$\mathbb{R} = \{0;\ \pm a_0, a_1 a_2 \dots\}$

$\pm$ segno
$a_0 \in \mathbb{N}_0$ parte intera
a_i $(i \geq 1)$ cifre
$a_i \in \{0; 1; \dots; 9\}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

$X \subseteq \mathbb{R} \qquad X \neq \emptyset$

$M \in X : \ M \geq x \ \ \forall x \in X \qquad M = \max X$
$m \in X : \ m \leq x \ \ '' \qquad m = \min X$

$k \in \mathbb{R}$ maggiorante per X se $k \geq x \ \ \forall x \in X$

$\bar{M}_X$ = ins dei magg.

$\max X \in \bar{M}_X$
$k \in \bar{M}_X,\ k' > k \Rightarrow k' \in \bar{M}_X$
$k \notin \bar{M}_X$ se $\exists x \in X : x > k$

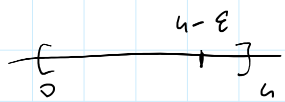
Def. X limitato super. se $\bar{M}_X \neq \emptyset$

Def. $\sup X = \begin{cases} \min \bar{M}_X \ (\text{si può dim che } \exists) & \text{se } \bar{M}_X \neq \emptyset \\ +\infty & \text{se } \bar{M}_X = \emptyset \end{cases}$

$\sup X$ è unico. Se $\sup X$ è un numero si hanno queste proprietà

$\sup X = L \iff$ 1) $L \geq x \ \ \forall x \in X$
2) $\forall \varepsilon > 0 \ \exists x \in X ;\ x > L - \varepsilon$

se $\exists M = \max X \Rightarrow \sup X = M$ infatti 1) vera
2) $\exists x \in X : x > M - \varepsilon$?
basta prendere $x = M$



se $\sup X \in X \Rightarrow \exists \max X = \sup X$ perché $\sup X$ è un magg e $\in X$

DEF. $h \in \mathbb{R}$ MINORANTE di X se $h \leq x \ \forall x \in X$

$\min X \in \underline{M}_X$

$\underline{M}_X$ = ins dei min.

$h \in \underline{M}_X,\ h' < h \Rightarrow h' \in \underline{M}_X$

$h \notin \underline{M}_X$ se $\exists x \in X : x < h$

DEF. X limitato infer. se $\underline{M}_X \neq \emptyset$

DEF. $\inf X = \begin{cases} \max \underline{M}_X \text{ (si dimostra che esiste)} & \text{se } \underline{M}_X \neq \emptyset \\ -\infty & \text{se } \underline{M}_X = \emptyset \end{cases}$

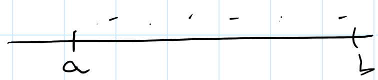
$\inf X$ è unico

$\inf X = \ell \in \mathbb{R} \Leftrightarrow$ 1) $\ell \leq x \ \forall x \in X$

2) $\forall \varepsilon > 0 \ \exists x \in X : x < \ell + \varepsilon$

DEF. X limitato se lo è sia super. che infer.

$\Leftrightarrow \exists a, b \in \mathbb{R} : X \subseteq [a, b]$

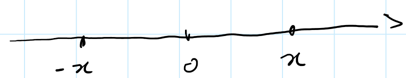


Proprietà del valore assoluto

$x = \pm a_0, a_1 a_2 \ldots$

$-x = \mp a_0, a_1 a_2 \ldots$ opposto

DEF $|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$



$|-x| = |x|$

$|x| \geq 0 \quad \forall x$

$|x-2| = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2$

$|x-2| \geq 0 \quad \forall x$

$|x-2| > 0 \quad x \neq 2$

$|x| \geq 0 \quad \forall x$

$= 0 \iff x = 0$

$|x| = \max(x, -x)$

$|x-2| \geq 0 \quad \forall x$

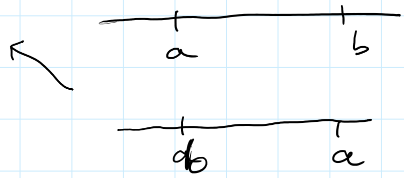
$|x-2| > 0 \quad x \neq 2$

$|x-2| < 0 \quad$ ness. sol.

