12 marzo 2025

mercoledì 12 marzo 2025 13:46

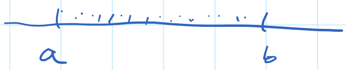
$X = (a,b)$ $\quad$ $\text{int}(X) =]a,b[$

$D(X) = [a,b]$

$\bar{X} = X \cup D(X)$

$X = (a,b) \cap \mathbb{Q}$ $\quad$ $\text{int}(X) = \emptyset$

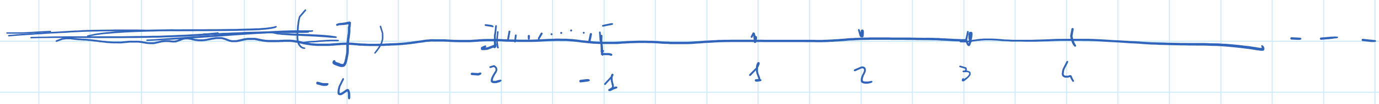
$X = (a,b) \cap (\mathbb{R} \setminus \mathbb{Q})$ $\quad$ $D(X) = [a,b]$



ESERCIZI

① $X =]-\infty, -4] \cup (]-2,-1[\cap \mathbb{Q}) \cup \mathbb{N}$

trovare $\text{int}(X)$, $D(X)$, $F(X)$, $\bar{X}$, $\inf X$, $\sup X$



$\text{int}(X) =]-\infty, -4[$

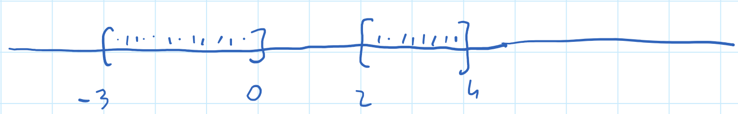
$D(X) =]-\infty, -4] \cup [-2,-1]$

$\bar{X} = X \cup D(X) =]-\infty,-4] \cup [-2,-1] \cup \mathbb{N}$

$F(X) = \{-4\} \cup [-2,-1] \cup \mathbb{N}$

$\inf X = -\infty$ $\quad$ $\sup X = +\infty$

② $X = \Big([-3,0] \cap (\mathbb{R} \setminus \mathbb{Q})\Big) \cup \Big([2,4] \cap \mathbb{Q}\Big)$



$\text{int}(X) = \emptyset$

$D(X) = [-3,0] \cup [2,4]$ $\quad$ $\inf X = -3$ non è min.

$F(X) = [-3,0] \cup [2,4]$

$D(X) = [-3,0] \cup [2,4]$ $\inf X = -3$ non è min.

$f(X) = [-3,0] \cup [2,4]$ $\sup X = 4 = \max X$

$\bar{X} = [-3,0] \cup [2,4]$

(3) $X = (]-\infty, 0[\cap \mathbb{Z}) \cup (]4, +\infty[\cap (\mathbb{R} - \mathbb{Q}))$ es. proposto

$\text{int}(X) =$ $\bar{X} =$

$D(X) =$ $\inf X =$

$F(X) =$ $\sup X =$

es. propost: TROVARE UN ESEMPIO DI INS. X tale che

1) $\sup X \in D(X)$

2) $\sup X \notin D(X)$

3) $\sup X \in \text{int}(X)$

4) $\sup X = \max X$

5) $\sup X$ non è $\max X$

6) X che abbia sia punti di accum.
che punti isolati.

$f: X \to \mathbb{R}$ $X \subseteq \mathbb{R}$ $f(X) = \{ f(x) : x \in X \}$

$f: (a,b) \to \mathbb{R}$ MONOTÒNA in (a,b) se è

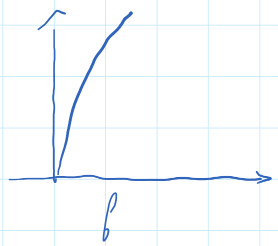
strett cresc $(x < y \Rightarrow f(x) < f(y))$
" decr
cresc (debolm.)
decr. "

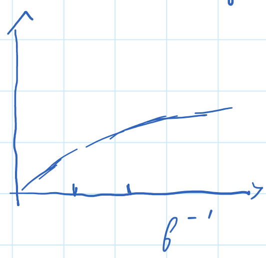
$f: (a,b) \to (\alpha, \beta)$ strett cresc.

Dimostrare che la sua inversa è strett. cresc.

