14 ottobre 2025_MZ

martedì 14 ottobre 2025 11:02

1. Trovare le prim in $]-\infty, +\infty[$ di

$$f(x) = \begin{cases} 4e^x & x < 0 \\ x^4 + 4 & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 4e^x + k_1 & x < 0 \\ \frac{x^5}{5} + 4x + k_2 & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$4 + k_1 = k_2$$

le prim. sono $f(x) = \begin{cases} 4e^x + k & x < 0 \\ \frac{x^5}{5} + 4x + k + 4 & x \geq 0 \end{cases}$ ←

cercare quella tale che $f(1) = 3$

$$\frac{1}{5} + 4 + k + 4 = 3 \qquad k = -\frac{26}{5}$$

$$f(x) = \begin{cases} 4e^x - \frac{26}{5} & x < 0 \\ \frac{x^5}{5} + 4x - \frac{26}{5} + 4 & x \geq 0 \end{cases}$$

2. Trovare le prim in $]-\infty, +\infty[$ di $f(x) = 2x^2 - |x^2 - 1| - x + 3$

$-1 \qquad 1$

$x \leq -1 \quad x \geq 1 \qquad f(x) = 2x^2 - (x^2 - 1) - x + 3 = x^2 - x + 4$

$-1 < x < 1 \qquad f(x) = 2x^2 - (1 - x^2) - x + 3 = 3x^2 - x + 2$

$$f(x) = \begin{cases} x^2 - x + 4 & x \leq -1 \\ 3x^2 - x + 2 & -1 < x < 1 \\ x^2 - x + 4 & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} \frac{x^3}{3} - \frac{x^2}{2} + 4x + k_1 & x \leq -1 \\ x^3 - \frac{x^2}{2} + 2x + k_2 & -1 < x < 1 \\ \frac{x^3}{3} - \frac{x^2}{2} + 4x + k_3 & x \geq 1 \end{cases}$$

$$\lim_{x \to (-1)^-} f(x) = \lim_{x \to (-1)^+} f(x) \qquad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$-\frac{29}{6} + k_1 = -\frac{7}{2} + k_2 \qquad \frac{5}{2} + k_2 = \frac{25}{6} + k_3$$

$$\Downarrow \qquad \Downarrow$$

$$k_2 = k_1 - \frac{29}{6} + \frac{7}{2} = k_1 - \frac{8}{6} = k_1 - \frac{4}{3} \qquad k_3 = k_2 + \frac{5}{2} - \frac{25}{6} = k_2 - \frac{10}{6} =$$

$$= k_2 - \frac{5}{3}$$

$$= k_1 - \frac{4}{3} - \frac{5}{3}$$

$$= k_1 - 3$$

$$f(x) = \begin{cases} \frac{x^3}{3} - \frac{x^2}{2} + 4x + k_1 \\ x^3 - \frac{x^2}{2} + 2x + k_1 - \frac{4}{3} \\ \frac{x^3}{3} - \frac{x^2}{2} + 4x + k_1 - 3 \end{cases}$$

Se errando avessi scritto

$$f(x) = \begin{cases} \frac{x^3}{3} - \frac{x^2}{2} + 4x + k_1 & \text{in }]-\infty, -1] \cup [1, +\infty[\\ x^3 - \frac{x^2}{2} + 2x + k_2 & \text{in }]-1, 1[\end{cases}$$

$$\lim_{x \to (-1)^-} = \lim_{x \to (-1)^+} \qquad \lim_{x \to 1^-} = \lim_{x \to 1^+}$$

$$-\frac{29}{6} + k_1 = -\frac{7}{2} + k_2 \qquad \frac{5}{2} + k_2 = \frac{25}{6} + k_1 \Rightarrow$$

$$\Downarrow \qquad \Rightarrow k_2 = k_1 + \frac{25}{6} - \frac{5}{2} = k_1 + \frac{10}{6}$$

$$k_2 = k_1 - \frac{29}{6} + \frac{7}{2} = k_1 - \frac{8}{6} = \qquad = k_1 + \frac{5}{3}$$

