$\mathbb{R}$ ha le seguente proprietà di separazione:

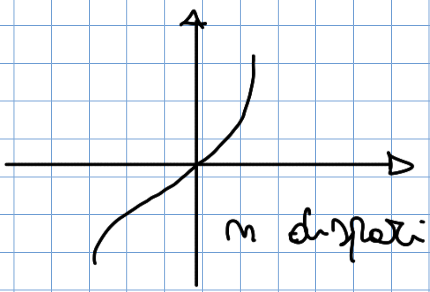
$a, b \in \mathbb{R}$ con $a \neq b$ $\exists\, r, s > 0$ $B(a,r) \cap B(b,s) = \emptyset$

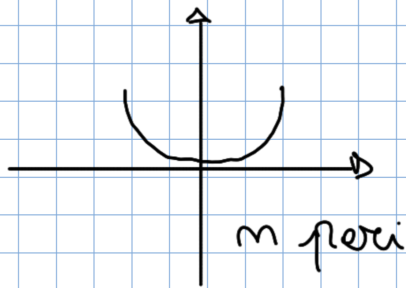
$a + r < b - s$

Funzioni elementari:

1) Funzione potenza

$n \in \mathbb{N}$ $f(x) = x^n$ $\forall x \in \,]-\infty ; +\infty[$





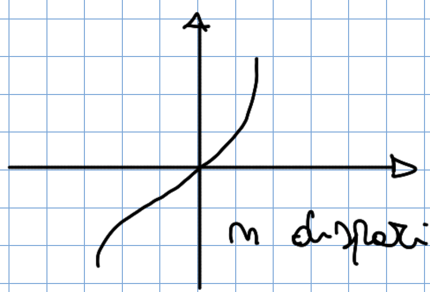
n dispari $\Rightarrow$ invertibile in $]-\infty ; +\infty[$

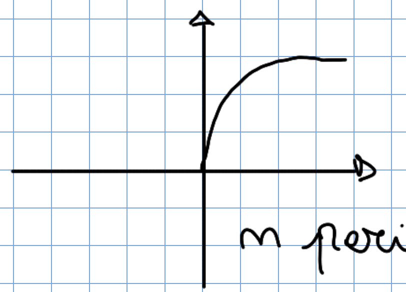
n pari $\Rightarrow$ // in $[0, +\infty[$ e $]-\infty, 0]$

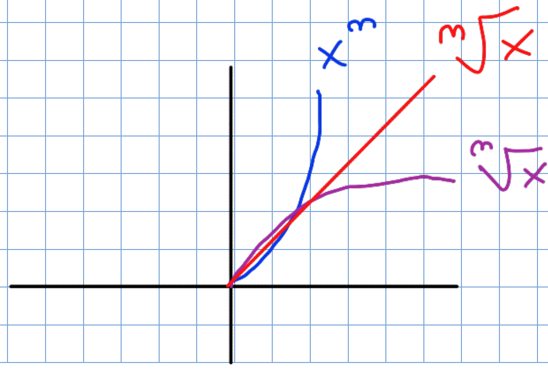
inversa: $\sqrt[n]{x}$

$f(x) = \sqrt[n]{x}$ $\forall x \geq 0$ se n pari

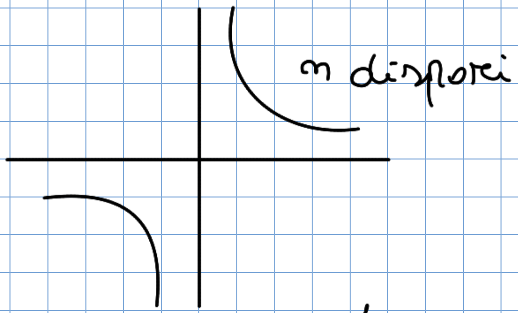
$\forall x \in \mathbb{R}$ se n dispari

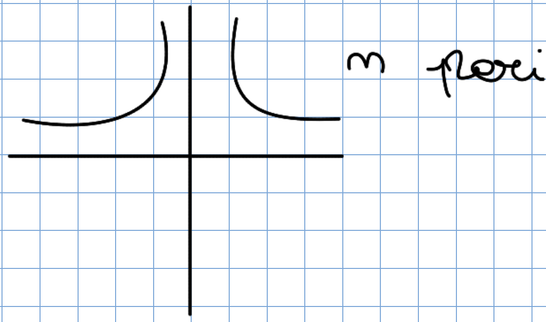






$n \in \mathbb{N}$ $\quad f(x) = x^{-n}$ $\quad \forall x \neq 0$ $\quad x^{-n} = \frac{1}{x^n}$