$|x-2| \leq 0 \quad x = 2$

$|a-b|$ = ampiezza dell'interv. (a,b)

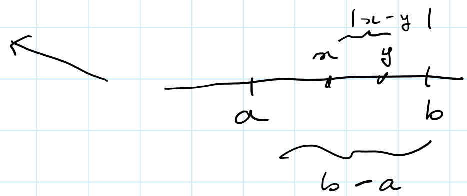
ampiezza di (a,b)



$b-a$

$a-b$

$$\left.\begin{array}{l} a < x < b \\ a < y < b \end{array}\right\} \Rightarrow |x-y| < b-a$$



$-|x| \leq x \leq |x|$

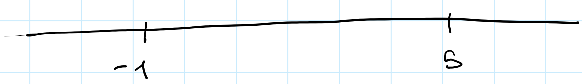
$|ab| = |a|\,|b|$

$|a+b| \leq |a| + |b|$

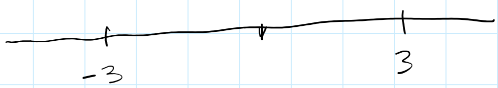
$\left|\frac{1}{a}\right| = \frac{1}{|a|} \qquad (a \neq 0)$

se $|x| < \varepsilon \quad \forall \varepsilon > 0 \Rightarrow x = 0$

X limitato $\iff \exists\, M > 0 : |x| \leq M \quad \forall x \in X$

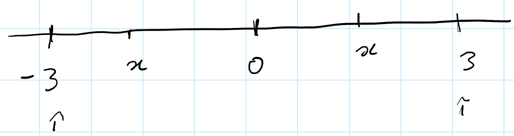


$M = 5$



$M = 3$

$|x| < a \quad (a > 0) \iff -a < x < a$

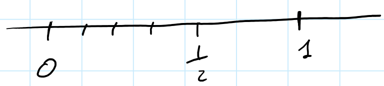


Determinare l'estremo inferiore e l'estremo superiore precisando se sono minimo e massimo

1) $a, b \in \mathbb{R}$ $\quad X = (a, b)$ $\quad \inf X = a$ $\quad \sup X = b$

$X = [a, b)$ $\quad \min X = a$ $\quad \sup X = b$

2) $X = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$



$\inf X = 0$ $\quad \begin{cases} 1) \quad 0 \le \frac{1}{n} \quad \forall n \ ? \quad \le 1 \\ 2) \quad \forall \varepsilon > 0 \ \exists \ n \in \mathbb{N} : \frac{1}{n} < \varepsilon \ ? \end{cases}$

$\Updownarrow$

$n > \frac{1}{\varepsilon}$ vero

$\sup X = \max X = 1$

3) $X = \left\{ -\frac{1}{n} : n \in \mathbb{N} \right\}$ $\qquad 0 < \frac{1}{n} \le 1 \quad \forall n \Rightarrow$

$\Rightarrow -1 \le -\frac{1}{n} < 0$

$\min X = -1$

$\sup X = 0$

4) $X = \left\{ (-1)^n \frac{n}{n+1} : n \in \mathbb{N} \right\}$ $\qquad (-1)^n = \begin{cases} 1 & n \text{ pari} \\ -1 & n \text{ disp} \end{cases}$

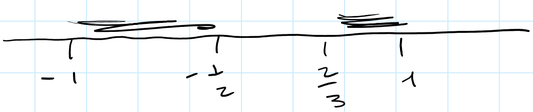
n pari $\qquad \frac{2}{3} < \frac{4}{5} < \dots < 1$ $\qquad \sup = 1$, $\min = \frac{2}{3}$

n disp. $\quad -\frac{n}{n+1}$ $\quad$ cons. $\frac{n}{n+1}$ $\qquad \frac{1}{2} < \frac{2}{3} < \dots < 1$

$\frac{1}{2} \le \frac{n}{n+1} < 1 \Rightarrow -1 < -\frac{n}{n-1} \le -\frac{1}{2}$

