Esercizi sul calcolo differenziale

$$y = f(c) + f'(c)\cdot(x - c)$$

1) $\underbrace{\text{eq tangente}}$ nei punti indicati

$$f(x) = (x^2 - 1) - 2x^2 + 3x \qquad c_1 = 0 \quad c_2 = 1 \quad c_3 = 2$$

$$f(x) = \begin{cases} -x^2 + 3x - 1 & x \le -1,\ x \ge 1 \\ 1 - 3x^2 + 3x & -1 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} -2x + 3 & x < -1,\ x > 1 \\ -6x + 3 & -1 < x < 1 \end{cases} \qquad \begin{array}{l} f'_+(1) = 1 \\ f'_-(1) = 3 \end{array} \quad \nexists f'(1)$$

$c_1 = 0 \qquad f(0) = 1 \qquad f'(0) = 3 \qquad t_1:\ y = 1 + 3x$

$c_2 = 1 \qquad f(1) = 1 \qquad f'_-(1) = 3,\ f'_+ = 1 \qquad t'_2:\ y = 1 - 3(x-1)$ da sx

$t''_2:\ y = 1 + (x - 1)$ da dx

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$c_3 = 2 \qquad f(2) = 1 \qquad f'(2) = -1 \qquad t_3:\ y = 1 - (x - 2)$

$$f(x) = \arctan \frac{2x}{(x+2)+1} \qquad c_1 = -3 \quad c_2 = -2 \quad c_3 = 0$$

$$f(x) = \begin{cases} -\arctan \dfrac{2x}{x+1} & x < -2 \\ \arctan \dfrac{2x}{x+3} & x \ge -2 \end{cases}$$

$$f'(x) = \begin{cases} -\dfrac{1}{1 + \frac{4x^2}{(x+1)^2}} \cdot \dfrac{2x+2-2x}{(x+1)^2} = -\dfrac{2}{4x^2+1} & x < -2 \qquad f'_-(-2) = -\dfrac{2}{17} \\ \dfrac{1}{1 + \frac{4x^2}{(x+3)^2}} \cdot \dfrac{2x+6-2x}{(x+3)^2} = \dfrac{6}{(x+3)^2 + 4x^2} & x > -2 \qquad f'_+(-2) = \dfrac{6}{17} \end{cases}$$

$\nexists f'(2)$

$c_1 = -3 \quad f(-3) = -\arctan 3 \quad f'(-3) = -\frac{2}{37}$

$t_1: y = \arctan 3 - \frac{2}{37}(x+3)$

$c_2 = -2 \quad f(-2) = -\arctan 4 \quad f'_-(-2) = -\frac{2}{17} \quad t_2': y = -\operatorname{arctg} 4 - \frac{2}{17}(x+2)$

$f'_+(-2) = \frac{6}{17} \quad \underbrace{t_2'': y = -\operatorname{arctg} 4 + \frac{6}{17}(x+2)}_{\text{P. ANGOLOSO}}$

$c_3 = 0 \quad f(0) = 0 \quad f'(0) = \frac{2}{3} \quad t_3: y = \frac{2}{3}x$

$f(x) = \log(|x-3|+1) \qquad c_1 = 1, \; c_2 = 3, \; c_3 = 4$

$$f(x) = \begin{cases} \log(4-x) & x < 3 \\ \log(x-2) & x \geq 3 \end{cases} \qquad f'(x) = \begin{cases} \frac{1}{x-4} & x < 3 \\ \frac{1}{x-2} & x > 3 \end{cases} \qquad \begin{array}{l} f'_-(3) = -1 \\ f'_+(3) = 1 \end{array} \; \nexists f'(3)$$

$c_1 = 1 \quad f(1) = \log 3 \quad f'(1) = -\frac{1}{3} \quad t_1: y = \log 3 - \frac{1}{3}(x-1)$

$c_2 = 3 \quad f(3) = 0 \quad f'_-(3) = -1, \; f'_+(3) = 1 \quad t_2': y = -(x-3)$ da sx

$t_2'': y = x - 3$ da dx

$c_3 = 4 \quad f(4) = \log 2 \quad f'(4) = \frac{1}{2} \quad t_3: y = \log 2 + \frac{1}{2}(x-4)$

Proposti:

