2-echelon lastmile delivery with lockers and occasional couriers

Mathematical Optimisation

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Sections

01

Problem Description

The paper

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2-echelon lastmile delivery with lockers and occasional couriers

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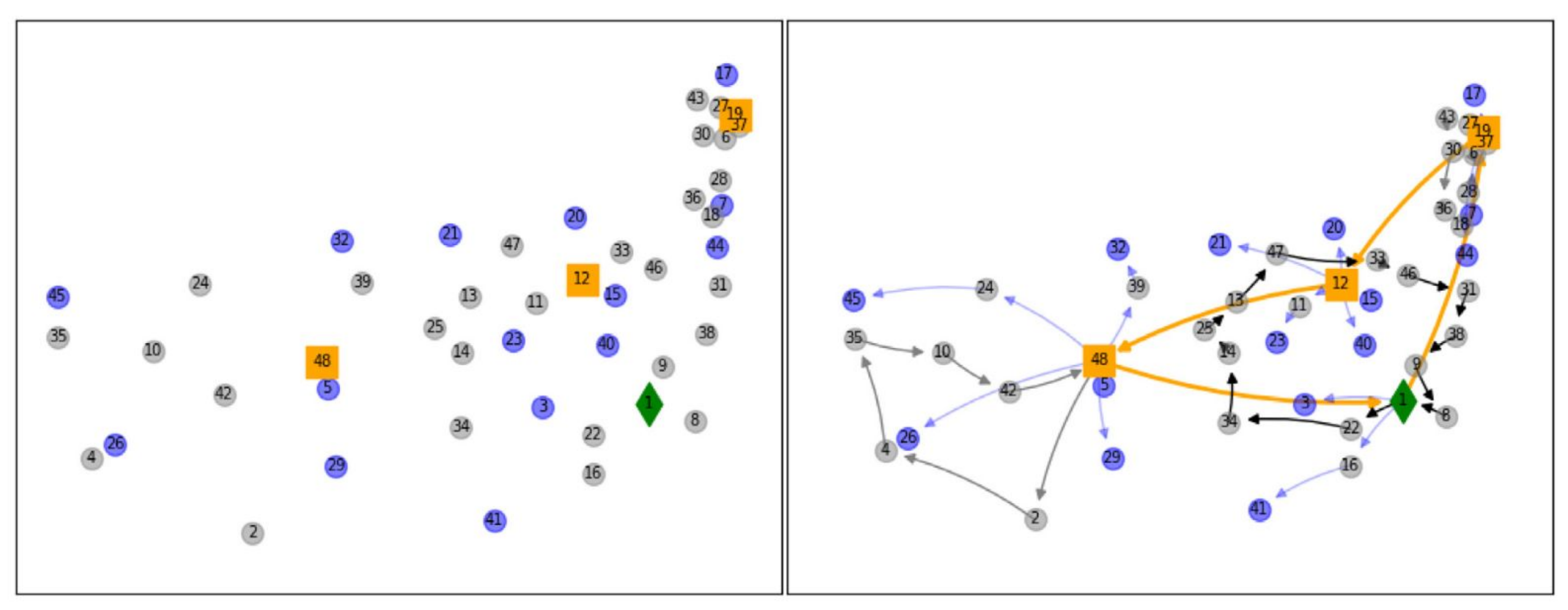
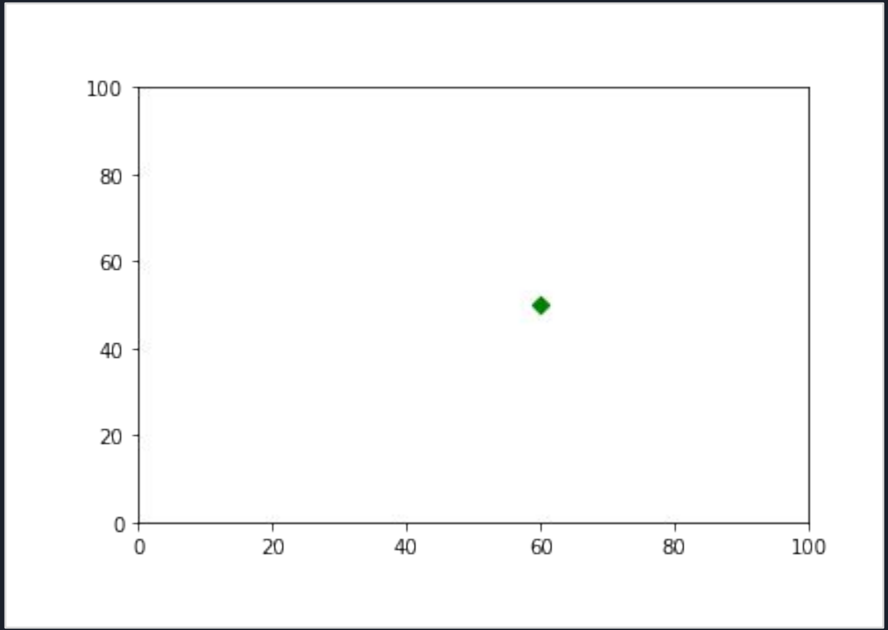
Delivery Problem

