SEMANTICA DINAMICA FUN

16/05/2014

$E ::= K \mid x \mid E_0 \text{ bop } E_1 \mid \text{uop } E \mid \text{if } E_0 \text{ then } E_1 \text{ else } E_2 \mid$
$\quad \text{let } D \text{ into } E \mid f(ae)$

$D ::= \text{nil} \mid x:\tau = E \mid D_0; D_1 \mid D_0 \text{ in } D_1 \mid D_0 \text{ and } D_1 \mid \text{function } f(\text{form}):\tau = E$
$\quad \text{form} = ae \mid \rho \mid \text{rec } D$

$\text{form} ::= \circ \mid x:\tau, \text{form}$

$ae ::= \circ \mid E, ae$

Formato semantica dinamica senza memoria

$$\rho \vdash e \to e' / k$$
$$\rho \vdash x \to \rho(x)$$

$$\frac{\rho \vdash E_0 \to E_0'}{\rho \vdash E_0 \text{ bop } E_1 \to E_0' \text{ bop } E_1} \qquad \frac{\rho \vdash E_1 \to E_1'}{\rho \vdash K_0 \text{ bop } E_1 \to K_0 \text{ bop } E_1'}$$

$$\cancel{\rho \vdash K_0 \text{ bop } K_1 \to K}, \; K_0 \text{ bop } K_1 = K$$

$$\frac{\rho \vdash E \to E'}{\rho \vdash \text{Uop } E \to \text{Uop } E'} \qquad \rho \vdash \text{uop } K \to K', \; \text{uop } K = K$$

$$\frac{\rho \vdash E_0 \to E_0'}{\rho \vdash \text{if } E_0 \text{ then } E_1 \text{ else } E_2 \to \text{if } E_0' \text{ then } E_1 \text{ else } E_2}$$

1. $\rho \vdash \text{if } tt \text{ then } E_1 \text{ else } E_2 \to E_1$
2. $\rho \vdash \text{if } ff \text{ then } E_1 \text{ else } E_2 \to E_2$

$$\frac{\rho \vdash D \to D'}{\rho \vdash \text{let } D \text{ into } E \to \text{let } D' \text{ into } E}$$

$$\frac{\rho[\rho_0] \vdash E \to E'}{\rho \vdash \text{let } \rho_0 \text{ into } E \to \text{let } \rho_0 \text{ into } E'}$$

$$\rho \vdash \text{let } \rho_0 \text{ into } k \to k$$

$\rho \vdash f(ae) \to$ let form $= ae$ into E , $\rho(f) = \lambda$ form : $\tau = E$

$\rho \vdash$ fonction $f($form$) : \tau = E \to [f = \lambda.$ form $: \tau . E']$

$$E' = \begin{cases} E & \text{scoping dinammico} \\ \text{let } \rho_{|FV(E) \setminus form} \text{ into } E & \text{scoping statico} \end{cases}$$

SEMANTICA DINAMICA DICHIARAZIONI

$$\frac{\rho \vdash E \xrightarrow{*}_{e} k}{\rho \vdash x:\tau = E \rightarrow [x = k]}$$

$$\frac{\rho \vdash D_0 \rightarrow D_0'}{\rho \vdash D_0 \text{ in } D_1 \rightarrow D_0' \text{ in } D_1}$$

$$\frac{\rho[\rho_0] \vdash D_1 \rightarrow D_1'}{\rho \vdash \rho_0 \text{ in } D_1 \rightarrow \rho_0 \text{ in } D_1'}$$

$$\rho \vdash \rho_0 \text{ in } \rho_1 \rightarrow \rho_1$$

$$\frac{\rho \vdash D_0 \rightarrow D_0'}{\rho \vdash D_0 ; D_1 \rightarrow D_0' ; D_1}$$

$$\frac{\rho[\rho_0] \vdash D_1 \rightarrow D_1'}{\rho \vdash \rho_0 ; D_1 \rightarrow \rho_0 ; D_1'}$$

$$\rho \vdash \rho_0 ; \rho_1 \rightarrow \rho_0[\rho_1]$$

$$\frac{\rho \vdash D_0 \rightarrow D_1'}{\rho \vdash D_0 \text{ and } D_1 \rightarrow D_0' \text{ and } D_1}$$

$$\frac{\rho \vdash D_1 \rightarrow D_1'}{\rho \vdash D_0 \text{ and } D_1 \rightarrow D_0 \text{ and } D_1'}$$

$$\rho \vdash \rho_0 \text{ and } \rho_1 \rightarrow \rho_0, \rho_1$$

$$\frac{\rho \backslash V_0 \vdash D \rightarrow D_1'}{\rho \vdash \text{rec}\, D \rightarrow \text{rec}\, D'} , \quad V_0 = BV(D) \cap FV(D)$$

