$R = \{0; \pm a_0, a_1 a_2 \ldots\}$

$\pm$ segno con a_0 in N_0 parte intera

a_i $(i \geq 1)$ cifre

$a_i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$N \subseteq Z \subseteq Q \subseteq R$

è numerico quindi è un sottoinsieme di R

$X \subseteq R$ $X \neq \emptyset$

$M \in X : M \geq m \ \forall m \in X$ $M = \max X$

$m \in X : m \leq n$ " $m = \min X$

$h \in R$ maggiorante per X se $h \geq x \ \forall x \in X$

$\bar{M}_X$ = ins dei magg

$\max \in \bar{M}_X$

$K \in \bar{M}_X$, $K' > K \Rightarrow K' \in \bar{M}_X$

$K \notin \bar{M}_X$ se $\exists x \in X : x > k$

Definizione: X limitato superiormente se $\bar{M}_X \neq 0$

Definizione: $\sup X = \begin{cases} \min \bar{M}_X & \text{se } \bar{M}_X \neq 0 \\ +\infty & \text{se } \bar{M}_X = \emptyset \end{cases}$

$\sup X$ è unico

Se $\sup X$ è un numero si hanno queste proprietà:

$\sup X = L \iff$ 1) $L \geq x \quad \forall x \in X$

2) $\forall \varepsilon > 0 \ \exists x \in X : x > L - \varepsilon$

se $\exists M = \max X \Rightarrow \sup X = M$ infatti 1) vero

2) $\exists x \in X : x > M - \varepsilon$?

Basta prendere $x = M$

se $\sup X \in X \Rightarrow \exists \max X = \sup X$

perché $\sup X$ è un maggioranti $\in X$

Definizione: $h \in \mathbb{R}$ minorante di X se $h \leq x \quad \forall x \in X$

$\underline{M}_X$ = insieme dei minoranti $\qquad \min X \in \underline{M}_X$

$h \in \underline{M}_X,\ h' < h \Rightarrow h' \in \underline{M}_X$

$h \notin \underline{M}_X$, se $\exists x \in X : x < h$

Definizione: X limitato inferiormente se $\underline{M}_X \neq \emptyset$

$$\inf X \begin{cases} \max \underline{M}_X & \text{se } \underline{M}_X \neq \emptyset \\ -\infty & \text{se } \underline{M}_X = \emptyset \end{cases}$$

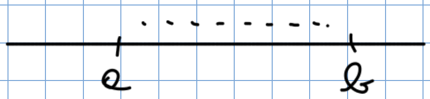
↳ è unico

$$\inf X = \underbrace{\ell \in \mathbb{R}}_{\text{numero reale}} \iff 1)\ \ell \leq x \quad \forall x \in X$$

$$2)\ \forall \varepsilon > 0\ \exists x \in X : x < \ell + \varepsilon$$

Definizione: X è limitato se $\underline{M}_X \neq 0$ e $M^{-}_X \neq 0$ cioè se è limitato superiormente e inferiormente

$\iff \exists a, b \in \mathbb{R} : X \subseteq [a, b]$



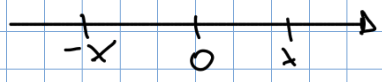
Proprietà del valore assoluto

$x = \pm a_0, a_1 a_2$

$-x = \mp a_0, a_1 a_2$

Definizione

$$|x| = \begin{cases} x & \text{se } x > 0 \\ -x & \text{se } x < 0 \end{cases}$$



$|-x| = |x|$

$|x|$ sempre $\geq 0 \quad \forall x$

$= 0 \Leftrightarrow x = 0$

$|x| = \max(x; -x)$

$-|x| \leq x \leq |x|$

$|x-2| = 0 \Rightarrow x-2=0 \Rightarrow x=2$

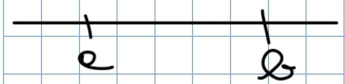
$|x-2| > 0 \quad \forall x$

$|x-2| \geq 0 \quad x \neq 2$

$|x-2| < 0 \quad \nexists x$

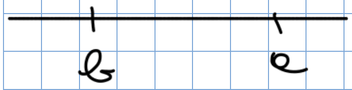
$|x-2| \leq 0 \quad x = 2$

ampiezza di (a, b)