Objective: minimize transport costs



Warehouse

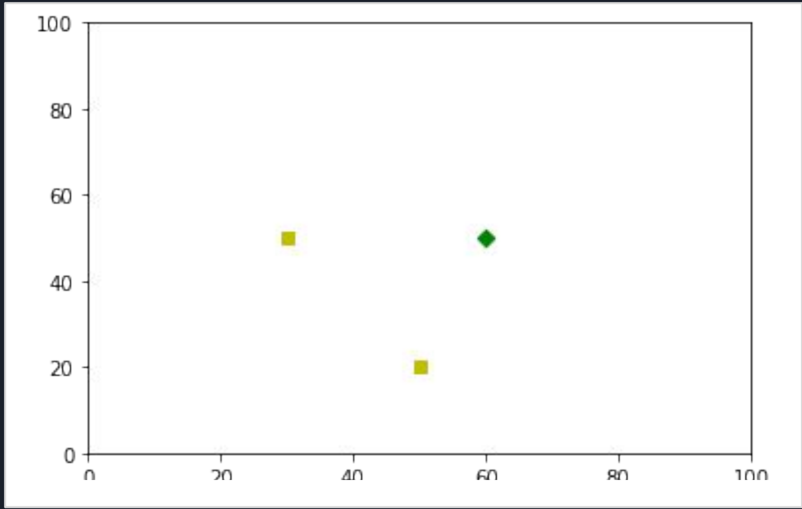
- Point of origin of parcels
- Unique
- Infinite capacity



Lockers

- Spot where to collect and help to distribute packages
- Limited capacity
- L = len(lockers)

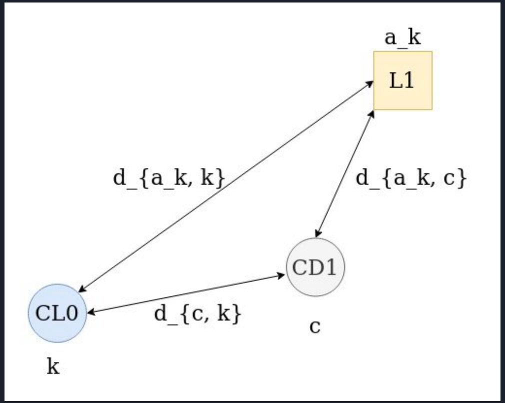
$$\text{locker capacity} = W_l = \left\lceil 0.8 \cdot \frac{|C|}{|L|} \right\rceil$$

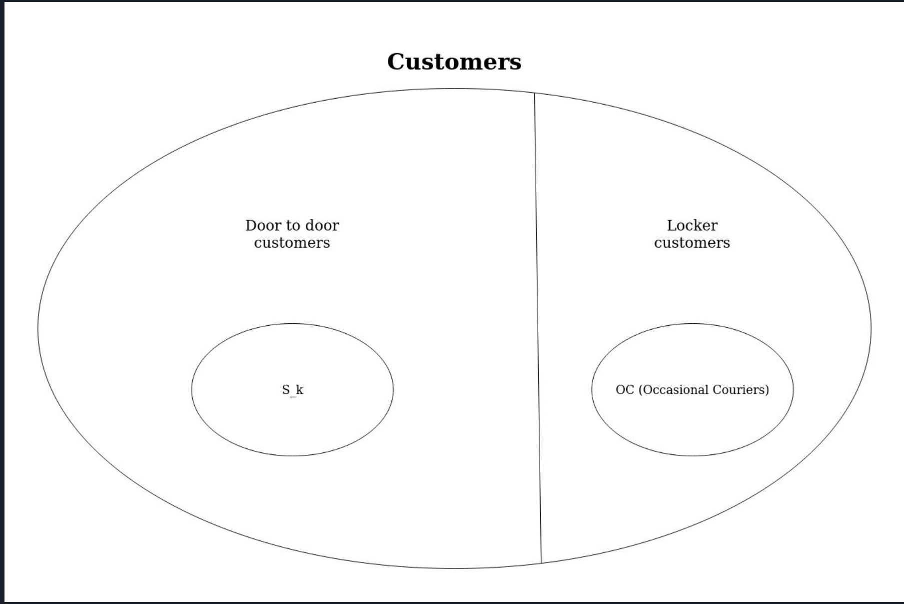


Customers

- Customers partition:
  - Locker Customer (CL)[30%]
    - Occasional Couriers (OC)
  - Door to door Customers (CD)[70%]
    - S_k : served by the OCs
- C = len(customers)

$$c \in S_k \text{ if and only if:}$$
$$d_{a_k,c} + d_{c,k} \leq 1.5 \cdot d_{a_k,k}$$





Vehicles

- Vehicles type:
  - Professional (PF)
  - Local Fleet (LF)
  - Locker supply

PF capacity	Q	$\lceil 0.5 \cdot \lvert C_D \rvert \rceil$
LF capacity	Q^l	$\lceil 0.6 \cdot W_l \rceil$
Locker supply capacity	Q^L	$\lceil 0.8 \cdot \sum_l W_l \rceil$

02

Mathematical Formulation

Parameters

Table 1
Default delivery costs, proportional to the distance d_{ij} associated to arc (i, j).

Type	Symbol	Standard cost
PF	c_{ij}	$\pi \times d_{ij}$, where $\pi = 1.00$
LF	c_{ij}^{l}	$\pi^{l} \times d_{ij}$, where $\pi^{l} = 0.85$
Supply	c_{ij}^{L}	$\pi^{L} \times d_{ij}$, where $\pi^{L} = 0.75$
Compensation	p_{ck}	$\rho \times d_{a_k,c}$, where $\rho = 0.50$

Table 3
Default values for customers' characterization.