$$= k_1 - \frac{4}{3} \qquad \longleftrightarrow \; ??$$

SBAGLIATO

esec trovare f primitiva in $]-\infty, +\infty[$ di

$f(x) = 3x^2 - |x^2 - 9| + 2x + 1$

e tale che $f(0) = -1$

FUNZIONI RAZIONALI FRATTE

$f(x) = \frac{A(x)}{B(x)}$ A pol.n di grado n $(n \in \mathbb{N}_0)$

B " m $(m \in \mathbb{N})$

A, B primi fra loro

Se $n \geq m$ f raz fratta non propria

Se $n < m$ " propria

NON PROPRIA → POLIN. + PROPRIA

$\exists\, Q, R$ (grado R < grado B) : $A(x) = B(x)\,Q(x) + R(x) \Rightarrow$

$$f(x) = \underbrace{Q(x)}_{\text{pol.}} + \underbrace{\frac{R(x)}{B(x)}}_{\text{propria}}$$

$$\frac{x^2+5}{\ } = \frac{x^2-4+9}{\ } = \frac{x^2-4}{\ } + \frac{9}{\ } = x + 2 + \frac{9}{\ }$$

$B(x)$

pol. propria

$$\frac{x^2+5}{x-2} = \frac{x^2-4+9}{x-2} = \frac{x^2-4}{x-2} + \frac{9}{x-2} = \underset{\text{pol.}}{x+2} + \underset{\text{propria}}{\frac{9}{x-2}}$$

$$\frac{x^2+6}{x+1} = \frac{x^2+2x+1-2x+5}{x+1} = \frac{x^2+2x+1}{x+1} - 2\,\frac{x-\frac{5}{2}}{x+1} =$$

$$= x+1-2\,\frac{x+1-\frac{7}{2}}{x+1} = x+1-2\,\frac{x+1}{x+1} + 7\,\frac{1}{x+1} =$$

$$= \underset{\text{pol.}}{x-1} + \underset{\text{propria}}{\frac{7}{x+1}}$$

FRATTE PROPRIE

- risolvere l'eq $B(x)=0$ trovando sol. reali $x=a$ e coppie di sol immag $x=b\pm ic$
- decomporre il denom. $(x-a_1)(x-a_2)^2(x-a_3)^4 \cdot$

$$[x-(b+ic)][x-(b-ic)] = [(x-b)-ic][(x-b)+ic] =$$
$$= (x-b)^2-(ic)^2 = \underbrace{(x-b)^2+c^2}_{\text{pol di II grado a coeff real}}$$

- decomporre f in "FRATTI SEMPLICI"

una sol. reale $x=a$ del denom., di molteplicità p, dà luogo a p fratti semplici

$$\frac{A_1}{x-a} \quad \frac{A_2}{(x-a)^2} \quad \dots \quad \frac{A_p}{(x-a)^p} \qquad (1)$$

una coppia di sol immag. $b\pm ic$, di molteplic. p, dà luogo

$$\frac{B_1x+C_1}{(x-b)^2+c^2} \quad \frac{B_2x+C_2}{((x-b)^2+c^2)^2} \quad \dots \quad \frac{B_px+C_p}{((x-b)^2+c^2)^p} \qquad (2)$$

iniziamo a integrare quelli di tipo (1) (li conosciamo già) e quelli di tipo 2 fino a $p=2$

es. $$\frac{3x^4+5x-6}{x^2(x-1)(x^2+2)(x^2+3)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{Dx+E}{x^2+2} +$$
$$\frac{Fx+G}{x^2+3} + \frac{Hx+K}{(x^2+3)^2} + \frac{Jx+L}{(x^2+3)^3}$$

$$\frac{2x+1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} = \frac{Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2}{x^2(x^2+1)} =$$

$$= \frac{Ax^3+Ax+Bx^2+B+Cx^3+Dx^2}{x^2(x^2+1)}$$

$$\begin{cases} A + C = 0 \\ B + D = 0 \\ A = 2 \\ B = 1 \end{cases}$$

$$\begin{cases} A = 2 \\ B = 1 \\ C = -2 \\ D = -1 \end{cases} \qquad \int \frac{2x}{x^2(x^2+1)}\,dx = \int \frac{2}{x}\,dx + \int \frac{1}{x^2}\,dx + \int \frac{-2x-1}{x^2+1}\,dx =$$

