$z = (a, b)$

Prodotto: $z \cdot w = (ac - bd, ad + bc)$

Somme: $z + w = (a + c, b + d)$

$$(a,0) + (0,1)(b,0) = (a,0) + (0 - 0, 0 + b) = (a,0) + (0,b) = (a,b) = z$$

a $\quad$ i $\quad$ b

$z = a + ib$ Forma algebrica

parte reale $\quad$ parte immaginaria

Re a $\quad$ i = unità imm. $\quad$ $b = \text{Im}\, z$

$z = \text{Re}\, z + i\, \text{Im}\, z$

numero alla seconda diventa negativo ed è una cosa fattibile nei numeri complessi

$$i^2 = (0,1)(0,1) = (0 - 1, 0 + 0) = -1$$

$$(2 + 6i)(1 - 4i) = 2 - 8i + 6i - 24 - (-1) = 26 - 2i$$

↳ uso le formule del prodotto

$z = 3 + i$ $\quad w = 4 - 2i$ $\quad$ Calc $\quad \text{Re}(zw) = 3 \cdot 4 + (-2) \cdot i^2 = 14$

Formule usate in questo esercizio

$|a + ib| = a^2 + b^2$ $\quad \bar{z} = |z|^2$ $\quad (a - b)(a + b) = a^2 - b$ $\quad i^2 = -1$

$$\frac{3 - i}{2 + 3i} = \frac{(3 - i)(2 - 3i)}{(2 + 3i)(2 - 3i)} = \frac{3 - 11i}{4 + 9} = \frac{3}{13} - \frac{11}{13} i$$

$(3 - i)(2 - 3i) \to 6 - 9i - 2i + 3i^2$

$(2 + 3i)(2 - 3i) \to 4 - 9i^2$

z $\quad \bar{z}$

Si moltiplicano numeratore e denominatore per il coniugato del denom

---

$$\frac{2 + 5i}{3i - 1} = \frac{(2 + 5i)(-1 - 3i)}{(3i - 1)(-1 - 3i)} = \frac{13 - 11i}{3^2 + (-1^2)} = \frac{13}{10} - \frac{11}{10} i$$

$$(2 - i)\, i - (2 + i)\, \frac{4}{5 - 3i} = 2i + 1 - \frac{(8 + 4i)(5 + 3i)}{34}$$

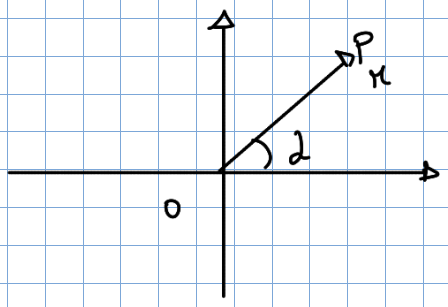
$$= 2i + 1 - \frac{\cancel{28}^{14}}{\cancel{34}_{17}} - \frac{\cancel{44}^{22}}{\cancel{34}_{17}} = \frac{3}{17} + \frac{12}{17}$$

Potenze di i:

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$\vdots$

} questi sono i valori ripetibili ciclicamente

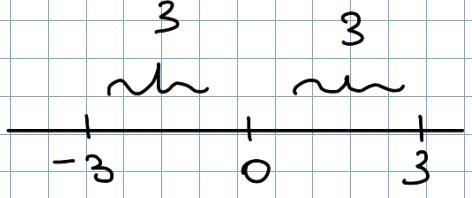
Risolviamo l'eq binomie $w^n = z$ nel campo complesso
Per farlo occorre introdurre una nuova espressione dei numeri complessi



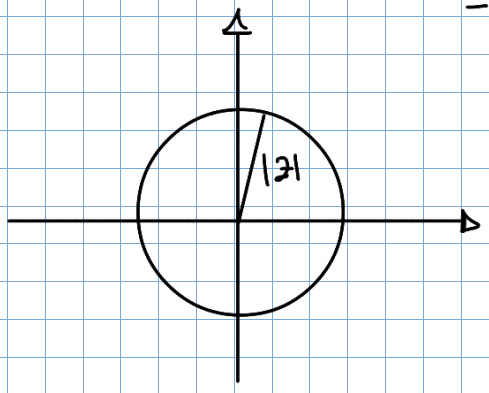
$z = a + ib \quad P(a,b)$
$\overline{OP} = |z|$
x = semiretta OP orientata da O verso P

α = misura in radianti del minimo angolo di cui deve ruotare in verso contrario al semiasse delle ascisse positive per sovrapporsi in direzione e verso a x