$f(x) = |x-2| + x^2 - 2x + 1 \qquad c_1 = 0, \; c_2 = 2, \; c_3 = 4$

$f(x) = \log((x+1)^2 + |x|) \qquad c_1 = -1, \; c_2 = 0, \; c_3 = 2$

$f(x) = \arctan \frac{|x|}{x+3} \qquad c_1 = -2 \quad c_2 = 0 \quad c_3 = 1$

② estremi assoluti nell'intervallo indicato

$$f(x) = |x^2 - 3x| + 2x^2 - x + 1 \qquad [-1, 4]$$

$A = \{ c \in]a, b[: f'(c) = 0 \} \to$ stazionari

$B = \{ c \in]a, b[: \nexists f'(c) \} \to$ dove non esiste f'

$C = \{ a; b \}$ estremi

$$f(x) = \begin{cases} 3x^2 - 4x + 1 & x \leq 0, x \geq 3 \\ x^2 + 2x + 1 & 0 < x < 3 \end{cases}$$

$$f'(x) = \begin{cases} 6x - 4 & x < 0, x > 3 \\ 2x + 2 & 0 < x < 3 \end{cases}$$

$f'_-(0) = -4 \quad f'_+(3) = 14$

$f'_+(0) = 2 \quad f'_-(3) = -4$

$\nexists f'(0), \nexists f'(3)$

$B = \{0; 3\} \quad C = \{-1; 4\}$

$$\left.\begin{array}{l} 6x - 4 = 0 \quad \text{per } x = \frac{2}{3} \notin]-\infty, 0[\cup]3, +\infty[\\ 2x + 2 = 0 \quad \text{per } x = -1 \notin]0, 3[\end{array}\right\} \Rightarrow A = \emptyset$$

$f(0) = 1 \quad f(3) = 16 \quad f(-1) = 8 \quad f(4) = 33$

$\min_{[1,4]} f = 1 = f(0) \qquad \max_{[-1,4]} = 33 = f(4)$

$$f(x) = \begin{cases} \log \dfrac{|x| + 1}{2x^2 + 3} & x < 0 \\ \log \dfrac{x + 1}{x^2 + 3} & x \geq 0 \end{cases} \qquad [-2, 3]$$

$$f'(x) \to \frac{x^2 + 3}{1 - x} \cdot \frac{-x^2 - 3 - 2x^2}{(x^2 + 3)^2} = \frac{1}{x - 1} \quad x < 0 \qquad f'_-(0) = -1$$

$$\to \frac{x^2 + 3}{x + 1} \cdot \frac{x^2 + 3 - 2x^2}{(x^2 + 3)^2} = \frac{3 - x^2}{(x + 1)(x^2 + 3)} \quad x > 0 \qquad f'_+(0) = 1$$

$\nexists f'(0)$

$B = \{0\}$ $\quad C = \{-2;\ 3\}$

$\frac{1}{x-1} \neq 0 \quad \forall x$

$\frac{3-x^2}{(x+1)(x^2+3)} = 0$ per $x = -\sqrt{3} \notin]0,+\infty[$ per $x = \sqrt{3} \in]0,+\infty[$

$A = \{\sqrt{3}\}$

$f(\sqrt{3}) = \log \frac{\sqrt{3}+1}{6}$ $\quad f(0) = \log \frac{1}{3}$ $\quad f(-2) = \log \frac{3}{7}$ $\quad f(3) = \log \frac{1}{3}$

$\log 3 < \log \frac{3}{7}$ $\quad \frac{\sqrt{3}+1}{6} < \frac{1}{3}$? $\quad \frac{\sqrt{3}+1}{6} < \frac{3}{7}$

$3\sqrt{3} + 3 < 6$ $\quad 7\sqrt{3} + 7 < 18$

$3\sqrt{3} < 3$ $\quad 7\sqrt{3} < 11$

$\sqrt{3} < 1$ $\quad 49 \cdot 3 < 110$

$\max_{[-1,3]} f = \log \frac{3}{7} = f(-2)$

$\min_{[-1,3]} f = \log \frac{1}{3} = f(0) = f(3)$

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(3) Calcolare $f'(x)$ e $f'(x)$, crescente? convessa?

$f(x) = 3x^4 + x - 2$ $\quad f''(2)$

$f'(x) = 12x^3 + 1$ $\quad f''(x) = 36x^2$

$f'(2) = 144 > 0 \Rightarrow x = 2$ min rel?