$n \in \mathbb{N}$ $\quad f(x) = x^{\frac{1}{n}}$ $\left(= \sqrt[n]{x} \right.$ se $\left. x \geq 0 \right)$

$r = \frac{m}{n} \in \mathbb{Q}$ $\quad f(x) = x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$

$s \in \mathbb{R} - \mathbb{Q}$ $\quad f(x) = x^s$ $\quad \forall x \geq 0$ se $s > 0$

$\forall x > 0$ se $s < 0$

x^r $(r \in \mathbb{Q})$, x^s $(s \in \mathbb{R} - \mathbb{Q})$ e l'inversa è $x^{\frac{1}{r}}$, $x^{\frac{1}{s}}$ sono strett crescenti in $]0, +\infty[$

2) Polinomi

$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$

$a_0, \dots a_n \in \mathbb{R}$ $\quad n \in \mathbb{N}_0$ $\quad a_1 \dots a_n$ coefficienti

$\forall x \in \mathbb{R}$

$n = 0$ polinomio costante $f(x) = a_0$ $\forall x \in \mathbb{R}$

se $a_0 = 0$ polinomio nullo

$f(x) = a_0 x^n + \dots + a_n$

$g(x) = b_0 x^p + \dots + a_p$

sono uguali (cioè $f(x) = g(x)$ $\forall x$) se e solo se sono due polinomi identici cioè $n = p$ → grado

(principio di identità del polinomio)

Dati 2 polinomi f, p $\exists$ $q(x), r(x)$ con grado di r < grado di p tali che $f(x) = p(x)\,q(x) + r(x)$

teorema di ruffini

il polinomio $f(x)$ è divisibile per $x - c \iff f(c) = 0$

3) Funzione razionale fratta

$$f(x) = \frac{a_0 x^n + \ldots + a_n}{b_0 x^p + \ldots + b_p}$$

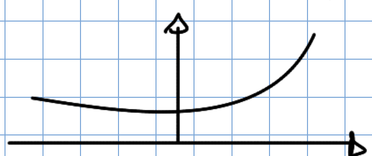
$a_0, \ldots, a_n, b_0 \ldots b_p \in \mathbb{R}$
$n \in \mathbb{N}_0$
$p \in \mathbb{N}$

f è definita in $\mathbb{R} \setminus \{ c \in \mathbb{R} : b_0 c^p + \ldots + b_p = 0 \}$

4) Funzione esponenziale

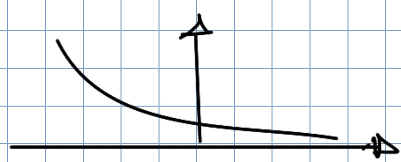
$a \in \mathbb{R}$ $\quad a > 0,\ a \neq 1$ $\quad f(x) = a^x$ $\quad \forall x \in \mathbb{R}$

$a^x > 0 \quad \forall x \in \mathbb{R}$



$a > 1$

strett. cresc.



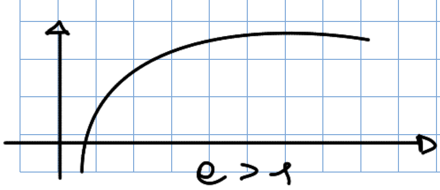
$0 < a < 1$

strett. decresc.

