$X = (a, b)$ $\quad \text{int}(x) =]a, b[$

$\qquad\qquad\qquad D(x) = [a, b]$

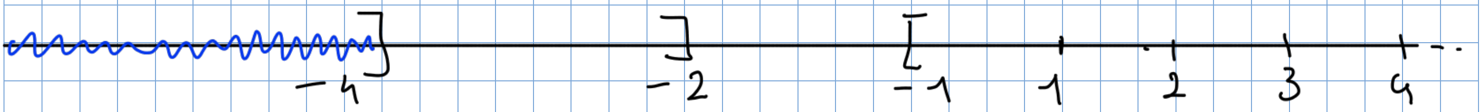
$\left.\begin{array}{l} X = (a, b) \cap \mathbb{Q} \\ X = (a, b) \cap (\mathbb{R} \setminus \mathbb{Q}) \end{array}\right\} \quad \begin{array}{l} \text{int}(x) = \emptyset \\ D(x) = [a, b] \end{array}$

$\overline{X} = X \cup D(x)$

Tipico esercizio

$X =]-\infty, -4[\; \cup \; (]-2, -1[\cap \mathbb{Q}) \cup \mathbb{N}$

trovare $\text{int}(x)$, $D(x)$, $F(x)$, $\inf x$ e $\sup x$



① $\text{int}(x) =]-\infty; -4[$

$D(x) =]-\infty; -4] \cup [-2; -1]$

$\overline{X} = X \cup D(x) =]-\infty; -4] \cup [-2, -1] \cup \mathbb{N}$

$F(x) = \{-4\}$

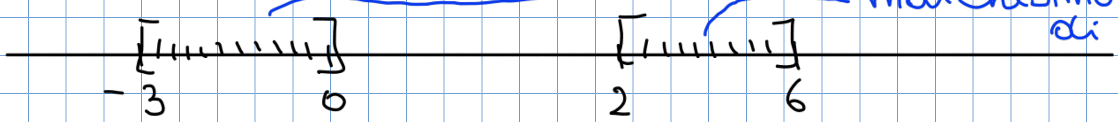
$\inf x = -\infty$

$\sup x = +\infty$

②

$X = \left([-3; 0] \cap (\mathbb{R} \setminus \mathbb{Q})\right) \cup \left([2, 4] \cap \mathbb{Q}\right)$

indichiamo solo i punti di Q



$\text{int}(x) = \emptyset$

$D(x) = [-3; 0] \cup [2, 4]$

$F(x) = [-3, 0] \cup [2, 4]$

$\overline{X} = X \cup D(x)$

$\inf x = -3$ non è min

$\sup x = 4$ non è max

(3)

$X = (]-\infty, 0[\cap \mathbb{Z}) \cup (]4,+\infty[\cap (\mathbb{R}\setminus\mathbb{Q}))$

Da fare per casa

Trovare un esempio di insieme X tale che

1) $\sup \in D(X)$
2) $\sup X \notin D(X)$
3) $\sup X \in \operatorname{int}(X)$
4) $\sup X = \max(X)$
5) $\sup X$ non è $\max(X)$
6) X che abbia sia punti di acc che punti isolati

$f: X \to \mathbb{R}$ $\quad X \subseteq \mathbb{R}$ $\quad f(X) = \{ f(x) : x \in X \}$

$f: (a,b) \to \mathbb{R}$ monotona in (a,b) se è

strett cres $\quad (x<y \Rightarrow f(x) < f(y))$

// decr

cresc (debolmente)

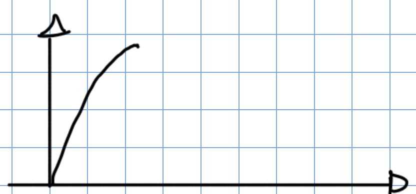
decres (//)

