8 aprile 2025 EAM1 FN

martedì 8 aprile 2025 08:00

1) $\begin{cases} a_1 = 4 \\ a_{n+1} = \sqrt{6+a_n} \end{cases}$ $\qquad a_2 = \sqrt{10} < 4$

$\{a_n\}$ è dec ?

$a_{n+1} < a_n$

$\sqrt{6+a_n} < a_n \Rightarrow 6 + a_n < a_n^2 \Rightarrow a_n^2 - a_n - 6 > 0$

$x^2 - x - 6 > 0 \qquad x = \frac{1 \pm 5}{2} \begin{matrix} \nearrow -2 \\ \searrow 3 \end{matrix} \qquad x < -2 \ \vee \ x > 3$

$a_n < -2 \ \forall n$? No

$a_n > 3 \ \forall n$?

$a_1 > 3$ vero

Se $a_n > 3 \Rightarrow a_{n+1} > 3$?

$\sqrt{6+a_n} > 3$

$6 + a_n > 9$

$a_n > 3$ vero

$a_n > 3 \Rightarrow a_n^2 - a_n - 6 > 0 \Rightarrow \{a_n\}$ dec $\Rightarrow a_n \to \inf a_n$

suppl $a_n \to \ell \Rightarrow a_{n+1} \to \ell$ ma $a_{n+1} = \sqrt{6+a_n} \to \sqrt{6+\ell}$

unic. del lim $\Rightarrow \ell = \sqrt{6+\ell} \Rightarrow \ell^2 - \ell - 6 > 0 \Rightarrow \ell = -2 \vee \ell = 3$

$a_n > 3 \ \forall n \Rightarrow 3 = \inf a_n = \lim a_n$

2) $\begin{cases} a_1 = 5 \\ a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right) \end{cases}$ $\qquad a_2 = \frac{1}{2}\left(5 + \frac{3}{5}\right) = \frac{14}{5} < 5$

$\{a_n\}$ dec ?

$\frac{1}{2}\left(a_n + \frac{3}{a_n}\right) < a_n$?

$\frac{a_n^2+3}{a_n} < 2a_n \Rightarrow a_n^2 + 3 < 2a_n^2 \Rightarrow a_n^2 > 3 \Rightarrow a_n > \sqrt{3}$?

$a_1 = 5 > \sqrt{3}$

$a_n > \sqrt{3} \Rightarrow a_{n+1} > \sqrt{3}$?

$\frac{1}{2}\left(a_n + \frac{3}{a_n}\right) > \sqrt{3} \Rightarrow \frac{a_n^2+3}{a_n} > 2\sqrt{3} \Rightarrow \underbrace{a_n^2 - 2\sqrt{3}a_n + 3}_{(a_n - \sqrt{3})^2} > 0$ vera

dunque $\{a_n\}$ dec $\Rightarrow a_n \to \inf a_n$ Se $\inf a_n = \ell$ allora

$a_n \to \ell \Rightarrow a_{n+1} \to \ell$ ma $a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right) \to \frac{1}{2}\left(\ell + \frac{3}{\ell}\right)$

$\ell = \frac{1}{2}\left(\ell + \frac{3}{\ell}\right) \Rightarrow \frac{\ell^2+3}{\ell} = 2\ell \Rightarrow \ell^2 + 3 = 2\ell^2 \Rightarrow \ell = \pm\sqrt{3}$

$\ell = -\sqrt{3} = \inf a_n$? No $\qquad \ell = \sqrt{3} = \inf a_n$? Sì

allora $a_n \to \sqrt{3}$

3) $(n^4 - n + 1)\sin^2\frac{2n}{n^3+2} = \dfrac{\sin^2\frac{2n}{n^3+2}}{\underbrace{\left(\frac{2n}{n^3+2}\right)^2}_{\to 1}} \underbrace{\left(\frac{2n}{n^3+2}\right)^2 (n^4 - n + 1)}_{\to 4} \to 4$

4) $(n+1)\log\left(1 + \frac{n}{n^2+1}\right) =$

$= \dfrac{\log\left(1 + \frac{n}{n^2+1}\right)}{\frac{n}{n^2+1}} \cdot \dfrac{n(n+1)}{n^2+1} \to 1$

