ESERCIZI SUL CALCOLO DIFFERENZIALE

(1) eq. delle tangenti nei punti indicati $\qquad y = f(c) + f'(c)(x-c)$

$f(x) = |x^2-1| - 2x^2+3x \qquad c_1 = 0 \qquad c_2 = 1 \qquad c_3 = 2$

$$f(x) = \begin{cases} -x^2+3x-1 & x \le -1,\ x \ge 1 \\ 1-3x^2+3x & -1 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} -2x+3 & x < -1,\ x > 1 \\ -6x+3 & -1 < x < 1 \end{cases} \qquad \begin{matrix} f'_+(1) = 1 \\ f'_-(1) = -3 \end{matrix} \qquad \nexists f'(1)$$

$c_1 = 0 \qquad f(0) = 1 \qquad f'(0) = 3 \qquad t_1 : y = 1+3x$

$c_2 = 1 \qquad f(1) = 1 \qquad f'_-(1) = 3,\ f'_+(1) = 1 \qquad t'_2 : y = 1 - 3(x-1)$ da sin $\qquad$ P. ANGOLOSO

$t''_2 : y = 1 + (x-1)$ da dx.

$c_3 = 2 \qquad f(2) = 1 \qquad f'(2) = -1 \qquad t_3 : y = 1-(x-2)$

$f(x) = \operatorname{arctg} \dfrac{2x}{|x+2|+1} \qquad c_1 = -3 \qquad c_2 = -2 \qquad c_3 = 0$

$$f(x) = \begin{cases} -\operatorname{arctg} \dfrac{2x}{x+1} & x < -2 \\ \operatorname{arctg} \dfrac{2x}{x+3} & x \ge -2 \end{cases}$$

$$f'(x) = \begin{cases} -\dfrac{1}{1+\frac{4x^2}{(x+1)^2}} \dfrac{2x+2-2x}{(x+1)^2} = -\dfrac{2}{4x^2+1} & x < -2 \qquad f'_-(-2) = -\dfrac{2}{17} \\ \dfrac{1}{1+\frac{4x^2}{(x+3)^2}} \dfrac{2x+6-2x}{(x+3)^2} = \dfrac{6}{(x+3)^2+4x^2} & x > -2 \qquad f'_+(-2) = \dfrac{6}{17} \end{cases} \qquad \nexists f'(-2)$$

$c_1 = -3 \qquad f(-3) = -\operatorname{arctg} 3 \qquad f'(-3) = -\dfrac{2}{37} \qquad t_1 : y = -\operatorname{arctg} 3 - \dfrac{2}{37}(x+3)$

$c_2 = -2 \qquad f(-2) = -\operatorname{arctg} 4 \qquad f'_-(-2) = -\dfrac{2}{17},\ f'_+(-2) = \dfrac{6}{17} \qquad t'_2 : y = -\operatorname{arctg} 4 - \dfrac{2}{17}(x+2)$ $\qquad$ PUNTO ANGOL.

$t''_2 : y = -\operatorname{arctg} 4 + \dfrac{6}{17}(x+2)$

$c_3 = 0 \qquad f(0) = 0 \qquad f'(0) = \dfrac{2}{3} \qquad t_3 : y = \dfrac{2}{3}x$

$c_3 = 0 \qquad f(0)=0 \qquad f'(0)=\frac{2}{3} \qquad t_3 : y = \frac{2}{3}x$

$y = f(c) + f'(c)(x-c)$

$f(x) = \log(|x-3|+1) \qquad c_1 = 1, \quad c_2 = 3, \quad c_3 = 4$

$$f(x) = \begin{cases} \log(4-x) & x<3 \\ \log(x-2) & x \geq 3 \end{cases} \qquad f'(x) = \begin{cases} \frac{1}{x-4} & x<3 \\ \frac{1}{x-2} & x>3 \end{cases}$$

$f'_-(3) = -1$

$\nexists f'(3)$

$f'_+(3) = 1$

$c_1 = 1 \qquad f(1) = \log 3 \qquad f'(1) = -\frac{1}{3} \qquad t_1 : y = \log 3 - \frac{1}{3}(x-1)$

$c_2 = 3 \qquad f(3) = 0 \qquad f'_-(3) = -1, \ f'_+(3) = 1 \qquad t'_2 : y = -(x-3)$ da sin.

$t''_2 : y = x-3$ da dx.

p. angol.