$f: (a,b) \to (\alpha,\beta)$ strett crescente

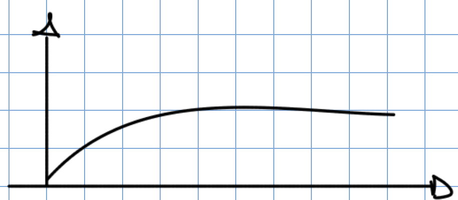
dimostrare che la sua inversa è strett cresc

$f^{-1}: (\alpha,\beta) \to (a,b)$

$y,z \in (\alpha,\beta)$ $\quad y<z \Rightarrow f^{-1}(y) < f^{-1}(z)$



f



f^{-1}

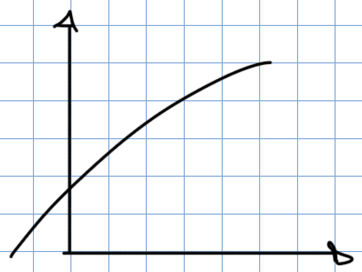
$f^{-1}(y) = x \in (a,b) : f(x) = y$

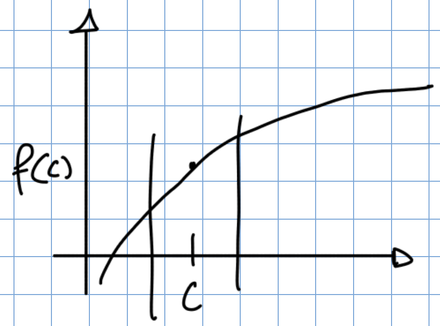
$f^{-1}(z) = x' \in (a,b) : f(x') = z$ devo provare che $x < x'$

se fosse $x \geq x'$ dato che f è strettamente cresc si avrebbe

$f(x) \geq f(x')$

$y \geq z$ falso





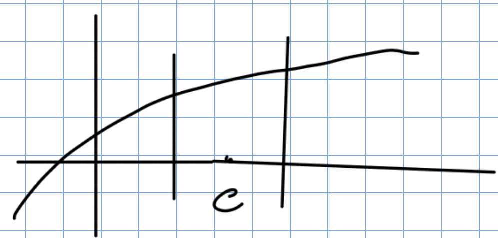
$f(a,b) \to \mathbb{R}$ $c \in (a,b)$ f crescente in c se $\exists r > 0$

se $x \in (a,b) \cap \,]c-r; c[$ si ha $f(x) < f(c)$

se $x \in (a,b) \cap \,]c, c+r[$ si ha $f(x) > f(c)$

Teorema

f strett cresc (strett decres) in $(a,b) \Leftrightarrow$ è cresc (decres) in ogni punto di (a,b)



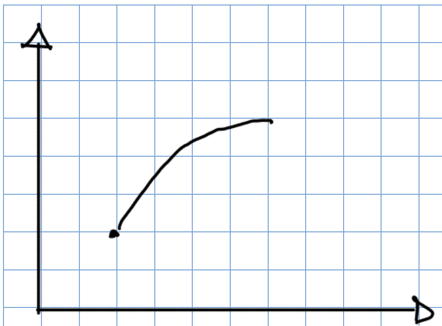
$f : (a,b) \to \mathbb{R}$ $c \in (a,b)$

$\forall x \in (a,b), x \neq c$ poniamo $r(x) = \dfrac{f(x) - f(c)}{x - c}$

Rapporto incrementale

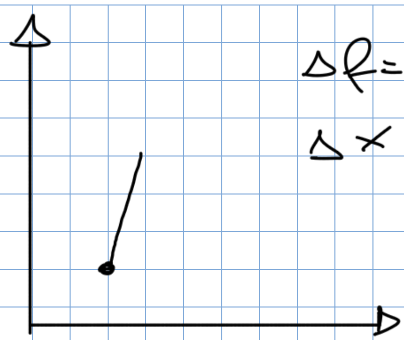
$\forall x \neq c$ $\Delta f = f(x) - f(c)$ incremento della funzione