$$= 2\log|x| - \frac{1}{x} - \int \frac{2x}{x^2+1}\,dx - \int \frac{dx}{x^2+1} =$$

$$= 2\log|x| - \frac{1}{x} - \log(x^2+1) - \operatorname{arctg} x + k$$

FRATTI SEMPLICI

tipo (1) $$I_m = \int \frac{dx}{(x-c)^m}$$

$$I_1 = \int \frac{dx}{x-c} = \log|x-c| + k$$

$m>1$ $$I_m = \int (x-c)^{-m}\,dx = \left[\int t^{-m}\,dt\right]_{t=x-c} = \frac{(x-c)^{-m+1}}{-m+1} + k$$

tipo (2) $$I_1 = \int \frac{px+q}{(x-a)^2+c^2}\,dx \qquad I_2 = \int \frac{px+q}{((x-a)^2+c^2)^2}\,dx$$

vediamo alcuni casi particolari

abbiamo studiato nella lez. prec. $\int \frac{dx}{(x^2+1)^2}$ $\quad \int \frac{dx}{x^2+1}$

vediamo ora $$I_1 = \int \frac{dx}{x^2+c^2} = \int \frac{1}{c^2}\,\frac{1}{\left(\frac{x}{c}\right)^2+1}\,dx = \frac{1}{c}\int \frac{1}{c}\,\frac{1}{}$$

$$= \frac{1}{c}\left[\int \frac{dy}{y^2+1}\right]_{y=\frac{x}{c}} = \frac{1}{c}\operatorname{arctg}\frac{x}{c} + k$$

$$\int \frac{dx}{x^2+16} = \frac{1}{4}\operatorname{arctg}\frac{x}{4}+k \qquad (c=4)$$

$$\int \frac{dx}{x^2+7} = \frac{1}{\sqrt7}\operatorname{arctg}\frac{x}{\sqrt7}+k \qquad c=\sqrt7$$

$$I_2=\int\frac{dx}{(x^2+c^2)^2} = \frac{1}{c^2}\int\frac{c^2}{(x^2+c^2)^2}dx = \frac{1}{c^2}\int\frac{c^2+x^2-x^2}{(x^2+c^2)^2}dx =$$

$$= \frac{1}{c^2}\int\frac{x^2+c^2}{(x^2+c^2)^2}dx + \frac{1}{c^2}\int\frac{-x^2}{(x^2+c^2)^2}dx =$$

$$D\left(\frac{1}{x^2+c^2}\right)=\frac{-2x}{(x^2+c^2)^2} \qquad = \frac{1}{c^2}I_1 + \frac{1}{2c^2}\int\frac{-2x}{(x^2+c^2)^2}\,x\,dx =$$

fg

$$= \frac{1}{c^2}I_1 + \frac{1}{2c^2}\frac{x}{x^2+c^2} - \frac{1}{2c^2}I_1$$

$$\int\frac{x+3}{x^2-x+4}dx$$ pol denom. di II grado con $\Delta<0$

es. $\int\frac{dx}{x^2-x+4}$ ci si riconduce al caso $\frac{\cdot\cdot}{(x-a)^2+c^2}$

metodo del completamento dei quadrati

$$x^2-x+4=\left(x-\frac12\right)^2+\frac{15}{4} \qquad x^2+4x+9=(x+2)^2+5$$

fratti semplici di tipo (2)

$$\int\frac{2x+3}{x^2+4x+9}dx =$$

- al numer la der del den.
- compl. dei quadrati

$$=\int\frac{2x+4-1}{x^2+4x+9}dx = \int\frac{2x+4}{x^2+4x+9}dx - \int\frac{dx}{x^2+4x+9} =$$

$$= \log(x^2+4x+9) - \frac{dx}{(x+2)^2+5} =$$

$$= \log(x^2+4x+9) - \left[\int\frac{dt}{t^2+5}\right]_{t=x+2} = \qquad c=\sqrt5$$

$$= \log(x^2+4x+9) - \frac{1}{\sqrt5}\operatorname{arctg}\frac{x+2}{\sqrt5}+k$$

LOG + ARCTG

$$\int\frac{x}{x^2+3x+4}dx = \frac12\int\frac{2x}{x^2+3x+4}dx = \frac12\int\frac{2x+3-3}{x^2+3x+4}dx =$$