$$\rho \vdash \text{rec}\, \rho_0 \rightarrow \{x = k \mid \rho_0(x) = k\} \cup \{f = (\lambda\, \text{form} : \tau .\ \text{let rec}\, \rho_0 \text{ in } E) \mid \rho_0(f) = \lambda\, \text{form} : \tau\, E\}$$

$$\frac{\rho \vdash E \xrightarrow{*}_{e} k}{\rho \vdash (E, ae) \xrightarrow[ae]{} (k, ae)}$$

$$\frac{\rho \vdash ae \rightarrow_{ae} ae'}{\rho \vdash (k, ae) \xrightarrow[ae]{} (k, ae')}$$

$$\frac{\rho \vdash ae \xrightarrow{*}_{ae} ak}{\rho \vdash \text{form} = ae \rightarrow_{d} \text{form} = ak}$$

$$\frac{ak \vdash \text{form} : \rho_0}{\rho \vdash \text{form} = ak \rightarrow \rho_0}$$

$\to$ let $[g = \lambda x{:}int.\ \text{let rec } g_0 \text{ into } \bar{e}_2]$ into
let $[x = 1]$ into
let $[g = \lambda x{:}int.\ \text{let rec } g_0 \text{ into } \bar{e}_2]$ into $\to$ *
$1 +$ let $[x = 0]$ into
let $[g : \lambda x{:}int.\ \text{let rec } g_0 \text{ into } \bar{e}_2]$ into 0

$\to$ let $[g = \lambda x{:}int.\ \text{let rec } g_0 \text{ into } \bar{e}_2]$ into
let $[x = 1]$ into
let $[g = \lambda x{:}int.\ \text{let rec } g_0 \text{ into } e_2]$ into $1 + 0$

$\downarrow$

1

$$E \left[\begin{array}{l} \text{let} \quad \left. \begin{array}{l} \text{rec function } g(x:\text{int}):\text{int} = \\ \underbrace{\text{if } x=0 \text{ then } 0 \text{ else } x+g(x-1)}_{E_2} \end{array} \right] D \\ \text{into} \quad \underbrace{g(1)}_{E_1} \end{array} \right.$$

$E \to$ let rec D into E_1 $\to$ let rec $\overbrace{[g = \lambda x:\text{int}.\ E_2]}^{\rho_0}$ into E_1

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into E_1 $\to$

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
let $x:\text{int} = 1$ into $\to$
let rec ρ_0 into E_2

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
let $[x=1]$ into $\to$
let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into E_2

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
let $[x=1]$ into $\to^2$
let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into $x+g(x-1)$

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
let $[x=1]$ into $\to$
let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into $1+g(1-1)$

$\to$ let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
let $[x=1]$ into $\to^*$
let $[g = \lambda x:\text{int}.\ \text{let rec } \rho_0 \text{ into } E_2]$ into
$1+$ let ~~rec~~ $[x=0]$ into
let rec ρ_0 into E_2

$$\star \quad \cfrac{\cfrac{0 : int}{\Delta_1 \vdash 0 : int} \qquad \cfrac{\Delta_1 \vdash x : int \ , \ \cfrac{\Delta_1 \vdash x : int, \ \Delta_1 \vdash 1 : int}{\Delta_1 \ g(x-1) : int} \ , \ \Delta_1(g) = \cancel{\#} int \to int}{\Delta_1 \vdash x + g(x-1)}}{}$$

$$\otimes \quad \cfrac{\cfrac{\Delta \vdash 1 : int}{\Delta \vdash g(1) : int} \ , \ \Delta(g) = int \to int}{}$$

$$\cfrac{\cfrac{\vdash D : [g : int \to \cancel{x} int] = \Delta}{\vdash rec \ D} \qquad \cfrac{\vdash D : \Delta \qquad \Delta \vdash D}{\emptyset \vdash rec \ D} \qquad \cfrac{\Delta \vdash x : int \ , \ \cfrac{\bullet : \emptyset}{\bullet : \Delta[x : int] = \Delta_1} \qquad \cfrac{\cfrac{\Delta_1 \vdash x : int, \ \Delta \vdash 0 : int}{\Delta_1 \vdash x = 0 : bool} \ \otimes}{\Delta_1 \vdash E_2}}{} \ \otimes}{\emptyset \vdash \sigma : int}$$

$$\underbrace{\left[\begin{array}{l}\text{let rec function } g(x:int):int = \\ \quad \underbrace{\text{if } x=0 \text{ then } 0 \text{ else } x+g(x-1)}_{E_2}\end{array}\right]}_{\Delta} \text{ into } \underbrace{g(1)}_{E_1}$$