Parameter	Symbol	Standard value
Demand	q_i	1
Locker	a_k	The closest to customer k
OC's coverage	δ	1.5, i.e., a max detour of 50%

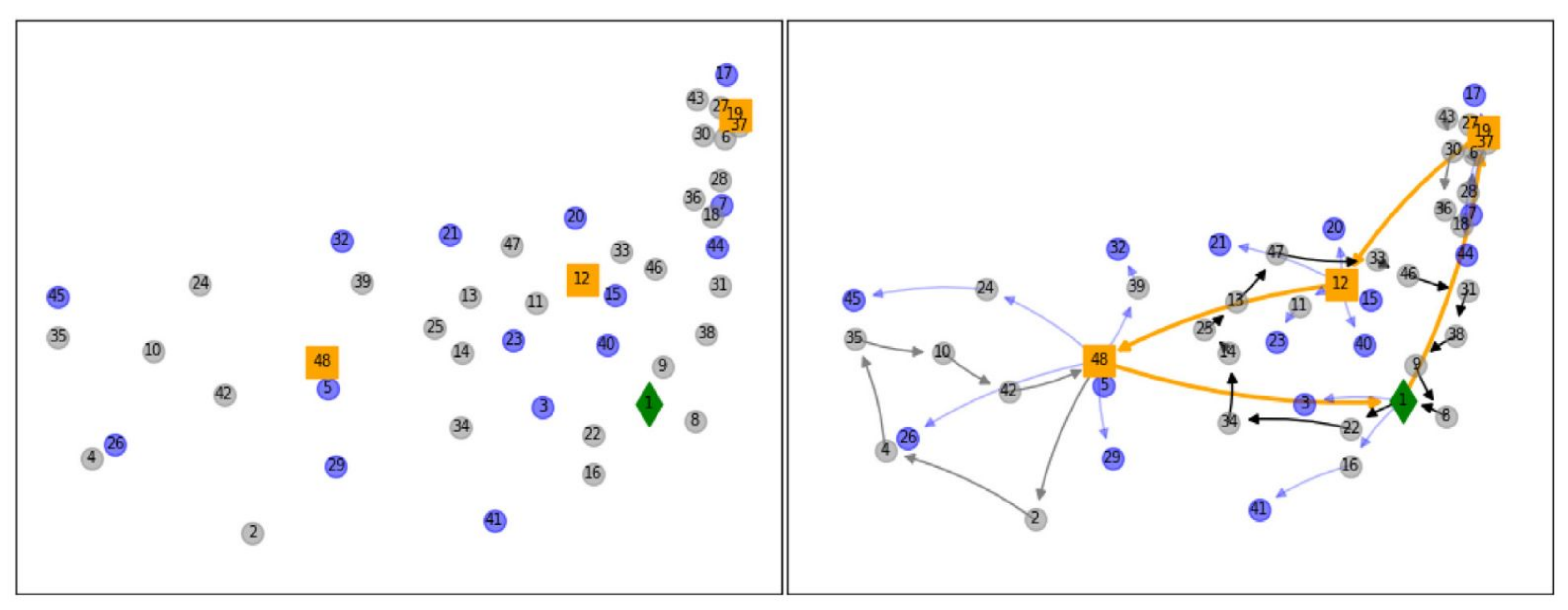
Decision Variables

The following decision variables are used:

- $x_{ij}, x_{ij}^L, x_{ij}^l$: 1, if arc (i, j) is traversed by the PF, the locker supplying vehicles or the LF associated to locker l, respectively; 0, otherwise.
- $y_{ij}, y_{ij}^L, y_{ij}^l$: load on each type of vehicle when traversing arc (i, j).
- z_c: 1, if customer c is served by the PF; 0, otherwise.
- z_c^l: 1, if customer c is served by the LF associated to locker l; 0, otherwise.
- z_l^L: 1, if locker l is served by a supply route; 0, otherwise.
- w_{ck}: 1, if customer c is outsourced to OC k; 0, otherwise.

Delivery Problem

Objective: minimize transport costs



Single Period Problem

Objective Function

$$\min \sum_{i,j \in O \cup L} c_{ij}^L x_{ij}^L + \sum_{k \in C_L} \sum_{c \in S_k} p_{ck} w_{ck} + \sum_{i,j \in O \cup C_D} c_{ij} x_{ij} + \sum_{l \in L} \sum_{i,j \in \{l\} \cup C_D} c_{ij}^l x_{ij}^l$$

$$\downarrow$$

$$\min \sum_{i,j \in O \cup L} c_{ij}^L x_{ij}^L + \sum_{k \in OC} \sum_{c \in S_k} p_{ck} w_{ck} + \sum_{i,j \in O \cup C_D} c_{ij} x_{ij} + \sum_{l \in L} \sum_{i,j \in \{l\} \cup C_D} c_{ij}^l x_{ij}^l$$

Single Period Problem

The constraints of the paper

Customers' service

$$z_c + \sum_{k \in C_L | c \in S_k} w_{ck} + \sum_{l \in L} z_c^l = 1, \qquad \forall c \in C_D$$

$$\sum_{c \in S_k} w_{ck} \leq 1, \qquad \forall k \in C_L$$

$$\sum_{k \in C_L | a_k = l} \left(q_k + \sum_{c \in S_k} q_c w_{ck} \right) + \sum_{c \in C_D} q_c z_c^l \leq W_l z_l^L, \qquad \forall l \in L$$

