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Granger Causality Analysis of the Interconnections between Equity Market and Cryptocurrencies

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Abstract

The advent of cryptocurrencies has significantly reshaped the financial landscape by offering decentralized alternatives to traditional currencies. Bitcoin, the pioneering cryptocurrency, started a wave of innovation, leading to the creation of numerous other digital assets. During the past decade, the market capitalization of cryptocurrencies has increased, driven by both retail and institutional investors seeking diversification and high returns. As of 2024, the total market capitalization of cryptocurrencies exceeds $2 trillion, analyzing their growing influence in financial markets.

This thesis investigates the causal relationships and dynamic interactions between equity markets and cryptocurrencies using advanced econometric models. Specifically, it examines whether there is a bidirectional causal relationship between major equity market indices and leading cryptocurrencies. The analysis employs Vector Autoregressive (VAR) models to capture linear interdependencies among multiple time series and Granger causality tests to explore predictive relationships.

The dataset includes daily closing prices for key equity indices alongside major cryptocurrencies, spanning from January 2019 to April 2024. The empirical findings from this research offer valuable insights into the interactions between equity markets and cryptocurrencies. By identifying potential bidirectional causality and the transmission of shocks between these markets, this study increases our understanding of how emerging digital assets integrate with traditional financial systems.

Our findings indicate that the bidirectional causality between cryptocurrencies and equity indices, estimated using a rolling-window of three months, is stronger during the COVID-19 pandemic when financial markets were more volatile. This period revealed intensified interactions, with market shocks in one asset class more likely to influence the other. Cryptocurrencies such as Bitcoin and Ethereum significantly influenced major equity indices throughout the five-year period. Bitcoin and Ethereum, known for its role in decentralized finance, both displayed strong correlations with traditional market indices. Conversely, the Nikkei 225, NASDAQ, and Russell 2000 emerged as the most influential indices affecting cryptocurrency prices. The NASDAQ, a hub for technology, aligns naturally with the tech-driven nature of cryptocurrencies, while the Russell 2000, representing smaller, volatile companies, mirrors the high-risk profile of many digital assets.

This thesis aims to investigate the mechanisms through which equity markets and cryptocurrencies influence each other, shedding light on the broader implications of cryptocurrency integration into the global financial system.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The financial landscape has undergone a huge transformation with the advent of cryptocurrencies, particularly Bitcoin. These digital assets, initially perceived as niche technological curiosities, have rapidly evolved into significant components of the global financial system. Bitcoin, representing the first decentralized digital currency based on blockchain technology, sparked the development of thousands of alternative cryptocurrencies. These digital assets aim to address various limitations and expand the applications of digital currencies, creating a diverse and dynamic market.

During the past decade, the market capitalization of cryptocurrencies has increased exponentially, driven by both retail and institutional investors seeking diversification and high returns. As of 2024, the total market capitalization of cryptocurrencies exceeded $2 trillion, confirming their growing importance in the financial markets. This substantial increase in value and market participation has elevated cryptocurrencies from speculative assets to integral components of the financial ecosystem, capable of impacting traditional financial markets significantly.

The rise of cryptocurrencies has been accompanied by significant macroeconomic developments. The global financial crisis of 2008 exposed vulnerabilities in traditional banking systems and weakened public trust in centralized financial institutions. This environment of uncertainty and distrust created fertile ground for the adoption of decentralized financial solutions like Bitcoin. Cryptocurrencies offered an alternative to traditional financial systems, promising increased security, transparency, and reduced reliance on centralized authorities.

In the following years, the proliferation of blockchain technology has extended beyond digital currencies to encompass a wide range of applications, including smart contracts, decentralized finance, and non-fungible tokens. These innovations have further integrated cryptocurrencies into the financial mainstream, attracting substantial venture capital investments and institutional interest. The increasing involvement of institutional investors, such as hedge funds, asset managers, and publicly traded companies, has not only legitimized cryptocurrencies but also amplified their impact on global financial markets.

The macroeconomic environment has also played a crucial role in the integration of cryptocurrencies into mainstream finance. Low-interest-rate policies adopted by central banks worldwide in response to economic slowdowns and the COVID-19 pandemic have driven investors to seek higher yields in alternative assets, including cryptocurrencies. Additionally, the proliferation of digital payment systems and the push towards a cashless economy have accelerated the adoption of digital currencies, positioning them as valuable complements to traditional financial instruments.

This thesis is motivated by the need to understand the interplay between these emerging digital assets and conventional equity markets. The dynamic nature of financial markets necessitates a accurate examination of how cryptocurrencies interact with traditional financial instruments, particularly in terms of causality and dynamic relationships. With the increasing integration of cryptocurrencies into mainstream finance, it is crucial to investigate their potential to influence equity markets, both as independent assets and through their interactions with traditional financial instruments.

The selection of this topic is driven by several key considerations. First, the growing prominence of cryptocurrencies in the financial system demands a rigorous analysis of their impact on traditional markets. The significant market capitalization and widespread adoption of cryptocurrencies indicate their potential to influence not only the financial behavior of retail investors but also the strategic decisions of institutional investors. Understanding these influences is essential for developing informed investment strategies and for ensuring market stability.

Second, the rapid technological advancements and regulatory developments surrounding cryptocurrencies create a dynamic and complex financial environment. This complexity necessitates a deeper exploration of how these digital assets interact with established financial instruments. By examining the causal relationships and dynamic interactions between cryptocurrencies and equity markets, this thesis aims to provide valuable insights into the mechanisms underlying these interactions, in order to contribute to the broader understanding of financial market behavior.

Third, the unique characteristics of cryptocurrencies, such as their decentralized nature and the underlying blockchain technology, present both opportunities and challenges for the financial system. These characteristics differentiate cryptocurrencies from traditional assets, making it imperative to investigate how they integrate with and potentially disrupt existing financial frameworks. This investigation will help identify potential risks and benefits, guiding both investors and policymakers in making informed decisions.

The central question driving this research is whether cryptocurrencies and equity markets exhibit bidirectional causality and how shocks in one market might propagate to the other. This question is crucial for several reasons. Understanding the direction and strength of causality between these markets can inform investment strategies. Investors seeking to optimize their portfolios need to know whether movements in cryptocurrency prices can predict changes in equity market indices and vice versa. Such knowledge can increase portfolio diversification and risk management strategies. Then, identifying how shocks in one market affect the other is vital for policymakers and financial regulators. If significant spillover effects exist, policymakers must consider these interdependencies when designing regulations and interventions to ensure financial stability. For instance, a sharp decline in cryptocurrency prices could potentially trigger a broader market sell-off, amplifying financial instability. Conversely, turmoil in equity markets could impact cryptocurrency valuations, affecting investors who hold both types of assets.

Given the rapid evolution of the cryptocurrency market and its growing importance in the global financial system, this thesis seeks to provide a comprehensive analysis of the causal relationships and dynamic interactions between equity markets and cryptocurrencies. By employing advanced econometric techniques, such as the Vector Autoregressive (VAR) model and Granger causality tests, this research aims to uncover the underlying mechanisms driving these interactions. Additionally, the study will use impulse response functions to quantify the impact of shocks and to understand the transmission channels between these markets.

This research is particularly timely and relevant due to the ongoing developments in the cryptocurrency space, including regulatory changes, technological advancements, and shifts in market sentiment. The findings of this thesis are expected to contribute to both academic literature and practical financial decision-making, providing a nuanced understanding of the evolving role of cryptocurrencies in global finance.

In summary, this thesis addresses a critical gap in the literature by exploring the bidirectional causality and dynamic interactions between equity markets and cryptocurrencies. The insights gained from this research will be valuable for investors seeking to optimize their portfolios and for policymakers aiming to ensure financial stability. The comprehensive analysis provided in this thesis will increase our understanding of how these two important components of the modern financial system interact, offering a foundation for future research and practical applications.

The body of literature on the relationship between equity markets and cryptocurrencies is growing, yet significant gaps remain. Early studies primarily examined the speculative nature of cryptocurrencies. Baur et al. (2018) [13] characterized Bitcoin as a speculative asset due to its high volatility. This study highlighted the risks and uncertainties of investing in cryptocurrencies, emphasizing their susceptibility to speculative behavior.

As the cryptocurrency market matured, researchers began exploring its broader financial implications. Corbet et al. (2019) [11] found that cryptocurrencies could serve as hedging instruments and diversifiers in financial portfolios, particularly during market stress. Dyhrberg (2016) [18] suggested that Bitcoin could hedge against traditional financial assets like gold and the US dollar, pointing to its potential role in portfolio diversification and risk mitigation.

Recent literature has focused on the dynamic interactions between cryptocurrencies and traditional financial markets. Balcilar et al. (2017) [4] used Granger causality tests to show the predictive power of cryptocurrency returns on equity market performance, supporting the idea of interdependencies between these asset classes. These studies advanced the understanding of how cryptocurrencies and traditional financial markets interact, suggesting that cryptocurrency price movements could forecast equity market trends.

Despite these advancements, gaps remain in fully understanding the causal relationships between equity markets and cryptocurrencies, particularly in light of recent market developments and regulatory changes. This thesis aims to bridge this gap by providing an econometric analysis of these interconnections, focusing on the period from 2019 to 2024, a timeframe marked by significant market and regulatory shifts.

By building on these studies, this thesis seeks to provide a more nuanced and up-to-date analysis of the causal relationships between these markets. The advanced econometric models employed in this research, including the VAR model and Granger causality tests, will help uncover the underlying mechanisms driving these interactions and assess their implications for investors and policymakers. Furthermore, the focus on the period from 2019 to 2024 allows for the examination of recent developments and regulatory changes, providing a contemporary perspective on the interconnections between equity markets and cryptocurrencies.

In summary, the existing literature offers valuable insights into the speculative nature, hedging capabilities, and dynamic interactions of cryptocurrencies with traditional financial markets. However, significant gaps remain, particularly regarding the causal relationships and the impact of recent market and regulatory changes. This thesis aims to address these gaps by providing a comprehensive econometric analysis of the interconnections between equity markets and cryptocurrencies, in order to contribute to the broader understanding of financial market behavior and informing investment strategies and policy decisions.

This thesis utilizes an extensive dataset covering major equity indices and cryptocurrencies from January 2019 to April 2024. The equity market data includes daily closing prices for key indices such as the S&P 500, FTSE 100, and Nikkei 225. Cryptocurrency data includes daily prices for major digital assets, including Bitcoin, Ethereum, and Tether. Data sources include reputable financial databases such as FirstRate Data and Yahoo Finance. These sources provide high-frequency data, ensuring the accuracy and reliability of the analyses. Additionally, macroeconomic indicators, such as interest rates, inflation rates, and GDP growth figures, are incorporated to contextualize the financial market dynamics. These indicators are sourced from official reports published by central banks and international financial organizations, ensuring the credibility and relevance of the data. The comprehensive nature of this dataset enables a detailed examination of the interconnections between equity markets and cryptocurrencies across different time periods and economic conditions.

Our findings indicate that the bidirectional causality between cryptocurrencies and equity indices, estimated using a rolling-window of three months, is particularly pronounced during the COVID-19 pandemic when financial markets experienced heightened volatility. During this period, the interconnectedness between these two asset classes intensified, suggesting that market participants responded to shocks in both markets more cohesively. The pandemic acted as a stress test, revealing the dynamic nature of the relationships between cryptocurrencies and equity indices under extreme market conditions.

Cryptocurrencies such as Bitcoin and Ethereum demonstrated a significant impact on major equity indices throughout the entire five-year study period. Bitcoin, often referred to as digital gold, showed strong correlations with fluctuations in traditional market indices, reflecting its growing role as both an investment asset and a speculative vehicle. Ethereum, with its broader application in decentralized finance and smart contracts, also exhibited substantial influence, likely due to its foundational role in the blockchain ecosystem and its appeal to institutional and retail investors alike.

Conversely, certain equity indices showed notable influence on cryptocurrency markets. The Nikkei 225, NASDAQ, and Russell 2000 were identified as the most influential indices affecting cryptocurrency prices. The Nikkei 225's influence may be attributed to Japan's progressive regulatory stance on cryptocurrencies, fostering an environment conducive to digital asset trading and integration. The NASDAQ, representing a hub of technology and innovation, shares a natural synergy with the tech-driven nature of cryptocurrencies, leading to mutual reinforcement of trends and investor sentiment between these markets. The

Russell 2000, reflecting the performance of smaller, more volatile companies, mirrors the high-risk, high-reward profile of many cryptocurrencies, thereby amplifying their interconnected behavior.

Moreover, our analysis highlights that the transmission of market shocks between cryptocurrencies and equity indices is not uniform across all time periods. During the pandemic, for instance, the rapid adoption of digital payment systems and increased interest in decentralized financial solutions caused cryptocurrencies to react more sensitively to changes in equity market conditions. This period saw an increased correlation, suggesting that investors viewed cryptocurrencies as viable alternatives or complements to traditional assets in times of uncertainty.

The rolling-window analysis further reveals temporal variations in these interactions. At certain intervals, particularly during significant policy announcements or major market events, the feedback loop between cryptocurrencies and equity indices either strengthened or weakened. This finding underscores the importance of considering temporal dynamics when evaluating the causality between these markets.

In summary, our findings underscore the complex and evolving nature of the relationship between cryptocurrencies and equity markets. The bidirectional causality is contingent on broader economic conditions, market sentiment, and specific characteristics of the indices and cryptocurrencies involved. These insights are crucial for investors and policymakers aiming to navigate the integrated landscape of traditional and digital finance.

The thesis is structured as follows: Chapter 2 discusses in-depth the existing body of research on the relationship between equity markets and cryptocurrencies, identifying key gaps and setting the stage for the empirical analysis. Chapter 3 details the VAR models and statistical tests employed in the analysis, including the impulse response functions. Chapter 4 presents the Grange causality test. Finally, Chapter 5 summarizes the key findings, discusses their implications for investors and policymakers, and suggests directions for future research.

In conclusion, the introduction of this thesis outlines the motivation for choosing the topic of interconnections between equity markets and cryptocurrencies, reviews the pertinent literature, and sets forth the research objectives. By leveraging advanced econometric techniques and a comprehensive dataset, this thesis aims to provide valuable insights into the dynamic interactions between these two critical components of the modern financial system. The findings are expected to contribute to both academic literature and practical financial decision-making, highlighting the evolving role of cryptocurrencies in global finance.

Chapter 2

Relationship between Equity and Cryptocurrency

2.1 Introduction to the Cryptocurrency Market

The advent of cryptocurrencies has significantly reshaped the financial landscape by offering decentralized alternatives to traditional currencies. Unlike conventional banking systems that rely on centralized authorities, such as banks and governments, cryptocurrencies operate on decentralized networks utilizing blockchain technology. This independence from central control provides numerous advantages, such as increased security, reduced transaction costs, and increased financial inclusivity.

2.1.1 Key Benefits of Decentralization

Decentralization is fundamental to the functioning of cryptocurrencies. Traditional financial systems rely on a central authority to oversee and validate transactions. In contrast, cryptocurrencies employ a distributed network of computers, or nodes, to verify and record transactions on a public ledger called the blockchain technology. This decentralized approach offers several key benefits:

- **Security:** Decentralization reduces the risk of fraud and hacking by making it difficult for any single entity to control or manipulate the network. The absence of a central point of failure makes the entire system more robust and resilient against attacks.

- **Transparency:** All transactions are recorded on the blockchain technology, which is publicly accessible and immutable. This transparency increases trust and accountability, as anyone can verify the history of transactions and ensure their validity.

- **Financial Inclusion:** Cryptocurrencies provide access to financial services for individuals without access to traditional banking systems, particularly in underdeveloped regions. This democratization of finance allows more people to participate in the global economy.

- **Efficiency and Speed:** Cryptocurrency transactions can be faster and more efficient compared to traditional banking systems, especially for cross-border transactions. This speed is achieved through the elimination of intermediaries and the streamlining of transaction processes.

- **Programmability:** Blockchain technology enables the creation of smart contracts, which are self-executing contracts with the terms directly written into code. These contracts facilitate automated and trustless transactions, reducing the need for intermediaries and lowering costs.

These characteristics have increasingly attracted potential users to cryptocurrencies. Previous literature, particularly the study by Baur et al. (2018) [13], has viewed Bitcoin more as a speculative asset than a digital currency, reflecting the early stage of the cryptocurrency market. This chapter aims to examine whether the connection of the cryptocurrency market with the financial market, specifically the stock market, has grown in recent years, as detailed in Chapter 5.

2.2 Focus on the first cryptocurrency: Bitcoin

Bitcoin, the pioneering cryptocurrency, was introduced in 2008 by an anonymous entity known as Satoshi Nakamoto through a white paper titled *"Bitcoin: A Peer-to-Peer Electronic Cash System"* [27]. This foundational document proposed a decentralized system allowing online payments to be transferred directly between parties without the need for a financial intermediary, fundamentally transforming the concept of digital currency.

2.2.1 Mechanics and impact of Bitcoin

Bitcoin officially launched on January 3, 2009, when Nakamoto mined the first block, known as the Genesis Block. This event marked the inception of the Bitcoin network and laid the groundwork for subsequent cryptocurrencies.

Bitcoin operates on blockchain technology, a type of distributed ledger. A blockchain technology consists of a series of blocks, each containing a list of transactions. These blocks are linked and secured using cryptographic hashes, ensuring the integrity and immutability of the transaction records. The process of adding transactions to the blockchain technology is known as mining, where miners use powerful computers to solve complex mathematical problems. This competitive process, known as Proof of Work (PoW), validates transactions and adds them to the blockchain technology. The first miner to solve the problem is rewarded with Bitcoin.

Bitcoin's introduction has had a profound impact on the financial world, paving the way for a broader cryptocurrency market. Key impacts include:

- **Alternative Investment:** Bitcoin has emerged as a new asset class, attracting investors seeking diversification and protection against traditional market volatility. Its limited supply and deflationary nature increase its appeal as a store of value.

- **Innovation in Finance:** Bitcoin's success has spurred the development of numerous other cryptocurrencies and blockchain-based technologies, leading to innovations such as smart contracts and decentralized finance (DeFi). These innovations have the potential to revolutionize various aspects of finance and beyond.

- **Regulatory Challenges:** The rise of cryptocurrencies poses significant challenges to regulators, who must balance the need for consumer protection with fostering innovation. This includes addressing concerns related to money laundering, tax evasion, and market manipulation while ensuring that regulations do not stifle technological advancements.

- **Market Volatility:** The cryptocurrency market is characterized by high volatility, presenting both opportunities and risks for investors. Price swings can be dramatic, influenced by market sentiment, technological developments, and regulatory news.

- **Global Accessibility:** Bitcoin and other cryptocurrencies are accessible to anyone with an internet connection, making them truly global financial instruments. This global reach enables people from different parts of the world to participate in the financial system.

The emergence of cryptocurrencies, starting with Bitcoin, introduced a revolutionary shift in the financial landscape by offering a decentralized, secure, and transparent alternative to traditional banking systems. Understanding Bitcoin's creation and functionality provides essential insights into the broader cryptocurrency market and its potential to reshape global finance.