E

$\downarrow$

let ρ_0 into E_1 $\longrightarrow$ let ρ_1 into E_1 $\longrightarrow$

let ρ_1 into
  let $x:int = 1$ into
    let rec ρ_0 into E_2 $\longrightarrow$ let ρ_1 into
      let $[x=1]$ into
        let rec ρ_0 into E_2 $\longrightarrow$

let ρ_1 into
  let $[x=1]$ into
    let ρ_1 into E_2 $\longrightarrow$ let ρ_1 into
      let $[x=1]$ into
        let ρ_1 into
          $x+g(x-1)$ $\xrightarrow{2}$

let ρ_1 into
  let $[x=1]$ into
    let ρ_1 into
      $1+g(1-1)$ $\longrightarrow$ let $x:int = * -1$ into
        let rec ρ_0 into E_2 $\xrightarrow{*}$

let ρ_1 into
  let $[x=1]$ into
    let ρ_1 into
      $1+$ let $[x=0]$ into
        let rec ρ_0 into E_2 $\xrightarrow{*}$ let ρ_1 into
          let $[x=1]$ into
            let ρ_1 into
              $1+$ let $[x=0]$ into
                let ρ_1 into 0 $\xrightarrow{*}$

let ρ_1 into
  let $[x=1]$ into
    let ρ_1 into $1+0$ $\xrightarrow{*}$ $\boxed{1}$

$$\dfrac{\dfrac{\emptyset \setminus \{g\} \vdash D \rightarrow g = [\lambda x : int . E_2]}{\emptyset \vdash rec\ D \rightarrow rec[g = \lambda x : int . E_2] = \rho_0}}{\emptyset \vdash let\ rec\ D\ into\ E_1 \rightarrow let\ rec\ \rho_0\ into\ E_1}$$

$$\dfrac{\emptyset \vdash rec\ \rho_0 \rightarrow [g = \lambda x : int .\ let\ rec\ \rho_0\ into\ E_2] = \rho_1}{\emptyset \vdash let\ rec\ \rho_0\ into\ E_1 \rightarrow let\ \rho_1\ into\ E_1}$$

$$\dfrac{\emptyset[\rho_1] \vdash E_1 \rightarrow let\ x : int = 1\ into\ let\ rec\ \rho_0\ into\ E_2 \ , \ \rho_1(g) = \lambda x : int .\ let\ rec\ \rho_0\ into\ E_2}{\emptyset \vdash let\ \rho_1\ into\ E_1 \rightarrow let\ \rho_1\ into\ let\ rec\ \rho_0\ into\ E_2}$$

$$\dfrac{\emptyset[\rho_1][x = 1] \vdash rec\ \rho_0 \rightarrow \rho_1}{let\ rec\ \rho_0\ into\ E_2 \rightarrow let\ \rho_1\ into\ E_2}$$

ESERCITI

19/05/14

Scrivere un programma in IMP/FUN che evidenti le differente tra scoping statico e dinamico. Motivare le risposte

var z:int = 1
var x:int = 3
procedure p (var w:int) z:= x+w
var x:int=2
p(1)
z?

- Scoping statico z = 4
- scoping dinamico z = 3

Per motivare la risposta fare l'albero di derivazione della semantica dinamica, evidenziando le strade dello scoping statico e dinamico.

$TRUE \triangleq \lambda xy.y$

$FALSE \triangleq \lambda xy.x$

$PAIR \triangleq \lambda xyf.fxy$

$FIRST \triangleq \lambda p.pTRUE$

$SECOND \triangleq \lambda p.pFALSE$

Valutare:

$$FIRST\left[SECOND\left(PAIR\ a\ (PAIR\ b\ c)\right)\right] \stackrel{?}{=} b$$

$$FIRST\left[SECOND\left(PAIR\ a\ (PAIR\ b\ c)\right)\right] = b$$

$$\left[SECOND\left(PAIR\ a\ (PAIR\ b\ c)\right)\right]TRUE = b$$

$$\left[PAIR\ a\ (PAIR\ b\ c)\ FALSE\right]TRUE = b$$

$$\left[FALSE\ a\ (PAIR\ b\ c)\right]TRUE = b$$

$$a\ TRUE \neq b$$

Dopo aver definito $=_\beta$ scrivere due λ-termini E_1, E_2 t.c. $E_1 =_\beta E_2$. Dimostrare l'equivalenza.