Single Period Problem

Our constraints

Customers' service

$$z_c + \sum_{k \in OC | c \in S_k} w_{ck} + \sum_{l \in L} z_c^l = 1 \qquad \forall c \in C_D$$

$$\sum_{c \in S_k} w_{ck} \leq 1 \qquad \forall k \in OC$$

$$\sum_{k \in C_L | a_k = l} q_k + \sum_{k \in OC | a_k = l} \sum_{c \in S_k} q_c w_{ck} + \sum_{c \in C_D} q_c z_c^l \leq W_l z_l^L \qquad \forall l \in L$$

Single Period Problem

The constraints of the paper

Professional fleet constraints

$$\sum_{j \in C_D \cup O} x_{ij} = \sum_{j \in C_D \cup O} x_{ji} = z_i, \qquad \forall i \in C_D$$

$$\sum_{j \in C_D} x_{oj} - \sum_{j \in C_D} x_{jo} = 0$$

$$\sum_{j \in C_D \cup O} y_{ji} - \sum_{j \in C_D \cup O} y_{ij} = q_i z_i, \qquad \forall i \in C_D$$

$$\sum_{j \in C_D} y_{jo} - \sum_{j \in C_D} y_{oj} = \sum_{i \in C_D} -q_i z_i$$

$$y_{ij} \leq Q x_{ij}, \qquad \forall i, j \in C_D \cup O$$

$$y_{io} = 0, \qquad \forall i \in C_D$$

Single Period Problem

The constraints of the paper

Supply routes constraints

$$\sum_{j \in L \cup O} x_{ij}^L = \sum_{j \in L \cup O} x_{ji}^L = z_i^L, \qquad \forall i \in L$$

$$\sum_{j \in L} x_{oj}^L - \sum_{j \in L} x_{jo}^L = 0$$

$$\sum_{j \in L \cup O} y_{ji}^L - \sum_{j \in L \cup O} y_{ij}^L = \sum_{k | a_k = i} \left(q_k + \sum_{c \in S_k} q_c w_{ck} \right) + \sum_{c \in C_D} q_c z_c^i, \qquad \forall i \in L$$

$$\sum_{j \in L} y_{jo}^L - \sum_{j \in L} y_{oj}^L = -\sum_{i \in L} \left(\sum_{k | a_k = i} \left(q_k + \sum_{c \in S_k} q_c w_{ck} \right) + \sum_{c \in C_D} q_c z_c^i \right)$$

$$y_{ij}^L \leq Q^L x_{ij}^L, \qquad \forall i, j \in L \cup O$$

$$y_{io}^L = 0, \qquad \forall i \in L$$

Single Period Problem
Our constraints

Supply routes constraints

$$\sum_{j\in L\cup O} x_{ij}^L = \sum_{j\in L\cup O} x_{ji}^L = z_i^L, \qquad \forall i \in L$$

$$\sum_{j\in L} x_{oj}^L - \sum_{j\in L} x_{jo}^L = 0$$

$$\sum_{j\in L\cup O} y_{ji}^L - \sum_{j\in L\cup O} y_{ij}^L = \sum_{k\in C_L|a_k=i} q_k + \sum_{k\in OC|a_k=i}\sum_{c\in S_k} q_c w_{ck} + \sum_{c\in C_D} q_c z_c^i \qquad \forall i \in L$$

$$\sum_{j\in L} y_{jo}^L - \sum_{j\in L} y_{oj}^L = -\sum_{i\in L}\Big(\sum_{k\in C_L|a_k=i} q_k + \sum_{k\in OC|a_k=i}\sum_{c\in S_k} q_c w_{ck} + \sum_{c\in C_D} q_c z_c^i\Big)$$

$$y_{ij}^L \le Q^L x_{ij}^L, \qquad \forall i,j \in L\cup O$$

$$y_{io}^L = 0, \qquad \forall i \in L$$

Single Period Problem

The constraints of the paper

Local fleet constraints $(\forall l \in L)$

$$\sum_{j \in C_D \cup \{l\}} x_{ij}^l = \sum_{j \in C_D \cup \{l\}} x_{ji}^l = z_i^l, \qquad \forall i \in C_D$$

$$\sum_{j \in C_D} x_{lj}^l - \sum_{j \in C_D} x_{jl}^l = 0$$

$$\sum_{j \in C_D \cup \{l\}} y_{ji}^l - \sum_{j \in C_D \cup \{l\}} y_{ij}^l = q_i z_i^l, \qquad \forall i \in C_D$$

$$\sum_{j \in C_D} y_{jl}^l - \sum_{j \in C_D} y_{lj}^l = \sum_{i \in C_D} -q_i z_i^l$$

$$y_{ij}^l \le Q^l x_{ij}^l, \qquad \forall i, j \in C_D \cup \{l\}$$

$$y_{il}^l = 0, \qquad \forall i \in C_D$$

3 new constraints

$$x_{ii} = 0 \qquad \forall i \in C_D \cup O$$

$$x_{ii}^{l} = 0 \qquad \forall l \in L \forall i \in C_D \cup l$$

$$x_{ii}^{L} = 0 \qquad \forall i \in L \cup O$$

Multi Period Problem

- C_L^l: set of locker customers of locker $l \in (L \cup O)$ that did not show up;
- C_D^l: set of door-to-door customers whose parcels remained in locker $l \in L \cup O$ (due to no-show of OCs); notice that C_D^o is the set of door-to-door customers that in a previous period were assigned to an OC associated to the warehouse that did not show-up;
- $\hat{C}_D^l \subseteq C_D^l$: subset of door-to-door customers that must be served in that period;