The predictability of Bitcoin's supply, capped at 21 million coins, and its deflationary nature further increase its appeal as a store of value. However, its future role as a medium of exchange versus an investment asset will depend on its acceptance and usage trends over time.

2.3 Current State of the Cryptocurrency Market

Bitcoin, though only one among thousands of existing cryptocurrencies, dominates the market. As illustrated in Figure 2.1, from 2019 to April 2024, the market has been largely controlled by a few major players. Notably, Bitcoin (in orange) maintains the largest share of the market capitalization throughout the period. Ethereum (in yellow) also holds a significant portion but exhibits more volatility compared to Bitcoin. Together, these two cryptocurrencies control approximately 60% of the total market capitalization.

Despite the absence of comprehensive public data due to the lack of regulatory frameworks governing cryptocurrencies, it is evident that the market is dominated by a few key players. This regulatory void makes it challenging to precisely determine the holders of these digital assets. Consequently, our analysis focuses on Bitcoin, as shown in Figure 2.2, which details Bitcoin ownership distribution sourced from `BitcoinTreasuries` website.

The Bitcoin Ownership Distribution chart from February 2024 indicates that individuals hold 57% (11.97 million BTC) of the total 21 million Bitcoin supply, reflecting its broad acceptance. Approximately 17.6% (3.7 million BTC) is presumed lost, in order to reduce the effective supply and improving scarcity. Satoshi Nakamoto retains 5.2% (1.1 million BTC),

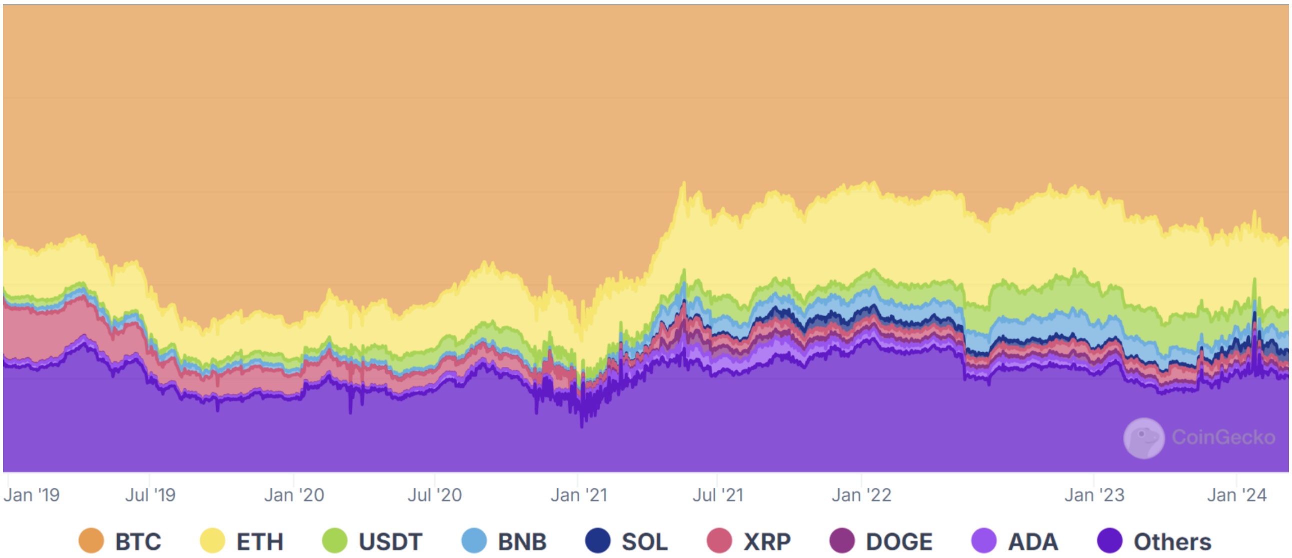


Figure 2.1: Evolution of the market cap share in the cryptocurrency market - Source: `CoinGecko`

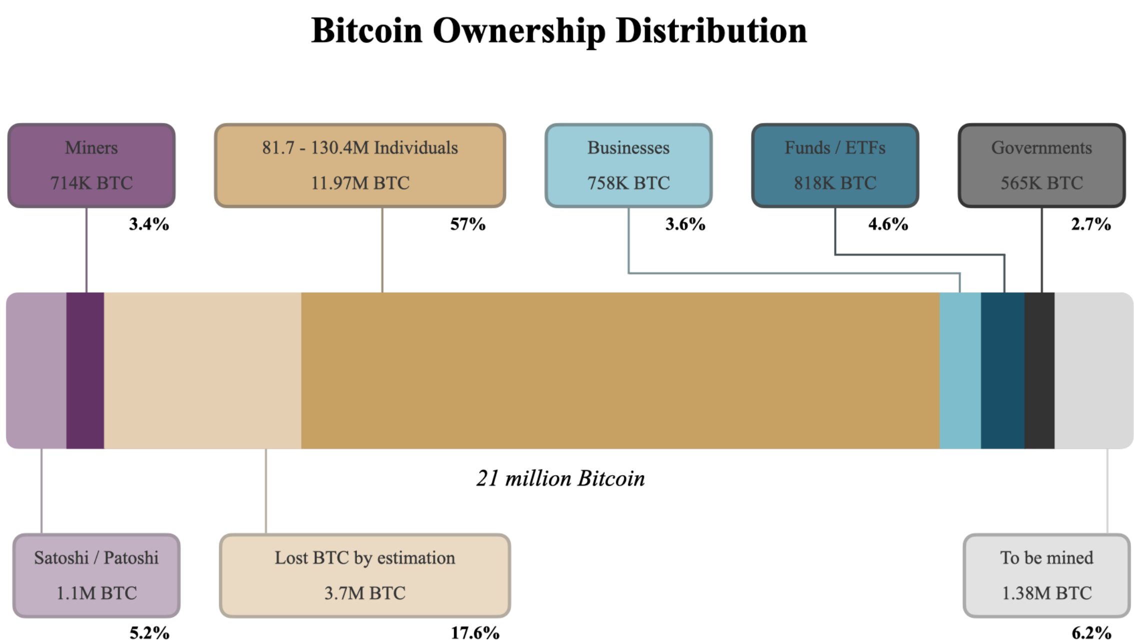


Figure 2.2: Bitcoin ownership distribution as of February 2024 - Source: `BitcoinTreasuries`

which remains untouched. Miners possess 3.4% (714,000 BTC), and businesses hold 3.6% (758,000 BTC), indicating significant corporate interest. Funds and ETFs account for 4.6% (818,000 BTC), while governments hold 2.7% (565,000 BTC). Additionally, 1.38 million BTC (6.2%) are yet to be mined.

This distribution significantly impacts market dynamics. The reduction in Bitcoin supply increases its scarcity, while substantial institutional holdings indicate strong confidence in Bitcoin as a store of value. Additionally, government involvement and regulatory measures also affect market sentiment. The study by Matta et al. [23] investigates whether information searches and social media activities can predict Bitcoin prices. By comparing Bitcoin's historical price with Google Trends data and tweet volumes, the authors explore the predictive power of these information sources. Although their dataset covers only 60 days, their study stands out by employing an automated sentiment analysis technique to measure user opinions, evaluations, sentiments, and attitudes on specific topics. They use a tool called "SentiStrength," which leverages a dictionary of sentiment words, each associated with a weight indicating sentiment strength. This tool assesses the sentiment strength in individual short messages and consolidates the results into a single value: positive, negative, or neutral sentiment. The study reveals a significant relationship between Bitcoin prices and the volumes of both tweets and Google searches. Understanding this ownership distribution is essential for analyzing market behavior and predicting trends within the evolving Bitcoin ecosystem. Therefore, we turn our attention to corporate Bitcoin holdings, as detailed in Table 2.1.

Rank	Company	Total Bitcoin	Estimated Value
1	MicroStrategy	226,331	$13.6B
2	Marathon Digital	17,857	$1.1B
3	Tesla	9,720	$583M
4	Coinbase	9,480	$571M
5	Hut 8 Mining Corp	9,109	$548M
6	Riot Platforms	9,084	$547M
7	Block	8,038	$484M
8	CleanSpark	6,154	$370M
9	Galaxy Digital	4,000	$240M
10	Bitcoin Group	3,589	$216M
11	HIVE Digital Technologies	2,451	$147M
12	Cipher Mining	2,142	$129M

Table 2.1: Top 12 corporate Bitcoin holders as of June 2024

Table 2.1 demonstrates that significant Bitcoin holdings extend beyond technology firms to a diverse range of companies, particularly those involved in cryptocurrency mining. This trend highlights the increasing institutional confidence in Bitcoin as a long-term investment and a crucial component of corporate treasury management. The data suggests that these companies perceive Bitcoin as a valuable asset for diversification, hedging against inflation, and potential capital appreciation.

The graphical representation of market capitalization and ownership distribution underscores the dominance of Bitcoin and Ethereum, the scarcity-driven value proposition of Bitcoin, and the growing institutional confidence, which collectively point to the cryptocurrency's increasing legitimacy and potential for future appreciation. Furthermore, the global participation and potential regulatory impacts highlighted in these graphs emphasize the dynamic and multifaceted nature of the cryptocurrency market, necessitating careful consideration by all stakeholders.

2.4 Regulation of cryptocurrencies: history and present

Cryptocurrencies have undergone a dynamic journey from their inception to the present, marked by rapid technological advancements, significant market growth, and an evolving regulatory landscape. The regulation of cryptocurrencies is a complex and multifaceted topic, influenced by various factors including economic stability, consumer protection, and technological innovation. This chapter explores the regulatory evolution of cryptocurrencies, highlighting key developments from the past and examining the current regulatory environment.

When Bitcoin was introduced, it operated in a regulatory vacuum. The blockchain technology and the decentralized nature of Bitcoin posed challenges to traditional regulatory frameworks. Initially, cryptocurrencies were largely ignored by governments and financial regulators, perceived as niche financial instruments used primarily by tech enthusiasts.

However, as Bitcoin's popularity grew, so did its use in illegal activities, notably in transactions on the dark-web marketplace Silk Road. This drew the attention of law enforcement and regulatory bodies. The first significant regulatory response came in 2013 when the U.S. Financial Crimes Enforcement Network (FinCEN) issued guidelines clarifying that the exchange of virtual currencies was subject to anti-money laundering (AML) laws. This marked the beginning of formal regulatory oversight of the cryptocurrency market.

Discussions about regulatory frameworks began to gain traction during the first blockchain conferences in 2016. The atmosphere was charged with both excitement and apprehension. As cryptocurrencies gained mainstream attention and more diverse digital assets emerged, regulatory bodies worldwide began to take a closer look.

In 2015, the New York State Department of Financial Services introduced the BitLicense framework, one of the first comprehensive regulatory regimes for cryptocurrencies. The BitLicense imposed strict requirements on businesses involved in the virtual currency industry, including AML, cybersecurity, and consumer protection measures.

During this period, other jurisdictions also started formulating their own approaches. The European Union, through the European Banking Authority, issued warnings about the risks associated with virtual currencies and proposed regulatory measures to mitigate these risks. In Asia, countries like Japan took a proactive stance, recognizing Bitcoin as a legal payment method and implementing a licensing regime for cryptocurrency exchanges.

The late 2010s witnessed an explosion of Initial Coin Offerings (ICOs), a novel fundraising mechanism that allowed startups to raise capital by issuing their own cryptocurrencies. The ICO boom of 2017 and 2018 brought significant innovation, but also rampant speculation and numerous scams. Many ICO projects failed to deliver on their promises, leading to substantial financial losses for investors.

The U.S. Securities and Exchange Commission (SEC) declared that many ICOs were, in fact, unregistered securities offerings and initiated enforcement actions against non-compliant projects. The SEC's stance was echoed by regulatory bodies worldwide, including the Financial Conduct Authority (FCA) in the UK and the Australian Securities and Investments Commission (ASIC). China and South Korea went further, implementing outright bans on ICOs and tightening controls on cryptocurrency exchanges.

Today, the regulatory landscape for cryptocurrencies is more developed, though it remains fragmented and varies significantly between jurisdictions. Major economies have adopted diverse approaches to balance innovation with risk management.

The EU is moving towards a unified regulatory framework with the proposed Markets in Crypto-Assets (MiCA) regulation. MiCA aims to provide legal certainty, promote innovation, and protect consumers across the EU. It includes provisions for AML, licensing of cryptocurrency service providers, and requirements for stablecoin issuers. Japan continues to lead with a progressive regulatory environment, promoting innovation while ensuring investor protection. In contrast, China has taken a hardline stance, banning all cryptocurrency transactions and mining activities. South Korea has implemented strict AML regulations and licensing requirements for exchanges.

There is increasing recognition of the need for international cooperation in the regulation of cryptocurrency. Organizations like the Financial Action Task Force (FATF) have issued guidelines to help countries develop cohesive AML and counter-terrorism financing (CTF) measures. The G20 has also highlighted the importance of monitoring and mitigating risks associated with digital assets. Despite significant progress, several challenges remain in the regulation of cryptocurrencies. These include the rapid pace of technological innovation, the global nature of digital assets, and the need to balance innovation with risk management. Privacy concerns, regulatory arbitrage, and the rise of decentralized finance further complicate the regulatory landscape.

In conclusion, the regulation of cryptocurrencies has evolved significantly from its early days of regulatory neglect to a complex and multifaceted framework. Although substantial progress has been made in developing regulatory approaches that promote innovation while managing risks, ongoing challenges and the rapid evolution of the cryptocurrency space require continuous adaptation and international cooperation. The journey of cryptocurrency regulation is far from over, and its trajectory will shape the future of digital finance.

2.4.1 Risk of regulatory gaps

The Financial Stability Board (FSB) in its report, [7], *Assessment of Risks to Financial Stability from Crypto-assets*, provides a detailed examination of the potential threats these digital assets pose to global financial stability. The FSB identifies several critical vulnerabilities, such as leverage, liquidity/maturity mismatch, and operational risks, which are reminiscent of traditional financial system fragilities but manifest uniquely within the cryptocurrency ecosystem. These vulnerabilities can potentially impact financial stability through various transmission channels, including financial sector exposures, wealth effects, confidence effects, and the increasing use of crypto-assets in payments and settlements.

2.5 Empirical evidence of market interconnections

The relationship between equity and cryptocurrency markets is influenced by several factors, including market sentiment, liquidity flows, macroeconomic indicators, portfolio diversification strategies, technological advancements, and regulatory frameworks.

Both equity and cryptocurrency markets are significantly impacted by market sentiment and investor behavior. Investor risk appetite plays a crucial role; when investors exhibit a higher tolerance for risk, they tend to allocate funds to higher-risk assets such as cryptocurrencies and speculative stocks. Conversely, during periods of heightened risk aversion, both markets often experience outflows as investors shift towards safer assets. Cryptocurrencies are frequently regarded as speculative investments, akin to high-growth or technology stocks, resulting in parallel behaviors where speculative fervor in one market can influence the other [5].

Liquidity flows establish a vital connection between these markets. Capital allocation decisions are predicated on expected returns and perceived risks. During periods of high liquidity and low interest rates, there is a tendency for increased capital to flow into both equity and cryptocurrency markets as investors seek higher returns. The growing presence of institutional investors further integrates these markets. As institutional investors enter the cryptocurrency market, the allocation of funds between these markets becomes more fluid, influenced by broader macroeconomic trends and opportunities. Institutions bring substantial capital, professional trading strategies, and risk management practices, improving market efficiency and stability [8].

Macroeconomic factors serve as another critical linkage. Central bank policies, particularly those related to interest rates and quantitative easing, significantly impact both markets. Lower interest rates typically lead to higher valuations in equities and cryptocurrencies as investors seek greater returns. Additionally, economic indicators such as growth, inflation, and employment data can affect investor confidence and behavior in both markets. Positive economic indicators tend to boost equities and cryptocurrency assets by improving future growth prospects and reducing perceived risks [14].

From the perspective of portfolio theory and diversification, cryptocurrencies represent a novel asset class. Historically, they have demonstrated a low correlation with traditional equities, in order to offer potential diversification benefits. However, it is important to note that this correlation can vary over time. Investors might utilize cryptocurrencies as a hedge against traditional market risks or inflation, which in turn influences capital allocation between equity and cryptocurrency markets [11].

In addition, the interdependencies among Bitcoin and Altcoin (expression that refers to any cryptocurrency that is not Bitcoin) markets have been examined for the period between 2013 and 2016. Ciaian et al. [10] found that these markets are interdependent, highlighting the interconnected nature of different types of cryptocurrencies. Another significant contribution to the literature is provided by Aliu et al. [15], who demonstrated the diversification benefits of incorporating cryptocurrencies into an equity portfolio. This is partly attributed to the high integration of equity indexes, which makes them more susceptible to global market trends. In contrast, cryptocurrencies often operate independently of traditional financial systems and are less influenced by governmental regulations, in order to provide a potential hedge against market fluctuations.

Technological and structural factors also forge connections between these markets. The underlying technology of cryptocurrencies, blockchain technology, has the potential to disrupt various industries, in order to affect equity valuations, particularly in technology sectors. Companies involved in blockchain technology or crypto-related services may see their stock prices influenced by developments in the cryptocurrency market. Moreover, the development of market infrastructure, such as exchanges and custody solutions, impacts both markets. Improved infrastructure facilitates greater liquidity and participation, leading to more integrated market dynamics. The advent of decentralized finance platforms is creating new financial ecosystems that intersect with traditional financial services, further blurring the lines between these markets [24].

The regulatory environment constitutes another significant link. Changes in regulation can affect both markets simultaneously. Regulatory clarity or restrictions on cryptocurrencies can influence investor sentiment and behavior in financial markets. As regulatory requirements for cryptocurrency investments increase, there is a growing overlap with traditional financial markets. This includes considerations for compliance, taxation, and reporting, affecting how investors allocate resources between equity and cryptocurrency markets. International regulatory cooperation is becoming increasingly important to address the cross-border nature of cryptocurrency markets and ensure cohesive regulatory standards [25].

From a macroeconomic perspective, both markets react to policy changes and economic indicators, influencing investor strategies across the board. The rise in institutional investment in cryptocurrencies has introduced more sophisticated trading strategies and a closer alignment with equity market movements. As the regulatory landscape evolves, it will further shape the interplay between these markets, making a comprehensive understanding of their connections increasingly crucial for investors and policymakers alike [16].

The theoretical connections between equity and cryptocurrency markets are complex and multifaceted, encompassing investor behavior, macroeconomic influences, diversification strategies, technological developments, and regulatory frameworks. Understanding these connections provides insight into the dynamics and potential co-movements between these two markets.

Empirical research provides substantial evidence of the growing interconnections between equity and cryptocurrency markets.

Lee et al. [22] investigated the causal relationships among cryptocurrencies, green bonds, and sustainable equity using Granger causality in quantile analysis. Their findings revealed bidirectional and quantile-specific causal relationships, highlighting the heterogeneous effects of these financial instruments across different market conditions. The study noted that these relationships were particularly strong at the tails of the distribution, suggesting a pronounced impact during periods of market stress or exuberance.

