

$$x = (a, b) \quad \begin{aligned} \text{int}(x) &=]a, b[\\ D(x) &= [a, b] \end{aligned}$$

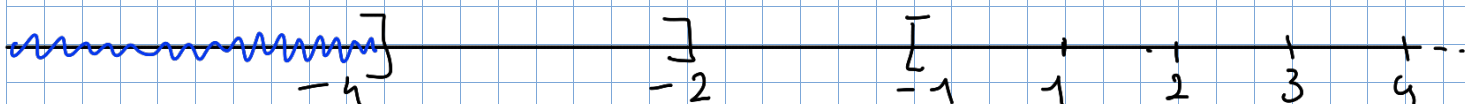
$$\left. \begin{aligned} x &= (a, b) \cap \mathbb{Q} \\ x &= (a, b) \cap (\mathbb{R} \setminus \mathbb{Q}) \end{aligned} \right\} \quad \begin{aligned} \text{int}(x) &= \emptyset \\ D(x) &= [a, b] \end{aligned}$$

$$\bar{x} = x \cup D(x)$$

Tipico esercizio

$$x =]-\infty, -4[\cup (]-2, -1[\cap \mathbb{Q}) \cup \mathbb{N}$$

trovare $\text{int}(x)$, $D(x)$, $F(x)$, $\inf x$ e $\sup x$



$$\textcircled{1} \text{int}(x) =]-\infty, -4[$$

$$D(x) =]-\infty, -4] \cup [-2, -1]$$

$$\bar{x} = x \cup D(x) =]-\infty, -4] \cup [-2, -1] \cup \mathbb{N}$$

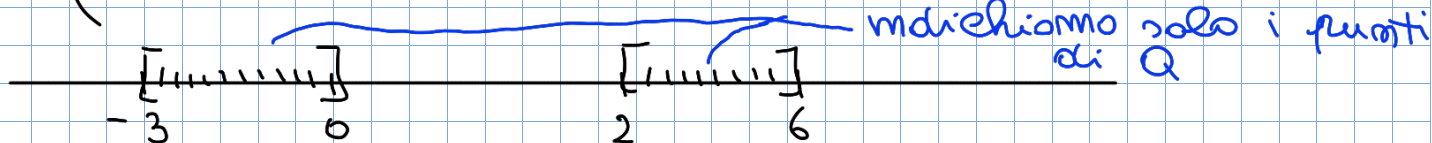
$$F(x) = \{-4\}$$

$$\inf x = -\infty$$

$$\sup x = +\infty$$

$\textcircled{2}$

$$x = ([-3, 0] \cap (\mathbb{R} \setminus \mathbb{Q})) \cup ([2, 4] \cap \mathbb{Q})$$



$$\text{int}(x) = \emptyset$$

$$D(x) = [-3, 0] \cup [2, 4]$$

$$F(x) = [-3, 0] \cup [2, 4]$$

$$\bar{x} = x \cup D(x)$$

$$\inf x = -3 \text{ non \acute{e} min}$$

$$\sup x = 4 \text{ non \acute{e} max}$$

③

$$X = (\exists - \infty, 0[\cap \mathbb{Z}) \cup (\exists 4, +\infty[\cap (\mathbb{R} \setminus \mathbb{Q}))$$

Da fare per cose

trovare un esempio di insieme X tale che

1) $\sup X \in D(X)$

$X = [a, b]$

2) $\sup X \notin D(X)$

$X = \{1, 2, 3\}$

3) $\sup X \in \inf(X)$

non esiste

4) $\sup X = \max(X)$ $]4, 5[$

5) $\sup X$ non è $\max(X)$

$X =]a, b[$

6) X che abbia sia punti di acc che punti isolati $X = [0, 1] \cup \{3\}$

$f: X \rightarrow \mathbb{R} \quad X \subseteq \mathbb{R} \quad f(X) = \{f(x) : x \in X\}$

$f: (a, b) \rightarrow \mathbb{R}$ monotona in (a, b) se e

strictly cresc $(x < y \Rightarrow f(x) < f(y))$

" decr

cresc (debolmente)