Multi Period Problem

Objective Function

$$\min \sum_{i,j \in O \cup L} c_{ij}^L x_{ij}^L + \sum_{k \in C_L \cup C_L^*} \sum_{c \in S_k} p_{ck} w_{ck} + \sum_{i,j \in O \cup C_D \cup C_D^o} c_{ij} x_{ij} + \sum_{l \in L} \sum_{i,j \in \{l\} \cup C_D \cup C_D^o \cup C_D^l} c_{ij}^l x_{ij}^l$$

Multi Period Problem
Objective Function

Table 8
Instances' size: total and per period.

ID	Base	Total				Days	Per day	
		N	L	C_L	C_D	D	C_L^d	C_D^d
R2	rat575	575	2	150	422	20	7–8	21–22
R3	rat575	575	3	180	391	20	9	19–20
D2	dsj1000	1000	2	200	797	30	6–7	26–27
D4	dsj1000	1000	4	240	755	30	8	25–26
N3	nrw1379	1379	3	375	1000	40	9–10	25
N4	nrw1379	1379	4	450	924	40	11–12	23–24

Multi Period Problem

Objective Function

$$\min \sum_{i,j \in O \cup L} c_{ij}^{L} x_{ij}^{L} + \sum_{k \in C_L \cup C_L^{*}} \sum_{c \in S_k} p_{ck} w_{ck} + \sum_{i,j \in O \cup C_D \cup C_D^{o}} c_{ij} x_{ij} + \sum_{l \in L} \sum_{i,j \in \{l\} \cup C_D \cup C_D^{o} \cup C_D^{l}} c_{ij}^{l} x_{ij}^{l}$$

$$\downarrow$$

$$Obj_{d_i} = min \sum_{i,j \in O \cup L} c_{ij}^{L} x_{ij}^{L} + \sum_{k \in OC^{d_i}} \sum_{c \in S_k^{d_i}} p_{ck} w_{ck} + \sum_{i,j \in O \cup C_D^{d_i} \cup C_D^{d_i - 1}} c_{ij} x_{ij} + \sum_{l \in L} \sum_{i,j \in l \cup C_D^{d_i} \cup C_D^{d_i - 1}} c_{ij}^{l} x_{ij}^{l}$$

Multi Period Problem

The constraints of the paper

$$z_c + \sum_{k \in C_L \cup C_L^* | c \in S_k} w_{ck} + \sum_{l \in L} z_c^l = 1, \qquad \forall c \in C_D \cup C_D^o$$

$$\sum_{k \in (C_L | a_k = l) \cup C_L^l | c \in S_k} w_{ck} + z_c^l = 1, \qquad \forall l \in L, c \in C_D^l$$

$$z_c^l = 1, \qquad \forall l \in L, c \in \hat{C}_D^l$$

$$z_c + \sum_{l \in L} z_c^l = 1, \qquad \forall c \in \hat{C}_D^o$$

$$\sum_{c \in S_k} w_{ck} \leq 1, \qquad \forall k \in C_L \cup C_L^*$$

$$\sum_{k \in C_L | a_k = l} \left(q_k + \sum_{c \in S_k} q_c w_{ck} \right) + \sum_{c \in C_D} q_c z_c^l + \sum_{k \in C_L^l} \sum_{c \in S_k} q_c w_{ck} \leq \left(W_l - \sum_{k \in C_L^l} q_k - \sum_{c \in C_D^l} q_c \right) z_l^L, \qquad \forall l \in L$$

Multi Period Problem
Our problem interpretation

- Day i

  OC may include CL of the previous day

  Sk include only the Sk of the current day

  ĈD are all the CD of the previous day

Multi Period Problem

The constraints of the paper

$$z_c + \sum_{k \in C_L \cup C_L^* | c \in S_k} w_{ck} + \sum_{l \in L} z_c^l = 1, \qquad \forall c \in C_D \cup C_D^o$$

$$\sum_{k \in (C_L | a_k = l) \cup C_L^l | c \in S_k} w_{ck} + z_c^l = 1, \qquad \forall l \in L, c \in C_D^l$$

$$z_c^l = 1, \qquad \forall l \in L, c \in \hat{C}_D^l$$

$$z_c + \sum_{l \in L} z_c^l = 1, \qquad \forall c \in \hat{C}_D^o$$

$$\sum_{c \in S_k} w_{ck} \leq 1, \qquad \forall k \in C_L \cup C_L^*$$

$$\sum_{k \in C_L | a_k = l} \left(q_k + \sum_{c \in S_k} q_c w_{ck} \right) + \sum_{c \in C_D} q_c z_c^l + \sum_{k \in C_L^l} \sum_{c \in S_k} q_c w_{ck} \leq \left(W_l - \sum_{k \in C_L^l} q_k - \sum_{c \in C_D^l} q_c \right) z_l^L, \qquad \forall l \in L$$