A study conducted by the International Monetary Fund (IMF) [31] found that the spillover effects of Bitcoin returns and volatility to equity markets have become more pronounced, indicating that developments in the cryptocurrency market can significantly impact traditional financial markets (IMF, 2021). Similarly, article of Wu S. [32] and the research of Zha et al. [29] reveal that investor sentiment in the cryptocurrency market often correlates with sentiment in the equity market (and also crude oil in the research of Zha et al.), further emphasizing the interconnected nature of these markets.

According to Cointelegraph, the return correlations between Bitcoin and stocks in certain Asian markets have increased tenfold since the onset of the pandemic. This growing integration poses risks to financial stability, as contagion could spread through investors holding both cryptocurrency and traditional assets. This highlights the need for clear regulatory guidelines to manage these risks.

[20] investigated the return and volatility spillovers among cryptocurrencies and their relationship with the US equity market. Using multivariate models, the study found that Bitcoin, Ethereum, Ripple, and Litecoin exhibited significant volatility spillovers with the equity market. The impact of these spillovers intensified during the COVID-19 pandemic, suggesting a stronger interdependence during periods of high market uncertainty.

Additionally, Billio et al. [6] demonstrated that during financial crises, financial institutions become more interconnected. For example, before and during the crisis of 1998 and the Financial Crisis of 2007-2009, the number of connections among financial institutions increased dramatically. This heightened interconnectedness facilitates the rapid spread of financial distress, contributing to systemic risk. Another study on the US equity market by Huang [30] validates the existence of bidirectional liquidity risk transfer between virtual and real markets from January 2019 to December 2022, proving that "*the US market is identified as a transmitter rather than a receiver of liquidity risk but may not escape cumulative liquidity shocks*".

The article of Mohd et al. [19] has examined Bitcoin's role as a hedging instrument against Asian stock indexes and the study found that Bitcoin has a long-run relationship with Asian stock indexes. Additionally, the volatility analysis indicated that Bitcoin is more suitable for long-term investments, with low short-term correlations with most of the equity indices.

Although in Chapter 5 we do not test the interconnections between cryptocurrencies, Civelli [1] shows that *"cryptocurrency returns are autocorrelated and significantly persistent for three weeks, in violation of market efficiency. The response of the stock returns to the cryptocurrency shock also remains positive for up to four weeks."*

These empirical studies collectively highlight the deepening ties between the equity and cryptocurrency markets. The growing influence of institutional investors, the role of cryptocurrencies in portfolio diversification, the sensitivity to macroeconomic news, and the impact of regulatory developments are all critical factors shaping the evolving dynamics of these interconnected financial markets.

Chapter 3

Vector Autoregressive (VAR) Model

The mathematical foundation of Granger causality lies in VAR models, which provide a statistical framework for evaluating predictive causality between time series.
The VAR model is a pivotal tool in econometrics and financial analysis, used to forecast interrelated variable systems and examine the dynamic effects of random disturbances on these systems. It is particularly adept at analyzing time series data in economic and financial contexts, where variables are often interdependent. The VAR model captures the linear interdependencies among multiple time series effectively.

3.1 Definition

Following the notation of Tsay's book [26], consider k endogenous variables represented as a vector $\mathbf{Y}_t$, where t denotes time. The VAR model of order p (VAR(p)) is expressed as:

$$\mathbf{y}_t = \phi_0 + \phi_1 \mathbf{y}_{t-1} + \phi_2 \mathbf{y}_{t-2} + \ldots + \phi_p \mathbf{y}_{t-p} + \varepsilon_t = \phi_0 + \sum_{i=1}^{p} \phi_i \mathbf{y}_{t-i} + \varepsilon_t \tag{3.1}$$

where:

- $\mathbf{y}_t$ is a $k \times 1$ vector of endogenous variables at time t.
- ϕ_0 is a $k \times 1$ vector of constants (intercepts).
- $\phi_1, \phi_2, \ldots, \phi_p$ are $k \times k$ coefficient matrices.
- ε_t is a $k \times 1$ vector of error terms at time t, assumed to be white noise with a mean of zero and a constant covariance matrix Σ.

To analyze the stability and dynamics of the VAR model, it is beneficial to represent it in the companion matrix form. This transformation allows the VAR(p) model to be expressed as a first-order VAR(1) model, making it easier to apply standard state-space methods and derive properties.
Consider the VAR(p) model described in Equation 3.1. We can rewrite this model using the companion matrix form, which involves stacking the lags of the time series into a single vector and transforming the system into a higher-dimensional VAR(1) representation. Define the $kp \times 1$ vector $\mathbf{Y}_t$ as:

$$\mathbf{Y}_t = \begin{pmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \vdots \\ \mathbf{y}_{t-p+1} \end{pmatrix} \tag{3.2}$$

The companion matrix Φ and the expanded vector of error terms ε_t are given by:

$$\Phi = \begin{pmatrix} \phi_1 & \phi_2 & \cdots & \phi_{p-1} & \phi_p \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{3.3}$$

Here, I_k denotes the $k \times k$ identity matrix, and the zeros are appropriately sized matrices of zeros.

Using this notation, the VAR(p) model can be reformulated as:

$$\mathbf{Y}_t = \Phi \mathbf{Y}_{t-1} + \varepsilon_t \tag{3.4}$$

This companion matrix form simplifies the analysis of the VAR model by converting it into a first-order system. The stability of the VAR model can be assessed by examining the eigenvalues of the companion matrix Φ. If all eigenvalues lie inside the unit circle, the system is stable.

In the bivariate case of a VAR(1) model, using matrix notation, the model is specified as follows. Let $\mathbf{y}_t$ be a 2×1 vector of endogenous variables, such that:

$$\mathbf{y}_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} \tag{3.5}$$

The VAR(1) model can be expressed as:

$$\mathbf{y}_t = \phi_0 + \phi_1 \mathbf{y}_{t-1} + \varepsilon_t \tag{3.6}$$

where:

- ϕ_0 is a 2×1 vector of intercepts:

$$\phi_0 = \begin{pmatrix} \phi_{0,1} \\ \phi_{0,2} \end{pmatrix}$$

- ϕ_1 is a 2×2 coefficient matrix:

$$\phi_1 = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$$

- ε_t is a 2×1 vector of error terms:

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}$$

where $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$ and Σ is the covariance matrix of the error terms.

Substituting these components into Equation 3.6, we get:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \phi_{0,1} \\ \phi_{0,2} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \tag{3.7}$$

This equation captures the dynamics of the two interrelated time series $y_{1,t}$ and $y_{2,t}$ in the bivariate VAR(1) model, allowing for the analysis of how past values of these series influence their current values.

3.2 Stationarity

The condition for stationarity in a VAR(1) model, as defined by 3.6, is centered around the characteristic roots (or eigenvalues) of its companion matrix. Assuming the mean is equal to 0:

$$E(\mathbf{y}_t) = (\mathbb{1} - \phi_1)^{-1}\phi_0 = 0 \tag{3.8}$$

provided that the matrix $\mathbb{1} - \phi_1$ is non-singular, where $\mathbb{1}$ is the $k \times k$ identity matrix.
Thus, 3.1 can be rewritten using substitutions as:

$$\mathbf{y}_t = \varepsilon_t + \phi_1\varepsilon_{t-1} + \phi_1^2\varepsilon_{t-2} + \phi_1^3\varepsilon_{t-3} + \dots \tag{3.9}$$

Equation 3.9 indicates that:

- $\mathrm{COV}(\varepsilon_t, \mathbf{y}_{t-1}) = 0$, indicating that ε_t is the *shock* or *innovation*.
- $\mathrm{COV}(\varepsilon_t, \mathbf{y}_t) = \Sigma$

The VAR(1) model can be succinctly expressed as:

$$\mathbf{y}_t = \Sigma\, \mathbf{y}_{t-1} + \varepsilon_t \tag{3.10}$$

where Σ is a block matrix composed of $\phi_1, \phi_2, \dots, \phi_p$ and identity matrices. The stationarity condition necessitates that all eigenvalues of Σ, denoted by λ, must have absolute values less than 1, as follows:

$$|\lambda_i| < 1, \text{ for all } i \tag{3.11}$$

This condition ensures that shocks to the system will have diminishing effects over time, allowing the process to revert to its mean.

3.2.1 Augmented Dickey-Fuller (ADF) Test

The Augmented Dickey-Fuller (ADF) [28] test is a statistical test used to determine whether a time series is stationary or contains a unit root, implying non-stationarity. The presence of a unit root indicates that shocks to the series have a permanent effect. Understanding stationarity is crucial in time-series analysis because many econometric models assume that the underlying time series are stationary.
A time series has a unit root if it can be represented as $\mathbf{y}_t = \mathbf{y}_{t-1} + \varepsilon_t$, where ε_t is a white noise error term. This implies that shocks to $\mathbf{y}_t$ have a lasting effect, making the series non-stationary.
Before explaining the ADF, the Dickey-Fuller test is a simplified version that tests for the presence of a unit root in a time series. From the famous article [12], the basic form of the test can be written as follows:

$$\Delta\mathbf{y}_t = \alpha + \beta t + \gamma\mathbf{y}_{t-1} + \varepsilon_t \tag{3.12}$$

where: $\Delta\mathbf{y}_t$ is the difference of $\mathbf{y}_t$ ($\Delta\mathbf{y}_t = \mathbf{y}_t - \mathbf{y}_{t-1}$), α is a constant (drift term), βt is a time trend, γ is the coefficient to be tested, and ε_t is a white noise error term.
The null hypothesis (H_0) is that $\gamma = 0$ indicating a unit root is present (non-stationary). The alternative hypothesis (H_1) is that $\gamma < 0$, indicating that the series is stationary.
The ADF test augments the Dickey-Fuller test to account for higher-order autoregressive processes. This is achieved by including lagged differences of the dependent variable to capture the autocorrelation structure of the time series. The ADF regression can be written as:

$$\Delta\mathbf{y}_t = \alpha + \beta t + \gamma\mathbf{y}_{t-1} + \sum_{i=1}^{p} \delta_i \Delta\mathbf{y}_{t-1} + \varepsilon_t \tag{3.13}$$

Where: $\Delta\mathbf{y}_{t-i}$ are the lagged differences of the dependent variable, and p is the number of lagged difference terms, chosen to ensure that ε_t is white noise.
Inclusion of the lagged difference terms $(\Delta\mathbf{y}_{t-i})$ helps to correct for serial correlation in the residuals.
In Chapter 5, we always test the stationarity with the ADF test.

3.2.2 Stability

Stability in VAR models is closely related to stationarity. A VAR model is stable if it is stationary, which means that it will revert to a stable equilibrium after a shock. The stability condition is often assessed through the roots of the characteristic polynomial derived from the VAR model's equations.
The VAR(1) model, given by 3.6, is stable if all the roots of this polynomial with respect to λ lie outside the unit circle in the complex plane.

$$\det(\mathbb{1} - \lambda\phi_1) = 0 \tag{3.14}$$

where $\mathbb{1}$ is the identity matrix. The VAR(1) model is stable if all the roots of this polynomial with respect to λ lie outside the unit circle in the complex plane. We can extend for the VAR(p) model, the stability condition involves checking the roots of the characteristic polynomial derived from the companion matrix Σ.

Implications of stationarity and stability

- **Forecasting:** stationary and stable VAR models produce forecasts that are not explosive, meaning forecasts do not diverge as the forecast horizon extends.

- **Impulse Responses:** the impulse response functions in a stationary and stable VAR model will converge to zero over time, indicating that the effect of a shock eventually dissipates.

- **Economic analysis:** stationary and stable VAR models allow for meaningful economic analysis since they reflect plausible economic dynamics where variables return to equilibrium after disturbances.

In practice, ensuring stationarity and stability often involves pre-testing the time series data for unit roots (e.g., using the Augmented Dickey-Fuller test) and differencing non-stationary series before fitting the VAR model. Additionally, the model specification, including the order p, should be chosen carefully based on criteria that balance model fit and complexity, such as the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC).

3.2.3 Estimation and inference

Estimation and inference in VAR models involve determining the coefficients that best fit the model to the observed time series data, and conducting statistical tests to infer the properties and relationships among the variables within the system. This process provides insights into the dynamics of the variables and allows for predictions and policy analysis. The steps and considerations in estimation and inference for VAR models are detailed below.

Estimation

The primary method for estimating the coefficients of a VAR model is ordinarily least squares (OLS). The estimation process involves the following steps:

1. **Model Specification:** determine the order p of the VAR model, indicating the number of lagged terms to include. This can be done using information criteria like AIC or the BIC, which balance model fit and complexity.

2. **OLS Estimation:** for a VAR(p) model with k variables, there are k equations. Each equation corresponds to one endogenous variable as the dependent variable and lags of all k endogenous variables (including its own lags) up to order p as independent variables. Estimate the coefficients of these equations separately using OLS by minimizing the sum of squared residuals.

3. **Vectorization:** although each equation is estimated separately, it is useful to think of the system in vectorized form, particularly for multivariate hypothesis testing and for calculating impulse responses and variance decompositions.

Inference

After estimating the model, statistical inference is conducted to test hypotheses about the relationships among the variables, the significance of lags, and the stability of the VAR model:

1. **Coefficient significance:** test the significance of individual coefficients using t-tests to understand the impact of lagged values of one variable on another. This sheds light on the dynamic interactions within the system.

2. **Model stability:** Check the stability condition of the VAR model by examining the roots of its characteristic equation. A stable VAR model requires all characteristic roots to lie outside the unit circle in the complex plane. Stability is crucial for meaningful impulse response analysis and forecasting.

3. **Granger causality tests:** use Granger causality tests to examine whether past values of one variable help in predicting another variable. A variable $\mathbf{X}$ is said to Granger-cause variable $\mathbf{Y}$ if $\mathbf{X}$ significantly improves the forecast of $\mathbf{Y}$ beyond what is possible by using past values of $\mathbf{Y}$ alone.

4. **Impulse Response Analysis (IRF):** Analyze the impulse responses to understand how a shock to one variable affects itself and other variables in the system over time. This provides insights into the transmission mechanisms within the VAR system and will be explained further in section 3.5.

5. **Variance decomposition:** perform variance decomposition to determine how much of the forecast error variance of each variable can be attributed to shocks to the other variables in the system. This helps in understanding the relative importance of each variable in explaining the movements in other variables.

6. **Integration and cointegration tests:** if the variables are non-stationary, test for integration and cointegration. Non-stationary variables require differencing or the estimation of Vector Error Correction Models (VECM) to capture long-run relationships.

In summary, estimation and inference in VAR models involve fitting the model to time-series data using OLS, followed by statistical tests and analyses to understand the dynamic relationships among the variables. These steps allow economists and financial analysts to forecast, analyze policy impacts, and gain insights into the structural interactions within the system.

3.3 Maximum lag selection

The selection of the appropriate number of past observations (lags) for each variable is paramount for the accuracy and reliability of any VAR model. The process of determining the optimal lag length involves balancing the inclusion of sufficient historical information to increase predictive accuracy against the risk of introducing noise and overfitting.

Incorporating too few lags may exclude significant historical data that could influence predictions, potentially leading to model misspecification and biased estimates. Conversely, including an excessive number of lags can introduce unnecessary complexity, increase estimation variance, and reduce the model's predictive power due to overfitting. Hence, the determination of an optimal lag length is crucial for the robustness of VAR model results.

Several statistical criteria are employed to identify the optimal lag length, with the AIC and the BIC being the most commonly used. Both AIC and BIC are likelihood-based criteria that penalize models for increased complexity, albeit to different extents.

Maintaining the same notation from before, the AIC for a VAR(i) model, under the normality assumption, is defined as:

$$\text{AIC}(i) = \ln(|\widetilde{\Sigma}_i|) + \frac{2k^2 i}{T} \tag{3.15}$$

where:

$$\widetilde{\Sigma}_i = \frac{1}{T} \sum_{t=i+1}^{T} \widehat{a}_t^{(i)} \left[\widehat{a}_t^{(i)}\right]^T \tag{3.16}$$

is the Maximum-Likelihood estimate of the residual covariance matrix for the VAR(i) model. Here, $|\widetilde{\Sigma}_i|$ represents the determinant of this covariance matrix, k denotes the number of endogenous variables in the system, and T is the total number of observations. The second term in the AIC formula, $\frac{2k^2 i}{T}$, serves as the *penalty function*, penalizing models based on the number of parameters ($k^2 i$) relative to the sample size.

Similarly, the BIC criterion for the same model is given by:

$$\text{BIC}(i) = \ln(|\widetilde{\Sigma}_i|) + \frac{2k^2 i \ln(T)}{T} \tag{3.17}$$

The BIC criterion includes a logarithmic adjustment to the penalty term, $\frac{2k^2 i \ln(T)}{T}$, which makes it more stringent than AIC, particularly in larger samples. Consequently, BIC often selects a model with fewer lags compared to AIC, as it imposes a heavier penalty for model complexity.

In practical applications, the choice between AIC and BIC depends on the specific context and objectives of the analysis. While AIC is more flexible and may be preferred for smaller sample sizes or when the primary concern is minimizing prediction error, BIC's more conservative approach is advantageous for identifying more parsimonious models that avoid overfitting, especially in larger datasets.

Ultimately, selecting the optimal lag length involves computing the AIC and BIC for various candidate models and choosing the model with the lowest criterion value. This approach ensures that the selected VAR model achieves an optimal balance between fit and parsimony, improving the reliability of subsequent inferences and predictions.

3.4 Cross-correlation matrix

Given the data $\{y_t | t = 1, \ldots, T\}$, where y_t refers to the returns, the cross-covariance matrix $\Gamma_\ell \equiv [\Gamma_{ij}(\ell)] = E[(y_t - \overline{y})(y_{t-\ell} - \overline{y})']$ can be estimated by:

$$\widehat{\Gamma}_\ell = \frac{1}{T} \sum_{t=\ell+1}^{T} (y_t - \overline{y})(y_{t-\ell} - \overline{y})^T, \quad \ell \geq 0, \tag{3.18}$$

where $\overline{y} = \frac{1}{T} \sum_{t=1}^{T} y_t$ is the vector of sample means. The (i, j)th element of $\widehat{\Gamma}_\ell$ represents the covariance between y_{it} and $y_{j,t-\ell}$. The cross-correlation matrix ρ_ℓ is estimated by:

$$\widehat{\rho}_\ell = \widehat{D}^{-1} \widehat{\Gamma}_\ell \widehat{D}^{-1}, \quad \ell \geq 0, \tag{3.19}$$

where σ_i^2 is the sample variance of the i-th component of y_t. The normalization process ensures that the elements of $\widehat{\rho}_\ell$ lie between -1 and 1, making it easier to interpret the strength and direction of the linear relationships between the time series at different lags.
Here, $\widehat{D}$ is a diagonal matrix where each diagonal element $\widehat{D}_{ii}$ is the square root of the sample variance of the i-th component of y_t. Specifically,

$$\widehat{D}_{ii} = \sqrt{\sigma_i^2}, \tag{3.20}$$

with σ_i^2 being the sample variance of the i-th component of y_t.
In addition, using return instead of prices is crucial for several reasons. Returns are generally stationary, meaning their statistical properties remain constant over time. This stationarity simplifies the analysis and modeling of time series data Returns facilitate the comparison of performance across different assets and they normalize the data by removing the scale effect of different asset prices, allowing for the aggregation and comparison of multiple time series on a common scale.
The cross-correlation matrix provides valuable insights into the dynamic interrelationships among multiple time series by normalizing the covariance, in order to enable a more straightforward interpretation of their linear dependencies over different time lags. This information is crucial for modeling and forecasting in multivariate time series analysis, as it helps identify the presence and extent of temporal dependencies between the variables.

