$z = (a, b)$

Prodotto: $z \cdot w = (ac - bd,\ ad + bc)$

Somma: $z + w = (a + c,\ b + d)$

$$\underset{a}{(a,0)} + \underset{i}{(0,1)}\underset{b}{(b,0)} = (a,0) + (0-0,\ 0+b) = (a,0) + (0,b) = (a,b) = z$$

$z = a + ib$ Forma algebrica $z = \operatorname{Re} z + i \operatorname{Im} z$

parte reale parte immaginaria

$\operatorname{Re} a$ i = unità imm.

$b = \operatorname{Im} z$

$$i^2 = (0,1)(0,1) = (0-1,\ 0+0) = -1$$

numero alla seconda diventa negativo ed è una cosa fattibile nei numeri complessi

$$\underbrace{(2+6i)(1-4i)}_{\text{uso la formula del prodotto}} = 2 - 8i + 6i - 24 - (-1) = \underbrace{26 - 2i}$$

$z = 3 + i$ $w = 4 - 2i$ Calc $R_e(zw) = 3 \cdot 4 + (-2) \cdot i^2 = 14$

Formule usate in questo esercizio

$|a + ib| = a^2 + b^2$ $z\bar{z} = |z|^2$ $(a-b)(a+b) = a^2 - b$ $i^2 = -1$

$$\frac{3-i}{2+3i} = \frac{\overbrace{(3-i)(2-3i)}^{6-9i-2i+3i^2}}{\underbrace{(2+3i)}_{z}\underbrace{(2-3i)}_{\bar{z}}} = \frac{3-11i}{\underbrace{4+9}_{4-9i^2}} = \frac{3}{13} - \frac{11}{13}i$$

Si moltiplicano numeratore e denominatore per il coniugato del denom

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$$\frac{2+5i}{3i-1} = \frac{(2+5i)(-1-3i)}{(3i-1)(-1-3i)} = \frac{13-11i}{3^2+(-1^2)} = \frac{13}{10} - \frac{11}{10}i$$

$$(2-i)i - (2+i)\frac{4}{5-3i} = 2i + 1 - \frac{(8+4i)(5+3i)}{34}$$

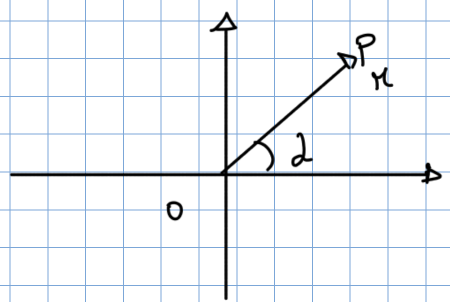
$$= 2i + 1 - \frac{\cancel{28}^{14}}{\cancel{34}_{17}} - \frac{\cancel{44}^{22}}{\cancel{34}_{17}} = \frac{3}{17} + \frac{12}{17}$$

Potenze di i:

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$\vdots$

questi sono i valori possibili ciclicamente

Risolviamo l'eq binomia $w^n = z$ nel campo complesso
Per farlo occorre introdurre una nuova espressione dei numeri complessi



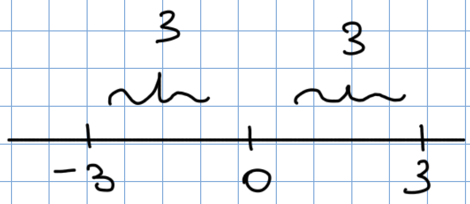
$z = a + ib \quad P(a, b)$

$\overline{OP} = |z|$

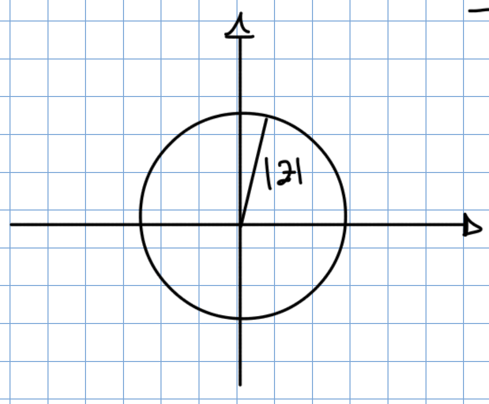
r = semiretta OP orientata da O verso P

α = misura in radianti del minimo angolo di cui deve ruotare in verso contrario al semiasse delle ascisse positive per sovrapporsi in direzion e verso r

