Data Strucures and Algorithms Homework 1

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1 Proof Theorem 1

Since the size of the graph is reduced by a constant factor at each level of recursion then the depth of recursion is O(log n).
The Algorithm findCut uses $O(n^2)$ time to reduce an n-vertex graph to a 2-vertex graph or to a disconnected graph.
So to find a cut on graph H1 and a cut on H2 it require $O(2n^2) = O(n^2)$.
Let $T(n)$ be the time complexity of the algorithm fastCut. Then:

$$T(n) = 2T\left(\left\lceil 1 + \frac{n}{\sqrt{2}} \right\rceil\right) + O(n^2)$$

Which have as solution:

$$T(n) = O(n^2 log n)$$

The proof of Theorem 1 is illustrated as Theorem 10.16 in the book [1].

2 Implement Contract and the FastCut

All the source code can be found in this GitHub repository https://github.com/AndreaGonzato/MinCut.
I chose to write the code in Java. I used the external library JGrapht to store a graph as a data structure.

3 Verify Theorem 2 experimentally

The proof of Theorem 2 is illustrated as Theorem 10.17 in the book [1].

References

[1] Motwani and Raghavan. *Randomized Algorithms*. chapter: 10 Graph Algorithms - The Min-Cut problem. URL: https://www.semanticscholar.org/paper/Randomized-Algorithms-Motwani-Raghavan/827814eafd1b26fbf64ae6ef1c5508912e9721 (accessed: 27.10.2021).