$$= \frac12\int\frac{2x+3}{x^2+3x+4}dx - \frac32\int\frac{dx}{\left(x+\frac32\right)^2+\frac74} \qquad c=\frac{\sqrt7}{2}$$

$$= \frac12\log(x^2+3x+4) - \frac32\cdot\frac{2}{\sqrt7}\operatorname{arctg}\frac{x+\frac32}{\frac{\sqrt7}{2}}+k$$

$$\int\frac{x+1}{(x^2+2x+6)^2}dx = \frac12\int\frac{2x+2}{(x^2+2x+6)^2}dx = \frac12\left[\int\frac{dt}{t^2}\right]_{t=x^2+2x+6} =$$

$$= -\frac12\,\frac{1}{x^2+2x+6}+k$$

Abbiamo visto il caso di polin. di I grado / pol di II gr. con $\Delta<0$

Vediamo ora il caso $\Delta=0$

$$I=\int\frac{2x+3}{x^2-8x+16}dx = \int\frac{2x+3}{(x-4)^2}dx$$

$$\frac{2x+3}{(x-4)^2} = \frac{A}{x-4}+\frac{B}{(x-4)^2} = \frac{Ax-4A+B}{(x-4)^2} \qquad \begin{cases}A=2\\-4A+B=3\end{cases} \Rightarrow \begin{cases}A=2\\B=11\end{cases}$$

$$I=\int\frac{2}{x-4}dx+\int\frac{11}{(x-4)^2}dx = 2\log|x-4| - \frac{11}{x-4}+k$$

$$I=\int\frac{x-3}{x^2+10x+25}dx = \int\frac{x-3}{(x+5)^2}dx$$

$$\frac{x-3}{(x+5)^2} = \frac{A}{x+5}+\frac{B}{(x+5)^2} = \frac{Ax+5A+B}{(x+5)^2} \qquad \begin{cases}A=1\\5A+B=-3\end{cases} \qquad \begin{cases}A=1\\B=-8\end{cases}$$

$$I=\log|x+5|+\frac{8}{x+5}+k$$

Sia ora $\Delta>0$

$$I=\int\frac{8x-3}{x^2+2x-3}dx \qquad x^2+2x-3=0 \quad x=-1\pm2 \;\begin{smallmatrix}-3\\1\end{smallmatrix}$$

$$\frac{8x-3}{x^2+2x-3} = \frac{A}{x+3}+\frac{B}{x-1} = \frac{(A+B)x-A+3B}{(x+3)(x-1)} \qquad \begin{cases}A+B=8\\-A+3B=-3\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}B=\frac54\\A=3B+3\end{cases} \Rightarrow \begin{cases}B=\frac54\\A=\frac{27}{4}\end{cases}$$

$$I=\frac{27}{4}\log|x+3|+\frac54\log|x-1|+k$$

Esercizi

$$I=\int\frac{x^2-1}{\quad}dx \qquad x^3-2x^2=x^2(x-2)$$

$x^3 - 2x^2$

$$\frac{x^2-1}{x^3-2x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} = \frac{Ax^2 - 2Ax + Bx - 2B + Cx^2}{x^2(x-2)}$$

$$\begin{cases} A + C = 1 \\ -2A + B = 0 \\ -2B = -1 \end{cases} \qquad \begin{aligned} C &= \tfrac{3}{4} \\ A &= \tfrac{1}{4} \\ B &= \tfrac{1}{2} \end{aligned}$$

$$I = \frac{1}{4} \log|x| - \frac{1}{2}\frac{1}{x} + \frac{3}{4}\log|x-2| + k$$

INTEGRAZIONE PER RAZIONALIZZAZIONE

$$\int \frac{e^x}{e^{2x}+1}\,dx = \left[\int \frac{dt}{t^2+1}\right]_{t=e^x} = \operatorname{arctg} e^x + k \qquad \frac{1}{t^2+1}$$

$$I = \int \frac{e^{2x}+3e^x}{e^{2x}+2e^x+4}\,dx = \int \underbrace{e^x}_{D(e^x)} \frac{e^x+3}{e^{2x}+2e^x+4}\,dx = \Bigg[\underbrace{\int \frac{t+3}{t^2+2t+4}\,dt}_{J}\Bigg]_{t=e^x}$$