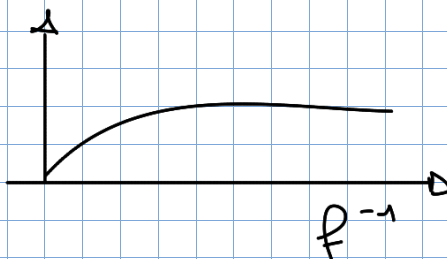
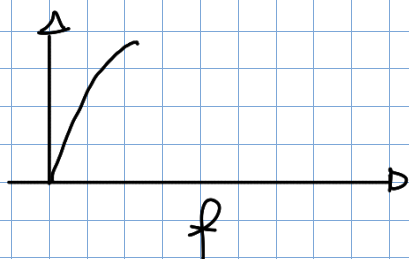
decr (")

$f: (a, b) \rightarrow (d, \beta)$ strictly crescente

dimostrare che la sua inversa è strictly cresc

$d^{-1}: (d, \beta) \rightarrow (a, b)$

$y, z \in (d, \beta) \quad y < z \Rightarrow f^{-1}(y) < f^{-1}(z)$



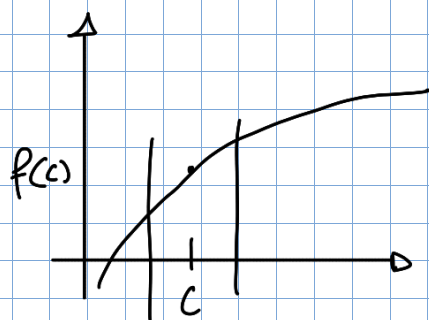
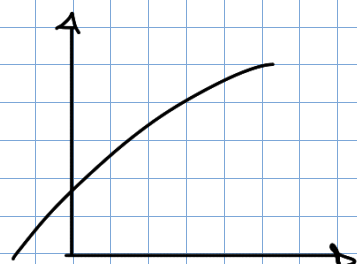
$$f^{-1}(y) = x \in (a, b): f(x) = y \quad \text{devo provare che } x < x'$$

$$f^{-1}(z) = x' \in (a, b): f(x') = z$$

ma forse $x \geq x'$ dato che f è strettamente crescente si avrebbe

$$f(x) \geq f(x')$$

$$y \geq z \quad \text{falso}$$



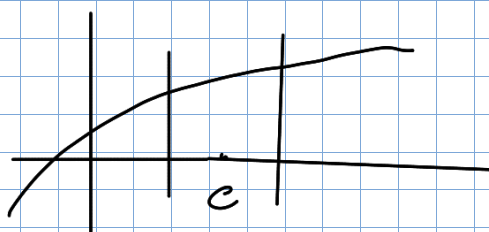
$f: (a, b) \rightarrow \mathbb{R}$ $c \in (a, b)$ f crescente in c se $\exists \eta > 0$

se $x \in (a, b) \cap]c - \eta, c[$ si ha $f(x) < f(c)$

se $x \in (a, b) \cap]c, c + \eta[$ si ha $f(x) > f(c)$

Teorema

f strett. cresc. (strett. decres.) in $(a, b) \Leftrightarrow f$ è cresc. (decres.) in ogni punto di (a, b)

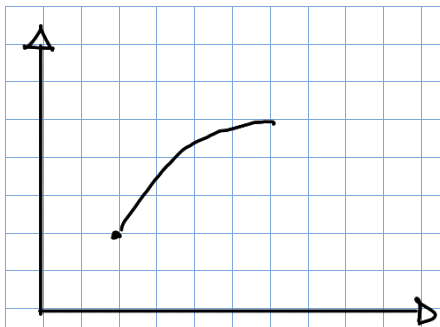


$f: (a, b) \rightarrow \mathbb{R}$ $c \in (a, b)$

$$\forall x \in (a, b), x \neq c \quad \text{rapporto } q(x) = \frac{f(x) - f(c)}{x - c}$$