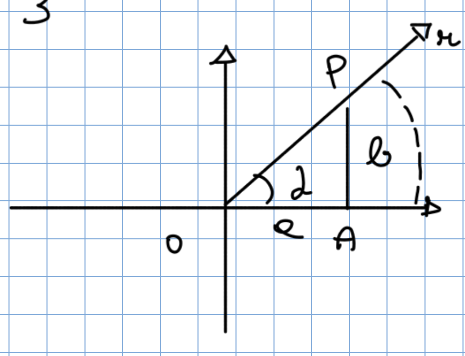
Definizione: $\arg z = \alpha + 2k\pi \quad (k \in \mathbb{Z})$ ARGOMENTO DI z



in $\mathbb{R}$



in $\mathbb{C}$



$OP = |z|$

$OA = a = OP\cos\alpha = |z|\cos(\arg z)$

$BA = b = OP\sin\alpha = |z|\sin(\arg z)$

$z = a + ib = |z|(\cos\alpha + i\sin\alpha)$ FORMA TRIGONOMETRICA

$z = |z|(\cos\alpha + i \operatorname{sen}\alpha)$

$w = |w|(\cos\beta + i\operatorname{sen}\beta)$

$$z = w \iff \begin{cases} |z| = |w| \\ \beta = \alpha + 2k\pi \end{cases}$$

$$w = |z|\left(\cos\alpha + i\sin\alpha\right)|w|\left(\cos\beta + i\sin\beta\right) =$$

$$= |z||w|\left(\cos\alpha\cos\beta + i\left(\cos\alpha\sin\beta + \sin\alpha\cos\beta\right)\right) =$$

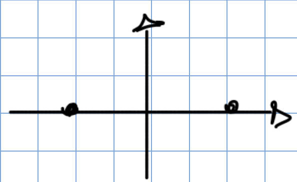
$$= |z||w|\left(\cos(\alpha+\beta) + i\operatorname{sen}(\alpha+\beta)\right) \Rightarrow |zw| = |z||w|$$

$\arg(zw) = \arg(z) + \arg(w)$

in modo simile si trova che

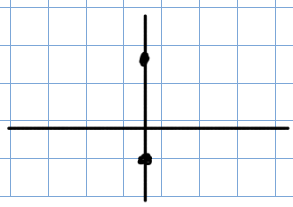
$$\left|\frac{z}{w}\right| = \frac{|z|}{|w|} \qquad \arg\frac{z}{w} = \arg z - \arg w$$

se $z = a \in \mathbb{R}$ $\quad |z| = |a|$ $\quad \arg z = \begin{cases} 0 & \text{se } a > 0 \\ \pi & \text{se } a < 0 \end{cases}$



se $z = ib$ $(b \in \mathbb{R})$

$|z| = |b|$ $\quad \arg z = \begin{cases} \frac{\pi}{2} & \text{se } b > 0 \\ -\frac{\pi}{2} & \text{se } b < 0 \end{cases}$



Formula di MOIVRE (potenze intere di un numero complesso)

$z \in \mathbb{C} \quad z \neq 0 \quad z = |z| (\cos \alpha + i \sin \alpha)$

$n \in \mathbb{Z} \quad z^n = |z|^n (\cos (n \cdot \alpha) + i \sin (n \cdot \alpha))$

es: $i^2 = -1 \quad i = |i| (\cos \alpha + i \sin \alpha) = 1 (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$

$i^2 = i^2 (\cos 2 \frac{\pi}{2} + i \sin 2 \frac{\pi}{2}) = 1 (\cos \pi + i \sin \pi) = -1$

Radice $z \in \mathbb{C} \quad n \in \mathbb{N}, n \geq 2$ si cerca $w \in \mathbb{C} : w^n = z$

se $z = 0 \Rightarrow w = 0$ è l'unica sol

se $z \neq 0 \Rightarrow z = |z| (\cos \alpha + i \sin \alpha)$

$w = 0$ non è sol, sia $w \neq 0$ una sol $\quad w = |w| (\cos \beta + i \sin \beta)$

$w^n = z \Leftrightarrow |w|^n \cdot (\cos (n\beta) + i \sin (n\beta)) =$

$= |z| (\cos \alpha + i \sin \alpha) \Rightarrow \begin{cases} |w|^n = |z| \\ n\beta = \alpha + 2k\pi \end{cases} \Rightarrow \begin{matrix} |w| = \sqrt[n]{|z|} \\ \beta = \frac{\alpha + 2k\pi}{n} \end{matrix}$

$\Rightarrow w$, se è sol., è del

$$w = \sqrt[n]{|z|} \left(\cos \frac{\alpha + 2k\pi}{n} + i \sin \frac{\alpha + 2k\pi}{n} \right)$$ per qualche $k \in \mathbb{Z}$

infatti $(w_k)^n = (\sqrt[n]{|z|})^n \left(\cos \cancel{n} \frac{\alpha + 2k\pi}{\cancel{n}} + i \sin \cancel{n} \frac{\alpha + 2k\pi}{\cancel{n}} \right)$

$= |z| (\cos \alpha + \sin \alpha) = z$

i numeri w_k sono distinti solo per n valori di k

$k \in I = \{ 0, 1, \dots n-1 \}$

$w^n = z$ ha le sol $w_0, w_1, \dots w_{n-1}$

$\forall k \in I \quad |w_k| = \sqrt[n]{|z|} \quad \arg w_k = \dfrac{\arg z + 2k\pi}{n}$