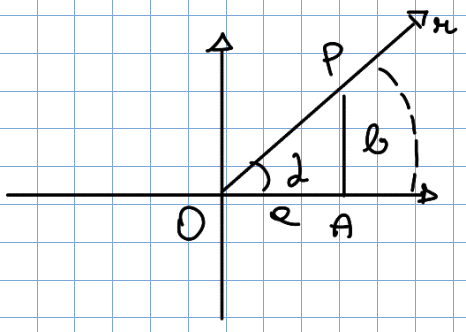
Definizione: $\arg z = \alpha + 2k\pi \quad (k \in \mathbb{Z})$ ARGOMENTO DI z



in $\mathbb{R}$



in $\mathbb{C}$



$OP = |z|$

$OA = a = OP \cos\alpha = |z| \cos(\arg z)$

$BA = b = OP \sin\alpha = |z| \sin(\arg z)$

$z = a + ib = |z|(\cos\alpha + i \sin\alpha)$ FORMA TRIGONOMETRICA

$z = |z|(\cos\alpha + i \operatorname{sen}\alpha)$

$w = |w|(\cos\beta + i \operatorname{sen}\beta)$

$$z = w \iff \begin{cases} |z| = |w| \\ \beta = \alpha + 2k\pi \end{cases}$$

$$w = |z|(\cos\alpha + i \sin\alpha)\,|w|(\cos\beta + i \sin\beta) =$$

$$= |z||w|(\cos\alpha\cos\beta + i(\cos\alpha \sin\beta + \sin\alpha\cos\beta) =$$

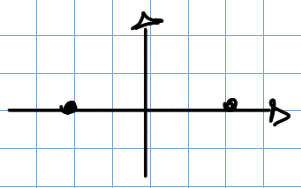
$$= |z||w|(\cos(\alpha+\beta) + i \operatorname{sen}(\alpha+\beta) \Rightarrow |zw| = |z||w|$$

$$\arg(zw) = \arg(z) + \arg(w)$$

In modo simile si trova che

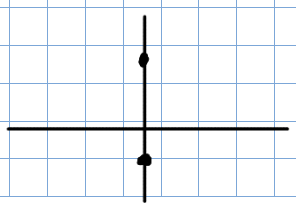
$$\left|\frac{z}{w}\right| = \frac{|z|}{|w|} \qquad \arg\frac{z}{w} = \arg z - \arg w$$

se $z = a \in \mathbb{R}$ $\quad |z| = |a|$ $\quad \arg z = \begin{cases} 0 & \text{se } a > 0 \\ \pi & \text{se } a < 0 \end{cases}$



se $z = ib$ $(b \in \mathbb{R})$

$|z| = |b|$



$$\arg z = \begin{cases} \frac{\pi}{2} & \text{se } b > 0 \\ -\frac{\pi}{2} & \text{se } b < 0 \end{cases}$$

Formule di MOIVRE (potenza intera di un numero complesso)

$z \in \mathbb{C} \quad z \neq 0 \quad z = |z|(\cos\alpha + i\sin\alpha)$

$n \in \mathbb{Z} \quad z^n = |z|^n (\cos(n\cdot\alpha) + i\sin(n\cdot\alpha))$

es: $i^2 = -1 \quad i = |i|(\cos\alpha + i\sin\alpha) = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$i^2 = i^2\left(\cos 2\frac{\pi}{2} + i\sin 2\frac{\pi}{2}\right) = 1(\cos\pi + i\sin\pi) = -1$

Radice $z \in \mathbb{C} \quad n_1 \in \mathbb{N},\ n \geq 2$ si cerca $w \in \mathbb{C} : w^n = z$

se $z = 0 \Rightarrow w = 0$ è l'unica sol

se $z \neq 0 \Rightarrow z = |z|(\cos\alpha + i\sin\alpha)$

$w = 0$ non è sol, sia $w \neq 0$ una sol $\quad w = |w|(\cos\beta + i\sin\beta)$

$w^n = z \iff |w|^n \cdot (\cos(n\beta) + i\sin(n\beta)) =$

$= |z|(\cos\alpha + i\sin\alpha) \Rightarrow \begin{cases} |w|^n = |z| \\ n\beta = \alpha + 2k\pi \end{cases} \Rightarrow \begin{matrix} |w| = \sqrt[n]{|z|} \\ \beta = \frac{\alpha + 2k\pi}{n} \end{matrix}$

$\Rightarrow w_1$ se è sol., è del

$w = \sqrt[n]{|z|}\left(\cos\frac{\alpha + 2k\pi}{n} + i\sin\frac{\alpha + 2k\pi}{n}\right)$ per qualche $k \in \mathbb{Z}$

infatti $(w_k)^n = \left(\sqrt[n]{|z|}\right)^n \left(\cos n\frac{\alpha + 2k\pi}{n} + i\sin n\frac{\alpha + 2k\pi}{n}\right)$

$= |z|(\cos\alpha + i\sin\alpha) = z$

i numeri w_k sono distinti solo per n valori di k

$k \in I = \{0, 1, \dots n-1\}$

$w^n = z$ ha le sol $w_0, w_1, \dots w_{n-1}$

$\forall k \in I \quad |w_k| = \sqrt[n]{|z|} \quad \arg w_k = \dfrac{\arg z + 2k\pi}{n}$