$c_3 = 4 \qquad f(4) = \log 2 \qquad f'(4) = \frac{1}{2} \qquad t_3 : y = \log 2 + \frac{1}{2}(x-4)$

PROPOSTI

$f(x) = |x-2| + x^2 - 2x + 1 \qquad c_1 = 0, \quad c_2 = 2 \quad c_3 = 4$

$f(x) = \log((x+1)^2 + |x|) \qquad c_1 = -1 \quad c_2 = 0 \quad c_3 = 2$

$f(x) = \arctg \frac{|x|}{x+3} \qquad c_1 = -2 \quad c_2 = 0 \quad c_3 = 1$

2 estremi assoluti nell'intervallo indicato

$f(x) = |x^2 - 3x| + 2x^2 - x + 1 \qquad [-1, 4]$

$A = \{c \in]a,b[: f'(c) = 0\}$

$B = \{c \in]a,b[: \nexists f'(c)\}$

$C = \{a; b\}$

$$f(x) = \begin{cases} 3x^2 - 4x + 1 & x \leq 0, \ x \geq 3 \\ x^2 + 2x + 1 & 0 < x < 3 \end{cases}$$

$$f'(x) = \begin{cases} 6x - 4 & x<0, \ x>3 \\ 2x+2 & 0<x<3 \end{cases}$$

$f'_-(0) = -4 \qquad f'_+(3) = 14$

$f'_+(0) = 2 \qquad f'_-(3) = -4$

$\nexists f'(0), \ \nexists f'(3)$

$6x - 4 = 0$ per $x = \frac{2}{3} \notin]-\infty, 0[\cup]3, +\infty[$

$2x + 2 = 0$ per $x = -1 \notin]0,3[$

$\Big\} \Rightarrow A = \emptyset$

$B = \{0; 3\} \qquad C = \{-1; 4\}$

$f(0) = 1 \qquad f(3) = 16 \qquad f(-1) = 8 \qquad f(4) = 33$

$\min f = 1 = f(0) \qquad \max f = 33 = f(4)$

$f(0) = 1$ $\quad$ $f(3) = 16$ $\quad$ $f(-1) = 8$ $\quad$ $f(4) = 33$

$\min\limits_{[-1,4]} f = 1 = f(0)$ $\quad$ $\max\limits_{[-1,4]} f = 33 = f(4)$

$f(x) = \log \dfrac{|x|+1}{x^2+3}$ $\quad$ $[-2, 3]$

$$f(x) = \begin{cases} \log \dfrac{1-x}{x^2+3} & x<0 \\ \log \dfrac{x+1}{x^2+3} & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{x^2+3}{1-x} \dfrac{-x^2-3-2x^2}{(x^2+3)^2} = \dfrac{1}{x-1} & x<0 \quad f'_-(0) = -1 \\ \dfrac{x^2+3}{x+1} \dfrac{x^2+3-2x^2}{(x^2+3)^2} = \dfrac{3-x^2}{(x+1)(x^2+3)} & x>0 \quad f'_+(0) = 1 \end{cases}$$

$\nexists f'(0)$

$B = \{0\}$ $\quad$ $C = \{-2; 3\}$

$\dfrac{1}{x-1} \neq 0 \quad \forall x$

$\dfrac{3-x^2}{(x+1)(x^2+3)} = 0$ per $x = -\sqrt{3} \notin]0, +\infty[$, per $x = \sqrt{3} \in]0, +\infty[$ $\quad$ $A = \{\sqrt{3}\}$

$f(\sqrt{3}) = \log \dfrac{\sqrt{3}+1}{6}$ $\quad$ $f(0) = \log \dfrac{1}{3}$ $\quad$ $f(-2) = \log \dfrac{3}{7}$ $\quad$ $f(3) = \log \dfrac{1}{3}$

$\log \dfrac{1}{3} < \log \dfrac{3}{7}$ $\quad$ $\dfrac{\sqrt{3}+1}{6} < \dfrac{1}{3}$? $\quad$ $\dfrac{\sqrt{3}+1}{6} < \dfrac{3}{7}$

$3\sqrt{3} + 3 < 6$ $\quad$ $7\sqrt{3} + 7 < 18$

$3\sqrt{3} < 3$ $\quad$ $7\sqrt{3} < 11$

$\sqrt{3} < 1$ NO $\quad$ $49 \cdot 3 < 121$ NO

$\max\limits_{[-2,3]} f = \log \dfrac{3}{7} = f(-2)$

$\min\limits_{[-2,3]} f = \log \dfrac{1}{3} = f(0) = f(3)$

(3) $\quad f(x) = 3x^4 + x - 2$ $\quad$ $f''(2)$

$f'(x) = 12x^3 + 1$ $\quad$ $f''(x) = 36x^2$ $\quad$ $f''(2) = 144 > 0 \Rightarrow x = 2$ min rel?