$$J = \frac{1}{2}\int \frac{2t+6}{t^2+2t+4}\,dt \pm \frac{1}{2}\int \frac{2t+2}{t^2+2t+4}\,dt + 2\int \frac{dt}{(t+1)^2+3} \qquad c = \sqrt{3}$$

$$= \frac{1}{2}\log(t^2+2t+4) + \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{t+1}{\sqrt{3}} + k$$

$$I = \frac{1}{2}\log(e^{2x}+2e^x+4) + \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{e^x+1}{\sqrt{3}} + k$$

$$I = \int \frac{e^x+2}{e^{2x}+4}\,dx = \int \underbrace{e^x}_{D(e^x)} \frac{e^x+2}{e^x(e^{2x}+4)}\,dx = \Bigg[\underbrace{\int \frac{t+2}{t(t^2+4)}\,dt}_{J}\Bigg]_{t=e^x}$$

$$\frac{t+2}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4} = \frac{At^2+4A+Bt^2+Ct}{t(t^2+4)}$$

$$\begin{cases} A + B = 0 \\ C = 1 \\ 4A = 2 \end{cases} \Rightarrow \begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{2} \\ C = 1 \end{cases}$$

$$J = \frac{1}{2}\log|t| + \int \frac{-\frac{1}{2}t+1}{t^2+4}\,dt = \frac{1}{2}\log|t| - \frac{1}{4}\int\frac{2t}{t^2+4}\,dt + \int\frac{dt}{t^2+4} =$$

$$= \frac{1}{2}\log|t| - \frac{1}{4}\log(t^2+4) + \frac{1}{2}\operatorname{arctg}\frac{t}{2} + k$$

$$I = \frac{1}{2}x - \frac{1}{4}\log(e^{2x}+4) + \frac{1}{2}\operatorname{arctg}\frac{e^x}{2} + k$$

$$I = \int \frac{\operatorname{tg} x + 3}{\operatorname{tg}^2 x + \operatorname{tg} x - 2}\,dx = \int \underbrace{(1+\operatorname{tg}^2 x)}_{D(\operatorname{tg} x)} \frac{\operatorname{tg} x + 3}{(\operatorname{tg}^2 x + 1)(\operatorname{tg}^2 x + \operatorname{tg} x - 2)}\,dx =$$

$$= \Bigg[\underbrace{\int \frac{t+3}{(t^2+1)(t^2+t-2)}\,dt}_{J}\Bigg]_{t=\operatorname{tg} x}$$

$$t^2 + t - 2 = 0 \quad t = \frac{-1 \pm 3}{2} \begin{array}{l} -2 \\ 1 \end{array}$$

$$\frac{t+3}{(t^2+1)(t^2+t-2)} = \frac{t+3}{(t^2+1)(t+2)(t-1)} = \frac{At+B}{t^2+1} + \frac{C}{t+2} + \frac{D}{t-1} =$$

$$= \frac{(At+B)(t^2+t-2) + C(t^2+1)(t-1) + D(t^2+1)(t+2)}{(t^2+1)(t+2)(t-1)}$$

$$= \frac{At^3 + At^2 - 2At + Bt^2 + Bt - 2B + Ct^3 - Ct^2 + Ct - C + Dt^3 + 2Dt^2 + Dt + 2D}{----}$$

$$\begin{cases} A + C + D = 0 \\ A + B - C + 2D = 0 \\ -2A + B + C + D = 1 \\ -2B - C + 2D = 3 \end{cases} \qquad \begin{cases} A = -C - D \\ B - 2C + D = 0 \\ B + 3C + 3D = 1 \\ -2B - C + 2D = 3 \end{cases} \qquad \begin{cases} A = -C - D \\ B = 2C - D \\ 5C + 2D = 1 \\ -5C + 4D = 3 \end{cases}$$

$$C = \frac{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}{30} = -\frac{1}{15} \qquad D = \frac{\begin{bmatrix} 5 & 1 \\ -5 & 3 \end{bmatrix}}{30} = \frac{2}{3}$$