$$E = E \qquad \frac{E_1 = E_2}{E_2 = E_1} \qquad \frac{E_1 = E_2 \quad , \quad E_2 = E_3}{E_1 = E_3}$$

$$\frac{E \to E'}{E = E'} \qquad \frac{E = E'}{\lambda x . E = \lambda x . E'} \qquad \frac{E = E'}{E E_0 = E' E_0} \qquad \frac{E = E'}{E_0 E = E_0 E'}$$

$\downarrow$

$$\frac{(\lambda x . x) y \to y}{(\lambda x . x) y =_\beta y}$$

Scrivere un programma IMP che evidenzi le differenze tra passaggio di parametri per valore e per riferimento. Motivare la risposta.

$$\left.\begin{array}{l}\text{var } x : int = 1 \\ \text{Procedure } p(\text{var } z : int, \text{ ref } y : int)\ \boxed{C_1}\end{array}\right] D$$

$$\left. p(x, x) \right] C_2$$

$$\emptyset \to_D^* \langle [x : l_x], D; C_2, [l_x = 1] \rangle \to$$

$$\langle \underbrace{[x : l_x][p : \lambda \text{var } z : int, \text{ ref } y : int. [x = l_x]; C_1]}_{\rho_0}, C_2, [l_x = 1] \rangle$$

$$\langle \rho_0; \text{var } z : int = 1, \text{ ref } y : int = l_x; C_1, [l_x = 1] \rangle$$

Dimostrare che:

$$E \xrightarrow{\alpha} E' \iff E' \xrightarrow{\alpha} E$$

con E, E' λ-termini

Esempio: perché E e E' sono generici non posso fare per casi.
Dimostrazione sulla struttura della sintassi:

$E ::= x \mid \lambda x. E \mid E_0\, E_1$ → per ipotesi induttiva posso assumere che vada bene per E_0 e E_1

$$\lambda x. E \xrightarrow{\alpha} \lambda z. E\{^z/_x\}, \quad z \notin FV(E)$$

$$\lambda x. E\{^x/_z\}' \xleftarrow{\alpha}$$

Scrivere due frammenti EUN Equivalenti. Dimostrarlo.

$E_1 = 1+2$

$E_2 = 2$

$$\frac{\rho \vdash E_0 \to E_0'}{\rho \vdash E_0 \text{ bop } E_1 \to E_0' \text{ bop } E_1}$$

$$\frac{\rho \vdash E_1 \to E_1'}{\rho \vdash E_0 \text{ bop } E_1 \to E_0 \text{ bop } E_1'}$$

$$\rho \vdash k \text{ bop } k_1 \to k, \quad k_0 \text{ bop } k_1 \to k$$

Dato il programma IMP

$$C_0 \left[\begin{array}{l} \left.\text{var } x{:}\text{int} = 1 \text{ in var } z{:}\text{int} = 1+x;\right] D_1 \\ \left.\text{procedure } p\,(\text{var } x{:}\text{int})\ \underbrace{z := z+x}_{C}\right] D_2 \\ \left.p(10^{13})\right] C_2 \end{array} \right] C_1$$

Scrivere un programma equivalente e dimostrare equivalenta.

$$C_0 \equiv C_1 \iff \forall \sigma_0 .\ Exec(C_0, \sigma_0) = Exec(C_1, \sigma_0)$$

Costruisco il sistema di transizione di C_0.
Il nuovo programma $C_1 = C_0; \text{nil}$, nil non modifica la memoria e quindi $Exec(C_0, \sigma_0) = Exec(C_1, \sigma_0)$ è vero.

SEMANTICA DINAMICA CON SCOPING STATICO E DINAMICO

$$\emptyset \to \langle C_0, \emptyset \rangle \to \langle D_1 ; C_1 , \emptyset \rangle \xrightarrow{*} \langle [z = l_z], C_1, [l_z = 2] \rangle$$

$$\langle [z = l_z], D_2 ; C_2 , [l_z = 2] \rangle \to$$

scoping statico

$$\langle [z = l_z][p : \lambda var\, x : int . [z = l_z]; c]; p(10^{19}), [l_z = 2] \rangle \to$$

$$\langle [z = l_z][p : \lambda var\, x : int . [z = l_z]; c]; var\, x : int = 10^{19}; c , [l_z = 2] \rangle \to$$

$$\langle [z = l_z][p : \lambda var\, x : int . [z = l_z]; c][x = l_x]; c , [l_z = 2][l_x = 10^{19}] \rangle \xrightarrow{*}$$

$$\langle [z = l_z][p : \lambda var\, x : int . [z = l_z]], [l_z = 10^{19} + 2] \rangle$$

scoping dinamico

$$\langle [z = l_z][p : \lambda var\, x : int . c]; p(10^{19}), [l_z = 2] \to$$

$$\langle [z = l_z][p : \lambda var\, x : int . c]; var\, x : int = 10^{19}; c , [l_z = 2] \rangle \to$$

$$\langle [z = l_z][p : \lambda var\, x : int . c][x = l_x]; c , [l_z = 2][l_x = 10^{19}] \rangle \xrightarrow{*}$$

$$\langle [z = l_z][p : \lambda var\, x : int . c], [l_z = 10^{19} + 2]$$

: differenza tra scoping statico e dinamico

Dimostrare che Left Most - Outer Most è deterministico.