Multi Period Problem

Our constraints

$$\sum_{k \in C_L^{d_i} | a_k = l} q_k + \sum_{k \in OC^{d_i} | a_k = l} \sum_{c \in S_k^{d_i}} q_c w_{ck} + \sum_{c \in C_D^{d_i}} q_c z_c^l \leqslant (W_l - \sum_{k \in C_L^{d_i - 1}} q_k - \sum_{c \in C_D^{d_i - 1}} q_{c)} z_l^L$$

$$\forall l \in L$$

03

Implementation

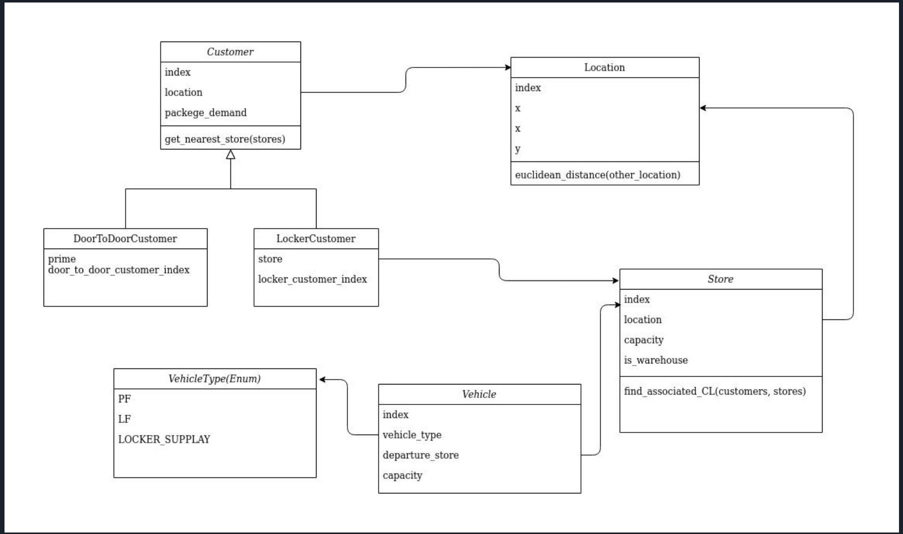
Tools

- python
- Solver
  - Xpress
  - Gurobi
- GitHub

Structure (1/2)

.py files

- Classes:
  - location
  - customer
    - door_to_door_customer
    - locker_customer
  - store
  - vehicle
  - vehicle_type (Enum)
- Executors:
  - single_period_executor
  - single_period_of_multi_period_executor
  - multi_period_executor



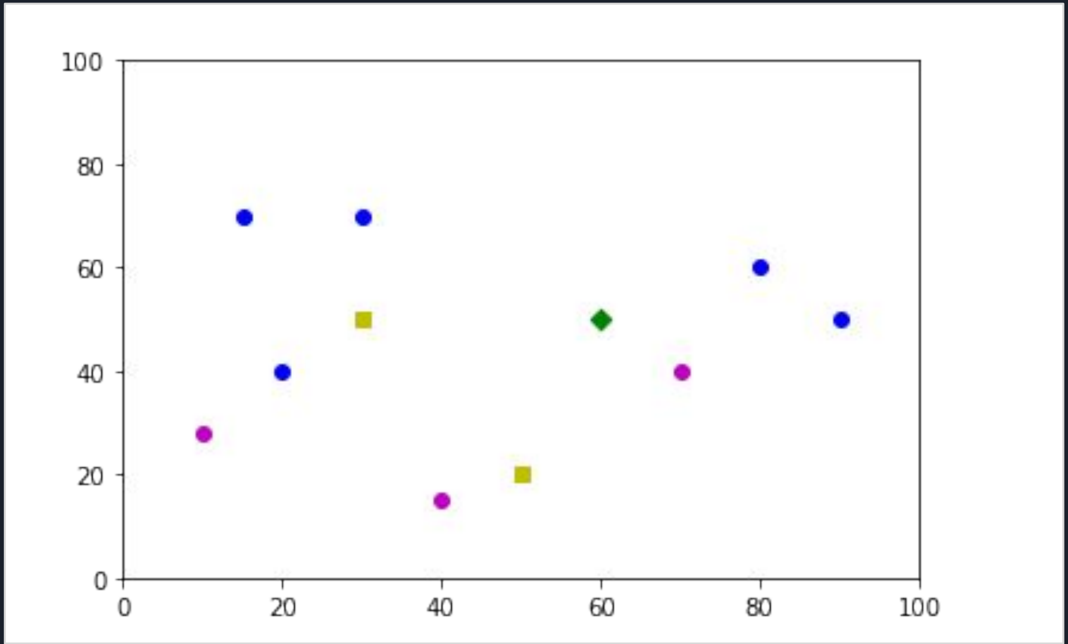
Structure (2/2)
python notebooks

- single period
  - single_period_problem_xpress
  - single_period_problem_gurobi
- multi period
  - multi_period_gurobi
- scalability analysis
  - scalability_single_period_analysis
  - scalability_multi_period_analysis