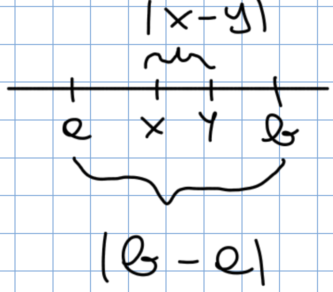


$b - a$

quindi $|a-b|$



$a - b$



$$\left.\begin{array}{l} a < x < b \\ a < y < b \end{array}\right\} \Rightarrow |x-y| < b-a$$

$|ab| = |a| \cdot |b|$

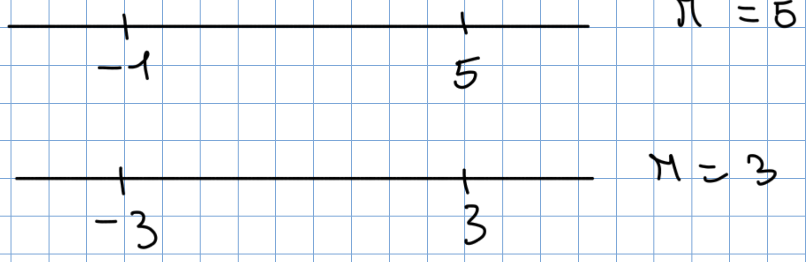
$|a+b| \leq |a| + |b|$

$\left|\frac{1}{a}\right| = \frac{1}{|a|} \quad (a \neq 0)$

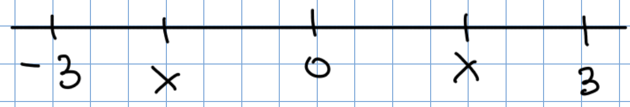
se $|x| < \varepsilon \ \forall \varepsilon > 0 \Rightarrow x = 0$

Massimo

X limitato $\Leftrightarrow \exists M > 0 : |x| \leq M \ \forall x \in X$



$|x| < a \quad (a > 0) \Leftrightarrow -a < x < a$



Esercizi:

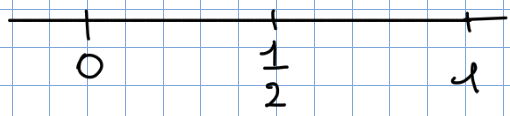
Determinare l'estremo inferiore e l'estremo superiore precisando se sono minimo e massimo

1) $a, b \in \mathbb{R}$ $\quad x = (a, b)$ $\quad \inf x = a$ $\quad \sup x = b$

$x = [a, b)$ $\quad \min x = a$ $\quad \sup x = b$

2)

$X = \left\{ \frac{1}{n} : n \in N \right\}$



$\inf x = 0$ $\quad \begin{cases} 1) \ 0 \leq \frac{1}{n} \ \forall n ? \quad Si \\ 2) \ \forall \varepsilon > 0 \ \exists n \in N : \frac{1}{n} < \varepsilon \Leftrightarrow n > \frac{1}{\varepsilon} \end{cases}$

$\sup x = \quad n < x \quad x = 1$

3) $x = \left\{ -\frac{1}{n} : n \in N \right\}$ $\quad 0 < \frac{1}{n} \leq 1 \quad \forall n \Rightarrow -1 \leq \frac{1}{n} < 0$

minimo $= -1$

$\sup x = 0$

4)

$x = \left\{ (-1)^n \frac{n}{n+1} : n \in N \right\}$ $\quad (-1)^n = \begin{cases} 1 & n \text{ pari} \\ -1 & n \text{ dispari} \end{cases}$

n pari

$\frac{2}{3} < \frac{4}{5} < \ \ldots \ < 1$ $\quad \sup = 1$

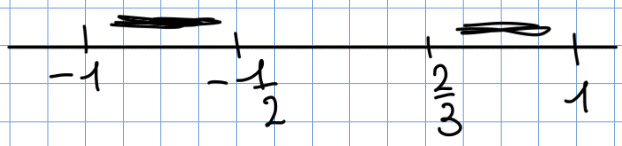
minimo $= \frac{2}{3}$

n dispari

$-\frac{n}{n+1}$ $\quad$ con $\quad \frac{n}{n+1}$ $\quad \frac{1}{2} < \frac{2}{3} < \ \ldots \ < 1$

