = FI(D_0) \cup FI(D_1)$

$$\boxed{\Delta \vdash_I D_0 : \Delta_0}$$

$$\Delta \vdash nill : \emptyset$$

$$\frac{\Delta \vdash_I E : \tau}{\Delta \vdash const\ x : \tau = E : [x, \tau]}$$

$$\frac{\Delta \vdash_I E : \tau}{\Delta \vdash var\ x : \tau = E : [x, \tau loc]}$$

$$\frac{\Delta \vdash D_0 : \Delta_0 \quad \Delta[\Delta_0] \vdash D_1 : \Delta_1}{\Delta \vdash D_0 ; D_1 : \Delta_0[\Delta_1]}$$

$\Delta[\Delta_0]$ : ~~esteso~~ $\Delta$ esteso con $\Delta_0$

$$\frac{\Delta \vdash D_0 : \Delta_0 \qquad \Delta[\Delta_0] \vdash D_1 : \Delta_1}{\Delta \vdash D_0 \text{ in } D_1 : \Delta_1}$$ $\longrightarrow$ tutto quello che è contenuto in $D_0$ è visibile solo in $D_1$, risultato l'ambiente ~~a di~~ $\Delta_1$

$$\frac{\Delta \vdash D_0 : \Delta_0 \qquad \Delta \vdash D_1 : \Delta_1}{\Delta \vdash D_0 \text{ and } D_1 : \Delta_0, \Delta_1} \; ; \; I_0 \cap I_1 = \emptyset$$

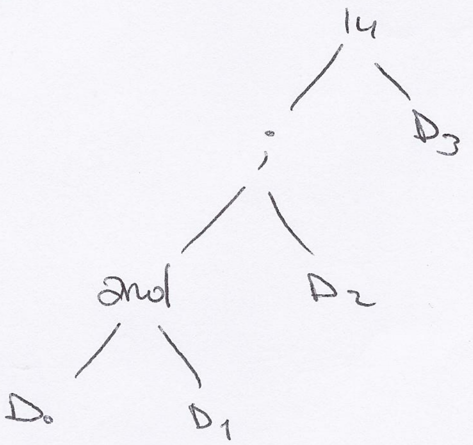
$$\Delta_0, \Delta_1 = \Delta_0[\Delta_1] = \Delta_1[\Delta_0]$$

es) è corretto staticamente?

$$d \left[ \overbrace{\left[ \left( \overbrace{\text{var } x : \text{int} = 1}^{D_0} \text{ and } \overbrace{\text{var } z : \text{int} = 2}^{D_1} \right) ; \overbrace{\text{var } y : \text{int} = \underbrace{x + z}_{E_1}}^{D_2} \right]}^{D_4} \right.$$

~~and~~ in

$$\left. \underbrace{\text{var } a : \text{bool} = (\overbrace{u = y}^{E_2})}_{D_3} \right]$$



$$\dfrac{\dfrac{(\Delta_0,\Delta_1)[\Delta_2] \vdash 4 : int \quad , \quad (\Delta_0 \Delta_1)[\Delta_2] \vdash y : int}{(\Delta_0,\Delta_1)(\Delta_2) \vdash E_2 : \tau_=(\tau_0,\tau_1) : bool}}{(\Delta_0,\Delta_1)[\Delta_2] \vdash D_3 : [a, bool\,loc]} \circledast$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\emptyset \vdash 1 : int}{\emptyset \vdash D_0 : [x, int\,loc] = \Delta_0} \quad \dfrac{\emptyset \vdash 2 : int}{\emptyset \vdash D_1 : [z, int\,loc] = \Delta_1}}{\emptyset \vdash D_0 \text{ and } D_1 : \Delta_0, \Delta_1} \quad \dfrac{\dfrac{\Delta_0,\Delta_1 \vdash x : int \quad \Delta_0,\Delta_1 \vdash z : int}{\Delta_0,\Delta_1 \vdash E_1 : \tau_+(\tau_0,\tau_1) : int}}{\Delta_0,\Delta_1 \vdash D_2 : [y : int\,loc] = \Delta_2}}{\emptyset \vdash D_4 ; D_2 : (\Delta_0,\Delta_1)[\Delta_2]} \circledast}{\emptyset \vdash D_5 \text{ in } D_3 : [a : bool\,loc]}$$

21/03/14

$D ::= nil \mid const\ x:\tau = E \mid var\ x:\tau = E \mid D_0; D_1 \mid D_0\ in\ D_1 \mid D_0\ and\ D_1$

$$\rho \vdash \langle D, \sigma \rangle \rightarrow \langle D', \sigma' \rangle \longrightarrow \langle \rho', \sigma \rangle$$