$f''(2) = 97 > 0 \Rightarrow$ NO

in $c = 2$ f è cresc e con

$f(x) = e^{\frac{x}{x^2+1}}$ $\quad f'(x) = e^{\frac{x}{x^2+1}} \frac{1-x^2}{(x^2+1)^2}$

$c = 0$

$c_2 = 1$

$$f''(x) = e^{\frac{x}{x^2+1}} \left(\frac{1-x^2}{(x^2+1)^2}\right) + e^{\frac{x}{x^2+1}} \frac{-2\cdot x(x^2+1)^{\cancel{2}} - 4x(x^2+1)(1-x^2)}{(x^2+1)^{\cancel{4}\,3}} =$$

$$\dots = e^{\frac{x}{x^2+1}} \frac{(1-x^2)^2 + (2x^3 - 6x)(x^2+1)}{(x^2+1)^4}$$

$f''(0) = 1 > 0 \Rightarrow$ c'è min rel? $f'(0) = 1 \Rightarrow$ NO

f in $c = 0$ è cresc e conv

$f''(1) = \sqrt{e} \dfrac{-8}{16} = -\dfrac{1}{2}\sqrt{e} < 0 \Rightarrow$ max rel?

$f'(1) = 0 \Rightarrow$ il p.to è stazionario

nel p.to $c = 1$ f ha un max rel

ⓐ Dimostrare che è invertibile in $]2;+\infty[$

trovare l'ins di def di f^{-1}

$f(x) = e^{\frac{1}{x-2}}$ $\dfrac{-1}{(x-2)^2} < 0$ $\forall x \in]2,+\infty[\Rightarrow f$ strett decresc $\Downarrow$ invertibile

$f :]2;+\infty[\Rightarrow]\inf f, \sup f[$
$]2,+\infty[$ $]2,+\infty[$

$\inf\limits_{]2;+\infty[} f = \lim\limits_{x \to +\infty} f(x) = 1$

$\sup\limits_{]2;+\infty[} f = \lim\limits_{x \to 2^+} f(x) = +\infty$

$f^{-1} :]1,+\infty[\to]2,+\infty[$

che $(f^{-1})'(e^2) = \dfrac{1}{f'(c)}$ dove c è tale che $f(c) = e^2$

risolvere l'eq $f(x) = e^2$

$e^{\frac{1}{x-2}} = e^2 \Leftrightarrow \dfrac{1}{x-2} = 2 \Rightarrow 1 = 2x - 4$

$x = \dfrac{5}{2} \in]2;+\infty[$

$f'\left(\frac{5}{2}\right) = -4e^2 \neq 0 \Rightarrow (f^{-1})'(e^2) = \dfrac{1}{4e^2}$

$f(x) = \arctan \sqrt{\left|\frac{x+1}{x-1}\right|}$ $\quad]-\infty, -1[\quad (f^{-1})'\left(\frac{\pi}{6}\right)$

in $]-\infty; -1[$ $\quad f(x) = \arctan \sqrt{\frac{x+1}{x-1}}$

$$f'(x) = \frac{1}{1+\frac{x+1}{x-1}} \cdot \frac{1}{2\sqrt{\frac{x+1}{x-1}}} \cdot \frac{x-1-x-1}{(x-1)^2} =$$

$$\frac{x-1}{2x} \cdot \frac{1}{2\sqrt{\frac{x+1}{x-1}}} \cdot \frac{-2}{(x-1)^2} = -\frac{1}{2x(x-1)}\sqrt{\frac{x-1}{x+1}} < 0$$

$\forall x \in]-\infty; -1[\Rightarrow f$ strett decr $\Rightarrow$ invert

$$\inf_{]-\infty,-1[} f = \lim_{x\to(-1)^-} f(x) = 0$$

$$\sup_{]-\infty;+1[} f = \lim_{x\to-\infty} f(x) = \frac{\pi}{4} \qquad f^{-1}:]0, \tfrac{\pi}{4}[\to]-\infty, -1[$$

risolvere l'eq $f(x) = \frac{\pi}{6}$ $\quad \arctan\sqrt{\frac{x+1}{x-1}} = \frac{\pi}{6} = \arctan\frac{1}{\sqrt{3}}$

$$\frac{x+1}{x-1} = \frac{1}{3} \Rightarrow 3x+3 = x-1 \Rightarrow x = -2 \in]-\infty, -1[$$

$$f'(-2) = -\frac{1}{12}\sqrt{3} \neq 0 \Rightarrow (f^{-1})'\left(\frac{\pi}{6}\right) = -\frac{12}{\sqrt{3}}$$

$f(x) = e^{\frac{x}{x^2+1}}$ $\quad]-1, 1[\quad (f^{-1})'(1)$ $\quad$ (proposto)

CAPITOLO I