$f'(2) = 97 > 0 \Rightarrow$ NO

in $c = 2$ f è cresc e conv.

$f(x) = e^{\frac{x}{x^2+1}}$ $\quad$ $f'(x) = e^{\frac{x}{x^2+1}} \dfrac{1-x^2}{(x^2+1)^2}$

$c_1 = 0$

$c_2 = 1$

$$f''(x) = e^{\frac{x}{x^2+1}} \left(\frac{1-x^2}{(x^2+1)^2} \right)^2 + e^{\frac{x}{x^2+1}} \frac{-2x(x^2+1)^{\cancel{2}} - 4x\cancel{(x^2+1)}(1-x^2)}{(x^2+1)^{\cancel{4}\,3}} =$$

$$= e^{\frac{x}{x^2+1}} \left(\frac{(1-x^2)^2}{(x^2+1)^4} + \frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3} \right)$$

$$= e^{\frac{x}{x^2+1}} \left(\frac{(1-x^2)^2}{(x^2+1)^4} + \frac{\ldots}{(x^2+1)^3} \right)$$

$$= e^{\frac{x}{x^2+1}} \frac{(1-x^2)^2 + (2x^3-6x)(x^2+1)}{(x^2+1)^4}$$

$f''(0) = 1 > 0 \Rightarrow$ c'è min rel ? $\quad f'(0) = 1 \Rightarrow$ No $\quad f$ in $c=0$ è cresc e conv

$f''(1) = \sqrt{e} \dfrac{-8}{16} = -\dfrac{1}{2}\sqrt{e} < 0 \Rightarrow$ max rel ?

$f'(1) = 0 \Rightarrow$ il p. è staz.

nel p. $c=1$ f ha un max rel

(h) $\quad f(x) = e^{\frac{1}{x-2}}$ $\quad$ dim. che è invert. in $]2, +\infty[$
trovare l'ins di def di f^{-1}
calc $(f^{-1})'(e^2)$

$f'(x) = e^{\frac{1}{x-2}} \dfrac{-1}{(x-2)^2} < 0 \quad \forall x \in]2, +\infty[\quad \Rightarrow f$ strett decr $\Rightarrow$ invert

$f :]2, +\infty[\to]\inf\limits_{]2,+\infty[} f, \sup\limits_{]2,+\infty[} f[\qquad \inf\limits_{]2,+\infty[} f = \lim\limits_{x\to+\infty} f(x) = 1$

$\sup\limits_{]2,+\infty[} f = \lim\limits_{x\to 2^+} f(x) = +\infty$

$f^{-1} :]1, +\infty[\to]2, +\infty[$

calc. $(f^{-1})'(e^2) = \dfrac{1}{f'(c)}$ dove c è tale che $f(c) = e^2$

risolvere l'eq. $f(x) = e^2 \qquad e^{\frac{1}{x-2}} = e^2 \iff \dfrac{1}{x-2} = 2 \Rightarrow 1 = 2x - 4$

$x = \dfrac{5}{2} \in]2, +\infty[$

$f'\left(\dfrac{5}{2}\right) = -4e^2 \neq 0 \Rightarrow (f^{-1})'(e^2) = -\dfrac{1}{4e^2}$

$f(x) = \operatorname{arctg} \sqrt{\left|\dfrac{x+1}{x-1}\right|} \qquad]-\infty, -1[\qquad (f^{-1})'\left(\dfrac{\pi}{6}\right)$

in $]-\infty, -1[\qquad f(x) = \operatorname{arctg} \sqrt{\dfrac{x+1}{x-1}}$

$$f'(x) = \frac{1}{1 + \frac{x+1}{x-1}} \frac{1}{2\sqrt{\frac{x+1}{x-1}}} \frac{x-1-x-1}{(x-1)^2} = \frac{\cancel{x-1}}{2x} \frac{1}{\cancel{2}\sqrt{\frac{x+1}{x-1}}} \frac{-\cancel{2}}{(x-1)^{\cancel{2}}} =$$

$= -\frac{1}{2x(x-1)}\sqrt{\frac{x-1}{x+1}} < 0 \quad \forall x \in]-\infty,-1[\;\Rightarrow f$ strett decr $\Rightarrow$ invert