3.4.1 Durbin-Watson test

The Durbin-Watson (DW) [21] test is a statistical test used to detect the presence of autocorrelation at lag 1 in the residuals of a regression analysis. The test statistic ranges from 0 to 4. A value near 2 indicates no autocorrelation; values approaching 0 suggest positive autocorrelation, and values towards 4 indicate negative autocorrelation. The Durbin-Watson statistic is calculated as:

$$\mathrm{DW} = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} \tag{3.21}$$

where e_t represents the residual at time t.

Under the null hypothesis H_0 of no autocorrelation, the Durbin-Watson statistic approximately follows a distribution that can be compared to tabulated critical values. These critical values depend on the sample size n and the number of regressors in the model. However, for large samples, the Durbin-Watson statistic can be approximated by a distribution involving the inverse of the Chi-Squared distribution.

In the context of VAR models, where you model multiple time series simultaneously, ensuring no autocorrelation in residuals is crucial for the validity of the model. The presence of autocorrelation can indicate misspecification in the model, such as omitted variables or incorrect lag length.

3.4.2 Ljung-Box test

The Ljung-Box (LB) [17] test is a type of statistical test of whether any of a group of autocorrelations of a time series are different from zero. Unlike the DW test, which only tests for autocorrelation at lag 1, the LB test can be used to test for autocorrelation at multiple lags simultaneously. The Ljung-Box statistic is calculated as:

$$\mathrm{Q} = n(n+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{n-k} \tag{3.22}$$

where n is the sample size, m is the number of lags being tested, and $\hat{\rho}_k$ is the sample autocorrelation at lag k.

Under the null hypothesis H_0 of no autocorrelation up to lag m, the Ljung-Box statistic follows an asymptotic Chi-Squared distribution with m degrees of freedom:

$$Q \sim \chi_m^2 \tag{3.23}$$

The DW test is effective for detecting first-order autocorrelation, while the Ljung-Box test provides a more accurate check for autocorrelation across multiple lags, which is particularly important for ensuring the validity of VAR models.

For VAR models, the LB test is useful for testing whether the residuals from each equation in the VAR model are white noise, which is a key assumption for the validity of inference in VAR analysis. In Chapter 5, both tests will be used.

3.5 Impulse Response Function (IRF)

The Impulse Response Function (IRF) in VAR models is a fundamental tool for analyzing how shocks propagate across variables over time. Understanding IRFs is crucial for interpreting the dynamic interactions and causal relationships within a system of interrelated time-series variables. This section provides a comprehensive explanation of the IRF's role and calculation within VAR models.

3.5.1 Definition and Interpretation

The IRF traces the effect of a one-time shock to one of the innovations (error terms) on the current and future values of the variables in the VAR system. Essentially, it addresses the question: "What happens to the system when there is an unexpected change in one variable?"
The IRF is described in Hansen's book [3] as the coefficients of the moving average representation:

$$y_t = \sum_{j=0}^{\infty} b_j e_{t-j} = b_0 e_t + b_1 e_{t-1} + b_2 e_{t-2} + \cdots \tag{3.24}$$

These coefficients, b_j, are known among economists as the IRF. Often, the IRF is scaled by the standard deviation of e_t. In linear models, the IRF is defined as the change in y_{t+j} due to a shock at time t, expressed as:

$$\frac{\delta y_{t+j}}{\delta e_t} = b_j \tag{3.25}$$

This indicates that the coefficient b_j can be interpreted as the magnitude of the impact of a shock at time t on the variable at time $t+j$. Plots of b_j can be used to assess the time propagation of shocks. A convenient method to calculate impulse responses b_j from the coefficients of an autoregressive model involves the linear AR(p) model. As Hansen describes, the coefficient b_j is simply the derivative:

$$b_j = \frac{\delta y_{t+j}}{\delta e_t} = \frac{\delta y_j}{\delta e_0} \tag{3.26}$$

We can calculate b_j by generating a history and perturbing the shock e_0. Since this calculation is unaffected by all other shocks, we can set $e_t = 0$ for $t \neq 0$ and set $e_0 = 1$. This leads to the recursion:

$$\begin{aligned} b_0 &= 1 \\ b_1 &= a_1 b_0 \\ &\vdots \\ b_j &= a_1 b_{j-1} + a_2 b_{j-2} + \ldots + a_p b_{j-p} \end{aligned} \tag{3.27}$$

This recursion can be conveniently calculated through the following simulation. Set $y_t = 0$ for $t \leq 0$. Set $e_0 = 1$ and $e_t = 0$ for $t \geq 1$. Generate y_t for $t \geq 0$ using $y_t = a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_p y_{t-p} + e_t$. Then $y_j = b_j$.

Analyzing IRFs

- **Zero response at time zero:** For all variables except the one that receives the shock, the immediate response at time zero is zero because other variables cannot change instantaneously due to the model's structure. The shocked variable itself increases by the size of the shock.

- **Dynamic responses:** In subsequent periods, the effects of the initial shock propagate through the system according to the estimated VAR coefficients. This propagation reflects both direct and indirect effects—direct effects from the shocked variable on others and indirect effects as those variables, in turn, affect each other.

- **Decay over time:** typically, the effects of a shock will decay over time, eventually diminishing to zero if the system is stable. Stability in this context means that the effects of temporary shocks do not persist indefinitely.

- **Plots:** IRFs are usually plotted as graphs showing the response of each variable over time. These plots provide intuitive insights into the system's dynamics. For example, they can illustrate whether a shock to one variable leads to oscillations, convergence, or divergence in another variable.

Interpretation

- **Economic significance:** by examining the IRFs, analysts can interpret the short-term and long-term effects of shocks. For instance, in macroeconomic analysis, an IRF can show how a shock to government spending affects output, inflation, and interest rates over time.

- **Policy analysis:** IRFs are also valuable for policy analysis, helping to predict the outcomes of policy interventions. For example, they can illustrate the expected impact of a monetary policy shock on various economic indicators.

Statistical considerations

It is essential to assess the statistical significance of the responses. This is usually done by computing confidence intervals for the impulse responses, often through bootstrapping, to ensure that the observed responses are not due to random variations in the data.

In summary, the Impulse Response Functions in VAR models offer a powerful method to visualize and analyze dynamic interactions and causal relationships among variables in a multivariate time series framework. They are indispensable for both theoretical analysis and practical policy-making.

An example of IRF is shown in the Figure 3.1. This graph illustrate the dynamic interactions between Bitcoin and FTSE MIB. A shock to Bitcoin significantly increases its value initially, which then decays and stabilizes. Conversely, a shock in FTSE MIB has a minimal and brief impact on Bitcoin. Bitcoin shocks have a modest initial effect on FTSE MIB, which diminishes over time. FTSE MIB's own shocks significantly impact its value initially, then decrease and stabilize.

IRFs trace the effect of shocks, showing immediate impacts and subsequent decay, indicating system stability. They provide insights into dynamic interactions and are useful for economic analysis and policy prediction.

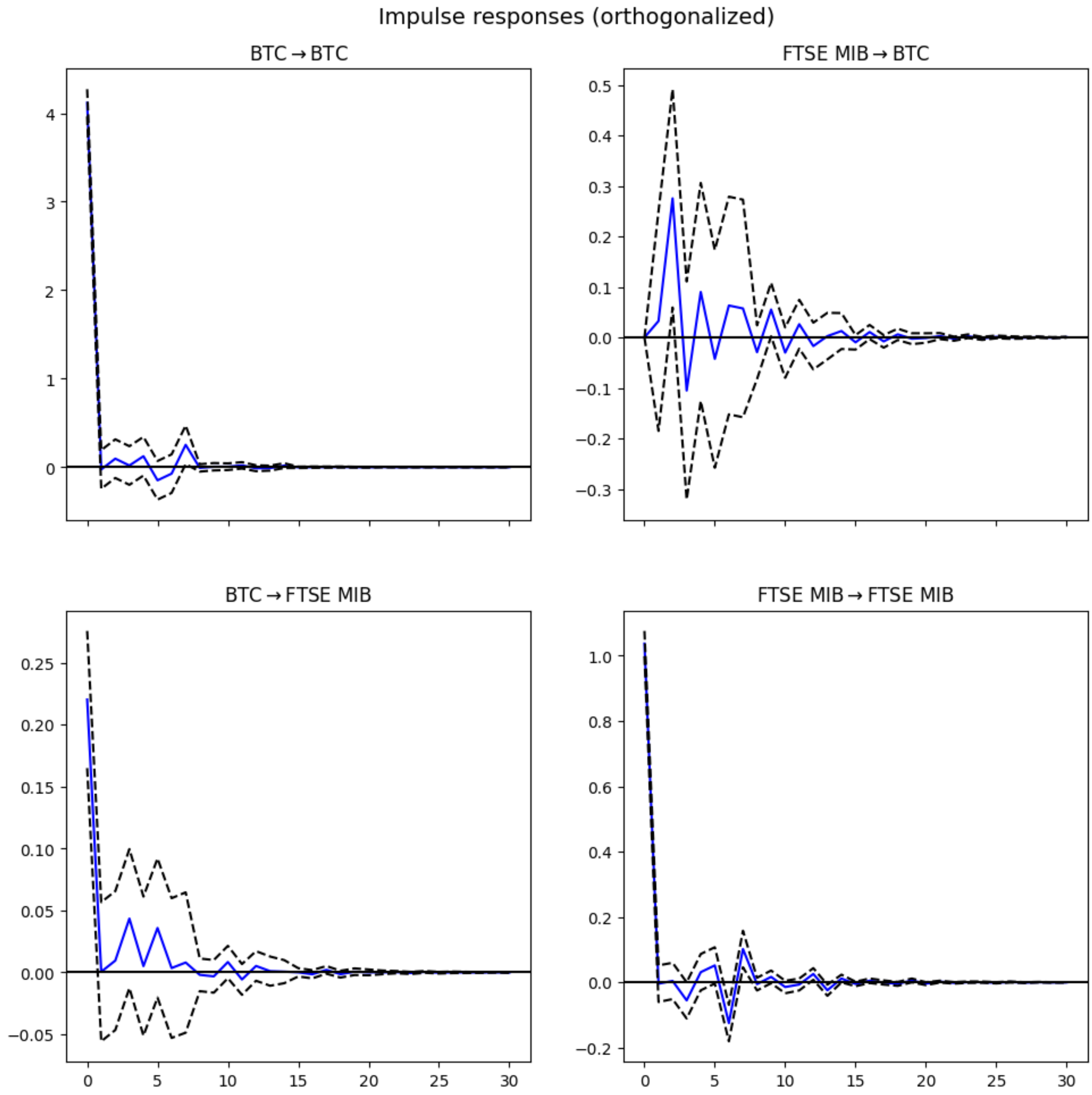


Figure 3.1: IRF for Bitcoin and FTSE MIB

3.6 Limitations of VAR models

Despite their versatility and popularity, VAR models have inherent limitations that must be acknowledged to accurately interpret the results they generate. This section discusses the primary limitations of VAR models, incorporating mathematical and statistical concepts to explain these points.
Each ϕ_i is a $k \times k$ matrix of coefficients. The total number of parameters to estimate in the model is $k^2 \cdot p + k$, where $k^2 \cdot p$ accounts for the coefficient matrices and k for the intercept terms. As the number of variables k increases, the number of parameters to estimate grows quadratically. This can lead to several issues:

- With a high number of parameters, the degrees of freedom for estimation decrease, especially if the sample size is not sufficiently large.

- The precision of parameter estimates diminishes, increasing the standard errors and reducing the reliability of the model.

- High correlation among explanatory variables can lead to unstable parameter estimates, complicating the interpretation of individual coefficients.

For a VAR model to provide meaningful insights and forecasts, the time series involved must be stationary. This means that their mean, variance, and autocovariance are constant over time. However, many economic and financial time series are non-stationary, exhibiting trends, cycles, and seasonality. Transforming non-stationary series to stationary form (e.g., via differencing) before modeling can result in: differencing and other transformations can obscure long-term trends and relationships inherent in the original series and the differenced series may be harder to interpret in an economic context, complicating the analysis of long-term dynamics.

VAR models are primarily empirical and do not require prior theoretical knowledge about the interrelationships among variables. While this flexibility is advantageous, without theoretical guidance, the model may capture relationships that are statistically significant but economically meaningless. Determining the appropriate lag length p relies on information criteria (e.g., AIC, BIC) rather than economic theory, which can lead to model misspecification if not carefully managed.

Standard VAR models assume linear relationships among variables. However, many economic relationships are non-linear, which the linear VAR model cannot capture. Although extensions like Threshold VAR (TVAR) and Smooth Transition VAR (STVAR) models address nonlinearity, they introduce increased complexity.

VAR models are designed to analyze the interdependence among a set of variables, treating all variables as endogenous. This can be limiting when external factors (exogenous variables) significantly influence the system.

VAR models are powerful tools for multivariate time series analysis, providing valuable insights into dynamics and potential forecasts. However, their limitations necessitate careful application and interpretation of the results.

Chapter 4

Granger causality

Granger causality, conceptualized by Clive W.J. Granger in 1969 [9], is a seminal method in statistical hypothesis testing used to determine whether one time series can predict another. This analytical tool has widespread applications across fields such as economics, neuroscience, and environmental science due to its effectiveness in revealing directional influences between sequential data points. This chapter delves into the nuances of Granger causality, exploring its foundational concepts, theoretical underpinnings, and practical implications.

4.1 Introduction

Granger causality is a robust tool in time series analysis used to determine whether historical information from one variable can predict future values of another. Unlike traditional causality, which implies a direct cause-and-effect relationship, Granger causality focuses on predictive causality, where one time series is useful in forecasting another. This section explaines the conceptual foundation of Granger causality, explaining its core principles, practical applications, and limitations.

The core principle behind Granger causality is that predictability serves as a proxy for causal inference. It operates under the assumption that if the past values of a variable $\mathbf{X}$ contain information that uniquely contributes to the prediction of future values of another variable $\mathbf{Y}$, then $\mathbf{X}$ can be considered to have a causal influence on $\mathbf{Y}$ within the Granger framework. This is not causality in the traditional sense, which often implies a direct or mechanistic influence, but rather a statistical indication of precedence and predictive power.

This concept emerged from the necessity to handle economic time series data, where understanding directional influences between variables (such as inflation rates, GDP growth, or unemployment rates) is crucial. Granger's innovation was to propose a systematic, data-driven approach to test for causality, moving beyond mere correlation to disentangle complex interdependencies in economic data.

Granger causality diverges from traditional deterministic notions of causality that dominate natural sciences and philosophy. Instead, it adopts a probabilistic viewpoint, rooted in the idea that causality can be inferred from the ability to predict future states. This stance acknowledges the limitations inherent in observing real-world systems, particularly in the social sciences, where controlled experimentation may not be feasible. Granger emphasized that his concept of causality is based on predictability rather than a physical or mechanistic understanding of causation. Thus, while Granger causality can suggest a causal link based on temporal precedence and information flow, it cannot establish causation in a definitive sense.

Practically, the value of Granger causality lies in its applicability to forecasting and policy-making. By identifying predictive relationships between variables, analysts and policymakers can make informed decisions and anticipate future trends.

Interpreting Granger causality tests requires caution. A finding that **X** Granger-causes **Y** does not imply that **X** is the sole or even the primary cause of changes in **Y**. Other variables and external factors may play significant roles. Moreover, Granger causality cannot capture simultaneous or instantaneous causality due to its reliance on temporal precedence.

The conceptual framework of Granger causality has broad applicability across various fields, including economics, finance, environmental studies, and neuroscience. Its utility in these domains stems from its straightforward implementation and the actionable insights it provides into the dynamics of complex systems.

In summary, the conceptual foundation of Granger causality represents a pivotal shift towards understanding causality in terms of predictability and information flow. This perspective has enriched the analytical toolbox available to researchers and practitioners, offering a nuanced approach to dissecting the intricate web of relationships that define our world.

4.2 Theoretical framework

4.2.1 Definition and Operational Criteria

Granger causality is based on the premise that if past values of a variable **X** contain information that helps predict future values of another variable **Y** better than using past values of **Y** alone, then **X** is said to Granger-cause **Y**. This concept operationalizes causality in terms of predictability, moving beyond mere correlation to examine the direction and information flow between variables.

The typical approach involves estimating a series of regression models, comparing the predictive accuracy of a model that includes past values of both **X** and **Y** against a restricted model that includes only past values of **Y**. The improvement in prediction, often measured through statistical tests on the coefficients of **X**'s past values, indicates Granger causality.

4.2.2 Lag length and model specification

Selecting the appropriate number of past observations (lags) of each variable to include in the model is crucial for the reliability of Granger causality tests. Including too few lags might miss significant historical information, while including too many lags might introduce noise, leading to overfitting. Methods like the AIC and the BIC are commonly used to select an optimal lag length that balances model complexity with goodness of fit.

Following [26] notation, the AIC for a Gaussian AR(ℓ) model is defined as:

$$\mathrm{AIC}(\ell) = \ln(\widetilde{\sigma_\ell^2}) + \frac{2\ell}{T} \tag{4.1}$$

where $\widetilde{\sigma_\ell^2}$ is the maximum likelihood estimate of σ_a^2, the variance of a_t. The second term is referred to as the *penalty function* of the criterion because it penalizes a candidate model by the number of parameters used.
The BIC criterion for the same Gaussian AR(ℓ) model is given by:

$$\mathrm{BIC}(\ell) = \ln(\widetilde{\sigma_\ell^2}) + \frac{\ell \ln(T)}{T} \tag{4.2}$$

Compared to AIC, BIC tends to select a lower-order AR model when the sample size is moderate or large.
The theoretical aspects of Granger causality include a systematic approach to testing predictive causality between time series variables. This involves careful model specification, including lag length selection and accommodating the dynamics of cointegrated series through error-correction modeling. These theoretical principles ensure the rigor and relevance of Granger causality analysis in empirical research, providing valuable insights into the temporal relationships and directional influences among economic, financial, and various other types of time series data.

4.3 Chi-Squared test

The Chi-Squared test used in the `grangercausalitytests`, function of Python from the package `statsmodels`, is based on the residual sum of squares (SSR). This test helps determine whether one time series can predict another, which is a key aspect of Granger causality. To test the Granger causality, we compare two models:

- **Restricted Model (H_0):** This model only includes past values of **Y** to predict the current value of **Y**.
- **Unrestricted Model (H_1):** This model includes past values of both **X** and **Y** to predict the current value of **Y**.

For a given lag length p, the models are formulated as follows:

- Restricted Model (H_0):

$$\mathbf{y}_t = a_0 + a_1\mathbf{y}_{t-1} + a_2\mathbf{y}_{t-2} + \cdots + a_p\mathbf{y}_{t-p} + \epsilon_t \tag{4.3}$$

- Unrestricted Model (H_1):

$$\mathbf{y}_t = a_0 + a_1\mathbf{y}_{t-1} + a_2\mathbf{y}_{t-2} + \cdots + a_p\mathbf{y}_{t-p} + b_1\mathbf{x}_{t-1} + b_2\mathbf{x}_{t-2} + \cdots + b_p\mathbf{x}_{t-p} + \epsilon_t \tag{4.4}$$

The SSR is calculated for both models:

- $\mathrm{SSR}_{\mathrm{restricted}}$: Sum of squared residuals from the restricted model.