04

Results & Conclusions

Custom instance problem representation



Different solvers: same solution

Gurobi and Xpress both have the same results with the custom instance problem

```
Objective: 1.978326002278e+02
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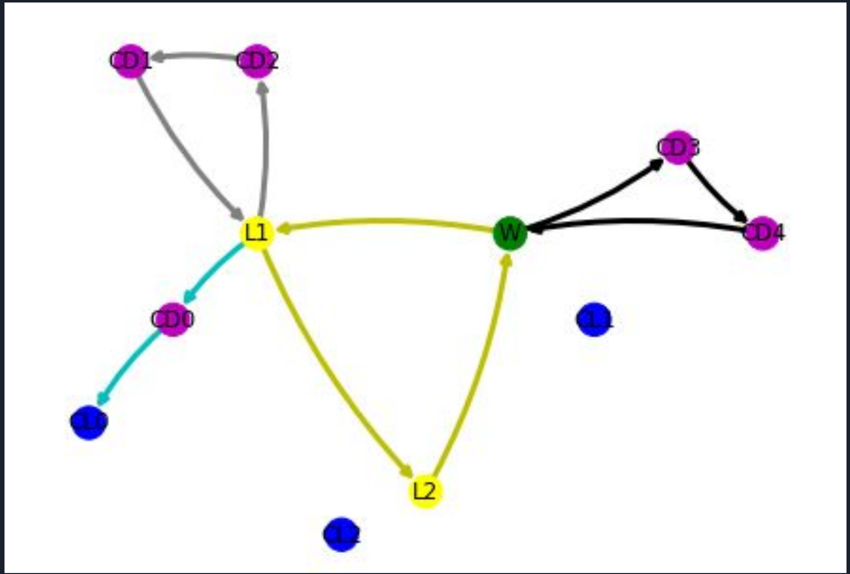
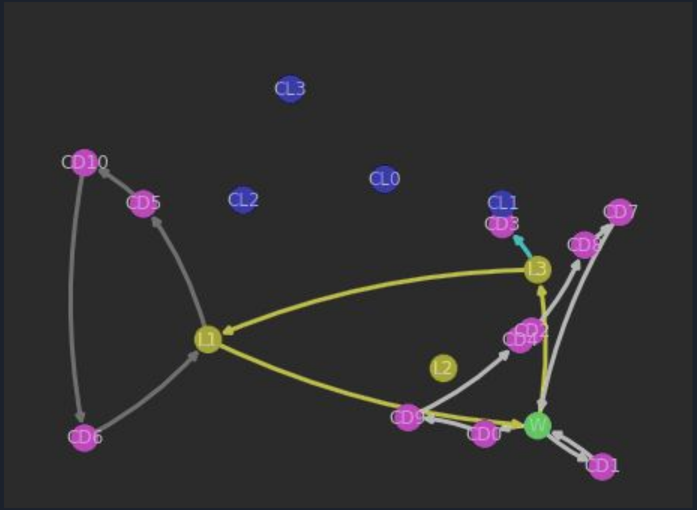
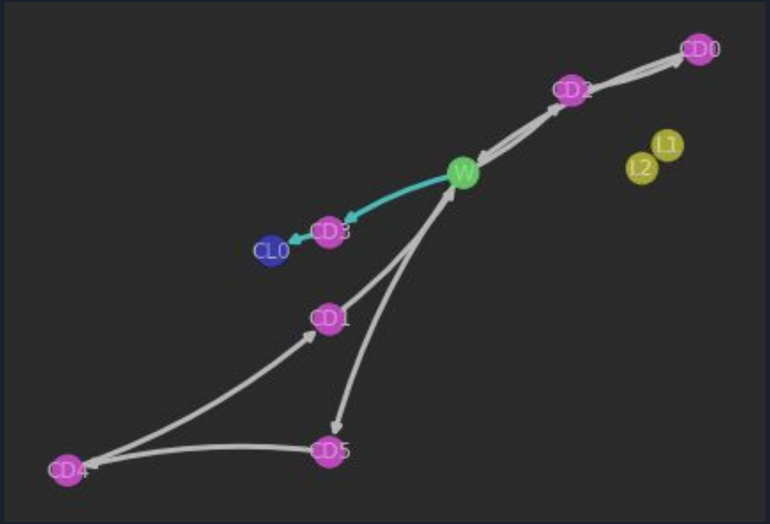
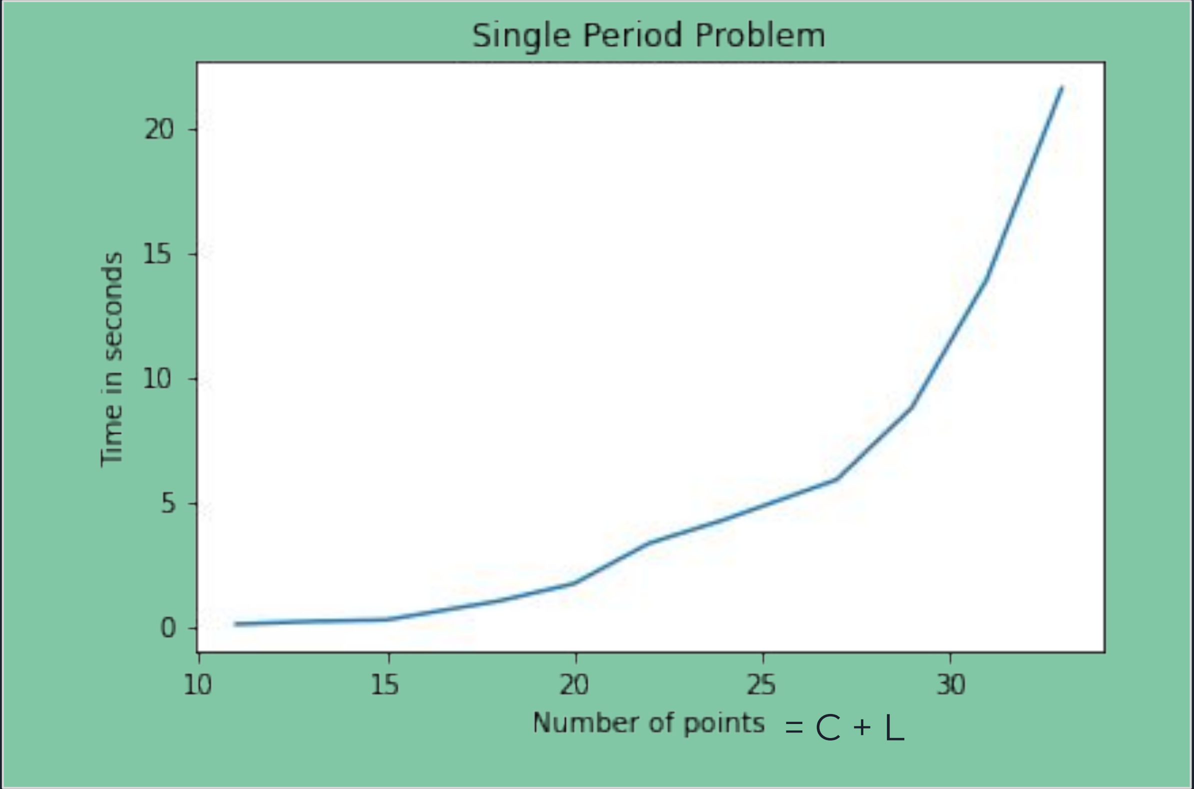


Not all lockers need to be supplied