$f:]-\infty; +\infty[\to]0, +\infty[$

Funzione logaritmica

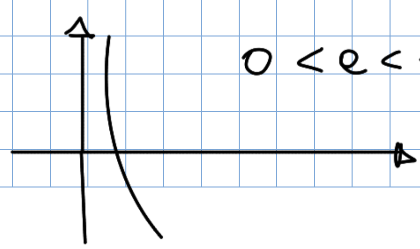
$f^{-1}:]0; +\infty[\to]-\infty; +\infty[$



$a > 1$

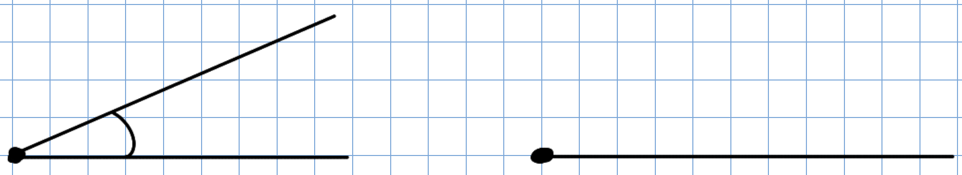
f^{-1} è la funzione logaritmo

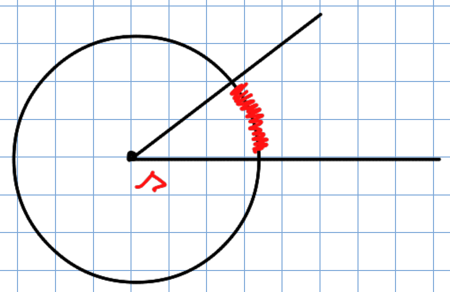
$0 < a < 1$

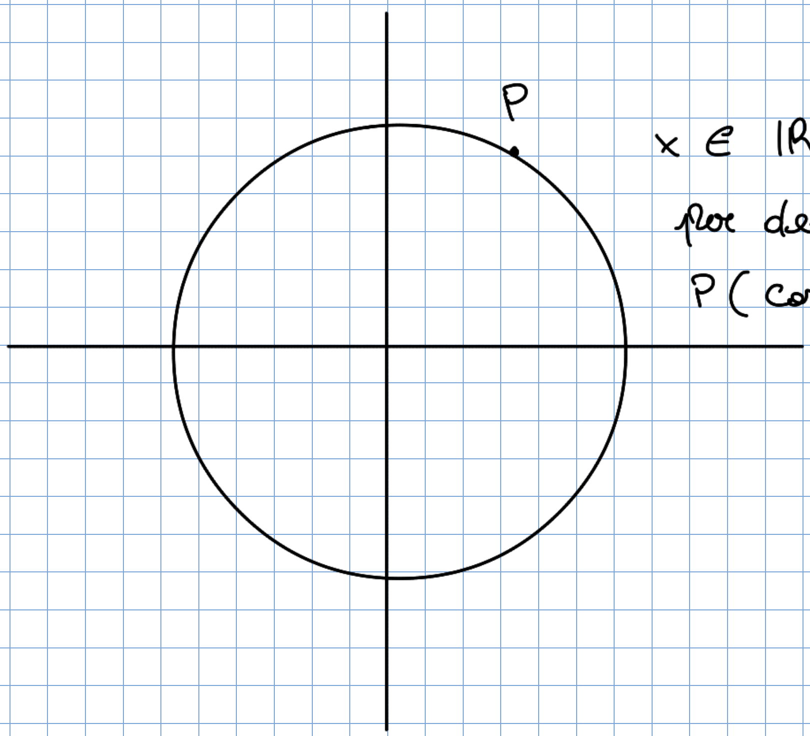


$f^{-1}(f(x)) = x \qquad f(f^{-1}(y)) = y$

5) funzioni trigonometriche







$x \in \mathbb{R}$

per definizione

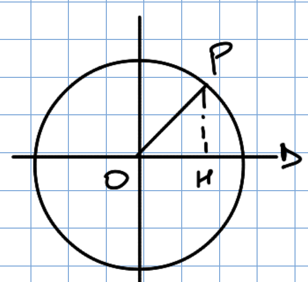
$P(\cos x, \sin x)$

Se $x \in \mathbb{R}$, $\cos x$ ($\operatorname{sen} x$) è l'ascisse (l'ordinate) del secondo estremo di un arco della circonferenza trigonometrica avente primo estremo $A(1,0)$ e misura in radianti x.