- $\text{SSR}_{\text{unrestricted}}$: Sum of squared residuals from the unrestricted model.

The test statistic for the Chi-Squared test is based on the difference in the SSR between the two models. The formula is:

$$\chi^2 = \frac{(\text{SSR}_{\text{restricted}} - \text{SSR}_{\text{unrestricted}})}{\sigma^2} \tag{4.5}$$

where σ^2 is the variance of the residuals from the unrestricted model, estimated as::

$$\sigma^2 = \frac{\text{SSR}_{\text{unrestricted}}}{(n - 2p - 1)} \tag{4.6}$$

Here, n is the number of observations, and p is the number of lags.
The null hypothesis (H_0) posits that $\mathbf{X}$ does not Granger-cause Y. If the test statistic exceeds the critical value from the Chi-Squared distribution with p degrees of freedom, we reject H_0. Otherwise, we do not reject H_0, indicating that $\mathbf{X}$ does not Granger-cause $\mathbf{Y}$.
In Chapter 5, we consider the minimum p-value across all lags. By considering the minimum p-value, we are assessing whether there is any lag within the range that provides significant evidence of causality. This approach can be more sensitive in detecting Granger causality since it does not rely on a single lag but considers multiple potential influences. However, by taking the minimum p-value from multiple tests, we may increase the risk of a Type I error (false positive).
To mitigate the risk of missing significant causality effects at shorter lags, which could be important in our analysis, we take into consideration all lags. This comprehensive approach ensures that we do not overlook any significant Granger causality relationships that may manifest at different lag lengths.

4.4 Methodological considerations

The methodological considerations for Granger causality testing delve into the technical nuances that ensure the robustness and validity of causality analysis. This section is critical for understanding the prerequisites and challenges associated with performing Granger causality tests.

4.4.1 Stationarity

Stationarity refers to a property of time series data where statistical properties such as mean, variance, and autocorrelation structure remain constant over time. Before applying Granger causality tests, it is imperative to test each time series for stationarity using tests like the Augmented Dickey-Fuller (ADF) test. If the series are non-stationary but cointegrated, meaning they share a long-term equilibrium relationship, an Error Correction Model (ECM) should be used to capture both short-term dynamics and long-term relationships, but we will see that the time series are stationary.
Before applying Granger causality tests, it is imperative to test each time series for stationarity. Techniques such as the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron test,

or the KPSS test are commonly used for this purpose, as applied in Chapter 5. These tests help identify the presence of unit roots, which indicate non-stationarity.
When time series are found to be non-stationary, they are often differenced to remove trends and stabilize the mean. However, the differencing process should be applied judiciously to avoid over-differencing, which can introduce artificial stationarity or eliminate meaningful relationships between variables.

Chapter 5

Data analysis

This chapter contains the original research of the thesis. It describes the datasets and the results of the statistical analyses conducted to investigate the causal relation between cryptocurrencies and equity markets. The data and the code used for this analysis, written in Python, are available in a GitHub repository. The repository can be accessed through the following link:
`https://github.com/AndreaGreggio/Granger-Causality--Thesis`.

5.1 Data

In this section, we detail the datasets used for our analysis. The analysis was conducted using Python 3.12.4. The dataset is categorized into two primary asset types: equity indexes and cryptocurrencies. The equity indexes include major financial markets across the American, European, and Japanese regions, covering various sectors. The specific indexes used are: S&P 500, Nikkei 225, Russell 2000, FTSE MIB, NASDAQ, Dow Jones, CAC 40, DAX, and FTSE 100. These indexes were selected to provide a comprehensive view of global equity markets.

The cryptocurrency dataset includes the most capitalized cryptocurrencies for which data is readily available. These are: Bitcoin, Ethereum, Tether (USDT), Ripple, DogeCoin, and Cardano. These cryptocurrencies were chosen due to their market significance and availability of historical data. Table 5.1 provides a summary of the variables along with their respective sources and descriptions.

We have retrieved the data from FirstRate Data, a website that is specialized on providing cryptocurrency data.

Table 5.2 provides a summary of the variables along with their respective sources.

Logo	Name	Market cap (USD bn)	Description / Objectives
	Bitcoin	1267	On-line payment without going through a financial institution
	Ethereum	427	Global platform for decentralized applications
	Tether (USDT)	112	Stablecoin pegged to the US dollar, providing stability in the volatile cryptocurrency market for trading and transactions
	Ripple	27	Open-source, permissionless and decentralized technology for payments, tokenization, DeFi and stablecoin
	DogeCoin	18	Fun, light-hearted cryptocurrency, with greater appeal beyond Bitcoin's core audience, since it was based on a dog meme
	Cardano	14	Proof-of-stake blockchain technology platform focused on building a scalable, secure and efficient decentralized network

Table 5.1: Market cap and short description of cryptocurrencies in scope, as of June 2024

Variable	Description	Source
Bitcoin	Daily price of Bitcoin in US dollars based on direct quote prices	FirstRate Data
Ethereum	Daily price of Ethereum in US dollars based on direct quote prices	FirstRate Data
Tether (USDT)	Daily price of Tether in US dollars based on direct quote prices	FirstRate Data
Ripple	Daily price of Ripple in US dollars based on direct quote prices	FirstRate Data
DogeCoin	Daily price of DogeCoin in US dollars based on direct quote prices	FirstRate Data
Cardano	Daily price of Cardano in US dollars based on direct quote prices	FirstRate Data
S&P 500	Equity Index comprised of 500 large-capitalization companies in the US	Yahoo Finance
Nikkei 225	Equity Index comprised of 225 large-capitalization companies in Japan	Yahoo Finance
Russell 2000	Equity Index comprised of 2000 small-capitalization companies in the US	Yahoo Finance
FTSE MIB	Equity Index comprised of 40 large-capitalization companies in Italy	Yahoo Finance
NASDAQ	Equity Index comprised of 100 non-financial large companies in the US	Yahoo Finance
Dow Jones	Equity Index comprised of 30 blue-chip companies in the US	Yahoo Finance
CAC 40	Equity Index comprised of 40 large-capitalization companies in France	Yahoo Finance
DAX	Equity Index comprised of 40 large-capitalization companies in Germany	Yahoo Finance
FTSE 100	Equity Index comprised of 100 large-capitalization companies in UK	Yahoo Finance

Table 5.2: Variable descriptions and data sources

The equity indexes data was obtained using the Python package `yfinance`, which facilitated downloading data from Yahoo Finance. The time period considered was from January 1, 2019, to April 30, 2024. We utilized daily closing prices for each index. On holidays specific to one market, we excluded data for that day from other markets to maintain consistency.
Cryptocurrency prices were sourced from FirstRate Data (https://firstratedata.com). The dataset includes hourly data, but for consistency with equity index data, we aligned the cryptocurrency data to match the daily closing prices of the equity indexes. For instance, since the NASDAQ closes at 4:00 PM US time, we compared the cryptocurrency prices at

the same time. This method helps to avoid introducing spurious correlations that might arise if we used closing prices from the cryptocurrency market's 24-hour cycle, particularly since the closing price at midnight could miss significant information from the preceding hours.

5.2 Descriptive statistics

We begin the Granger causality test by creating a VAR model for each combination of cryptocurrencies and equity indices. The percentage return formula is defined as:

$$y_t = \frac{p_t - p_{t-1}}{p_{t-1}} \cdot 100\%$$

where p_t represents the price at time t.

Tables below present the descriptive statistics of daily returns, including the mean, standard deviation, minimum, maximum, median (all in percentage values), as well as skewness and kurtosis for different time periods. Specifically, Table 5.3 covers the full sample period, Table 5.4 covers the period from January 2019 to December 2020, and Table 5.5 covers the period from January 2021 to April 2024.

Asset	Mean	St. Dev.	Min	Median	Max	Skew.	Kurt
Bitcoin	0.29	4.05	-27.41	0.11	24.01	0.08	4.94
Ethereum	0.36	5.23	-43.60	0.20	36.07	-0.08	7.41
USDT	0.00	0.16	-3.39	0.00	1.75	-4.92	157.53
Ripple	0.21	6.38	-41.05	0.00	73.08	2.63	30.04
DogeCoin	1.23	15.87	-68.25	0.00	202.63	5.98	62.31
Cardano	0.34	5.79	-25.64	0.00	29.52	0.49	3.04
S&P 500	0.05	1.37	-16.93	0.11	8.93	-1.83	23.13
Nikkei 225	0.04	1.26	-12.93	0.04	11.37	-0.58	21.51
Russell 2000	0.05	1.25	-12.28	0.09	8.39	-0.79	12.88
FTSE MIB	0.02	1.07	-10.87	0.05	9.05	-0.91	15.06
NASDAQ	0.05	1.28	-12.24	0.07	10.98	-0.44	13.68
Dow Jones	0.06	1.29	-11.98	0.06	9.38	-0.53	14.22
CAC 40	0.07	1.53	-12.32	0.09	9.35	-0.43	7.08
DAX	0.05	1.18	-6.08	0.00	8.04	0.14	3.84
FTSE 100	0.04	1.67	-14.27	0.02	9.39	-0.69	9.08

Table 5.3: Descriptive statistics of daily returns for the full sample

Asset	Mean	St. Dev.	Min	Median	Max	Skew.	Kurt.
Bitcoin	0.48	4.31	-27.41	0.18	24.01	0.11	6.88
Ethereum	0.46	5.66	-43.60	0.19	22.10	-0.40	8.59
USDT	0.00	0.26	-3.39	0.00	1.75	-3.27	64.41
Ripple	0.08	6.29	-41.05	0.00	70.09	2.30	34.42
DogeCoin	1.01	13.44	-33.33	0.00	66.67	1.61	7.49
Cardano	0.45	5.99	-25.64	0.00	28.45	0.30	2.81
S&P 500	0.05	1.67	-16.93	0.07	8.93	-2.45	26.13
Nikkei 225	0.07	1.70	-12.93	0.10	11.37	-0.62	16.31
Russell 2000	0.04	1.55	-12.28	0.09	8.39	-1.08	12.79
FTSE MIB	0.00	1.39	-10.87	0.05	9.05	-0.96	12.91
NASDAQ	0.06	1.57	-12.24	0.07	10.98	-0.70	13.55
Dow Jones	0.09	1.61	-11.98	0.12	9.38	-0.70	14.77
CAC 40	0.14	1.70	-12.32	0.20	9.35	-0.80	11.20
DAX	0.07	1.26	-6.08	0.00	8.04	0.36	7.31
FTSE 100	0.09	1.99	-14.27	0.08	9.39	-1.15	11.48

Table 5.4: Descriptive statistics of daily returns from January 2019 to December 2020

Asset	Mean	St. Dev.	Min	Median	Max	Skew.	Kurt.
Bitcoin	0.17	3.89	-20.50	0.01	21.21	0.04	3.10
Ethereum	0.29	4.95	-26.32	0.21	36.07	0.20	5.95
USDT	0.00	0.05	-0.37	0.00	0.44	0.15	15.35
Ripple	0.29	6.43	-30.43	0.10	73.08	2.83	27.77
DogeCoin	1.36	17.17	-68.25	-0.16	202.63	7.10	70.97
Cardano	0.27	5.67	-22.66	-0.14	29.52	0.63	3.22
S&P 500	0.06	1.16	-6.24	0.12	6.95	-0.47	4.04
Nikkei 225	0.03	0.90	-3.94	0.02	3.70	-0.18	1.68
Russell 2000	0.05	1.04	-4.97	0.09	7.13	-0.07	4.60
FTSE MIB	0.03	0.83	-3.88	0.05	3.92	-0.40	3.47
NASDAQ	0.04	1.06	-4.41	0.07	7.92	0.13	5.18
Dow Jones	0.04	1.06	-4.32	0.00	5.54	-0.15	1.94
CAC 40	0.03	1.42	-5.16	0.01	7.35	-0.09	1.45
DAX	0.04	1.13	-3.99	0.01	3.94	-0.06	0.44
FTSE 100	0.01	1.44	-4.76	0.00	6.11	0.04	0.51

Table 5.5: Descriptive statistics of daily returns from January 2021 to April 2024

As shown in the descriptive statistics, and similar to the previous work of Billio et al. [6], the full sample means are positive for most assets, with the exception of one cryptocurrency, reflecting the significant market recovery post-COVID-19. This is especially evident in the financial and technology sectors. Cryptocurrencies, in particular, saw an increase in attention and prices following the initial pandemic wave in late 2020, driven by increasing popularity. The descriptive statistics indicate the presence of positive excess kurtosis and negative skewness across the datasets. These statistical characteristics are well-documented stylized facts in financial returns data, reflecting the propensity for extreme values and asymmetry in return distributions. Positive excess kurtosis suggests a higher likelihood of outliers or extreme returns, while negative skewness indicates a longer or fatter tail on the left side of the

distribution. Such features are critical in understanding the underlying risk and return profiles of the assets under study, offering valuable insights for risk management and portfolio optimization strategies.
Figure 5.1 illustrates the full range of prices, highlighting the evolution of both asset classes.

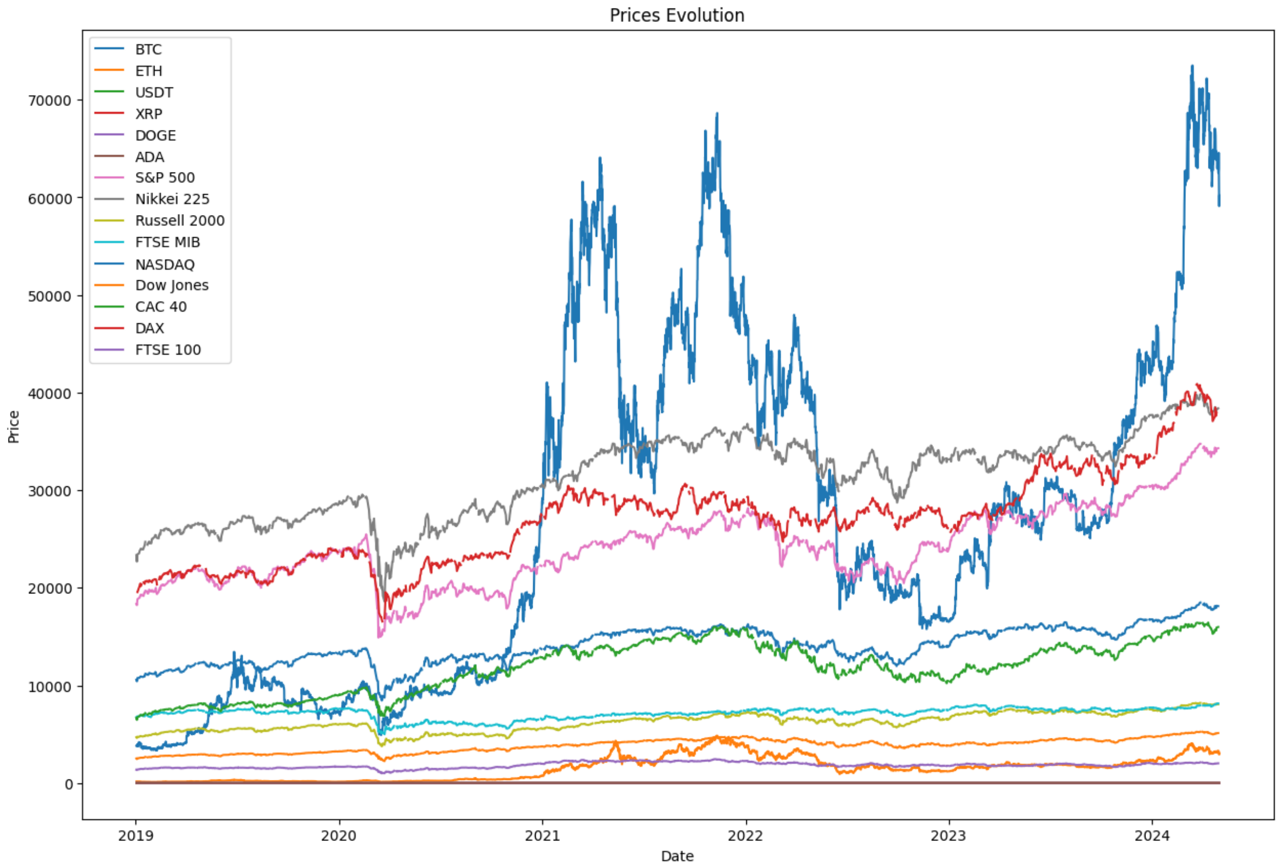


Figure 5.1: Evolution of prices

The equity indexes generally exhibit stability, while cryptocurrencies display higher volatility and significant price increases over the past five years. Key market events and trends are evident.
For instance, Bitcoin and Ethereum experienced major peaks in late 2020 and early 2021, driven by institutional adoption and the rise of decentralized finance and non-fungible tokens (NFTs). Ethereum's growth was further supported by critical network upgrades.
Stock market indices such as the S&P 500, NASDAQ, and Dow Jones saw steady growth, interrupted by a sharp decline in early 2020 due to the COVID-19 pandemic, followed by a recovery fueled by government stimulus and vaccine rollouts.
DogeCoin's notable rise in early 2021, driven by social media hype and celebrity endorsements, followed by a decline, underscores cryptocurrency market volatility. Ripple faced significant price fluctuations due to the SEC lawsuit against Ripple Labs in late 2020, highlighting regulatory risks.
Overall, the graph captures the impact of technological advances, regulatory changes, and global events on financial markets over the past five years.
Below, we plot the percentage daily returns derived from the price time series, as described in equation 5.2:

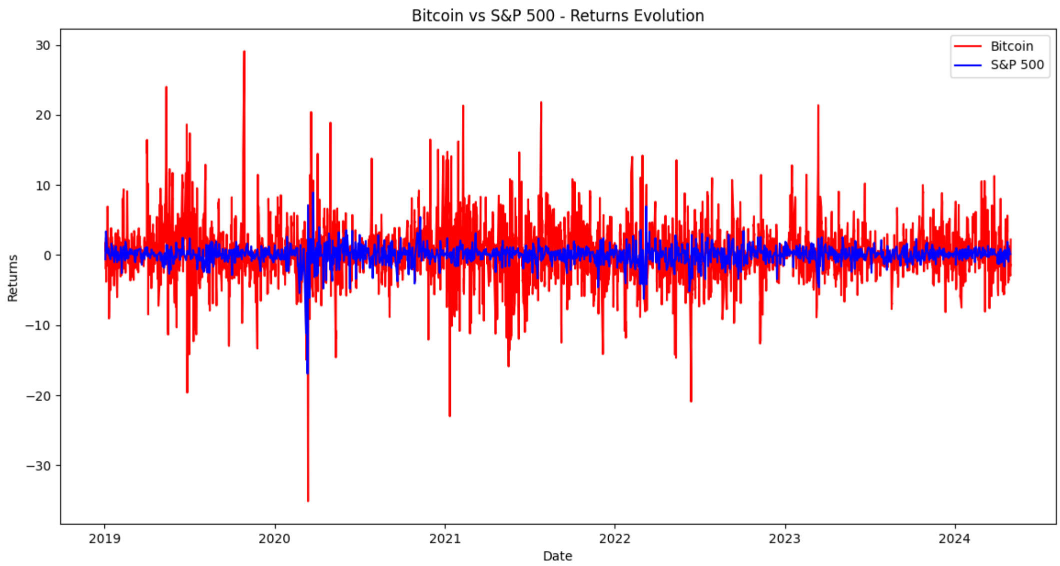


Figure 5.2: Bitcoin vs S&P 500: evolution of returns

As illustrated, Bitcoin exhibits higher volatility with significant peaks and drawdowns, while equity indexes have remained relatively stable over the past five years, despite the COVID-19 crisis.