$$B = -\frac{2}{15} - \frac{2}{3} = -\frac{12}{15} = -\frac{4}{5}$$

$$A = \frac{1}{15} - \frac{2}{3} = -\frac{9}{15} = -\frac{3}{5}$$

$$J = \int \frac{-\frac{3}{5}t - \frac{4}{5}}{t^2+1}\,dt - \frac{1}{15}\log|t+2| + \frac{2}{3}\log|t-1|$$

$$= -\frac{3}{10}\log(t^2+1) - \frac{4}{5}\operatorname{arctg} t - \frac{1}{15}\log|t+2| + \frac{2}{3}\log|t-1| + k$$

$$I = [J]_{t=\operatorname{tg} x}$$

$$I \int \frac{\sin 2x}{\cos^2 x + 2\cos x + 1}\,dx = 2\int \sin x \frac{\cos x}{\cos^2 x + 2\cos x + 1}\,dx =$$

$$= -2\int(-\sin x)\frac{\cos x}{(\cos x + 1)^2}\,dx =$$

$$= -2\Bigg[\underbrace{\int \frac{t}{(t+1)^2}\,dt}_{J}\Bigg]_{t=\cos x}$$

$$\frac{t}{(t+1)^2} = \frac{A}{t+1} + \frac{B}{(t+1)^2} = \frac{At+A+B}{(t+1)^2} \qquad \begin{cases} A = 1 \\ A+B = 0 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = -1 \end{cases}$$

$$J = \log|t+1| + \frac{1}{t+1} + k$$

$$I = \log(\cos x + 1) + \frac{1}{\cos x + 1} + k$$

$$I = \int \frac{\log x + 1}{x(\log^2 x + 2)}\, dx = \Big[\underbrace{\int \frac{t+1}{t^2+2}\, dt}_{J}\Big]_{t=\log x}$$

$$J = \int \frac{t}{t^2+2}\, dt + \int \frac{dt}{t^2+2} = \frac{1}{2}\int \frac{2t}{t^2+2} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} =$$

$$= \frac{1}{2} \log(t^2+2) + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + k$$

$$I = \frac{1}{2} \log(\log^2 x + 2) + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{\log x}{\sqrt{2}} + k$$

ESERCIZIO

$$I = \int \frac{dx}{e^{2x} - 4e^x + 4} = \int e^x \frac{1}{e^x(e^x-2)^2}\, dx = \Big[\underbrace{\int \frac{dt}{t(t-2)^2}}_{J}\Big]_{t=e^x}$$

$$\frac{1}{t(t-2)^2} = \frac{A}{t} + \frac{B}{t-2} + \frac{C}{(t-2)^2} = \frac{A(t-2)^2 + Bt(t-2) + Ct}{t(t-2)^2} =$$

$$= \frac{At^2 - 4At + 4A + Bt^2 - 2Bt + Ct}{t(t-2)^2} \qquad \begin{cases} A + B = 0 \\ -4A - 2B + C = 0 \\ 4A = 1 \end{cases}$$

$$\begin{cases} A = \frac{1}{4} \\ B = -\frac{1}{4} \\ C = \frac{1}{2} \end{cases} \qquad \begin{aligned} J &= \frac{1}{4}\log|t| - \frac{1}{4}\log|t-2| + \frac{1}{2}\,\frac{-1}{t-2} + k \\ I &= \frac{1}{4}x - \frac{1}{4}\log|e^x - 2| - \frac{1}{2}\,\frac{1}{e^x-2} + k \end{aligned}$$

Seconda formula di integr. per sostitut.

IP $\quad f : (a,b) \to \mathbb{R}$ dot di prim.

$\quad g : (c,d) \to (a,b)$ su tutto, deriv. e invert.

TS $\quad \int f(x)\,dx = \Big[\int f(g(t))\, g'(t)\, dt\Big]_{t = g^{-1}(x)}$

DIM. $\quad \int f(g(t))\, g'(t)\, dt = \Big[\int f(x)\,dx\Big]_{x = g(t)}$ per la I form.

$$\Downarrow$$

$$\Big[\int f(g(t))\, g'(t)\, dt\Big]_{t=g^{-1}(x)} = \Big[\int f(x)\,dx\Big]_{x = g(g^{-1}(x)) = x} \Rightarrow \text{TS}$$