$[x : l]$

ambiente, memoria

Definizione della semantica dinamica delle dichiarazioni:

<u>BASE</u>:

$$\rho \vdash \langle nil, \sigma \rangle \rightarrow_d \langle \emptyset, \sigma \rangle$$

$$\frac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle \overset{K}{\cancel{E'}}, \sigma \rangle}{\rho \vdash \langle const\ x:\tau = E, \sigma \rangle \longrightarrow_d \langle const\ x:\tau = E', \sigma \rangle}$$

$$\rho \vdash \langle const\ x:\tau = K, \sigma \rangle \rightarrow_d \langle [x : l], \sigma \rangle$$

---

$$\frac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e \langle E', \sigma \rangle}{\rho \vdash \langle var\ x:\tau = E, \sigma \rangle \rightarrow_d \langle var\ x:\tau = E', \sigma \rangle}$$

$$\rho \vdash \langle var\ x:\tau = K, \sigma \rangle \rightarrow_d \langle [x : l], \sigma[x : l] \rangle$$

$$\frac{\rho \vdash \langle E, \tilde{\sigma}_0 \rangle \xrightarrow{}_e^* \langle k, \tilde{\sigma}_0 \rangle}{\rho \vdash \langle \text{const } x:\tau = E, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle [x:l], \tilde{\sigma}_0 \rangle}$$

è equivalente alla prima ma si assume la valutazione delle espressioni in "n" passi.

$$\frac{\rho \vdash \langle E, \tilde{\sigma}_0 \rangle \xrightarrow{}_e^* \langle k, \tilde{\sigma}_0 \rangle}{\rho \vdash \langle \text{var } x:\tau = E, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle [x:l], \tilde{\sigma}_0[x:k] \rangle}$$

Dichiarazione composta:

$$\frac{\rho \vdash \langle D_0, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle D_0', \tilde{\sigma}_0' \rangle}{\rho \vdash \langle D_0 \underset{\text{in}}{;} D_1, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle D_0' \underset{\text{in}}{;} D_1, \tilde{\sigma}_0' \rangle} \Rightarrow \frac{\rho[\rho_0] \vdash \langle D_1, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle D_1', \tilde{\sigma}_0' \rangle}{\rho \vdash \langle \rho_0 \underset{\text{in}}{;} D_1, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle \rho_0 \underset{\text{in}}{;} D_1', \tilde{\sigma}_0' \rangle}$$

$$\Downarrow$$

$$\rho \vdash \langle \rho_0 ; \rho_1, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle \rho_0[\rho_1], \tilde{\sigma}_0 \rangle$$

- Dichiarazione privata.

$$\rho \vdash \langle \rho_0 \text{ in } \rho_1, \tilde{\sigma}_0 \rangle \xrightarrow{}_d \langle \rho_1, \tilde{\sigma}_0 \rangle$$

Dichiarazione Simultanee:

$$\frac{\rho \vdash \langle D_0, \tilde{\sigma} \rangle \xrightarrow{d} \langle D_0', \tilde{\sigma}' \rangle}{\rho \vdash \langle D_0 \text{ and } D_1, \tilde{\sigma} \rangle \xrightarrow{d} \langle D_0' \text{ and } D_1, \tilde{\sigma}' \rangle} \Rightarrow \frac{\rho \vdash \langle D_1, \tilde{\sigma} \rangle \xrightarrow{d} \langle D_1', \tilde{\sigma}' \rangle}{\rho \vdash \langle \rho_0 \text{ and } D_1, \tilde{\sigma} \rangle \xrightarrow{d} \langle \rho_0 \text{ and } D_1', \tilde{\sigma}' \rangle}$$

$$\rho \vdash \langle \rho_0 \text{ and } \rho_1, \tilde{\sigma} \rangle \xrightarrow{d} \langle \rho_0, \rho_1, \tilde{\sigma} \rangle$$

NB: le suddette regole sono sbagliate perché nella dichiarazione della sintassi manca $\rho$ ($\rho_0$) ma il programmatore non può utilizzarlo.

$$Elab(D, \tilde{\sigma}) = \rho \Leftrightarrow \langle D, \tilde{\sigma} \rangle \xrightarrow{*}_{d} \langle \rho, \tilde{\sigma}' \rangle$$

$$D_0 \equiv D_1 \Longleftrightarrow \forall \rho, \tilde{\sigma} \,.\, Elab(D_0, \tilde{\sigma}) = Elab(D_1, \tilde{\sigma})$$