5.2.1 Linear Correlations

To further understand the relationships within our dataset, we compute the correlation matrices after building the variance-covariance matrices.
Initially, we focus on linear correlations to understand how the returns are correlated. This analysis aims to highlight the differences in correlations between our two dataset periods, demonstrating the increasing integration of the cryptocurrency market with the equity market.
Before calculating the correlation matrix, we first determine the variance-covariance matrix, as detailed in the appendix for the three data samples. The ij-th element of the variance-covariance matrix is calculated as follows:

$$\mathrm{Cov}(\mathbf{y}_i, \mathbf{y}_j) = \frac{1}{n-1}\sum_{k=1}^{n}(\mathbf{y}_{ik} - \overline{\mathbf{y}}_i)(\mathbf{y}_{jk} - \overline{\mathbf{y}}_j) \tag{5.1}$$

Tables in Appendix, 5.16, 5.17 and 5.18, show the covariance between cryptocurrency and equity indexes. Subsequently, the ρ_{ij} element of the correlation matrix is derived as:

$$\rho_{ij} = \frac{\mathrm{Cov}(\mathbf{y}_i, \mathbf{y}_j)}{\sigma_i \sigma_j} = \frac{\frac{1}{n-1}\sum_{k=1}^{n}(\mathbf{y}_{ik} - \overline{\mathbf{y}}_i)(\mathbf{y}_{jk} - \overline{\mathbf{y}}_j)}{\sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(\mathbf{y}_{ik} - \overline{\mathbf{y}}_i)^2}\sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(\mathbf{y}_{jk} - \overline{\mathbf{y}}_j)^2}} \tag{5.2}$$

Table 5.6 compares the return correlations of selected cryptocurrencies with equity indexes for the periods 2019-2020 and 2021-2024:

Crypto	Equity index	2019–20	2021–24
Bitcoin	S&P 500	0.22	0.21
	Nikkei 225	0.09	0.21
	Russell 2000	0.20	0.22
	FTSE MIB	0.17	0.25
Ethereum	S&P 500	0.31	0.24
	Nikkei 225	0.26	0.21
	Russell 2000	0.25	0.24
	FTSE MIB	0.18	0.28
DogeCoin	S&P 500	0.01	0.09
	Nikkei 225	-0.02	0.00
	Russell 2000	-0.01	0.08
	FTSE MIB	0.17	0.24

Table 5.6: Return correlations comparison

We observe an increase in the correlation of Bitcoin, Ethereum, and DogeCoin from the first period to the second, indicating the growing integration of cryptocurrencies into the broader investment landscape. The complete correlation matrices for the periods 2019-2020 and 2021-2024 are presented in the Appendix (Tables 5.19 and 5.20).

The Table 5.7 displays the correlation matrix for the entire dataset.

	Crypto						**Equity Index**								
Asset	BTC	ETH	USDT	XRP	DOGE	ADA	S&P	Nikkei	Russell	FTSE MIB	NASDAQ	Dow Jones	CAC	DAX	FTSE 100
BTC	1.00	0.74	0.24	0.51	0.28	0.67	0.21	0.25	0.20	0.18	0.20	0.29	0.31	0.06	0.31
ETH	0.74	1.00	0.16	0.53	0.27	0.68	0.27	0.31	0.24	0.22	0.24	0.34	0.35	0.06	0.34
USDT	0.24	0.16	1.00	0.09	0.03	0.16	0.07	0.14	0.03	0.06	0.02	0.14	0.12	0.00	0.11
XRP	0.51	0.53	0.09	1.00	0.20	0.59	0.14	0.18	0.12	0.13	0.14	0.21	0.23	0.05	0.22
DOGE	0.28	0.27	0.03	0.20	1.00	0.28	0.05	0.05	0.04	0.04	0.05	0.06	0.06	-0.01	0.06
ADA	0.67	0.68	0.16	0.59	0.28	1.00	0.21	0.23	0.19	0.18	0.19	0.26	0.29	0.09	0.29
S&P	0.21	0.27	0.07	0.14	0.05	0.21	1.00	0.61	0.90	0.80	0.89	0.59	0.51	0.26	0.58
Nikkei	0.25	0.31	0.14	0.18	0.05	0.23	0.61	1.00	0.63	0.61	0.62	0.96	0.84	0.23	0.86
Russell	0.20	0.24	0.03	0.12	0.04	0.19	0.90	0.63	1.00	0.86	0.94	0.60	0.51	0.33	0.60
FTSE MIB	0.18	0.22	0.06	0.13	0.04	0.18	0.80	0.61	0.86	1.00	0.83	0.56	0.45	0.34	0.56
NASDAQ	0.20	0.24	0.02	0.14	0.05	0.19	0.89	0.62	0.94	0.83	1.00	0.61	0.53	0.33	0.60
Dow Jones	0.29	0.34	0.14	0.21	0.06	0.26	0.59	0.96	0.60	0.56	0.61	1.00	0.95	0.22	0.87
CAC	0.31	0.35	0.12	0.23	0.06	0.29	0.51	0.84	0.51	0.45	0.53	0.95	1.00	0.19	0.83
DAX	0.06	0.06	0.00	0.05	-0.01	0.09	0.26	0.23	0.33	0.34	0.33	0.22	0.19	1.00	0.21
FTSE 100	0.31	0.34	0.11	0.22	0.06	0.29	0.58	0.86	0.60	0.56	0.60	0.87	0.83	0.21	1.00

Table 5.7: Correlation matrix for the full dataset

Our analysis reveals that cross-correlations are generally higher within the same asset class, with equity indexes showing stronger correlations amongst themselves compared to the cryptocurrency market. Furthermore, the correlation between the equity and cryptocurrency markets is generally weaker than the correlation within the same market. Previous studies, such as Baur et al. study [13], have shown that cryptocurrencies often exhibit weaker correlations with global currencies, with some even displaying negative correlations. This suggests that cryptocurrencies can serve as an effective diversification asset within equity portfolios.

Next, we analyze lagged correlations to determine whether statistically significant relationships exist between the two time series at different lags. The lagged correlation is defined as:

$$\rho_k = \frac{\mathrm{Cov}(\mathbf{x}_t, \mathbf{y}_{t-k})}{\sqrt{\mathrm{Var}(\mathbf{x}_t)\mathrm{Var}(\mathbf{y}_{t-k})}} \tag{5.3}$$

Figure 5.3 shows the lagged correlation function for the S&P 500 against selected cryptocurrencies from 2019 to 2020.

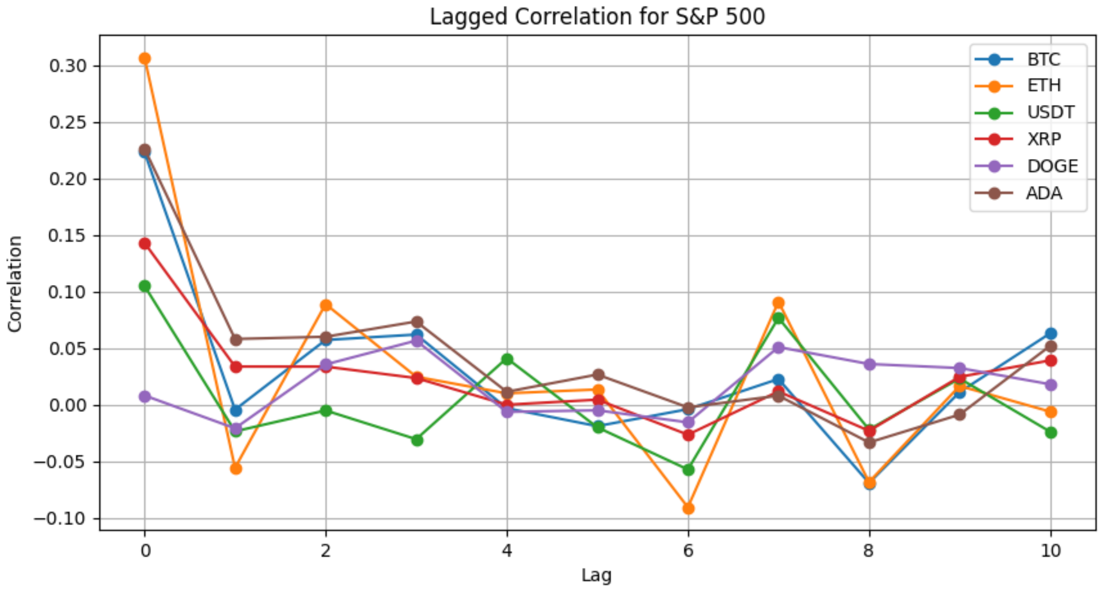


Figure 5.3: Lagged Correlation for S&P 500 (from 2019 to 2020)

The plot illustrates that the highest correlations predominantly occur at lag 0, indicating immediate or very short-term relationships. Beyond lag 0, the correlations tend to fluctuate around zero, suggesting weaker and less consistent relationships over longer lag periods. Notably, when comparing Figure 5.3 with Figures 5.4 and 5.5 we observe that the correlations and lagged correlations during the first two years of the dataset are generally higher than those of the full dataset and the 2021-2024 period. This phenomenon can be attributed to the risk spillover discussed in Chapter 2, which explains that during the COVID-19 pandemic, the financial market was highly interconnected and volatile.

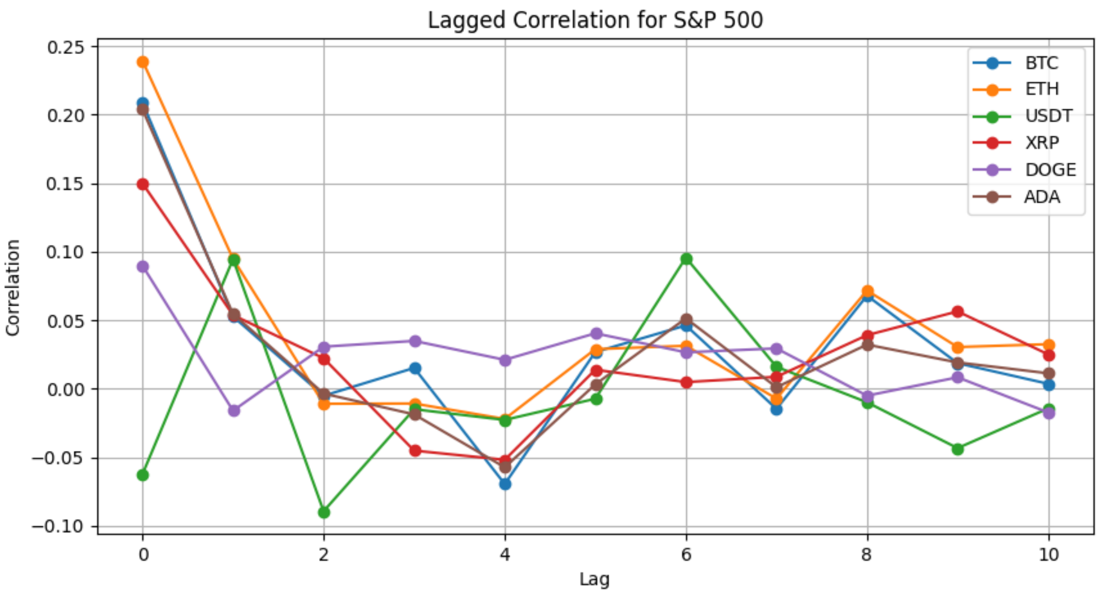


Figure 5.4: Lagged Correlation for S&P 500 (from 2021 to 2024)

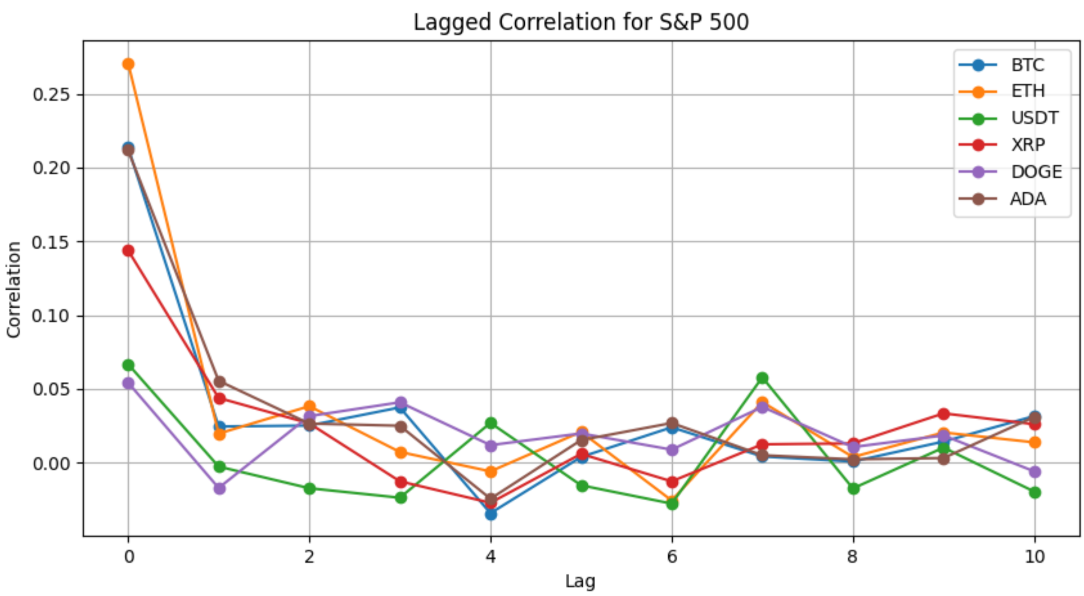


Figure 5.5: Lagged Correlation for S&P 500 (from 2019 to 2024)

Similar patterns are observed for other indices across both the sub-datasets and the full range dataset, as demonstrated in the Appendix (Figures 5.16, 5.17 and 5.18).

5.3 VAR estimation

VAR models are designed to capture the linear interdependencies among multiple time series. As described in Chapter 3, these models account for the simultaneous effects that each market may have on the other over time. In a VAR model, all variables are treated as en-

dogenous, meaning the model does not distinguish between dependent and independent variables.
To perform the Granger causality test, we first create a VAR model for each combination of cryptocurrency and equity index. This results in 6 (cryptocurrencies) x 8 (indexes) = 48 VAR models. We use the same approach discussed earlier, aligning hourly cryptocurrency data with the closing prices of the equity indexes.
The distribution of the parameters is shown in Figure 5.6. The maximum lag is set at 10, meaning the model can incorporate information from the previous 2 trading weeks. We note that the VAR model often selects lag 8, 9, or 10, indicating a strong connection between current and short-term past data. Typically, higher lags are chosen for the dataset from 2019-2020 (shown in blue), reflecting high interconnections in the market during the COVID-19 pandemic. For the dataset from 2021-2024 (shown in green) the model chooses frequently small lags, probably due to a less interconnected market compared to the previous one. The full range dataset (shown in red), has been choosing higher lags compared to the early dataset while around one third of the models chooses 2 lags.

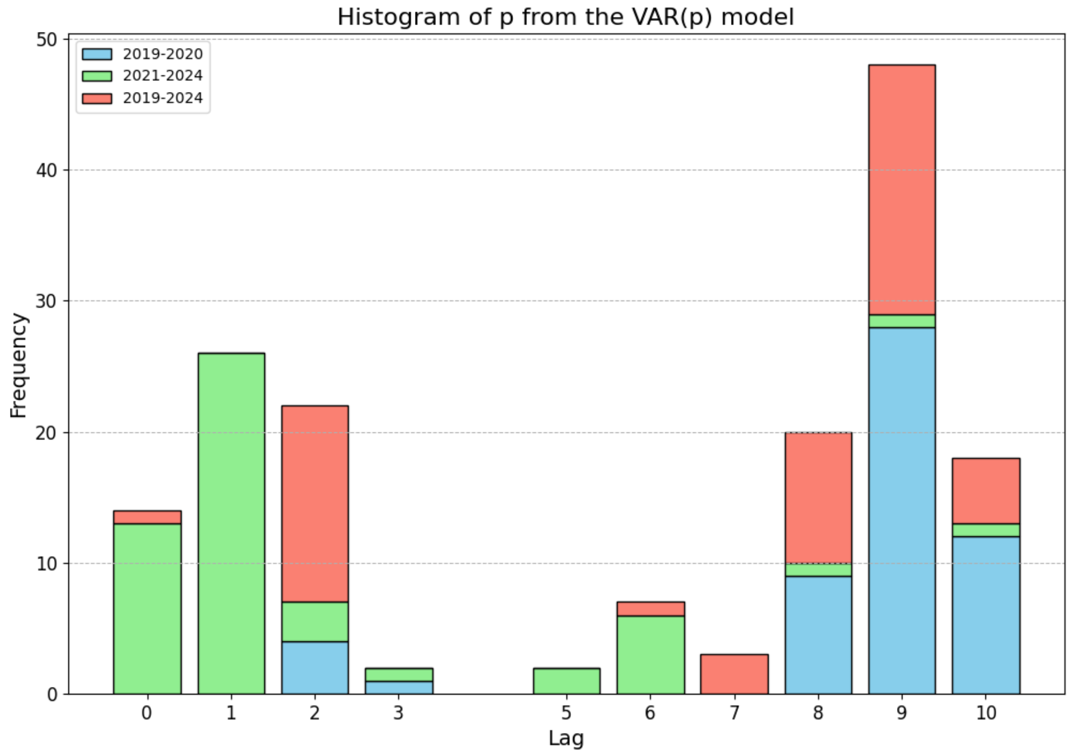


Figure 5.6: Distribution of VAR parameters for the 3 datasets

Now, we proceed to test the stationarity of the residuals. Given that the series have been transformed from prices to returns, we anticipate them to exhibit stationarity. As outlined in Chapter 3, we employ the Augmented Dickey-Fuller (ADF) test [28], utilizing the AIC for optimal lag selection, with a maximum of 10 lags. We reject the null hypothesis of non-stationarity for all series with a confidence level of 5%, in order to confirm strong evidence of stationarity (p-value $\approx$ 1). The detailed results are presented in Appendix, Table 5.21.

Additionally, we conduct further diagnostics to check for the presence of autocorrelation and partial autocorrelation, using the methods described in Sections 3.4.1 and 3.4.2. The Durbin-Watson test results indicate the absence of autocorrelation; however, due to the lack of a direct p-value in the Durbin-Watson test, we also perform the Ljung-Box test. The Ljung-Box test results corroborate the absence of autocorrelation at all lags.