Dimostrazione sulla struttura della sintassi.

$$E ::= x \mid \lambda x.E \mid E_0 E_1$$

$$lo(x) = \{x\}$$

$$lo(\lambda x.E) = \begin{cases} \lambda x.E & \text{con } M \\ lo(E) & \text{senza } M \end{cases}$$

$$lo(E_0E_1) = \begin{cases} E_0E_1 & E_0 = \lambda x.E \\ lo(E_0) & E_0 \neq \lambda x.E \\ lo(E_1) & lo(E_0) \neq \emptyset \end{cases}$$

Corrispondenza tra Semantica Statica e dinamica
↳ SUBJECT REDUCTION

SUBJECT REDUCTION SULLE ESPRESSIONI

$$\Delta \vdash e : \tau \;\wedge\; \rho \vdash \langle e, \sigma \rangle \xrightarrow[e]{} \langle e', \sigma \rangle \Rightarrow e' : \tau$$

<u>BASE</u>

$- e = K$ SS $\Delta \vdash K : \tau$

$- e = x$ SS $\Delta \vdash x : \tau \quad , \Delta(x) \in \{\tau, \tau loc\}$

SD $\rho \vdash \langle x, \sigma \rangle \xrightarrow[e]{} \langle K, \sigma \rangle$

<u>PI</u>

$- e_1 \text{ bop } e_2$ SS $$\frac{\Delta \vdash e_1 : \tau_1 \quad , \quad \Delta \vdash e_2 : \tau_2}{\Delta \vdash e_1 \text{ bop } e_2 \; : \; \tau_{bop}(\tau_1, \tau_2)}$$

SD $$\frac{\rho \vdash \langle e_1, \sigma \rangle \xrightarrow[e]{} \langle e_1 \quad , \sigma \rangle}{\rho \vdash \langle e_1 \text{ bop } e_2 \rangle \xrightarrow[e]{} \langle e_1' \text{ bop } e_2, \sigma \rangle}$$

$$\rho \vdash \langle K \text{ bop } K', \sigma \rangle \xrightarrow[e]{} \langle K'', \sigma \rangle$$

$$\xrightarrow[e]{} K'' = K \text{ bop } K'$$

SUBJECT REDUCTION DELLE DICHIARAZIONI

$$\Delta \vdash d : \Delta' \ \wedge \ \rho \vdash \langle d, \sigma\rangle \xrightarrow[d]{} \langle d', \sigma'\rangle \Rightarrow \Delta \vdash d' : \Delta'$$

<u>BASE</u>

- $d = \text{const } x : \tau = e$

SS
$$\frac{\Delta \vdash e : \tau}{\Delta \vdash \text{const } x : \tau = e \ : [x : \tau]}$$

SD
$$\frac{\rho \vdash \langle e, \sigma\rangle \xrightarrow[e]{} \langle e', \sigma\rangle}{\rho \vdash \langle \text{const } x : \tau = e, \sigma\rangle \xrightarrow[d]{} \langle \text{const } x : \tau = e', \sigma\rangle}$$

<u>PI</u>

- $d_0 ; d_1$

SS
$$\frac{\Delta \vdash d_0 : \Delta_0 \quad , \quad \Delta \vdash d_1 : \Delta_1}{\Delta \vdash \langle d_0 ; d_1\rangle : \Delta_0[\Delta_1]}$$

SD
$$\frac{\rho \vdash \langle d_0, \sigma\rangle \xrightarrow[d]{} \langle d_0', \sigma'\rangle}{\rho \vdash \langle d_0 ; d_1, \sigma\rangle \xrightarrow[d]{} \langle d_0' ; d_1, \sigma'\rangle}$$

Sapendo che applicando la semantica statica d_0' genera Δ_0 e d_1 genera Δ_1, e conoscendo l'assioma:

$$\rho \vdash \langle \rho_0 ; \rho_1, \sigma\rangle \xrightarrow[d]{} \langle \rho_0[\rho_1], \sigma\rangle$$