$\inf_{]-\infty,-1[} f = \lim_{x\to(-1)^-} f(x) = 0 \qquad \sup_{]-\infty,-1[} f = \lim_{x\to-\infty} f(x) = \frac{\pi}{4} \qquad f^{-1}:]0,\frac{\pi}{4}[\to]-\infty,-1[$

risolvere l'eq. $f(x) = \frac{\pi}{6} \qquad \operatorname{arctg}\sqrt{\frac{x+1}{x-1}} = \frac{\pi}{6} = \operatorname{arctg}\frac{1}{\sqrt{3}}$

$\frac{x+1}{x-1} = \frac{1}{3} \Rightarrow 3x+3 = x-1 \Rightarrow x = -2 \in]-\infty,-1[$

$f'(-2) = -\frac{1}{12}\sqrt{3} \neq 0 \Rightarrow (f^{-1})'\left(\frac{\pi}{6}\right) = -\frac{12}{\sqrt{3}}$

$f(x) = e^{\frac{x}{x^2+1}} \qquad]-1,1[\qquad (f^{-1})'(1)$ (proposto)

I CAPITOLO

$z = a+ib \qquad z\bar{z} = |z|^2 = a^2+b^2 \qquad i^2 = -1$

$\bar{z} = a - ib$

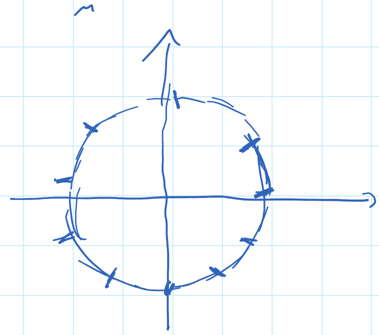
$|z| = \sqrt{a^2+b^2}$

$$\frac{2-i}{3i+5} = \frac{(2-i)(5-3i)}{(5+3i)(5-3i)} = \frac{7-11i}{25+9} = \frac{7}{34} - \frac{11}{34}i$$

$\sqrt[n]{z} = \{w_1, \dots, w_n\} \qquad w_k = \sqrt[n]{|z|}\left(\cos\frac{\overset{\alpha}{\arg z} + 2k\pi}{n} + i\sin \text{——}\right)$

$w_0 = \sqrt[n]{|z|}\left(\cos\frac{\alpha}{n} + i\sin\frac{\alpha}{n}\right)$

$w_1 = \sqrt[n]{|z|}\left(\cos\left(\frac{\alpha}{n} + \frac{2\pi}{n}\right) + i\sin \dots\right)$



$\sqrt{z} = \pm\sqrt{|z|}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right) \qquad \sqrt{1+6i} \qquad |z| = \sqrt{37} \qquad \cos\alpha = \frac{1}{\sqrt{37}}$

$\sqrt{1+6i} = \pm\sqrt{\sqrt{37}}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right) \qquad \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} \qquad \sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$

$\sqrt{-16} = \pm 4i \qquad az^2 + bz + c = 0 \qquad z = \frac{-b \pm \sqrt{\Delta}}{} \quad$ se $\Delta < 0$

$2a$ significa $\dfrac{-b \pm i\sqrt{-\Delta}}{2a}$

$$(1+2i)\,z^2 + 5z = \frac{5i}{1-2i}$$

$$(1+2i)(1-2i)\,z^2 + 5(1-2i)\,z - 5i = 0$$

$$\cancel{5}z^2 + \cancel{5}(1-2i)z - \cancel{5}i = 0 \qquad z = \frac{2i-1 \pm \sqrt{1-4-\cancel{4i}+\cancel{4i}}}{2} =$$

$$= \frac{2i-1 \pm i\sqrt{3}}{2} = \begin{cases} -\frac{1}{2} + \left(1+\frac{\sqrt{3}}{2}\right)i \\ -\frac{1}{2} + \left(1-\frac{\sqrt{3}}{2}\right)i \end{cases}$$

II cap.