The IRFs illustrated in Figure 5.7 offer valuable insights into the dynamic interactions between the S&P 500 index and Bitcoin. Focusing on the model selected by the AIC, which suggests a VAR model of order 6 (VAR(6)), we analyze the responses of each variable to shocks in itself and in the other variable. The orthogonalized IRFs, computed using Cholesky decomposition, are designed to account for contemporaneous correlations between the shocks, ensuring that the responses are to purely uncorrelated shocks.

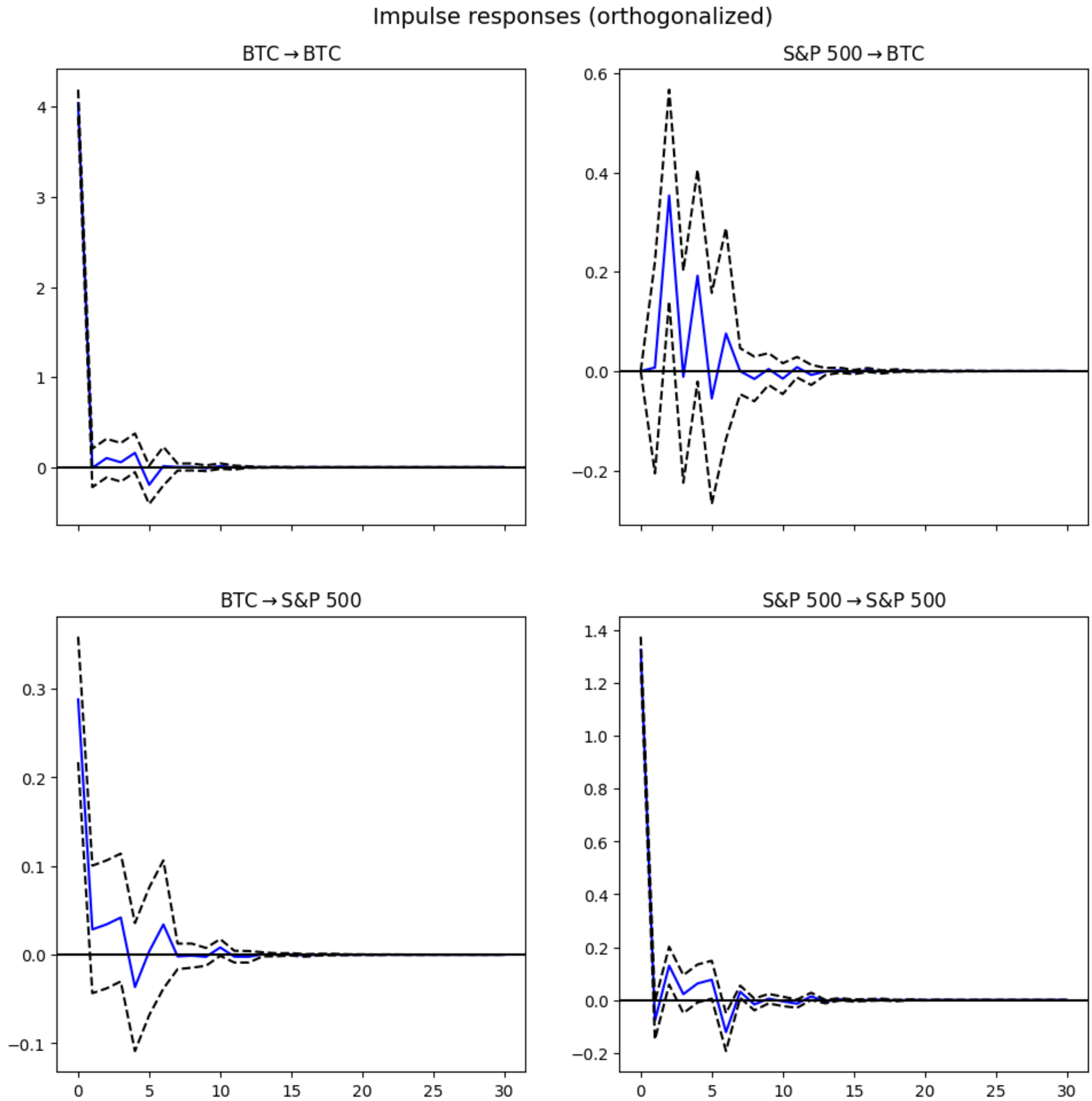


Figure 5.7: Impulse Response Functions of S&P 500 and Bitcoin

The top left panel shows Bitcoin's response to its own shocks. Initially, there is a substantial positive impact, which rapidly diminishes and stabilizes around zero. This suggests that shocks to Bitcoin have a pronounced but short-lived effect on its own returns. The top right panel shows Bitcoin's response to shocks originating from the S&P 500. The initial impact is positive, followed by oscillations before the response stabilizes near zero after several periods. This indicates that while the S&P 500 can influence Bitcoin in the short term, its long-term effect is negligible. The bottom left panel illustrates the response of the S&P 500 to shocks in Bitcoin. Here, the initial response is positive, followed by fluctuations before converging to zero. This suggests that Bitcoin has a short-term impact on the S&P 500, but like the reverse, it does not sustain long-term effects. The bottom right panel shows the S&P 500's response to its own shocks. The initial impact is significantly positive, which then declines and stabilizes around zero. This behavior indicates that the S&P 500's self-induced shocks have a strong immediate effect that diminishes quickly over time.

Empirically, these IRFs suggest that both Bitcoin and the S&P 500 are primarily influenced by their own shocks rather than those of each other. The observed short-term oscillations in cross-variable impacts (Bitcoin to S&P 500 and vice versa) imply that any influence between these markets is quickly absorbed and does not persist in the long term. This rapid stabilization highlights the resilience of each market to shocks from the other, reinforcing the notion of limited long-term interdependence between Bitcoin and traditional equity markets like the S&P 500.

5.4 Granger causality

In this section, we conduct a Granger causality analysis with a maximum lag of 10, effectively capturing information from the preceding two weeks. The analysis is detailed in the six bipartite graphs, corresponding to Figures 5.8 to 5.13, each graph is showing the direction of causality—either from cryptocurrency to equity or from equity to cryptocurrency—over three distinct time periods: January 2019 to December 2020, January 2021 to April 2024, and the entire period combined.

The Tables from 5.8 to 5.13 present p-values from the Granger causality tests, performed using the `grangercausalitytests` function from the `statsmodels` package in Python, our threshold is the 5% level significance. This function employs the Chi-Squared (χ^2) test, which is based on the sum of squared differences and derived from the likelihood ratio test. This test compares the likelihoods of the full and restricted models, with the null hypothesis being the absence of Granger causality.

As discussed in Section 4.3, we adopt the minimum p-value across all lags rather than focusing solely on the p-value of the last lag. This approach ensures that we do not overlook potential causality from earlier lags. The Chi-Squared test was chosen over the F-test due to its robust performance with large sample sizes [2]. However, it also means that our Granger causality network might incorporate some false positives.