$-1 \leq \cos x \leq 1$

$-1 \leq \sin x \leq 1 \qquad \forall x \in \mathbb{R}$

$\cos^2 x + \sin^2 x = 1 \qquad \forall x \in \mathbb{R}$

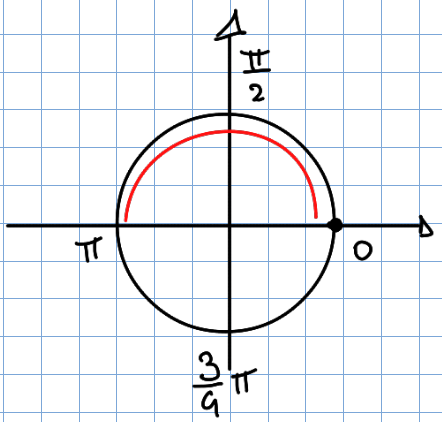


$PH = \sin x$

$OH = \cos x$

$OP = 1$

$$\left.\begin{array}{l}\cos(x+2k\pi)=\cos x\\ \sin(\;\;//\;\;)=\sin x\end{array}\right\}\ \forall x\in\mathbb{R},\ \forall k\in\mathbb{R}$$



$\cos 0 = 1$ in $[0,\pi]$ $f(x)=\cos x$ è

$\cos\pi=-1$ strett. decr. e assume

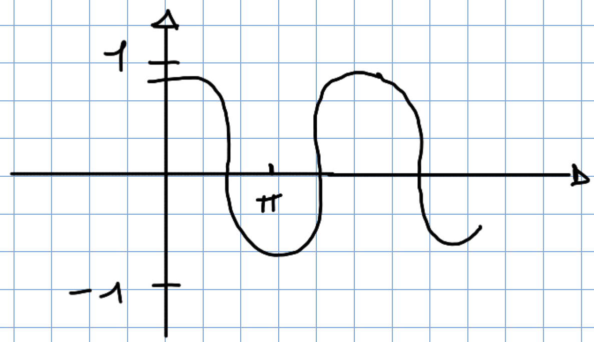
tutti i valori tra -1 e 1 quindi è

invertibile e abbiamo

$\cos:[0,\pi]\to[-1,1]$

$\arccos:[-1,1]\to[0,\pi]$

$\forall d\in[-1;1]$ def $\arccos d =$ l'unico $x\in[0,\pi]$: $\cos x = d$



$\cos x = d$ se $d\in[-1;1]$ sol $\pm\arccos d + 2k\pi$

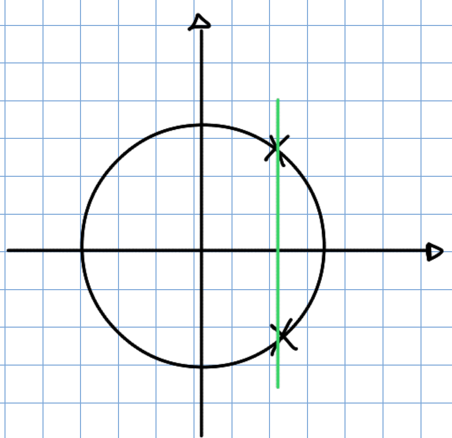
se $d>1$ oppure $d<-1$ non sol.

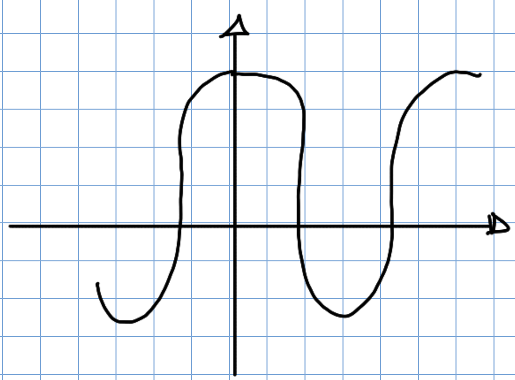
$\cos x > d$ se $-1\le d<1$ sol: $-\arccos d+2k\pi<x<\arccos d+2k\pi$