Se $z \in \mathbb{R}$ le eventuali soluzioni reali sono fra queste

es: $z = 16 \quad n = 4 \quad w_k = \sqrt[4]{16}\left(\cos \dfrac{0+2k\pi}{4} + i \sin \dfrac{0+2k\pi}{4}\right)$

$k = 0, 1, 2, 3$

$w_0 = 2(\cos 0 + i \sin 0) = 2 \quad w_1 = 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 2i$

$w_2 = 2(\cos \pi + i \sin \pi) = -2 \quad w_3 = 2\left(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi\right) = -2i$

$z = +16 \quad n = 2 \quad w_k = \sqrt{16} = \left(\cos \dfrac{0+2k\pi}{2} + i \sin \dfrac{0+2k\pi}{2}\right)$

$k = 0, 1$

$w_0 = 4(\cos 0 + i \sin 0) = 4 \quad w_1 = 4(\cos \pi + i \sin \pi) = -4$

$z = -16 \quad n = 2 \quad w_k = \sqrt{16}\left(\cos \dfrac{\pi + 2k\pi}{2} + i \sin \dfrac{\pi + 2k\pi}{2}\right.$

$w_0 = 4\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 4i \quad w_1 = -4i \dots$

in generale se $a \in \mathbb{R}, a < 0 \quad \sqrt{a} = \pm i \sqrt{-a}$

cons l'eq di II grado con $\Delta < 0 \quad \dfrac{-b \pm \sqrt{\Delta}}{2a} = -\dfrac{b \pm i\sqrt{\Delta}}{2a}$

$x^2 + x + 4 = 0 \quad \Delta = -15$

$x = \dfrac{-1 \pm \sqrt{-15}}{2} = -\frac{1}{2} + \frac{\sqrt{15}}{2} i \; ; \; -\frac{1}{2} - \frac{\sqrt{15}}{2} i$

$\sqrt[3]{8}$ $\quad |z| = 2 \quad \arg z = 0 \qquad w_k = \sqrt[3]{8}\left(\cos \frac{0+2k\pi}{3} + i \sin \frac{0+2k\pi}{3}\right.$

$k = 0, 1, 2$

$w_0 = 2 \qquad w_1 = 2\left(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi\right)$

$w_2 = 2\left(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi\right)$

$\sqrt[4]{i} \qquad |z| = 1 \qquad \arg z = \frac{\pi}{2} \qquad w_0 = \left(\cos \frac{\frac{\pi}{2}}{4} + i \sin \frac{\frac{\pi}{2}}{4}\right)$

$w_1 = \cos \frac{3}{8}\pi + i \sin \frac{3}{8}\pi$

CASO $n = 2 \qquad w_k = \sqrt{|z|}\left(\cos\left(\frac{\alpha+2k\pi}{2} + i \sin \frac{\alpha+2k\pi}{2}\right) \quad k=0,1\right.$

$w_0 = \sqrt{|z|}\left(\cos \frac{\alpha}{2} + i \sin \frac{\alpha}{2}\right) \qquad w_1 = \sqrt{|z|}\left(\cos\left(\frac{\alpha}{2}+\pi\right) + i \sin\left(\frac{\alpha}{2}+\pi\right)\right) = -w_0$

Osservazione

$z = a + ib \qquad z = |z|(\cos \alpha + i \sin \alpha)$

$a = |z| \cos \alpha \qquad b = |z| \operatorname{sen} \alpha$

Form. alg. $\rightarrow$ Form trig

$|z| = \sqrt{a^2+b^2} \qquad \cos \alpha = \frac{a}{|z|} \qquad \operatorname{sen} \alpha = \frac{b}{|z|}$

$\sqrt[3]{1+i}$ $\qquad z = 1+i \qquad |z| = \sqrt{2} \qquad 1 = |z| \cos \alpha$

$1 = |z| \operatorname{sen} \alpha$

$\Rightarrow \cos \alpha = \dfrac{1}{\sqrt{2}}, \operatorname{sen} \alpha = \dfrac{1}{\sqrt{2}} \qquad \alpha = \dfrac{\pi}{4}$

$w_0 = \sqrt[6]{2} \left(\cos \dfrac{\frac{\pi}{4}}{3} + i \sin \dfrac{\pi}{2} \right) \qquad w_1 = \sqrt[6]{2} \left(\cos \underset{\underset{\frac{5}{k}\pi}{\parallel}}{\dfrac{\frac{\pi}{4} + \pi}{3}} + i \right.$

$\left. \sin \underline{\qquad} \right)$

$w_2 = \sqrt[6]{2} \left(\cos \dfrac{\frac{\pi}{4} + 2k}{3} + i \sin \underline{\quad} \right)$

eq di II grado: la formula risolutiva vale anche nel campo complesso

$i z^2 + 2z - 2i = 0 \qquad z = \dfrac{-1 \pm \sqrt{1-2}}{i} = \dfrac{-1 \pm \sqrt{-1}}{i}$

$= \dfrac{-1 \pm \sqrt{-1}}{i} = \dfrac{-1}{i} \pm \dfrac{i}{i} = i \pm i$

le sol sono: $1+i$; $-1+i$