$\searrow 1 \qquad \downarrow 1$

$a_n \to 0$

$\frac{\log(1+a_n)}{a_n} \to 1$

$\left(1 + \frac{1}{a_n}\right)^{a_n} \to e$

$a_n \to \infty$

5) $\left(\frac{(n+2)^2}{n^2+3}\right)^{n-2} = \left(\frac{n^2+4n+4}{n^2+3}\right)^{n-2} = \left(\frac{n^2+3+4n+1}{n^2+3}\right)^{n-2} =$

$\left[\left(1 + \ldots\right)^{\frac{n^2+3}{4n+1}}\right]^{\frac{4n+1}{n^2+3}(n-2)} \qquad \frac{4n+1}{n^2+3}(n-2) \to 4$

$$\left(\quad n^2+3 \quad\right) \qquad \left(\quad n^2+3 \quad\right)$$

$$= \left[\left(1 + \frac{1}{\frac{n^2+3}{4n+1}} \right)^{\frac{n^2+3}{4n+1}} \right]_{\to e}^{\frac{4n+1}{n^2+3}(n-2) \to 4} \longrightarrow e^4$$

6) $$\left((-1)^n + \frac{n^2+1}{n} \right)^{2n-1} = \begin{cases} \left(1+\frac{n^2+1}{n}\right)^{2n-1} = \left(\frac{n^2+n+1}{n}\right)^{2n-1} \to +\infty & n \text{ p.} \\ \left(-1+\frac{n^2+1}{n}\right)^{2n-1} = \left(\frac{n^2-n+1}{n}\right)^{2n-1} \to +\infty & n \text{ d.} \end{cases}$$

$\downarrow$ $+\infty$

7) $$\left((-1)^n + \frac{5}{4} \right)^{3n+2} = \begin{cases} \left(\frac{9}{4}\right)^{3n+2} \to +\infty & n \text{ p.} \\ \left(\frac{1}{4}\right)^{3n+2} \to 0 & n \text{ d.} \end{cases} \qquad \text{oscill.}$$

8) $$\log_{\left((-1)^n + \frac{6}{5}\right)} \frac{n^2+2(-1)^n n+1}{(n-1)^2} = \begin{cases} \log_{\frac{11}{5}} \frac{(n+1)^2}{(n-1)^2} \to 0 & n \text{ p.} \\ \log_{\frac{1}{5}} \frac{(n-1)^2}{(n-1)^2} = 0 & n \text{ d.} \end{cases} \to 0$$

$(n \geq 2)$

9) $$(-1)^n \frac{n^3+2n-3}{n+5} \quad \begin{cases} \to +\infty & n \text{ p.} \\ \to -\infty & n \text{ d.} \end{cases} \qquad \text{oscill.}$$

10) $$(-1)^n \frac{2n^2+1}{(3n-2)^2} \quad \begin{cases} \to \frac{2}{9} & n \text{ p.} \\ \to -\frac{2}{9} & n \text{ d.} \end{cases} \qquad \text{oscill.}$$

11) $$(-1)^n \frac{2n^2+1}{(3n-2)^4} \to 0$$

12) $$2^{\frac{n^4-2n+3}{1-n^5}} \to 1$$

13) $$\log_{\frac{1}{4}} \frac{3n-1}{n^3+3} \to +\infty \qquad \left(\frac{1}{4} < 1\right)$$

14) $$\left(\frac{1}{e}\right)^{\frac{3n^3-2n}{n^2+1}} \to 0 \qquad \left(\frac{1}{e} < 1\right)$$

17) sia $a_n > 0 \ \forall n$

Affinché $\{a_n\}$ sia conv., la condizione "$\{\log_{\frac{1}{2}} a_n$ è conv.$\}$" è

A) N e S
B) N ma non S
C) S ma non N ←
D) né N né S

se $\log_{\frac{1}{2}} (a_n) \to \ell \Rightarrow a_n = \left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} a_n} \to \left(\frac{1}{2}\right)^{\ell}$ quindi la condiz. **è S**

se $a_n \to \ell \Rightarrow \log_{\frac{1}{2}} a_n \to \log_{\frac{1}{2}} \ell$? solo se $\ell > 0$

se $a_n \to 0 \Rightarrow \log_{\frac{1}{2}} a_n \to +\infty$ quindi la condiz. **non è N**