se $d\geq 1$ nessuna soluzione

se $d\le -1$ $\forall x\in\mathbb{R}$ è sol

$\cos x < d$

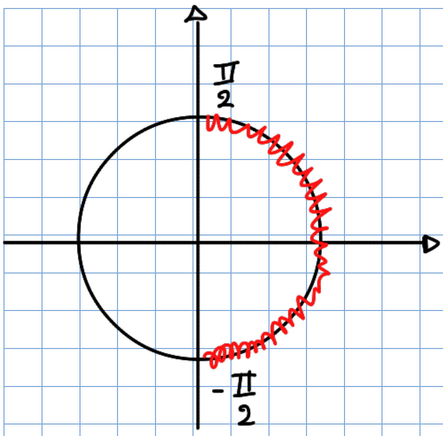




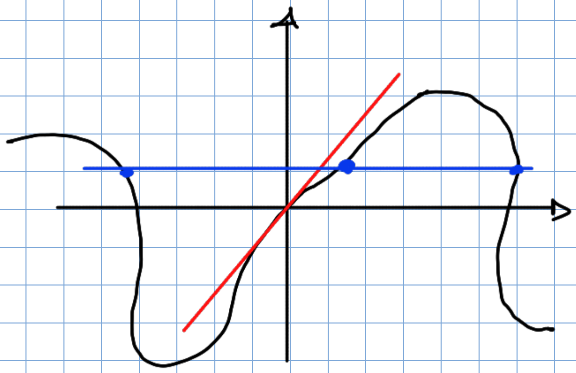
se $-1<d<1$ $\arccos d < x < 2\pi-\arccos d$

se $d\le -1$ nessuna soluzione

se $d>1$ $\forall x\in\mathbb{R}$ è sol



$\operatorname{sen}(-x) = -\operatorname{sen} x \quad \forall x$ (funz dispari)



$\sin\left(-\frac{\pi}{2}\right) = -1 \qquad \operatorname{sen}\left(\frac{\pi}{2}\right) = 1$

in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ è strett cresc e assume tutti valori fra -1 e 1

$\sin: \left[\frac{\pi}{2}, -\frac{\pi}{2}\right] \to [-1, 1]$

$\arcsin: [-1; 1] \to \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\operatorname{sen} x = \alpha$ se $-1 \leq \alpha \leq 1$

sol $x = \arcsin + 2k\pi$

$\pi - \arcsin + 2k\pi$

$\operatorname{sen} x < \alpha$

$-\pi - \arcsin\alpha + 2k < x < \arcsin\alpha + 2k\pi$ se $-1 < \alpha \leq 1$

$\forall x \in \mathbb{R}$ è sol se $\alpha > 1$

non sol. se $\alpha \leq -1$

$-\pi - \arcsin\alpha + 2k\pi < x < \pi - \arcsin + 2k\pi$ se $-1 < \alpha < 1$

non. sol se $\alpha \geq 1$

$\forall x \in \mathbb{R}$ è sol se $\alpha < -1$

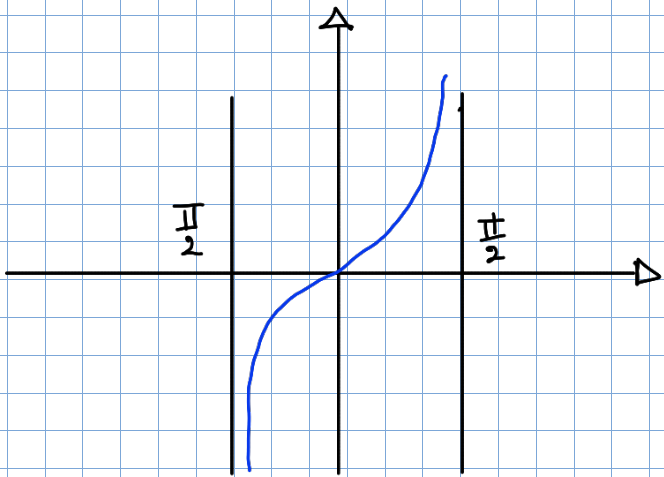
Formule di bisezione (Che cose sono?)