$\Delta x = x - c$ variabile



$\Delta f = 3$

$\Delta x = 4$



$\Delta f = 3$

$\Delta x = 1$

teor f cresc. in $c \iff \exists\, \delta > 0$ $\quad r(x) > 0$

$r(x) < 0$

$\forall x \in \,]c-\delta, c+\delta[\,, \ x \neq c$

Dim Se f è cresc $\Rightarrow \exists\, \delta > 0 : f(x) < f(c)$ in $]c-\delta; c[$

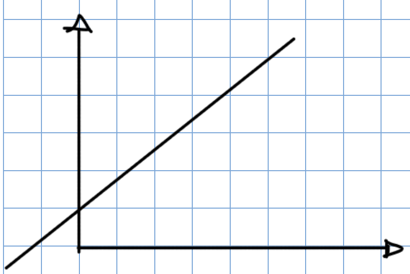
$f(x) > f(c)$ in $]c, c+\delta[$

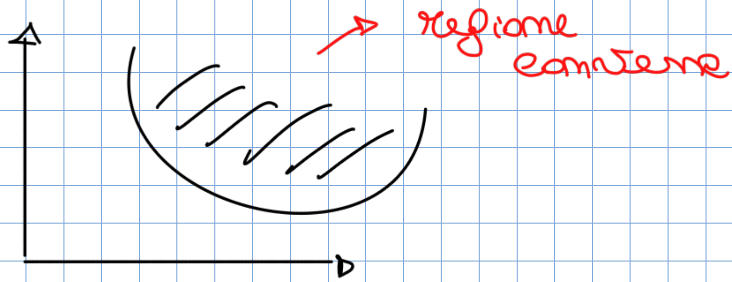
in $]c-\delta, c[$ $\quad \dfrac{f(x) - f(c) < 0}{x - c \quad > 0} \Rightarrow r(x) > 0$

Viceversa se $r(x) > 0$ in $]c-\delta, c+\delta[$

$$\left.\begin{array}{l} \text{in }]c-\delta, c[\quad \dfrac{r(x) > 0}{x - c < 0} \Rightarrow f(x) - f(c) < 0 \\ \\ \text{in }]c, c+\delta[\quad \dfrac{r(x) > 0}{x - c > 0} \Rightarrow f(x) - f(c) > 0 \end{array}\right\} f \text{ cresc in } c$$

Funz convesse

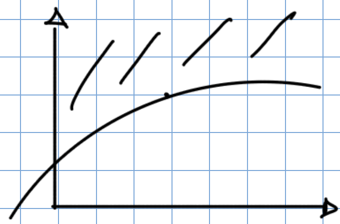




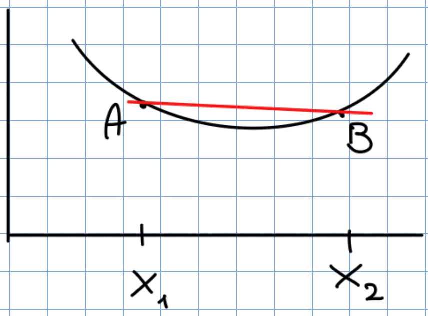
$X \in \mathbb{R}^2$ convesso se $X = \emptyset$ oppure $X = \{P\}$ oppure se contiene due punti, contiene il segmento che li comprende

$f:(a,b)\to\mathbb{R}$ $\quad \text{epi}(f) = \{(x,y)\in\mathbb{R}^2 : y \ge f(x)\}$

↓
epigrafico



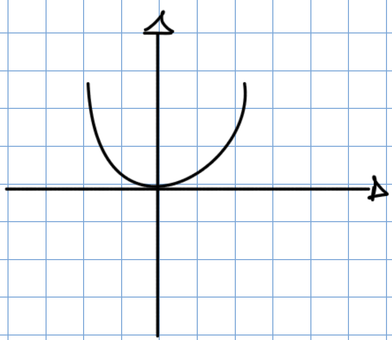
se epi(f) è convesso, f si dice convessa