$\rho_0 : \Delta_0$ e $\rho_1 : \Delta_1 \Rightarrow \rho_0[\rho_1] : \Delta_0[\Delta_1] = \Delta'$

SUBJECT REDUCTION DEI COMANDI

$$\Delta \vdash c \ \wedge \ \rho \vdash \langle c, \sigma \rangle \xrightarrow[c]{} \langle c', \sigma' \rangle \rightarrow \Delta \vdash c'$$

BASE

- $c = nil$

- $c = x := e$

SS $\dfrac{\Delta \vdash e : \tau}{\Delta \vdash x := e} \ \Delta(x) = \tau\, loc$

SD $\dfrac{\rho \vdash \langle e, \sigma \rangle \xrightarrow[e]{} \langle e', \sigma \rangle}{\rho \vdash \langle x := e, \sigma \rangle \xrightarrow[c]{} \langle x := e', \sigma \rangle}$

PI

- $c = c_0 ; c_1$

SS $\dfrac{\Delta \vdash c_0 \quad , \quad \Delta \vdash c_1}{\Delta \vdash c_0 ; c_1}$

SD $\dfrac{\rho \vdash \langle c_0, \sigma \rangle \xrightarrow[c]{} \langle c_0', \sigma \rangle}{\rho \vdash \langle c_0 ; c_1, \sigma \rangle \xrightarrow[c]{} \langle c_0' ; c_1, \sigma \rangle}$

- $c = \text{while } e \text{ do } c_0$

SS $\dfrac{\Delta \vdash e : bool \quad , \quad \Delta \vdash c_0}{\Delta \vdash \text{while } e \text{ do } c_0}$

SD $\dfrac{\rho \vdash \langle e, \sigma \rangle \xrightarrow[e]{*} \langle tt, \sigma \rangle}{\rho \vdash \langle \text{while } e \text{ do } c_0 ; \sigma \rangle \xrightarrow[c]{} \langle c_0 ; \text{while } e \text{ do } c_0, \sigma \rangle}$

- $C = \text{if } e \text{ then } C_0 \text{ else } C_1$

SS
$$\frac{\Delta \vdash e : bool \quad , \quad \Delta \vdash C_0 \quad , \quad \Delta \vdash C_1}{\Delta \vdash \text{if } e \text{ then } C_0 \text{ else } C_1}$$

SD
$$\frac{\rho \vdash \langle e, \sigma \rangle \xrightarrow{*}_{e} \langle tt, \sigma \rangle}{\rho \vdash \langle \text{if } e \text{ then } C_0 \text{ else } C_1, \sigma \rangle \rightarrow_{c} \langle C_0, \sigma \rangle}$$

$$\frac{\rho \vdash \langle e, \sigma \rangle \xrightarrow{*}_{e} \langle ff, \sigma \rangle}{\rho \vdash \langle \text{if } e \text{ then } C_0 \text{ else } C_1, \sigma \rangle \rightarrow_{c} \langle C_1, \sigma \rangle}$$

- $C = d ; C$

SD
$$\frac{\rho \vdash \langle d, \sigma \rangle \rightarrow_{d} \langle d', \sigma \rangle}{\rho \vdash \langle d ; C, \sigma \rangle \rightarrow_{c} \langle d' ; C ; \sigma \rangle}$$

$$\frac{\rho \vdash \langle C, \sigma \rangle \rightarrow_{c} \langle C', \sigma' \rangle}{\rho \vdash \langle \rho_0 ; C, \sigma \rangle \rightarrow_{c} \langle \rho_0 ; C', \sigma' \rangle}$$

C' è ben formato perché ottenuto da C (ben formato per Hp induttiva) tramite una regola di semantica dinamica.

<u>scoping statico/dinamico</u>: come binda le variabili dentro una funzione

<u>passaggio di parametri per valore/riferimento</u>: alla chiamata di funzione passo una variabile che viene assegnata ad un tipo "ref x : τ" nella funzione.

$$\emptyset \to \langle C_0, \emptyset \rangle \to \langle D_1; C_1, \emptyset \rangle \to$$

$$\langle [z = l_z]; C_1, [l_z = 2] \rangle \to$$

$$\langle [z = l_z]; D_2; C_2, [l_z = 2] \rangle \to$$

$$\langle [z = l_z][p : \lambda\, \mathrm{var}\ x : \mathrm{int}, \circ . [z = l_z]; C]; C_2, [l_z = 2] \rangle \to$$

$$\langle [z = l_z][p : \lambda\, \mathrm{var}\ x : \mathrm{int}, \circ . [z = l_z]; C]; \mathrm{var}\ x : \mathrm{int} = 10^9; C, [l_z = 2] \rangle$$

$$\langle [z = l_z][p : \lambda\, \mathrm{var}\ x : \mathrm{int}, \circ . [z = l_z]; z := z + x][x = l_x]; z := z + x, [l_z = 2][l_x = 10^9]$$

$$\langle [z = l_z][p : \lambda\, \mathrm{var}\ x : \mathrm{int}, \circ . [z = l_z]; C], [l_z = 10^9 + 2] \rangle$$