# Subject Reduction delle dichiarazioni

$$\Delta \vdash D_0 : \Delta_0 \wedge \rho \vdash \langle D_0, \sigma_0 \rangle \xrightarrow[d]{} \langle D_0', \sigma_0' \rangle, \rho : \Delta \Rightarrow \Delta \vdash D_0' : \Delta_0$$

$$\frac{\Delta \vdash E : \tau}{\Delta \vdash \text{const } x : \tau = E : [x : \tau]}$$

Esercizi:

Estendere $\Delta$ con $x==y$ con semantica intuitiva.
$x$ è associato alla locazione di $y$

① Definire $FI(x==y)$
$=\{y\}$

Semantica statica: ②

$$\frac{\Delta \vdash y : \tau_i}{\Delta \vdash x==y : [x : \tau loc]} \quad \text{se } \Delta(y) = \tau loc$$

Semantica Dinamica:

$$g \vdash \langle x==y, \sigma_0 \rangle \rightarrow_d \langle [x : l_1], \sigma_0 \rangle, \; l_1 = g(y)$$

$$\frac{\rho \vdash \langle D, \sigma \rangle \longrightarrow_d \langle \rho_0, \sigma_0 \rangle}{\rho \vdash \langle D; C, \sigma \rangle \longrightarrow_c \langle \rho_0; C, \sigma_0 \rangle} \qquad \frac{\rho[\rho_0] \vdash \langle C, \sigma \rangle \longrightarrow_c \langle C', \sigma' \rangle}{\rho \vdash \langle \rho_0; C, \sigma \rangle \longrightarrow_c \langle \rho_0; C', \sigma' \rangle}$$

$$\frac{\rho[\rho_0] \vdash \langle C, \sigma \rangle \longrightarrow_c \sigma'}{\rho \vdash \langle \rho_0; C, \sigma \rangle \longrightarrow_c \sigma'}$$

NB! il while viola il principio di induzione: si ha una catena crescente.

DEFINIZIONE EQUIVALENZA $\mathcal{L}, \mathcal{M}, \rightarrow, \equiv$

$\text{Exec}(C, \sigma) = \sigma' \iff \langle C, \sigma \rangle \longrightarrow_c^{*} \sigma'$

$C_0 \equiv C_1 \iff \forall \sigma.\ \text{Exec}(C_0, \sigma) = \text{Exec}(C_1, \sigma)$

SUBJECT REDUCTION // ben formato $\Delta \vdash C \wedge \rho : \Delta \wedge \rho \vdash \langle C, \sigma \rangle \longrightarrow_c \langle C', \sigma' \rangle \Rightarrow \Delta \vdash$

- nil ✓
- $\dfrac{\Delta \vdash E' : \tau}{\Delta \vdash x := E}$, $\Delta(x) = \tau loc$ // unione modificato con $E'$
- $\dfrac{\Delta \vdash C_0' \quad \Delta \vdash C_1}{\Delta \vdash C_0'; C_1}$ // vale per $C_0'$
- uguale per if E then $C_0$ else $C_1$
- $\dfrac{\Delta \vdash E : bool \quad \Delta \vdash C}{\Delta \vdash \text{while } E \text{ do } C}$ $\qquad \dfrac{\rho \vdash \langle E, \sigma \rangle \longrightarrow_e \langle E', \sigma \rangle}{\rho \vdash \langle \text{while } E \text{ do } C, \sigma \rangle \longrightarrow_c \langle \text{while } E' \text{ do } C, \sigma \rangle}$

$$\frac{\rho \vdash \langle E, \sigma \rangle \longrightarrow_e \langle ff, \sigma \rangle}{\rho \vdash \langle \text{while } E \text{ do } C, \sigma \rangle \longrightarrow_c \sigma} \qquad \rho \vdash \langle \text{while } E \text{ do } C, \sigma \rangle \longrightarrow \langle C; \text{while } E \text{ do } C, \sigma \rangle$$

$$\frac{\Delta \vdash C \qquad \Delta \vdash \text{while } E \text{ do } C}{\Delta \vdash C; \text{while } E \text{ do } C}$$

Definisco Mod : C → Ide

$\text{Mod}(\text{nil}) = \emptyset$

$\text{Mod}(x := E) = \{x\}$

$\text{Mod}(C_0; C_1) = \text{Mod}(C_0) \cup \text{Mod}(C_1)$

# COMANDI

$C ::= nil \mid x := E \mid C_0 ; C_1 \mid \text{if } E \text{ then } C_0 \text{ else } C_1 \mid \text{while } E \text{ do } C \mid D ; C$