$f^{-1} : (\alpha, \beta) \to (a, b)$

$y, z \in (\alpha, \beta) \qquad y < z \quad \Rightarrow \quad f^{-1}(y) < f^{-1}(z)$





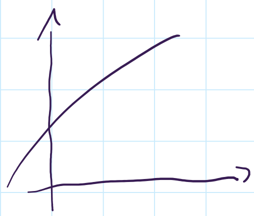
$f^{-1}(y) = x \in (a, b) : f(x) = y$

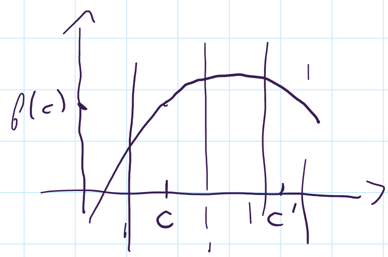
$f^{-1}(z) = x' \in (a, b) : f(x') = z$ devo provare che $x < x'$

se fosse $x \geq x'$ dato che f è strett cresc si avrebbe

$f(x) \geq f(x')$

$y \geq z$ falso





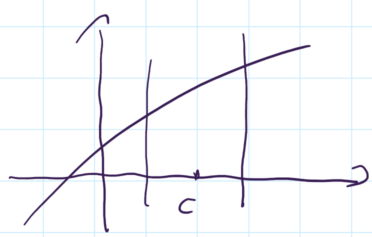
$f : (a, b) \to \mathbb{R} \quad c \in (a, b)$ f crescente in c se $\exists\, r > 0$:

se $x \in (a, b) \cap]c - r, c[$ si ha $f(x) < f(c)$

" $(a, b) \cap]c, c + r[$ " $f(x) > f(c)$

f decrescente in c se ... (def. simmetrica)

TEOR. f strett cresc (strett decr) in (a, b) $\Leftrightarrow$ è cresc (decr) in ogni punto di (a, b)

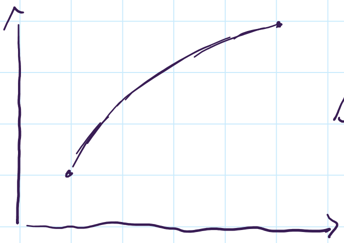


$f : (a, b) \to \mathbb{R} \quad c \in (a, b)$

$f(x) - f(c)$

$\forall x \in (a,b),\ x \neq c$ poniamo $r(x) = \dfrac{f(x) - f(c)}{x - c}$ RAPPORTO INCREMENTALE

$\forall x \neq c$ $\Delta f = f(x) - f(c)$ incremento della funzione
$\Delta x = x - c$ " variabile



$\Delta f = 3$
$\Delta x = 6$

$\Delta f = 3$
$\Delta x = 1$

TEOR. f cresc. in $c \iff \exists\ \delta > 0 :$ $r(x) > 0$ $\forall x \in\ ]c-\delta, c+\delta[,\ x \neq c$
decr. $r(x) < 0$

DIM. Se f è cresc. in $c \Rightarrow \exists\ \delta > 0 :$ $f(x) < f(c)$ in $]c-\delta, c[$
$f(x) > f(c)$ in $]c, c+\delta[$

in $]c-\delta, c[$ $\quad f(x) - f(c) < 0$ $\Rightarrow r(x) > 0$
$x - c < 0$

in $]c, c+\delta[$ $\quad f(x) - f(c) > 0$ $\Rightarrow r(x) > 0$
$x - c > 0$

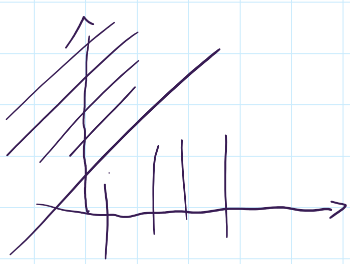
Viceversa se $r(x) > 0$ in $]c-\delta, c+\delta[$

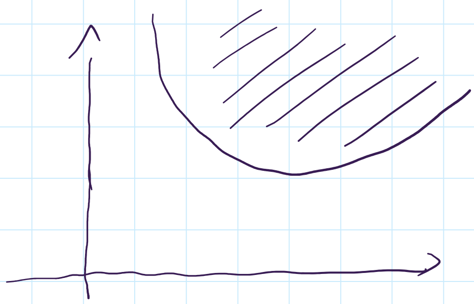
in $]c-\delta, c[$ $\quad r(x) > 0$ $\Rightarrow f(x) - f(c) < 0$
$x - c < 0$

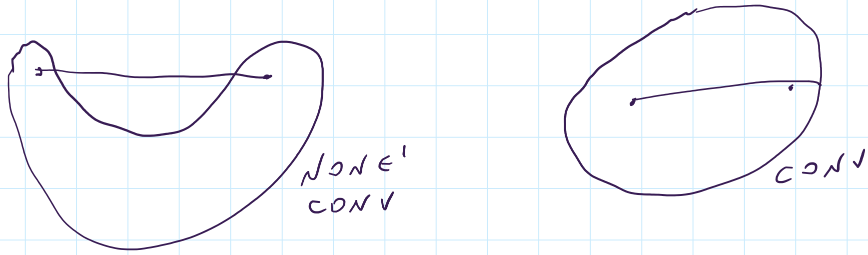
in $]c, c+\delta[$ $\quad r(x) > 0$ $\Rightarrow f(x) - f(c) > 0$
$x - c > 0$