$$(a_0 x^n + \dots + a_n) \to \begin{cases} +\infty & \text{se } a_0 > 0 \\ -\infty & \text{se } a_0 < 0 \end{cases}$$

$$\frac{2-n^4}{n^3+2n+1} \to -\infty \qquad \frac{2-n^4}{2n-n^3+3} \to +\infty$$

$$a^{x_n} \quad (a>0,\ a\neq 1) \qquad 2^{\frac{3n^2+1}{2-n}} \to 0$$

$$\log_a x_n \quad '' \qquad \log_{\frac{1}{3}}\left(\frac{3n^2+1}{n^4+3}\right) \to +\infty \qquad \frac{3n^2+1}{n^4+3} \to 0$$

$$\frac{\sin a_n}{a_n} \to 1 \quad \text{se } a_n \to 0 \qquad \frac{1-\cos a_n}{a_n^2} \to \frac{1}{2}$$

$$(2n^2+1)\,\mathrm{tg}\,\frac{n}{n^3+2} = \frac{\mathrm{tg}\,\frac{n}{n^3+2}}{\frac{n}{n^3+2}} \; \frac{n}{n^3+2}(2n^2+1) \to 2$$

$$\frac{\mathrm{tg}\,\frac{n}{n^3+2}}{\frac{n}{n^3+2}} \to 1 \qquad \frac{n}{n^3+2}(2n^2+1) \to 2$$

DEF $\quad \lim \left(1+\frac{1}{n}\right)^n = \sup\left(1+\frac{1}{n}\right)^n = e \qquad \left(1+\frac{1}{x_n}\right)^{x_n} \to e \quad \text{se } x_n \to \infty$

$$\left(\frac{n^2+2}{(n+3)^2}\right)^{2n-1} = \left(\frac{n^2+6n+9-6n-7}{n^2+6n+9}\right)^{2n-1} = \left[\left(1+\frac{1}{\frac{n^2+6n+9}{-6n-7}}\right)^{\frac{n^2+6n+9}{-6n-7}}\right]^{\frac{(-6n-7)(2n-1)}{n^2+6n+9}} \to e^{-12}$$

$\frac{n^2+6n+9}{-6n-7} \to -\infty$, $[\ldots] \to e$

CAPITOLO 3

$f: X \to \mathbb{R} \quad c \in D(X)$

$$\lim_{x\to c} f(x) = \ell \iff \forall \{x_n\} \subseteq X - \{c\} : x_n \to c \text{ si ha } f(x_n) \to \ell$$

$f: (a,b) \to \mathbb{R} \quad c \in (a,b) \quad f$ cont. in c se $\lim_{x\to c} f(x) = f(c)$

f cont. in $[a,b] \Rightarrow$ PVI
PVI + monot. $\Rightarrow$ cont.

$f(x) = \frac{2x^3+1}{x-3}$ ha solo as. vert.

$f(x) = \frac{2x^2+1}{x-3}$ ha as. vert. e as. obl.

$f(x) = \frac{2x^2+1}{x^4+5}$ ha solo as. orizz.

$f(x) = \frac{2x^2+1}{x^2-4}$ ha as. vert. e orizz

$f(x) = \sqrt{x^2-3x}$

as. sin. $\quad \frac{f(x)}{x} = \frac{\sqrt{x^2-3x}}{x} = -\frac{\sqrt{}}{-x} = -\sqrt{\frac{x^2-3x}{x^2}} \to -1$

$\sqrt{x^2} = -x$

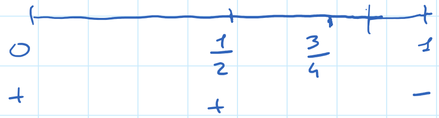
$$f(x) - (-1)x = \sqrt{x^2-3x} + x = \frac{\cancel{x^2} - 3x - \cancel{x^2}}{\sqrt{x^2-3x} - x} = \frac{3}{\sqrt{\frac{x^2-3x}{x^2}} + \frac{-x}{-x}} \to \frac{3}{2}$$

$y = -x + \frac{3}{2}$ as. obl. sin.

teor. di es. degli zeri per $f(x) = e^x - 5x^2$ in $[0,1]$

$f(0) = 1 > 0 \qquad f(1) = e - 5 < 0$

$f(0) = 1 > 0 \qquad f(1) = e - 5 < 0$



$f\left(\frac{1}{2}\right) = \sqrt{e} - \frac{5}{4} > 0$

$\sqrt{e} > \frac{5}{4}$

$e > \frac{25}{16}$

$16e > 25$ vero

$[a_1, b_1] = \left[\frac{1}{2}, 1\right]$

$f\left(\frac{3}{4}\right) = \sqrt[4]{e^3} - \frac{45}{16}$

$\sqrt[4]{e^3} > \frac{45}{16}$?