IDENTIFICATORI LIBERI

- $FI(nil) = \emptyset$
- $FI(x := E) = \{x\} \cup FI(E)$
- $FI(C_0 ; C_1) = FI(C_0) \cup (FI(C_1) \setminus BI(C_0))$
- $FI(\text{if } E \text{ then } C_0 \text{ else } C_1) = FI(E) \cup FI(C_0) \cup FI(C_1)$
- $FI(\text{while } E \text{ do } C) = FI(E) \cup FI(C)$
- $FI(D ; C) = FI(D) \cup (FI(C) \setminus BI(D))$

SEMANTICA STATICA - Devo verificare che un comando sia ben formato

$\Delta \vdash C : \underbrace{bf}_{\text{superfluo}}$

- $\Delta \vdash nil$
- $\dfrac{\Delta \vdash E : \tau}{\Delta \vdash x := E}, \; \Delta(x) = \tau loc$
- $\dfrac{\Delta \vdash C_0 \qquad \Delta \vdash C_1}{\Delta C_0 ; C_1}$
- $\dfrac{\Delta \vdash E : bool \qquad \Delta \vdash C_0 \qquad \Delta \vdash C_1}{\Delta \vdash \text{if } E \text{ then } C_0 \text{ else } C_1}$
- $\dfrac{\Delta \vdash E : bool \qquad \Delta \vdash C}{\Delta \vdash \text{while } E \text{ do } C}$
- $\dfrac{\Delta \vdash D : \Delta_0 \qquad \Delta[\Delta_0] \vdash C}{\Delta \vdash D ; C}$

SEMANTICA DINAMICA $\quad \rho \vdash \langle C, \sigma \rangle \rightarrow^* \sigma'$

- $\rho \vdash \langle nil, \sigma \rangle \rightarrow \sigma$
- $\dfrac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle k, \sigma}{\rho \vdash \langle x := E, \sigma \rangle \rightarrow_c \sigma[l = k]}, \; \rho(x) = l$
- $\dfrac{\rho \vdash \langle C_0, \sigma \rangle \rightarrow_c \langle C_0', \sigma' \rangle}{\rho \vdash \langle C_0 ; C_1 ; \sigma \rangle \rightarrow_c \langle C_0' ; C_1 , \sigma' \rangle}$ continuo finche' $\dfrac{\rho \vdash \langle C_0 , \sigma \rangle \rightarrow_c \sigma'}{\rho \vdash \langle C_0, C_1, \sigma \rangle \rightarrow_c \langle C_1, \sigma}$
- $\dfrac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle tt, \sigma \rangle}{\rho \vdash \langle \text{if } E \text{ then } C_0 \text{ else } C_1, \sigma \rangle \rightarrow_c \langle C_0, \sigma \rangle}$ $\dfrac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle ff, \sigma \rangle}{\rho \vdash \langle \text{if } E \text{ then } C_0 \text{ else } C_1, \sigma \rangle \rightarrow_c \langle C_1,}$
- $\dfrac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle ff, \sigma \rangle}{\rho \vdash \langle \text{while } E \text{ do } C \rangle \rightarrow_c \sigma}$ $\dfrac{\rho \vdash \langle E, \sigma \rangle \rightarrow_e^* \langle tt, \sigma \rangle}{\rho \vdash \langle \text{while } E \text{ do } C, \sigma \rangle \rightarrow \langle C ; \text{while } E \text{ do } C, \sigma \rangle}$

/28/03/19/

(1)
Estendere i C con

$$do\ E\ times\ C$$

con semantica intuitiva di ripetere C finché $E \neq 0$

(2)
Dimostrare

$$\text{if } E \text{ then } C_0 \text{ else } C_1$$
$$\equiv$$
$$\text{If } E \text{ then } C_0 \text{ ; If not} E \text{ then } C_1$$

Dopo aver definito:

$$\text{If } E \text{ then } C$$

(3)
Estendere espressioni con:

$$\text{begin } C \text{ return } E$$

con semantica intuitiva; esecuzione di C restituisce il valore di E

(4)
Estendere C con

$$C_0 \| C_1$$

con ~~sem~~ semantica intuitiva, eseguire $C_0$ e $C_1$ indipendentemente e in qualunque ordine

1

- $FI(\text{do } E \text{ times } C) = FI(E) \cup FI(C)$

- $$\frac{\Delta \vdash E : int, \quad \Delta \vdash C}{\Delta \vdash \text{do } E \text{ times } C}$$

- $$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow{*}_{e} \langle 0, \sigma_0 \rangle}{\rho \vdash \langle \text{do } E \text{ times } C, \sigma_0 \rangle \rightarrow \sigma_0}$$

$$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow{*}_{e} \langle k, \sigma_0 \rangle \quad k \neq 0}{\rho \vdash \langle \text{do } E \text{ times } C, \sigma_0 \rangle \rightarrow \langle C, \text{do } E \text{ times } C, \sigma_0 \rangle}$$