Definizione tangente

$$\tan x = \frac{\operatorname{sen} x}{\cos x} \qquad x \neq \frac{\pi}{2} + k\pi \, , \; k \in \mathbb{Z}$$

$$\tan(x + k\pi) = \tan x \qquad \forall x \, , \; \forall k \in \mathbb{Z}$$

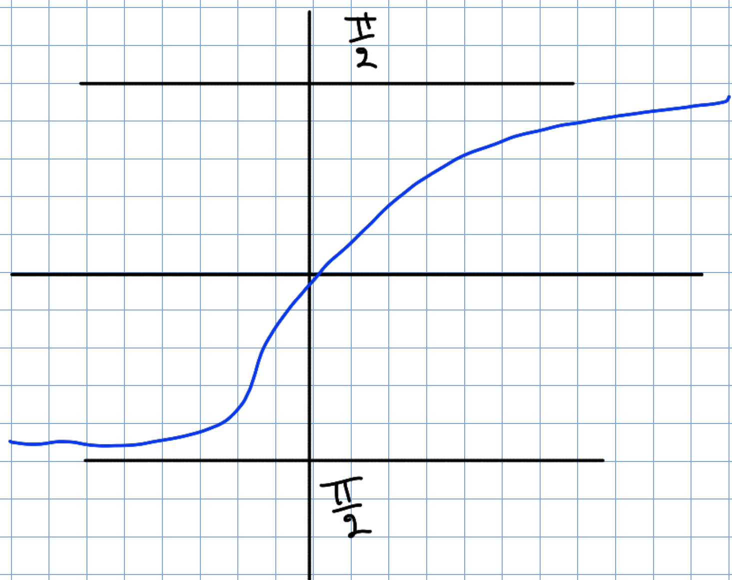
$$\tan(-x) = \frac{\operatorname{sen}(-x)}{\cos(-x)} = \frac{-\operatorname{sen} x}{\cos x} = -\tan x$$



in $]-\frac{\pi}{2}, \frac{\pi}{2}[$ $f(x) = \tan x$ è strett. cresc. e assume tutti i valori reali

$$\tan : \left]-\frac{\pi}{2} ; \frac{\pi}{2}\right[\to \left]-\infty ; +\infty\right[$$

$$\operatorname{arctan} : \left]-\infty ; +\infty\right[\to \left]-\frac{\pi}{2} , \frac{\pi}{2}\right[$$



$\tan x = \alpha \qquad x = \operatorname{arctan} \alpha + k\pi \qquad \forall \alpha \in \mathbb{R}$

$\tan x > \alpha \qquad \operatorname{arctan} \alpha + k\pi < x < \frac{\pi}{2} + k\pi$

$\tan x < \alpha \qquad -\frac{\pi}{2} + k\pi < x < \operatorname{arctan} + k\pi$

$f(x)=\sqrt{\arctan x - \frac{x}{x^2-1}}$ e' funz elementare

Esercizi sull'insieme di definizione

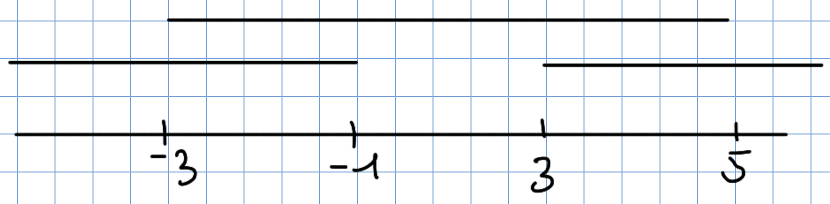
$$f(x) = 2^{\frac{\sqrt{x-1}}{|x|-3}} \qquad \begin{cases} x-1 \geq 0 \\ |x|-3 \neq 0 \end{cases} \qquad \begin{cases} x \geq 1 \\ x \neq 3 \,;\, x \neq -3 \end{cases}$$

$$f: \left([1,3[\,\cup\,]3,+\infty[\right) \to R$$

$$f(x) = \arcsin(|x-1|-3)$$

$$-1 \leq |x-1|-3 \leq 1 \to 2 \leq |x-1| \leq 4$$

$$\begin{cases} |x-1| \geq 2 \\ |x-1| \leq 4 \end{cases} \quad \begin{cases} x-1 \leq -2 \quad \vee \quad x-1 \geq 2 \\ -4 \leq x-1 \leq 4 \end{cases} \Rightarrow \begin{cases} x \leq -1 \,\vee\, x \geq 3 \\ -3 \leq x \leq 5 \end{cases}$$