SUBJECT REDUCTION SULLE ESPRESSIONI

$$\Delta \vdash e : \tau \;\wedge\; \rho \vdash \langle e, \sigma \rangle \to_e \langle e', \sigma \rangle \Rightarrow e' : \tau$$

$e = k$ $\quad$ SS $\quad \Delta \vdash k : \tau$

$e = x$ $\quad$ SS $\quad \Delta \vdash x : \tau, \; \Delta(x) \in \{\tau, \tau_{loc}\}$

$e_1 \text{ bop } e_2$ $\quad$ SS $\quad \dfrac{\Delta \vdash e_1 : \tau_1, \; \Delta \vdash e_2 : \tau_2}{\Delta \vdash e_1 \text{ bop } e_2 : \tau_{bop}(\tau_1, \tau_2)}$

SD $\quad \dfrac{\rho \vdash \langle e_1 \cancel{\text{bop } e_2}, \sigma \rangle \to_e \langle e_1' \cancel{\text{bop } e_2}, \sigma \rangle}{\rho \vdash \langle e_1 \text{ bop } e_2, \sigma \rangle \to_e \langle e_1' \text{ bop } e_2, \sigma \rangle}$

$$\rho \vdash \langle k \text{ bop } k', \sigma \rangle \to_e \langle k'', \sigma \rangle$$

$$\to_e \; k'' = k \text{ bop } k'$$

AND TRUE FALSE = FALSE

$$(\lambda pq.\, pqp)(\lambda xy.x)(\lambda xy.y) = (\lambda xy.y)$$

$$(\lambda xy.x)(\lambda xy.y)(\lambda xy.x) = (\lambda xy.y)$$

$$(\lambda xy.y) = (\lambda xy.y)$$

SUBJECT REDUCTION DELLE DICHIARAZIONI

$$\Delta \vdash d : \Delta' \quad \wedge \quad \rho \vdash \langle d, \sigma \rangle \xrightarrow[d]{} \langle d', \sigma' \rangle \Rightarrow \Delta \vdash d' : \Delta'$$

const $x : \tau = e$

SS $$\frac{\Delta \vdash e : \tau}{\Delta \vdash \text{const } x : \tau = e : [x : \tau]}$$

SD $$\frac{\rho \vdash \langle e, \sigma \rangle \xrightarrow[e]{} \langle e', \sigma \rangle}{\rho \langle \text{const } x : \tau = e, \sigma \rangle \xrightarrow[d]{} \langle \text{const } x : \tau = e', \sigma \rangle}$$

$d_0 ; d_1$

SS $$\frac{\Delta \vdash d_0 : \Delta_1 \,,\quad \Delta \vdash d_1 : \Delta_1}{\Delta \vdash d_0 ; d_1 : \Delta_0[\Delta_1]}$$

SD $$\frac{\rho \vdash \langle d_0 ; \cancel{d_1}, \sigma \rangle \rightarrow_d \langle d_0', \sigma' \rangle}{\rho \vdash \langle d_0 , d_1 , \sigma \rangle \xrightarrow[d]{} \langle d_0', d_1, \sigma \rangle}$$

Sapendo che $\rho \vdash \langle \rho_0 ; \rho_1 , \sigma \rangle \xrightarrow[d]{} \langle \rho_0[\rho_1], \sigma \rangle$

$$\rho_0 : \Delta_0 \text{ e } \rho_1 : \Delta_1 \Rightarrow \rho_0[\rho_1] : \Delta_0[\Delta_1] = \Delta'$$

$D\left[\begin{array}{l}\text{var } z : int = 1; \\ C\left[\begin{array}{l} D_1\left[\begin{array}{l}(\text{var } x : int = 4 \text{ and} \\ \text{procedure } p(\text{var } y : int)\ \underbrace{z := x + y}_{C_2});\end{array}\right. \\ C_1\left[p(2) \right.\end{array}\right.\end{array}\right.$

$\boxed{SS}$

$\emptyset \vdash \langle D; C \rangle \xrightarrow{②} \langle [z = l_z], D_1; C_1, [l_z = 1] \rangle \longrightarrow^*$