Correzione

$FI(\text{do } E \text{ times } C) = FI(E) \cup FI(C)$

$$\frac{\Delta \vdash E : int, \quad \Delta \vdash C}{\Delta \vdash \text{do } E \text{ times } C}$$

$$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow{*}_{e} \langle k, \sigma_0 \rangle \quad k \neq 0}{\rho \vdash \langle \text{do } E \text{ times } C, \sigma_0 \rangle \rightarrow_{c} \langle C; \text{do } E \text{ times } C, \sigma_0 \rangle}$$

$$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow{*}_{e} \langle 0, \sigma_0 \rangle}{\rho \vdash \langle \text{do } E \text{ times } C, \sigma_0 \rangle \rightarrow_{c} \sigma_0}$$

(2)

- $FI(\text{if } E \text{ then } c) = FI(E) \cup FI(c)$

- $$\frac{\Delta \vdash E : bool \quad , \quad \Delta \vdash c}{\Delta \vdash \text{if } E \text{ then } c}$$

- $$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow[e]{*} \langle ff, \sigma_0 \rangle}{\rho \vdash \langle \text{if } E \text{ then } c, \sigma_0 \rangle \longrightarrow \sigma_0}$$

$$\frac{\rho \vdash \langle E, \sigma_0 \rangle \xrightarrow[e]{*} \langle tt, \sigma_0 \rangle}{\rho \vdash \langle \text{if } E \text{ then } c, \sigma_0 \rangle \longrightarrow \langle c, \sigma_0 \rangle}$$

Correzione

$x = tt$

if $x$ then $x := \text{not } x$ ; if not $x$ then $C_1$ $] \equiv C_0 ; C_1$

$\not\equiv$

if $x$ then $x = \text{not } x$ else $C_1$ $] \equiv C_0$

non sono equivalenti

③

- $FI(\text{Begin } c \text{ return } E) = FI(c) \cup (FI(E) \setminus FI(c))$

- $$\frac{\Delta \vdash c : \Delta_0 \quad , \quad \Delta[\Delta_0] \vdash E : \tau}{\Delta \vdash \text{begin } c \text{ return } E : \tau}$$

- $$\frac{\rho \vdash \langle c, \sigma_0 \rangle \longrightarrow_c \sigma_0'}{\rho \vdash \langle \text{begin } c \text{ return } E, \sigma \rangle}$$

Correzione

- $FI(\text{begin } c \text{ return } E) = FI(c) \cup (FI(E) \setminus BI(c))$

- $$\frac{\Delta \vdash c : \Delta_0 \qquad \Delta[\Delta_0] \vdash E : \tau}{\Delta \vdash \text{begin } c \text{ return } E : \tau}$$

- $$\frac{\rho \vdash \langle c, \sigma_0 \rangle \longrightarrow \langle c', \sigma_0' \rangle}{\rho \vdash \langle \text{begin } c \text{ return } E, \sigma_0 \rangle \longrightarrow_c \langle \text{begin } c' \text{ return } E, \sigma_0' \rangle}$$

$$\frac{\rho \vdash \langle c, \sigma_0 \rangle \longrightarrow_c \sigma_0'}{\rho \vdash \langle \text{begin } c \text{ return } E, \sigma_0 \rangle \longrightarrow_c \langle E, \sigma_0' \rangle}$$

4

$$FI(C_0 \| C_1) = FI(C_0) \cup FI(C_1)$$

$$\frac{\Delta \vdash C_0 \qquad \Delta \vdash C_1}{\Delta \vdash C_0 \| C_1}$$

$$\forall \pi_0 .\ Exec(C_0 \| C_1, \pi_0) = Exec(C_0 ; C_1, \pi_0) = Exec(C_1 ; C_0, \pi_0)$$

Per induzione sul numero di passi:

NB ma non si riesce a dimostrarlo

$$Exec^1(C, \pi_0) = (C^1, \pi_0^1) \Longleftrightarrow \langle C, \pi_0 \rangle \longrightarrow \langle C^1, \pi_0^1 \rangle$$

$$Exec^n(C, \pi_0) = Exec^{n-1}(C, \pi_0) \longrightarrow \langle C', \pi_0' \rangle$$

⇒ Si aggiunge alla semantica statica

$$Id(C_0) \cap Mod(C_1) \neq \emptyset$$
$$Id(C_1) \cap Mod(C_0) \neq \emptyset$$

Id restituisce tutti gli identificatori di quel comando