$\inf = -1$

$\max = -\frac{1}{2}$



$\inf X = -1$

$\sup X = 1$

5) $X =]0,1[\cup \{3\}$ $\quad \inf X = 0 \quad \max X = 3$

6) $X = \{ x \in \mathbb{R} : |x+2| < 3 \}$

$$\Downarrow$$

$$-3 < x+2 < 3 \Rightarrow -5 < x < 1$$

$\inf X = -5$
$\sup X = 1$

7) $X = \{ x \in \mathbb{R} \setminus \mathbb{Q} : x^2 - |x| \leq 0 \}$

$$\begin{cases} x \geq 0 \\ x^2 - x \leq 0 \end{cases} \qquad \begin{cases} x < 0 \\ x^2 + x \leq 0 \end{cases}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{cases} x \geq 0 \\ 0 \leq x \leq 1 \end{cases} \qquad \begin{cases} x < 0 \\ -1 \leq x < 0 \end{cases}$$

$$0 \leq x \leq 1 \qquad -1 \leq x < 0$$

$X_1 = \{ x \in \mathbb{R} \setminus \mathbb{Q} : -1 \leq x \leq 1 \}$

$\inf X = -1$ (NON È MIN)
$\sup X = 1$ (" MAX)

$X = \{ x \in \mathbb{Q} : x^2 - |x| \leq 0 \}$

$\min X = -1$
$\max X = 1$

8) L'ins. $X = \{ x \in \mathbb{R} : \big| |x-1| - 2x \big| < x+3 \}$

A) è lim. solo inf
B) " sup
C) " inf e sup
D) non è lim. nè inf nè sup

$$\big| |x-1| - 2x \big| < x+3$$

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ 1-x & x < 1 \end{cases}$$

$$\begin{cases} x < 1 \\ |1-x-2x| < x+3 \end{cases} \vee \begin{cases} x \geq 1 \\ |x-1-2x| < x+3 \end{cases}$$

$$\begin{cases} x < 1 \\ |3x-1| < x+3 \end{cases} \qquad \begin{cases} x \geq 1 \\ |x+1| < x+3 \end{cases}$$

$\{ x \geq 1$

$$\left\{ |3x-1| < x+3 \right.$$

$$\begin{cases} x < \frac{1}{3} \\ 1-3x < x+3 \end{cases} \qquad \begin{cases} \frac{1}{3} \le x < 1 \\ 3x-1 < x+3 \end{cases}$$

$$\begin{cases} x < \frac{1}{3} \\ x > -\frac{1}{2} \end{cases} \qquad \begin{cases} \frac{1}{3} \le x < 1 \\ x < 2 \end{cases}$$

$$-\frac{1}{2} < x < \frac{1}{3} \qquad \frac{1}{3} \le x < 1$$

$$-\frac{1}{2} < x < 1$$

$$\left\{ |x+1| < x+3 \right.$$

$$\begin{cases} x \ge 1 \\ x+1 < x+3 \end{cases}$$

$$x \ge 1$$

$$X = \left]-\frac{1}{2}, +\infty\right[$$

(A)

ESERCIZI PROPOSTI (con le stesse opzioni A, B, C, D)

$$X = \{ x \in \mathbb{R} : |x-3| - 2 < |x| \}$$

$$X \quad \{ x \in \mathbb{R} : |2|x-1| - x + 3| < 4 \}$$

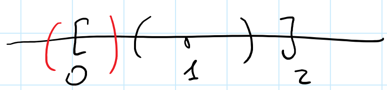
$c \in \mathbb{R}$ $\quad r > 0$ $\quad]c-r, c+r[$ intorno di c di raggio r

$$\overset{\shortparallel}{I_r(c)} = B(c, r)$$

$X \subseteq \mathbb{R}$ $\quad c \in \mathbb{R}$

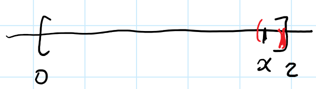
se $c \in X$ $\quad$ c è interno ad X se $\exists\, r > 0 :]c-r, c+r[\subseteq X$

$X = [0, 2]$



1 è interno ad X

0 NON è int. ad X



$]x-r, x+r[\subseteq [0, 2]$

$x + r < 2 \implies r < 2 - x$

$\mathrm{int}(X) =$ ins dei suoi p. interni

$\mathrm{int}\,(a, b) =]a, b[$

X aperto se $X = \emptyset$ opp. tutti i suoi punti sono interni

X aperto $\Leftrightarrow$ $X = \text{int}(X)$

osserv. $\emptyset$ è ap.

$\mathbb{R}$ è ap.

ESERC. trovare un ins. X : $\text{int}(X) = \emptyset$