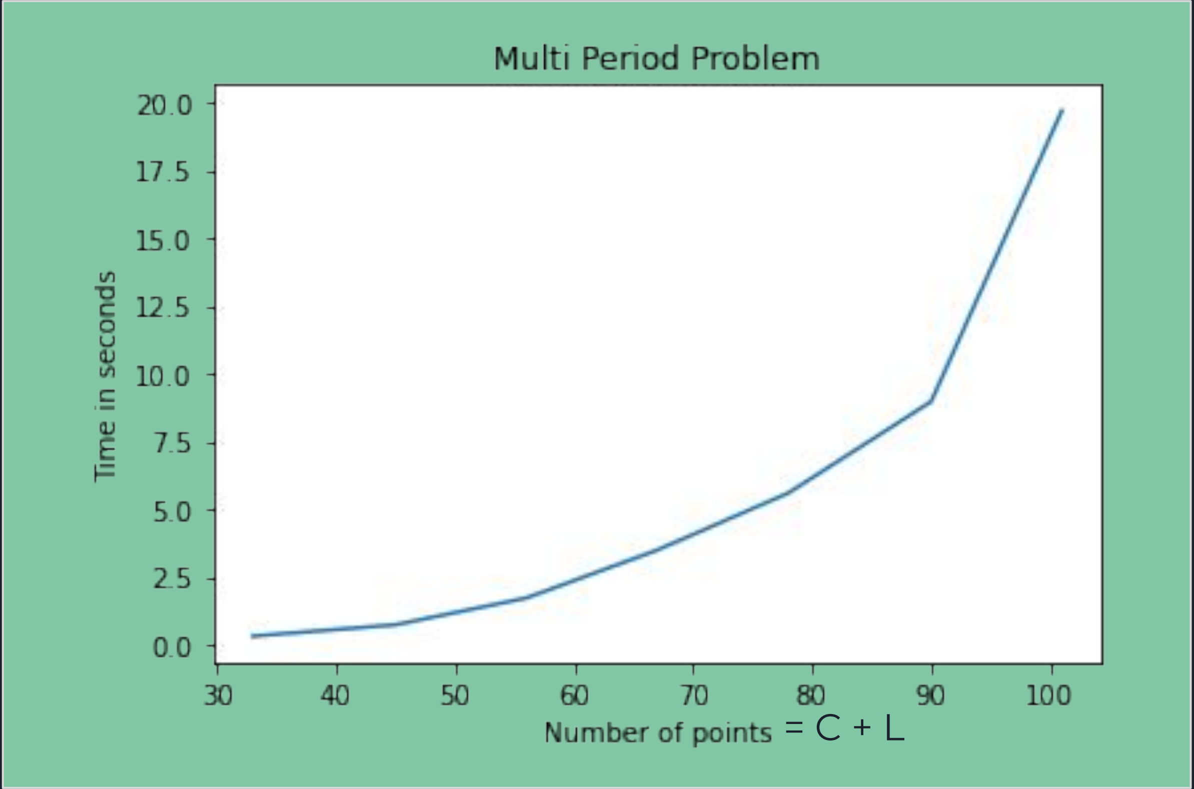




Single period model scalability



Multi-period model scalability



Conclusions

- Occasional couriers (OC) reduce transport costs
- Resolution of the model has an exponential complexity

References

[1] source code : https://github.com/AndreaGonzato/package_delivery_optimisation_problem

[2] paper: https://www.sciencedirect.com/science/article/pii/S1366554522001053

Thanks for the attention