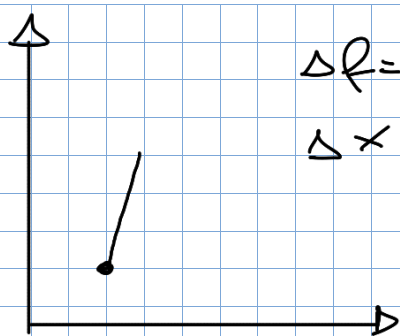
Rapporto incrementale

$\forall x \neq c \quad \Delta f = f(x) - f(c)$ incremento della
funzione variabile
 $\Delta x = x - c$



$$\Delta f = 3$$

$$\Delta x = 4$$



$$\Delta f = 3$$

$$\Delta x = 1$$

una f è c.v. in $c \iff \exists \delta > 0 \quad \kappa(x) > 0$

$$\kappa(x) < 0$$

$$\forall x \in]c-\delta, c+\delta[, x \neq c$$

Dim. Se f è c.v. in $c \Rightarrow \exists \delta > 0: f(x) < f(c)$ in $]c-\delta, c[$
 $f(x) > f(c)$ in $]c, c+\delta[$

$$\text{in }]c-\delta, c[\quad \frac{f(x) - f(c)}{x - c} < 0 \Rightarrow \kappa(x) > 0$$

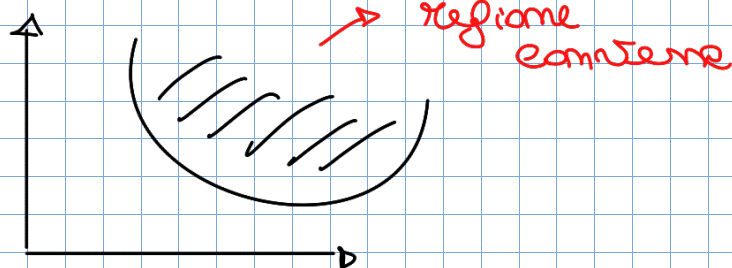
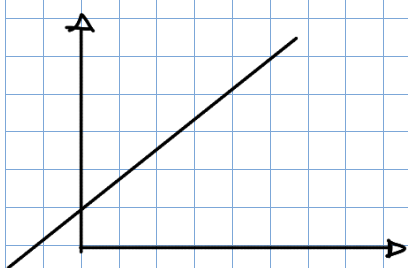
viceversa se $\kappa(x) > 0$ in $]c-\delta, c+\delta[$

$$\text{in }]c-\delta, c[\quad \kappa(x) > 0 \Rightarrow \frac{f(x) - f(c)}{x - c} < 0$$

$$\text{in }]c, c+\delta[\quad \kappa(x) > 0 \Rightarrow \frac{f(x) - f(c)}{x - c} > 0$$

} f
c.v. in c

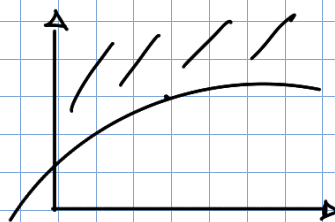
Funz. Convesse



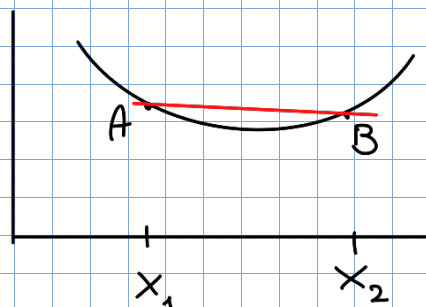
$x \in \mathbb{R}^2$ convesso se $x = \emptyset$ oppure $x = \{P\}$ oppure se
 contiene due punti, contiene il segmento che li
 congiunge

$$f: (a, b) \rightarrow \mathbb{R} \quad \text{epi}(f) = \{(x, y) \in \mathbb{R}^2 : y \geq f(x)\}$$

\downarrow
epigrafico



se $\text{epi}(f)$ è convesso, f si dice convessa

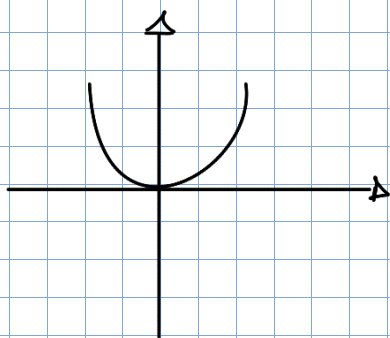


Le pte di grafico
relativo all'interv.