$\langle \underbrace{[z = l_z][x = l_x]}_{\rho_0}[p = \lambda y : int . \rho_0 . C_2], C_1, [l_z = 1][l_x = 4] \rangle$

$\langle [z = l_z][x = l_x][p = \lambda y : int . \rho_0 . C_2], \text{var } y : int = 2, C_2, [l_z = 1][l_x = 4] \rangle$

$\langle \rho_0, [p = \lambda y : int . \rho_0 . C_2][y = l_y], z := x + y, [l_z = 1][l_x = 4][l_y = 2] \rangle$

$\langle [z = l_z][x = l_x][p = \lambda y : int . \rho_0 . C_2], [l_z = 6][l_x = 4] \rangle$

$E = x \mid \lambda x . E \mid E_0 E_1$

$l_0(x) = \{x\}$

$l_0(\lambda x . E) = \begin{cases} \lambda x . E & \text{con ny} \\ l_0(E) & \text{senza ny} \end{cases}$

$l_0(E_0 E_1) = \begin{cases} E_0 E_1 & E_0 = \lambda x . E \\ l_0(E_0) & E_0 \neq \lambda x . E \\ l_0(E_1) & l_0(E_0) = \emptyset \end{cases}$

①

var x : int = 1 ;
var ~~y~~ z : int = 2 ;] D_0

procedure p(var z : int) $\overbrace{z := x+1}^{C}$;

var z : int = 3] D_1
p(5)] C_1

] D_2] C_2] C_3] C_4

z ? in scoping statico e dinamico

$\boxed{z=3}$ per scoping statico e dinamico

var z : int = 1
var z : int = 2 in var x : int = 1 + z
procedure p(z : int) z = x+1
var z : int = z+1
p(~~z~~+1)

z ? → z = 2 per scoping statico e dinamico

$$\frac{\Delta_5 \vdash 5 : int}{\Delta_5 \vdash 5, \bullet : int, \bullet} \quad , \Delta_5(p) = int, \bullet \ proc$$

$$\frac{\bullet : \emptyset}{var\ z : int, \bullet : [z : int\,loc] : \Delta_2} \qquad \frac{\Delta[\Delta_1][\Delta_2] \vdash x : int}{\Delta[\Delta_1][\Delta_2] \vdash Corpo} \qquad \frac{\Delta[\Delta_1][\Delta_2] \vdash 3 : int}{\Delta[\Delta_1][\Delta_2] \overset{=\Delta_4}{\vdash} D_1 : \Delta_1[z : int\,loc]}$$

Δ_5

$$\Delta[\Delta_1] \vdash D ; C$$

$$\frac{\emptyset \vdash 1 : int}{\emptyset \vdash var\ x : int = 1 : [x : int\,loc] : \Delta} \qquad \frac{\dfrac{\Delta \vdash 2 : int}{\Delta \vdash var\ z : int = 2 : [z : int\,loc] : \Delta_1}}{\Delta \vdash D ; C_c}$$

*

$$\emptyset \vdash D_i ; C_2$$

$$\emptyset \vdash C_4$$

1

$\emptyset \to \langle [x=l_x][z=l_z] ; D , [l_x=1][l_z=2] \rangle \to$

$\boxed{5.5}$

$\langle \underbrace{[x=l_x][z=l_z]}_{\rho_0}[\rho : \lambda\, var\ z : int . \rho_0 ; c] ; D_1 ; C_1 , [l_x=1][l_z=2] \rangle$

$\langle [x=l_x][z=l_z][\rho : \lambda\, var\ z : int . \rho_0 ; c][z=l_z'] ; C_1 , [l_x=1][l_z=2][l_z'=?] \rangle$

$\langle [x=l_x][z=l_z][\rho : \lambda\, var\ z : int . \rho_0 ; c][z=l_z'] ; var\ z : int = 5 ; c ,$
$[l_x=1][l_z=2][l_z'=3] \rangle$

$\langle [x=l_x][z=l_z][\rho : \lambda\, var\ z : int . \rho_0 ; c][z=l_z'][z=l_z''] ; c , [l_x=1][l_z=2] ,$
$[l_z'=3][l_z''=5] \rangle$

$\langle [x=l_x][z=l_z][\rho : \lambda\, var\ z : int . \rho_0 ; c][z=l_z'][z=l_z''] , [l_x=1][l_z=2] ,$
$[l_z'=3][l_z''=2] \rangle$

$\langle [x=l_x][z=l_z][\rho : \lambda\, var\ z : int . \rho_0 ; c][z=l_z'] , [l_x=1][l_z=2][l_z'=3] \rangle$