$$f\left([-3,-1] \cup [3,5] \right) \to R$$

$$f(x) = \sqrt[4]{\sqrt[3]{x-2}-1}$$

$$\sqrt[3]{x-2} - 1 \geq 0 \to \sqrt[3]{x-2} \geq 1 \to x-2 \geq 1 \to x \geq 3$$

$$f: [3; +\infty[\to R$$

e^x $\qquad \boxed{\log x = \log_e x}$ $\qquad e > 1$

$$f(x) = \log \sqrt{\frac{|x|-2}{x^2+3}}$$

$$\begin{cases} \sqrt{\dfrac{|x|-2}{x^2+3}} > 0 \\ \dfrac{|x|-2}{x^2+3} \geq 0 & \Longrightarrow |x| - 2 > 0 \\ & \Downarrow \\ x^2 + 3 \neq 0 & x > 2 \ \vee \ x < -2 \end{cases}$$

$$f: (\,]-\infty, -2[\, \cup \,]2, +\infty[\to \mathbb{R}$$

Numeri Complessi

$C = \{(a,b) : a, b \in \mathbb{R}\}$ coppie ordinate

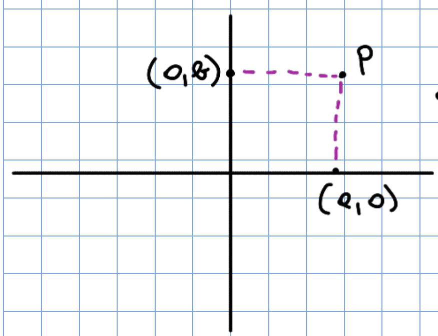
$\boxed{(0,0) = 0 \quad \text{zero}}$

$z = (a,b)$ $\qquad -z = (-a,-b)$ opposto

la nostra nuova x $\qquad \bar{z} = (a, -b)$ coniugato

$(a,0) \longrightarrow$ numero complesso reale

$(0,b) \longrightarrow$ numero immaginario puro



$P(a,b) \to z = (a,b)$

2 numeri complessi z, w sono uguali se: $\qquad z = (a,b)$, $w = (c,d)$

Definizione: $z = w$ se $a = c$, $b = d$

se $z \neq w$ non c'è un ordine

$i = (0,1)$ unità immaginaria

$1 = (1,0)$ unità reale

$z = (a,b)$
$w = (c,d)$

Definizione somma

$z + w = (a+c \;\; b+d)$ → vale la prop commutativa

$z + 0 = 0$

$z + \bar{z} = (a+a, b-b) = (2a, 0)$ è un numero complesso reale

Definizione moltiplicazione

$z \cdot w = (ac - bd, ad + bc)$

$z\bar{z} = (a, b)(a, -b) = (a^2+b^2, 0)$ è un numero complesso reale

$z = (a \;\; b)$ $\quad |z| = \sqrt{a^2+b^2}$ modulo di z

se $z = (a,0)$ $\quad |z| = \sqrt{a^2+0} = |a|$

Dimostrare che ponendo $a \to (a,0)$ si ottiene una corrispondenza biunivoca fra $\mathbb{R}$ e l'insieme complessi reali che conserva le operazioni

quindi si può scrivere $\mathbb{R} \subseteq \mathbb{C}$ $\quad a = (a,0)$ identificazione

$$\begin{array}{ccccc} 2 & + & 4 & = & 6 \\ \downarrow & & \downarrow & & \downarrow \\ (2,0) & + & (4,0) & = & (6,0) \end{array}$$

$$\begin{array}{ccccc} 2 & \cdot & 4 & = & 8 \\ \downarrow & & & & \downarrow \\ (2,0) & \cdot & (4,0) & = & (8,0) \\ \| & & & & \end{array}$$

$$(2 \cdot 4 - 0 \cdot 0, 2 \cdot 0 + 0 \cdot 4) = (8,0)$$

$z \cdot \bar{z} = |z|^2 = |\bar{z}^2| = |-z|^2$