$[x_1, x_2]$ sta al
di sotto del segmento
 \overline{AB}

GRAFICO: $f: (a, b) \rightarrow \mathbb{R}$

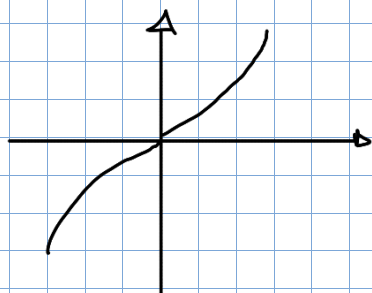
$$\rho(f) = \{(x, f(x)) : x \in (a, b)\}$$



simmm. rispetto a \vec{y} cioè $(x, y) \in \rho(f)$

$$\Rightarrow (-x, y) \in \rho(f)$$

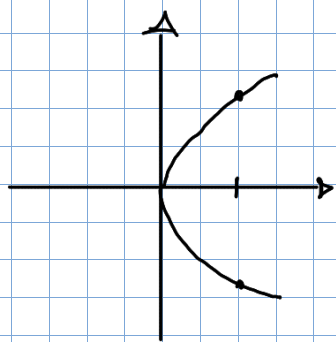
$$f(-x) = f(x) \quad \text{funzione pari}$$



simmm. rispetto all'origine

$$\text{se } (x, y) \in \rho(f) \Rightarrow (-x, -y) \in \rho(f)$$

$$f(-x) = -f(x) \quad (\text{funzione dispari})$$



il grafico non è mai simmetrico rispetto a \vec{x}

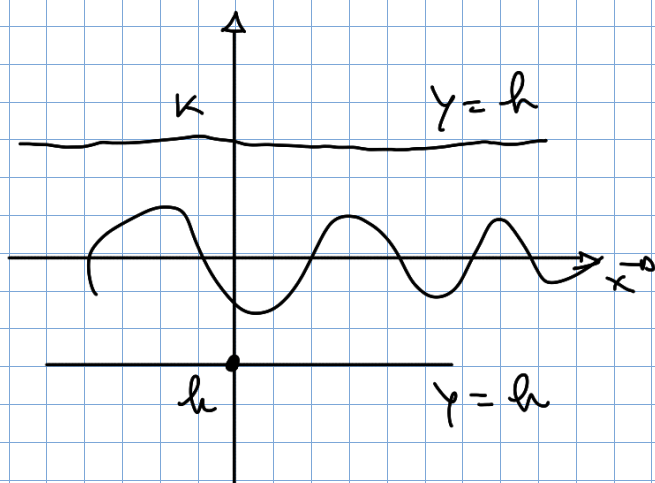
MARCELLINI - SBOADONE
Esercitazione di matematica

$f:]-\infty; +\infty[\rightarrow \mathbb{R}$ ne $\exists T > 0: f(x+T) = f(x) \forall x \in \mathbb{R}$
 f periodica di periodo T

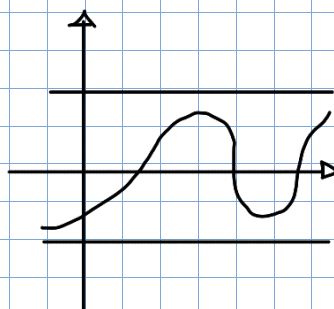
f periodica di periodo $T \Leftrightarrow f(x+hT) = f(x) \forall x \in \mathbb{R}$
 $\forall h \in \mathbb{Z}$

$$f(x+2T) = f(x+T+T) = f((x+T)+T) = f(x+T) = f(x)$$

↑
periodo



oscillazione = $\sup f - \inf f = \sup \{ |f(x) - f(y)| : x, y \in (a, b) \}$
 (se limitate)



} oscillazione di f

se f non è limitate o se $f = +\infty$