$e^3 > \left(\frac{45}{16}\right)^4$ - - - -

$$\lim_{x \to +\infty} \frac{a^x}{x^n} \qquad a > 1$$

$$\text{R.D.} \quad \frac{a^x \log a}{n x^{n-1}} \qquad \text{R.D.} \quad \frac{a^x \log^2 a}{n(n-1) x^{n-2}} \qquad - - - -$$

dopo n passi R.D. $\dfrac{a^x \log^n a}{n!} \to +\infty$

trovare le formule della derivata di fgh essendo f, g, h derivabili

$$F(x) = f(x)\, g(x)\, h(x) = f(x)\big(g(x)\, h(x)\big)$$

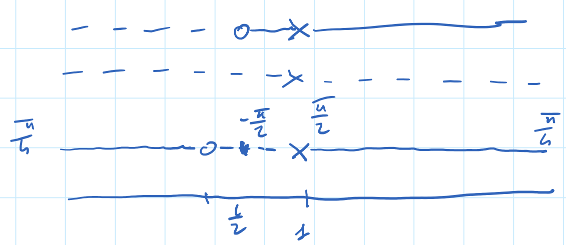
$$F'(x) = f'(x)\, g(x)\, h(x) + f(x)\big(g'(x)\, h(x) + g(x)\, h'(x)\big)$$

Studio della funzione

$$f(x) = \operatorname{arctg} \frac{x}{x-1} \qquad f : \big(]-\infty, 1[\,\cup\,]1, +\infty[\big) \to \mathbb{R}$$

f è cont. nel suo ins. di def

$x = 1$ p. di disc di I specie



$f\left(\frac{1}{2}\right) = -\frac{\pi}{4}$

f è limitata

f è derivabile

$$f'(x) = \frac{1}{1 + \frac{x^2}{(x-1)^2}} \cdot \frac{\cancel{x}-1-\cancel{x}}{(x-1)^2} = -\frac{1}{(x-1)^2 + x^2} < 0 \quad \forall x \in]-\infty, 1[\,\cup\,]1, +\infty[$$

$$f'(x) = \frac{1}{1+\frac{x^2}{(x-1)^2}} \cdot \frac{\cancel{x}-1-\cancel{x}}{(x-1)^2} = -\frac{1}{(x-1)^2+x^2} < 0 \quad \forall x \in \,]-\infty, 1[\, \cup \,]1, +\infty[$$

f decr. in $]-\infty, 1[$ e in $]1, +\infty[$

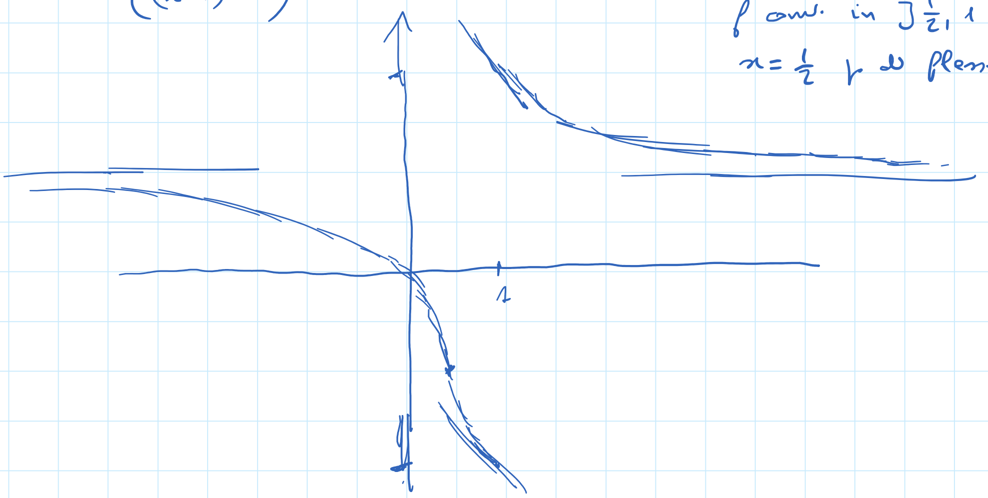
$$\lim_{x \to 1} f'(x) = -1$$

$$f''(x) = \frac{2(2x-1)}{((x-1)^2+x^2)^2} \gtrless 0 \iff x \gtrless \frac{1}{2}$$

f conc. in $]-\infty, \frac{1}{2}[$

f conv. in $]\frac{1}{2}, 1[$ e in $]1, +\infty[$

$x = \frac{1}{2}$ p. di flesso



proposto: studio di $f(x) = \log \frac{x-1}{x}$