$\frac{1}{2} \leq \frac{n}{n+1} < 1 \quad \Rightarrow -1 < -\frac{n}{n-1} \leq -\frac{1}{2}$ $\quad \inf = -1$

$\max x = -\frac{1}{2}$



$\inf X = -1$

$\sup X = 1$

5)

$X =]0,1[\cup \{3\}$ $\quad \inf X = 0 \quad \max X = 3$

6) $X = \{x \in \mathbb{R} : |x+2| < 3\}$

$$\Downarrow$$

$-3 < x+2 < 3 \Rightarrow -5 < x < 1$ $\quad \inf X = -5$

$\sup X = 1$

7) $X = \{x \in \mathbb{R} \setminus \mathbb{Q} : x^2 - |x| \leq 0\}$

$\begin{cases} x \geq 0 \\ x^2 - x \leq 0 \end{cases}$ $\qquad$ $\begin{cases} x < 0 \\ x^2 + x \leq 0 \end{cases}$

$\Downarrow$ $\qquad$ $\Downarrow$

$\begin{cases} x \geq 0 \\ 0 \leq x \leq 1 \end{cases}$ $\qquad$ $\begin{cases} x < 0 \\ -1 \leq x < 0 \end{cases}$

$0 \leq x \leq 1$ $\qquad$ $-1 \leq x < 0$

$X = \{x \in \mathbb{R} - \mathbb{Q} : -1 \leq x \leq 1\}$

$\inf X = -1$

$\sup X = 1$

$\min X = -1$

$\max X = 1$

8)

$X = \{ x \in \mathbb{R} : ||x-1| - 2x| < x+3 \}$

A) lim inf

B) lim sup

C) = sup e inf

D) non = sup e inf

$$||x-1| - 2x| < x+3 \qquad |x-1| = \begin{cases} x-1 \to x \geq 1 \\ 1-x \to x < 1 \end{cases}$$

$$\begin{cases} x < 1 \\ |1-x-2x| < x+3 \end{cases} \vee \begin{cases} x \geq 1 \\ |x-1-2x| < x+3 \end{cases}$$

$$\begin{cases} x < 1 \\ |3x-1| < x+3 \end{cases}$$

$$\begin{cases} x < \frac{1}{3} \\ 1-3x < x+3 \end{cases} \qquad \begin{cases} \frac{1}{3} \leq x < 1 \\ 3x-1 < x+3 \end{cases}$$

$$\begin{cases} x < \frac{1}{3} \\ x > -\frac{1}{2} \end{cases} \qquad \begin{cases} \frac{1}{3} \leq x < 1 \\ x < 2 \end{cases}$$

$$\Downarrow \qquad \Downarrow$$

$$-\frac{1}{2} < x < 1 \qquad \frac{1}{3} \leq x < 1$$

$$\Downarrow$$

$$\begin{cases} x \geq 1 \\ (x+1) < x+3 \end{cases}$$

$$\begin{cases} x \geq 1 \\ x+1 < x+3 \end{cases}$$

$$x \geq 1$$

$$X =]-\frac{1}{2}, +\infty[$$

A

Esercizi per casa (sempre con le risposte multiple)

$X = \{ x \in \mathbb{R} : |x-3| - 2 < |x| \}$

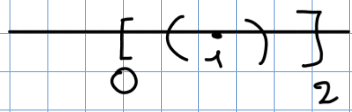
$X = \{ x \in \mathbb{R} : |2|x-1| - x + 3| < 4 \}$

$c \in \mathbb{R}$ $\quad r > 0$ $\quad]c-r\,; c+r[$ intorno di c di raggio r

$X \subseteq \mathbb{R}$ $\quad c \in \mathbb{R}$

se $c \in X$ c è interno ad X se $\exists r > 0 :]c-r, c+r[\subseteq X$

$X = [0,2]$



1 è interno ad X

0 non è interno ad X

$]x-r, x+r[\subseteq [0,2]$

$x + r < 2 \Rightarrow r < 2 - x$

int X = ins dei suoi interni

$\text{int}(a,b) =]a,b[$

X aperto se $X = \emptyset$ $\overset{\text{oppure}}{\vee}$ tutti i suoi punti sono interni

X aperto $\iff X = \text{int}(X)$

oss: $\emptyset$ è aperto

$\mathbb{R}$ è aperto

trovare un insieme tale che $\text{int}(X) = \emptyset$