La parte di grafico relativo all'interv. $[x_1, x_2]$ sta al di sotto del segmento $\overline{AB}$

GRAFICO: $f:(a,b)\to\mathbb{R}$

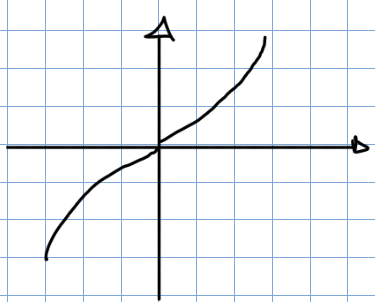
$G(f) = \{(x, f(x)) : x \in (a,b)\}$



simm. rispetto a y cioè $(x,y)\in G(f)$

$\Rightarrow (-x,y)\in G(f)$

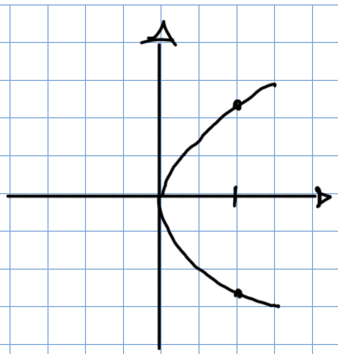
$f(-x) = f(x)$ funzione pari



simm. rispetto all'origine

se $(x,y)\in G(f) \Rightarrow (-x,-y)\in G(f)$

$f(-x) = -f(x)$ (funzione dispari)



il profilo non è mai simmetrico rispetto a $\vec{x}$

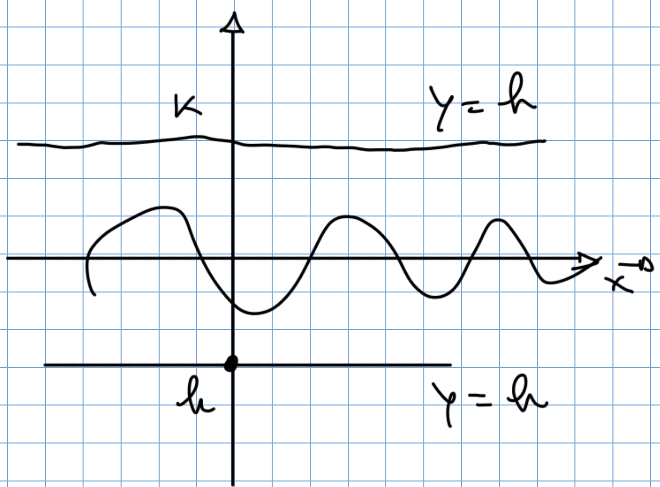
MARCELLINI - SBORDONE
Esercitazione di matematica

$f:]-\infty ; +\infty[$ se $\exists T > 0 : f(x+T) = f(x) \ \forall x \in \mathbb{R}$
f periodica di periodo T

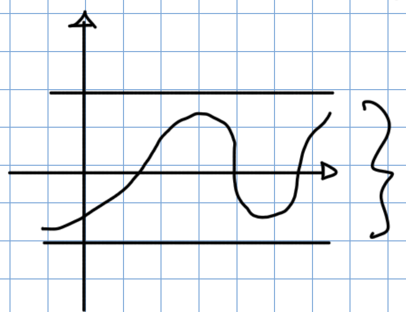
f periodica di periodo $T \Leftrightarrow f(x+hT) = f(x) \ \forall x \in \mathbb{R}$
$\forall h \in \mathbb{Z}$

$$f(x+2T) = f(x+T+T) = f((x+T)+T) = f(x+T) = f(x)$$

periodo



oscillazione = sup f − inf f = sup $\{|f(x) - f(y)| : x, y \in (a, b)\}$
(se limitata)



oscillazione di f

se f non è limitata osc $f = +\infty$