$\Rightarrow f$ cresc in c

FUNZ CONVESSE



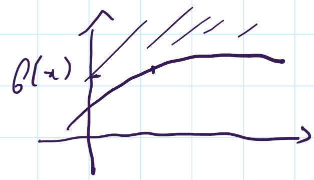




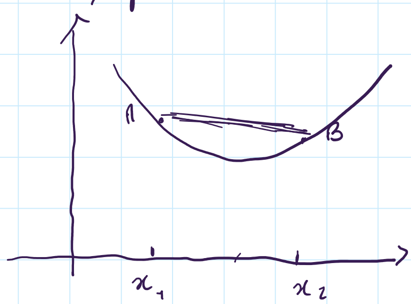
$X \subseteq \mathbb{R}^2$ convesso se: $X = \emptyset$ opp $X = \{P\}$ opp. se contiene due punti, contiene il segmento che li congiunge

$f : (a,b) \to \mathbb{R}$ $\quad epi(f) = \{(x,y) \in \mathbb{R}^2 : \ y \geq f(x)\}$

(EPIGRAFICO)



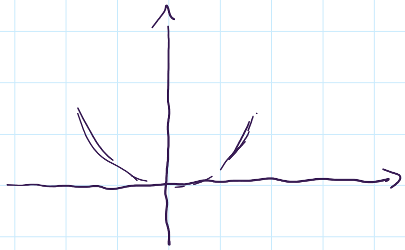
se $epi(f)$ è convesso, f si dice convessa



La parte di grafico relativa all'interv. $[x_1, x_2]$ sta al di sotto del segm. AB

GRAFICO $\quad f : (a,b) \to \mathbb{R}$

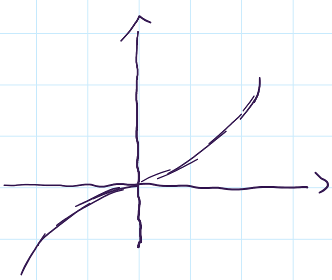
$gr(f) = \{(x, f(x)) : \ x \in (a,b)\}$



simm. risp. $\vec{y}$ cioè

se $(x,y) \in gr(f) \Rightarrow (-x, y) \in gr(f)$

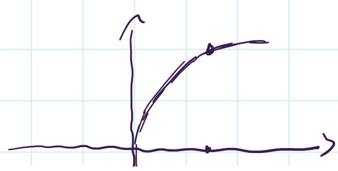
$f(-x) = f(x)$ (funzione pari)



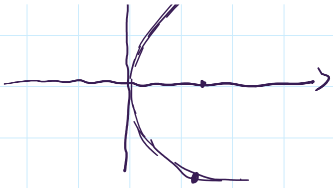
simm. risp. all'origine

se $(x,y) \in gr(f) \Rightarrow (-x,-y) \in gr(f)$

$f(-x) = -f(x)$ (funzione dispari)



il grafico non è MAI simm. risp. a $\vec{x}$



il grafico non è MAI simm. risp. a $\vec{x}$

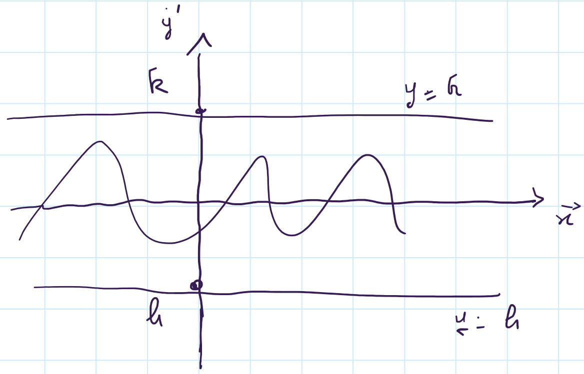
MARCELLINI - SBORDONE
ESERCITAZIONI DI MATEMATICA
vol I PARTE I

$f: \,]-\infty, +\infty[\, \to \mathbb{R}$ se $\exists \, T > 0 : f(x+T) = f(x) \quad \forall x \in \mathbb{R}$

f periodica di periodo T

f per. di periodo $T \iff f(x + kT) = f(x) \quad \forall x \in \mathbb{R}, \ \forall k \in \mathbb{Z}$

es. $f(x+2T) = f(x+T+T) = f((x+T) \underset{\text{per.}}{+} T) = f(x+T) = f(x)$

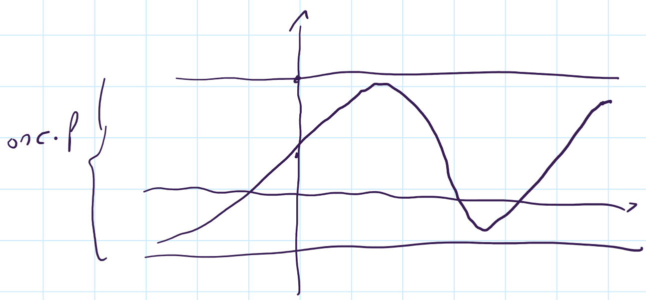


funz. limitata

oscillazione di f

$\sup f - \inf f = \sup\{|f(x) - f(y)| : x, y \in (a,b)\}$

(se f è limitata)



se f non è limitata $\operatorname{osc} f = +\infty$