Data Structures and Algorithms (INFO-F413)
Assignment 1: Karger's Algorithm

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We consider the minimum cut problem: Given an input graph G, partition its set of vertices into two nonempty parts so that the number of edges across the two parts is the smallest.

Karger's Algorithm for Minimum Cut

A simple version of Karger's algorithm for the minimum cut problem, that we will refer to as **Contract**, was described during a lecture. Given a simple graph G on n vertices, the **Contract** algorithm iteratively picks an edge at random and contracts it, until only two vertices are left. It then simply outputs the cut corresponding to those two vertices. We proved that the **Contract** algorithm computes a minimum cut with probability at least $2/(n(n-1))$. The figure on the final page of this document illustrates this process. Note that after a contraction, we can obtain parallel edges. A graph with parallel edges is called a *multigraph*.

We consider the following algorithm, that aims at improving the success probability:

Algorithm FastCut:

Input: A multigraph G.

Output: A cut C.

1. Let n be the number of vertices of G.
2. If $n < 6$, then compute the minimum cut by brute force. Otherwise,
   (a) Let $t \leftarrow \lceil 1 + n/\sqrt{2} \rceil$.
   (b) Using algorithm **Contract**, perform two independent contraction sequences to obtain graphs H_1 and H_2, each with t vertices.
   (c) Recursively compute cuts in each of H_1 and H_2.
   (d) return the smaller of the two cuts.

Note that in Step 2b, the execution of the **Contract** algorithm is stopped as soon as only t vertices are left. The following results are known.

Theorem 1. *The running time of the **FastCut** algorithm is* $O(n^2 \log n)$.

Theorem 2. *The **FastCut** algorithm succeeds in finding a minimum cut with probability* $\Omega(1/\log n)$.

The success probability is therefore much higher that for the simpler **Contract** algorithm.

Your Work

We ask you to do the following:

- Prove Theorem 1.
- Write programs, in your favorite programming language, that implement the **Contract** and the **FastCut** algorithms.
- Use your programs to compare the success probabilities of both algorithms for the same time budget, and verify Theorem 2 experimentally.

Requirements:

1. A technical report with the details of your proof, an outline of the main points of your implementation, your experimental plan, and your conclusions regarding the outcome of the experiments.
2. The source code of the programs.

You are encouraged to experiment on various, families of graphs with different properties. In particular, we draw your attention to the fact that random graphs may give biased results.

Further Readings

- Karger, David (1993). ”Global Min-cuts in RNC and Other Ramifications of a Simple Mincut Algorithm”. Proc. 4th Annual ACM-SIAM Symposium on Discrete Algorithms. (available at `people.csail.mit.edu/karger/Papers/mincut.ps.gz`).
- *Randomized Algorithms*, Motwani and Raghavan, Section 10.2.1, “The contraction algorithm revisited”.

Deadline

Thursday October 28, 2021.

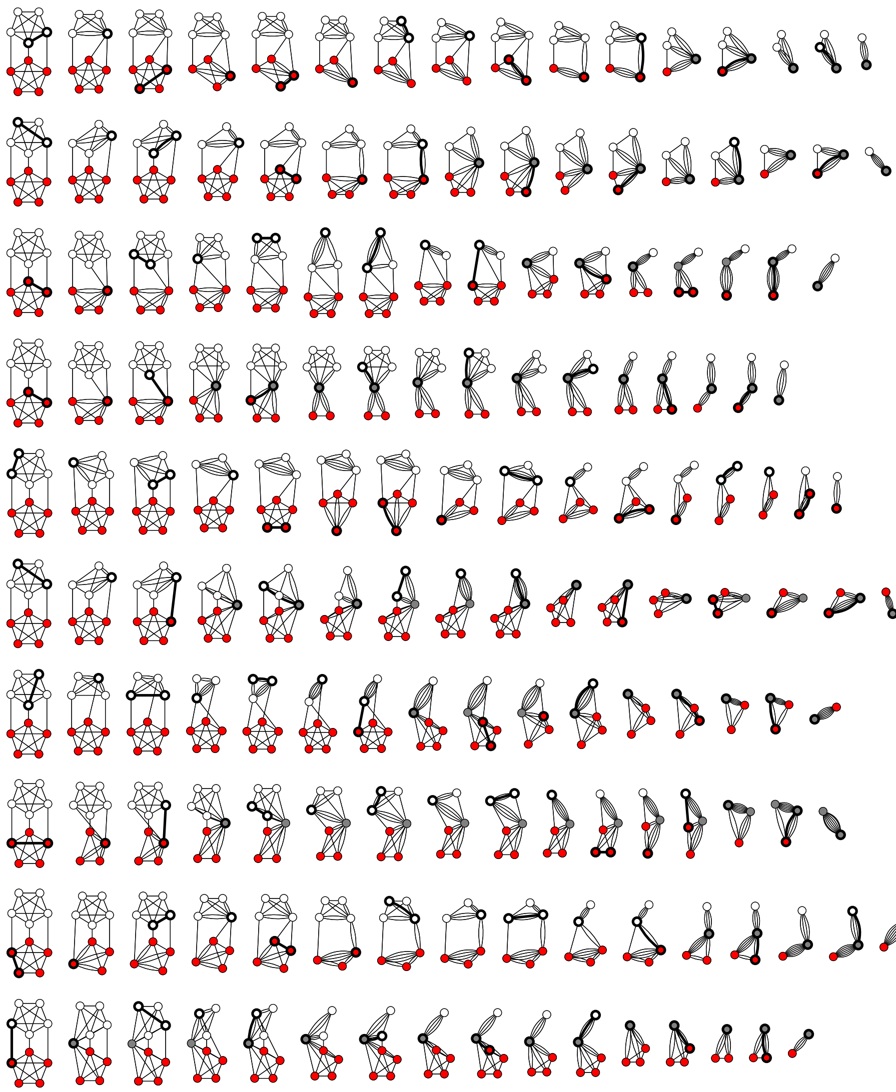

Figure 1: 10 repetitions of the Contract algorithm (Thore Husfeldt – Creative Commons).