Se $z \in \mathbb{R}$ le eventuali soluzioni reali sono fra queste

es: $z = 16 \quad n = 4 \quad w_k = \sqrt[4]{16}\left(\cos\dfrac{0+2k\pi}{4} + i\sin\dfrac{0+2k\pi}{4}\right)$

$k = 0, 1, 2, 3$

$w_0 = 2(\cos 0 + i\sin 0) = 2 \qquad w_1 = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$

$w_2 = 2(\cos\pi + i\sin\pi) = -2 \qquad w_3 = 2\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right) = -2i$

$z = +16 \quad n = 2 \quad w_k = \sqrt{16} = \left(\cos\dfrac{0+2k\pi}{2} + i\sin\dfrac{0+2k\pi}{2}\right)$

$k = 0, 1$

$w_0 = 4(\cos 0 + i\sin 0) = 4 \qquad w_1 = 4(\cos\pi + i\sin\pi) = -4$

$z = -16 \quad n = 2 \quad w_k = \sqrt{16}\left(\cos\dfrac{\pi+2k\pi}{2} + i\dfrac{\sin\pi+2k\pi}{2}\right)$

$w_0 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 4i \qquad w_1 = -4i \dots$

In generale se $a \in \mathbb{R}, a < 0 \quad \sqrt{a} = \pm i\sqrt{-a}$

come l'eq di II grado con $\Delta < 0 \quad \dfrac{-b \pm \sqrt{\Delta}}{2a} = -\dfrac{b \pm i\sqrt{\Delta}}{2a}$

$x^2 + x + 4 = 0 \qquad \Delta = -15$

$x = \dfrac{-1 \pm \sqrt{-15}}{2} = -\frac{1}{2} + \frac{\sqrt{15}}{2}i \quad ; \quad -\frac{1}{2} - \frac{\sqrt{15}}{2}i$

$\sqrt[3]{8}$ $\quad |z| = 2 \quad \arg z = 0 \quad w_k = \sqrt[3]{8}\left(\cos\frac{0+2k\pi}{3} + i\sin\frac{0+2k\pi}{3}\right)$

$k = 0, 1, 2$

$w_0 = 2 \qquad w_1 = 2\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)$

$w_2 = 2\left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right)$

$\sqrt[4]{i} \quad |z| = 1 \quad \arg z = \frac{\pi}{2} \quad w_0 = \left(\cos\frac{\frac{\pi}{2}}{4} + i\sin\frac{\frac{\pi}{2}}{4}\right)$

$w_1 = \cos\frac{3}{8}\pi + i\sin\frac{3}{8}\pi$

CASO $n = 2 \quad w_k = \sqrt{|z|}\left(\cos\left(\frac{\alpha+2k\pi}{2} + i\sin\frac{\alpha+2k\pi}{2}\right)\right) \quad k = 0, 1$

$w_0 = \sqrt{|z|}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right) \qquad w_1 = \sqrt{|z|}\left(\cos\left(\frac{\alpha}{2}+\pi\right) + i\sin\left(\frac{\alpha}{2}+\pi\right)\right) = -w_0$

Osservazione

$z = a + ib \qquad z = |z|(\cos\alpha + i\sin\alpha)$

$a = |z|\cos\alpha \qquad b = |z|\operatorname{sen}\alpha$

Form. alg. $\rightarrow$ Form trig

$|z| = \sqrt{a^2+b^2} \qquad \cos\alpha = \frac{a}{|z|} \qquad \operatorname{sen}\alpha = \frac{b}{|z|}$

$\sqrt[3]{1+i}$ $\quad z = 1+i \quad |z| = \sqrt{2} \quad 1 = |z| \cos\alpha$

$1 = |z| \operatorname{sen}\alpha$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{2}}, \operatorname{sen}\alpha = \frac{1}{\sqrt{2}} \quad \alpha = \frac{\pi}{4}$

$w_0 = \sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}}{3} + i \operatorname{sen}\frac{\pi}{2}\right) \quad w_1 = \sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}+\pi}{3} + i \operatorname{sen}\frac{\frac{5}{4}\pi}{k}\right)$

$w_2 = \sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}+2k}{3} + i \operatorname{sen}\right)$

eq di II grado: la formula risolutiva vale anche nel campo complesso

$iz^2 + 2z - 2i = 0 \qquad z = \frac{-1 \pm \sqrt{1-2}}{i} = \frac{-1 \pm \sqrt{-1}}{i}$

$= \frac{-1 \pm \sqrt{-1}}{i} = \frac{-1}{i} \pm \frac{i}{i} = i \pm i$

le sol sono: $1+i \; ; \; -1+i$