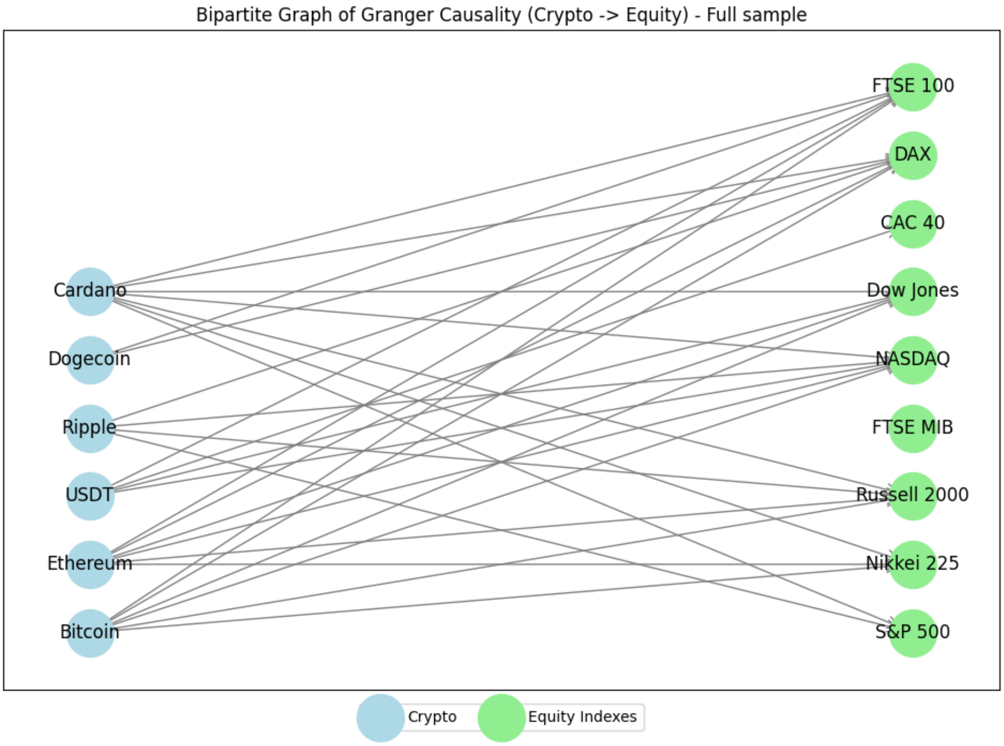


Figure 5.8: GC bipartite network - crypto to equity (2019 - 2024) - 5% Level

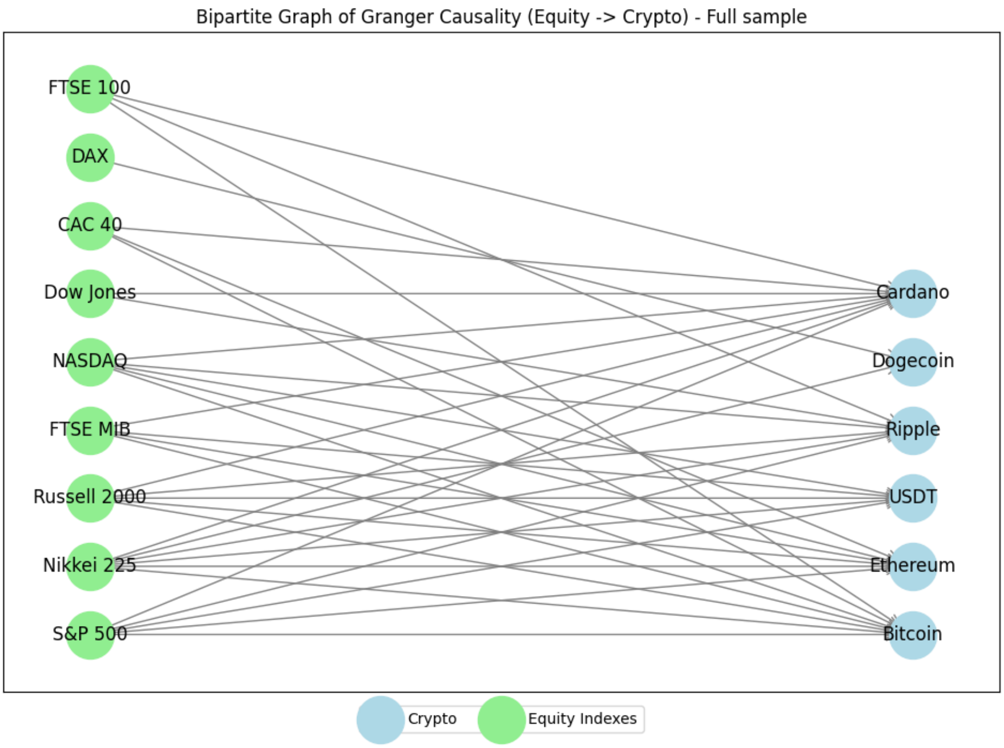


Figure 5.9: GC bipartite network - equity to crypto (2019 - 2024) - 5% Level

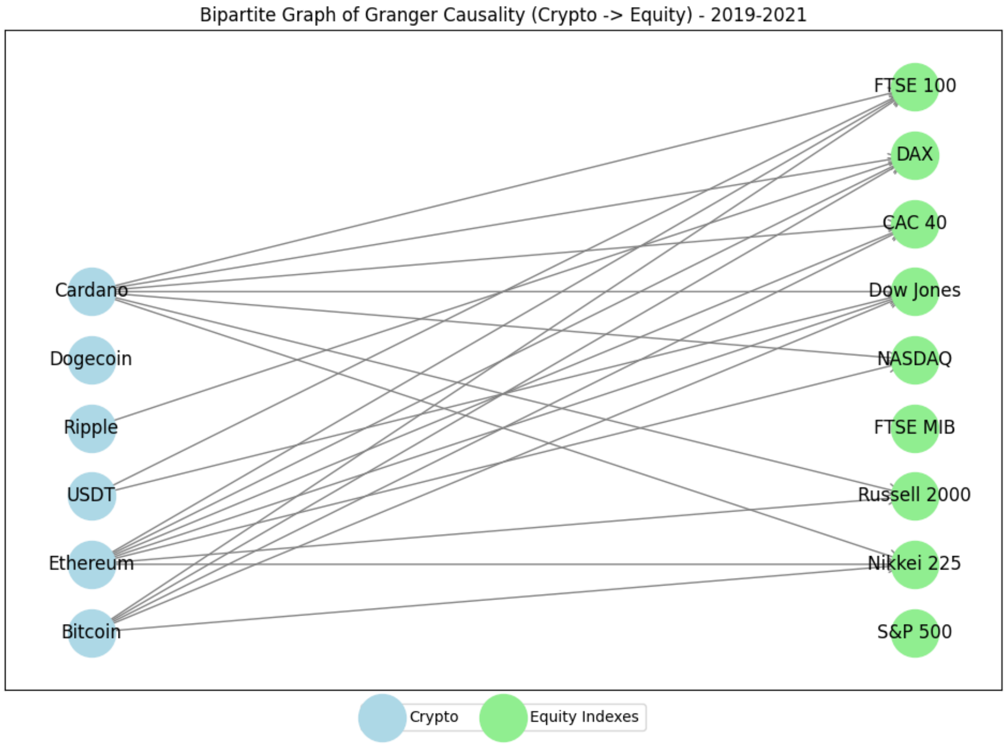


Figure 5.10: GC bipartite network - crypto to equity (2019 - 2020) - 5% Level

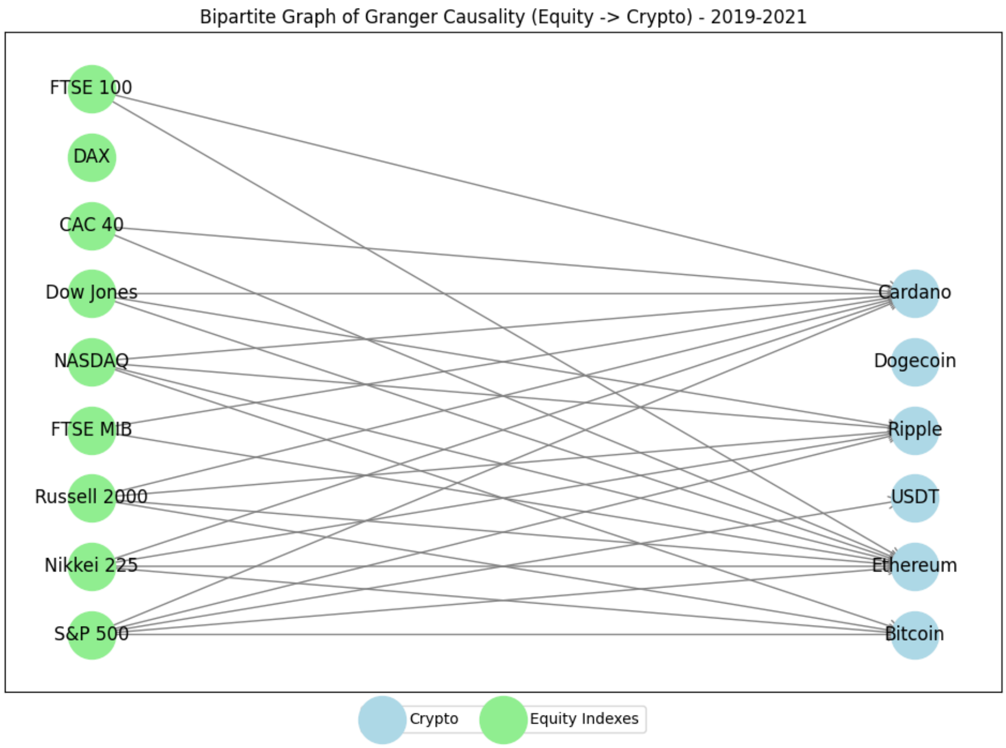


Figure 5.11: GC bipartite network - equity to crypto (2019 - 2020) - 5% Level

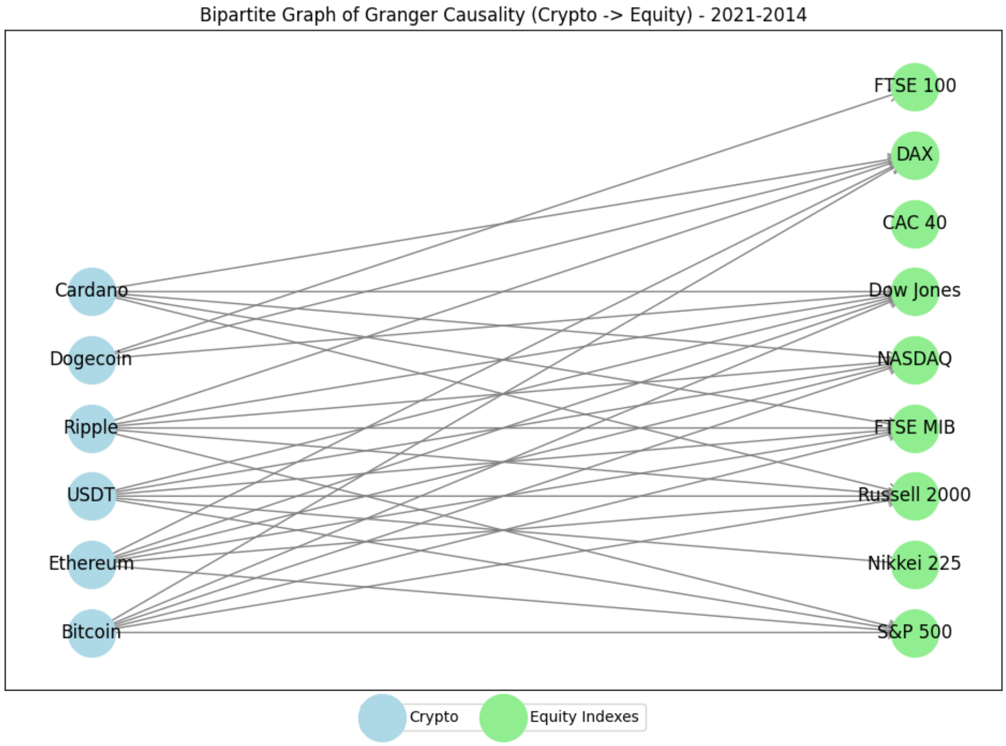


Figure 5.12: GC bipartite network - crypto to equity (2021 - 2024) - 5% Level

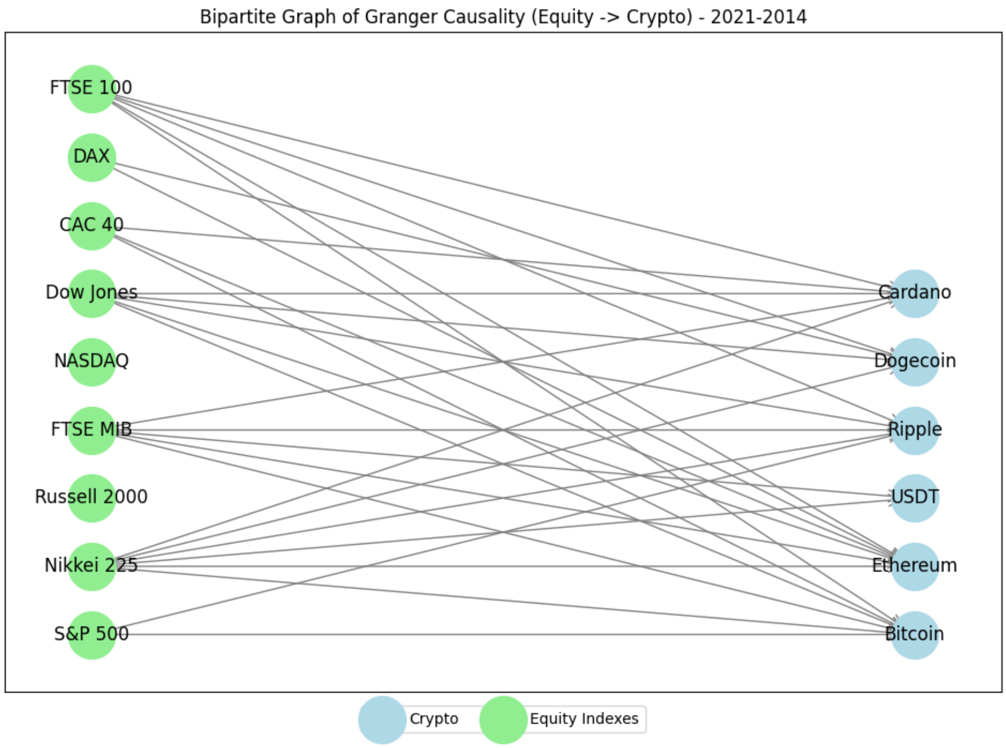


Figure 5.13: GC bipartite network - equity to crypto (2021 - 2024) - 5% Level

Figures 5.10 through 5.9 exhibit the Granger causality bipartite networks, elucidating the direction and strength of causality between cryptocurrencies and equity indexes over distinct time periods. These visualizations increase our comprehension of the intricate relationships between these financial instruments. Each graph delineates the direction of causality, either from equity indexes to cryptocurrencies or vice versa.

The initial set of graphs represents the Granger causality from equity indexes to cryptocurrencies over the entire sample period (2019-2024). In these visualizations, equity indexes such as the FTSE 100, DAX, CAC 40, and others are denoted by green nodes on the left, while cryptocurrencies like Bitcoin, Ethereum, and Ripple are illustrated by blue nodes on the right. The connecting lines indicate potential causality relationships, suggesting that movements in equity indexes may predict subsequent fluctuations in cryptocurrency prices. The dense network of connections indicates a substantial level of interdependence, with multiple equity indices exerting influence over various cryptocurrencies. This underscores the growing integration between traditional financial markets and digital assets.

Conversely, the second graph for the full sample period illustrates the reverse causality, from cryptocurrencies to equity indexes. Here, cryptocurrencies are positioned on the left and equity indexes on the right. The connections suggest that price movements in cryptocurrencies could forecast changes in equity indexes. This graph similarly displays a complex web of relationships, indicating that the cryptocurrency market can exert predictive influence on traditional equity markets. This further emphasizes the bidirectional nature of these financial instruments.

Focusing on the first half of the sample period, from 2019 to December 2020, the graph showing causality from equity indexes to cryptocurrencies shows fewer connections compared to the full sample period. This suggests a more limited influence of equity indexes on cryptocurrencies during this initial timeframe. Nevertheless, key equity indexes such as the FTSE 100 and NASDAQ still exhibit multiple connections to cryptocurrencies, indicating their significant role in predicting cryptocurrency price movements even in the earlier period.

The graph illustrating causality from cryptocurrencies to equity indexes during the first half of the sample period also reveals fewer connections. This indicates that the influence of cryptocurrencies on equity indexes was less pronounced in the initial period. Nonetheless, prominent cryptocurrencies like Bitcoin and Ethereum maintain several connections to major equity indexes, suggesting that even early on, movements in these digital assets could provide insights into future equity market trends.

Examining the second half of the sample period, from 2021 to 2024, the graph showing equity indexes' influence on cryptocurrencies reveals a more intricate network of connections than in the first half. This highlights an increased interdependence as the markets evolved. Equity indexes such as the DAX and S&P 500 display numerous connections to cryptocurrencies, underscoring their growing predictive power over digital asset movements in recent years.

The corresponding graph for the second half period, illustrating the causality from cryptocurrencies to equity indexes, also shows an increased number of connections compared to the first half. This suggests that as the cryptocurrency market matured, its influence on traditional equity markets became more substantial. Cryptocurrencies like Bitcoin and

Ethereum show extensive connections to various equity indexes, reinforcing the notion that digital assets have become critical indicators of market trends.

In conclusion, the bipartite graphs provide a comprehensive visual summary of the dynamic and evolving relationships between equity indexes and cryptocurrencies. The full sample graphs underscore a high degree of interconnectedness, while the temporal breakdown into first and second half periods highlights the increasing interdependence and influence between these markets over time. The analysis reveals a bidirectional causality, with both equity indexes and cryptocurrencies exerting significant predictive power over each other, reflecting the integrated nature of modern financial markets.

To further explain the findings, we present a series of tables detailing the p-values obtained from the Granger causality tests. Lower p-values (closer to 0) provide stronger evidence of Granger causality between the variables (cryptocurrency to equity or vice versa), while higher p-values (closer to 1) indicate a lack of causality evidence. We flag in green the p-values above 0.05 while we flag in red the p-values below 0.05.

Asset	Bitcoin	Ethereum	USDT	Ripple	DogeCoin	Cardano
S&P 500	0.163	0.175	0.215	0.050	0.249	0.010
Nikkei 225	0.000	0.006	0.208	0.067	0.386	0.000
Russell 2000	0.004	0.001	0.088	0.002	0.158	0.003
FTSE MIB	0.175	0.175	0.594	0.184	0.124	0.151
NASDAQ	0.003	0.002	0.049	0.002	0.153	0.000
Dow Jones	0.030	0.032	0.004	0.274	0.356	0.040
CAC 40	0.106	0.289	0.043	0.440	0.214	0.245
DAX	0.000	0.000	0.385	0.000	0.000	0.000
FTSE 100	0.001	0.004	0.002	0.379	0.017	0.009

Table 5.8: P-values of GC tests - cryptocurrency to equity (2019 - 2024)

Asset	S&P 500	Nikkei	Russell	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.003	0.000	0.005	0.037	0.004	0.060	0.039	0.562	0.000
Ethereum	0.005	0.000	0.006	0.003	0.010	0.066	0.044	0.295	0.077
USDT	0.000	0.039	0.021	0.004	0.037	0.058	0.084	0.544	0.420
Ripple	0.006	0.000	0.002	0.397	0.005	0.010	0.086	0.424	0.025
DogeCoin	0.110	0.006	0.117	0.394	0.100	0.448	0.249	0.000	0.315
Cardano	0.004	0.000	0.016	0.048	0.043	0.042	0.005	0.915	0.005

Table 5.9: P-values of GC tests - equity to cryptocurrency (2019 - 2024)

Asset	Bitcoin	Ethereum	USDT	Ripple	DogeCoin	Cardano
S&P 500	0.624	0.396	0.514	0.319	0.363	0.086
Nikkei 225	0.001	0.002	0.450	0.080	0.226	0.001
Russell 2000	0.076	0.011	0.289	0.131	0.085	0.019
FTSE MIB	0.076	0.397	0.523	0.481	0.195	0.098
NASDAQ	0.057	0.006	0.224	0.064	0.091	0.005
Dow Jones	0.017	0.000	0.028	0.265	0.359	0.002
CAC 40	0.031	0.034	0.113	0.145	0.239	0.009
DAX	0.003	0.000	0.544	0.016	0.263	0.003
FTSE 100	0.000	0.004	0.015	0.337	0.131	0.007

Table 5.10: P-values of GC tests - cryptocurrency to equity (2019 - 2020)

Asset	S&P 500	Nikkei	Russell	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.000	0.001	0.002	0.073	0.001	0.065	0.303	0.581	0.326
Ethereum	0.000	0.001	0.002	0.001	0.003	0.041	0.019	0.442	0.038
USDT	0.003	0.354	0.060	0.112	0.099	0.199	0.180	0.805	0.465
Ripple	0.000	0.000	0.001	0.265	0.002	0.015	0.089	0.461	0.498
DogeCoin	0.137	0.372	0.316	0.091	0.138	0.165	0.214	0.167	0.177
Cardano	0.001	0.000	0.005	0.011	0.011	0.014	0.011	0.245	0.031

Table 5.11: P-values of GC tests - equity to cryptocurrency (2019 - 2020)

Asset	Bitcoin	Ethereum	USDT	Ripple	DogeCoin	Cardano
S&P 500	0.000	0.001	0.003	0.000	0.643	0.056
Nikkei 225	0.444	0.298	0.002	0.639	0.570	0.888
Russell 2000	0.002	0.000	0.013	0.003	0.818	0.037
FTSE MIB	0.000	0.000	0.026	0.000	0.126	0.000
NASDAQ	0.006	0.000	0.048	0.007	0.595	0.015
Dow Jones	0.000	0.004	0.002	0.000	0.527	0.031
CAC 40	0.000	0.021	0.000	0.006	0.266	0.007
DAX	0.001	0.000	0.107	0.003	0.000	0.001
FTSE 100	0.002	0.000	0.000	0.009	0.414	0.001

Table 5.12: P-values of GC tests - cryptocurrency to equity (2021 - 2024)

Asset	S&P 500	Nikkei	Russell	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.078	0.105	0.000	0.264	0.226	0.069	0.033	0.105	0.007
Ethereum	0.019	0.182	0.000	0.013	0.000	0.232	0.020	0.035	0.001
USDT	0.003	0.047	0.025	0.008	0.037	0.049	0.001	0.001	0.819
Ripple	0.000	0.000	0.581	0.000	0.387	0.000	0.054	0.094	0.002
DogeCoin	0.380	0.001	0.154	0.427	0.060	0.000	0.268	0.000	0.000
Cardano	0.209	0.000	0.242	0.000	0.505	0.000	0.032	0.366	0.009

Table 5.13: P-values of GC tests - equity to cryptocurrency (2021 - 2024)

Tables 5.8 through 5.13 in this section examine Granger causality between cryptocurrencies and major equity indices from 2019 to 2024, using p-values to reveal significant interdependencies. Lower p-values indicate strong causality, highlighting how movements in one market can predict changes in the other.

From 2019 to 2024, notable findings include Bitcoin's significant impact on the Nikkei 225 and Cardano's influence on various indices. This suggests that cryptocurrencies play an increasingly important role in predicting equity market trends. Conversely, equity indices such as the S&P 500 and the Nikkei 225 have strong predictive power over Bitcoin and Ethereum prices, indicating mutual influence between these markets.

During the 2019 to 2020 period, the results show that cryptocurrencies like Bitcoin and Ethereum significantly influenced equity markets. This era coincided with the increase in interest and adoption of cryptocurrencies, driven by increasing institutional investments and the advent of decentralized finance (DeFi). At the same time, the global equity markets were affected by macroeconomic events such as the US-China trade war and the onset of the COVID-19 pandemic. These events caused substantial market volatility, highlighting how traditional market dynamics were crucial for understanding crypto movements.

From 2021 to 2024, the data show continued strong bidirectional causality. Bitcoin and Ethereum maintained significant influence over equity indices, while traditional markets increasingly predicted cryptocurrency trends. This period was marked by significant economic recovery post-pandemic, massive fiscal stimuli, and the acceleration of technological adoption in finance. Notable stock market events included the meme stock craze and the unprecedented market reactions to interest rate changes by central banks.

The significant causality between cryptocurrencies and equities during these years underscores the importance of monitoring both markets. For example, the stock market's reactions to Federal Reserve policies, economic growth reports, and geopolitical tensions influenced cryptocurrency prices. In contrast, cryptocurrency market developments, such as regulatory news and major technological advancements, impacted equity markets.

These findings highlight the necessity of incorporating both cryptocurrency and equity markets in investment strategies. Bidirectional causality underscores the need for diversified approaches, as developments in one market can significantly impact the other. As cryptocurrencies continue to integrate further with traditional finance, their role in global markets will likely expand, presenting new opportunities and challenges for investors.

In conclusion, understanding the interplay between these markets during significant financial and macroeconomic events from 2019 to 2024 provides valuable insights for investors aiming to navigate the complex landscape of interconnected financial systems.

Figure 5.14 shows the Granger causality link ratios of the two sub-datasets and the full range dataset. The ratios shown in each cell reflect the number of links drawn when the asset is the *cause* (as opposed to the *effect*) divided by all possible combinations. Despite the high volatility during the 2019 to 2020 period, the sub-dataset from 2021 to 2024 shows a 10% increase in the number of links compared to the previous years. The ratio of total Granger causality links for the full-range dataset is the highest.

A particular result is the absence of links from the Russell 2000 and NASDAQ for the 2021 to 2024 sub-dataset. This could be due to market volatility, structural changes in the market, or other external factors affecting small-cap stocks represented by the Russell 2000 index. Similarly, this might be due to sector-specific events, regulatory changes, or shifts in investor sentiment affecting the tech-heavy NASDAQ index. Assets like Bitcoin, Ethereum, and Cardano have relatively high Granger causality link ratios across all periods, indicating their significant role in predicting other equity indexes in the dataset.

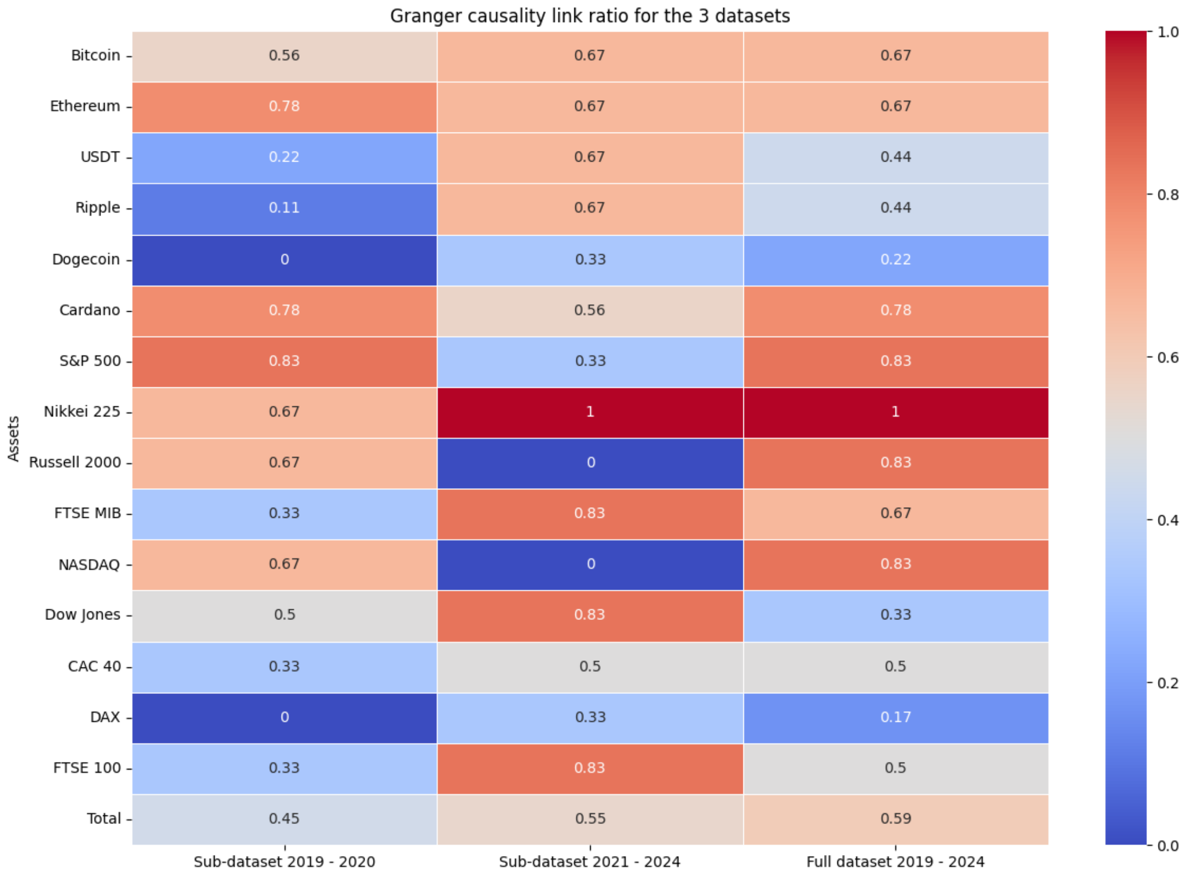


Figure 5.14: GC link ratio for the 3 datasets - 5% Level

Comparing these ratios with the article by Billio et al. [6], our results suggest a higher number of Granger causality connections. This difference can be attributed to several factors.

First, we built the Granger causality tests to take the minimum p-value across all lags, aiming to retain any potential connections, as explained in Section 4.3. This method may create false positive links. Second, the assets in scope are different from the standard financial companies used in Billio's article (banks, insurers, brokers, and hedge funds) and reflect a different environment (Billio's dataset spanned from 1994 to 2008). Cryptocurrencies have become increasingly popular in the last five years, and we incorporated the spikes of cryptocurrencies, which sometimes resulted in cryptocurrencies Granger-causing equity indices or individual stocks (e.g., Tesla's $140 billion loss in 2022 due to Bitcoin depreciation).

5.4.1 Granger causality estimation with rolling-window

To effectively anticipate and incorporate structural changes within the estimation process, techniques such as sample splitting and the introduction of dummy variables are often employed. However, these methods come with the disadvantage of introducing pre-test bias. To address issues of parameter non-constancy and to circumvent pre-test bias, we propose the utilization of the rolling-window sub-samples Granger causality test.

There are two principal justifications for employing the rolling estimation technique. Firstly, the rolling-window method aligns with the reality that causal relationships between variables evolve over time. Secondly, rolling estimation allows for the detection of instability across various sub-samples, which can be attributed to the occurrence of structural changes. In the study by Billio et al. [6], the authors applied a linear Granger causality test using a 36-month rolling-window to track the evolution of Granger causality among 100 institutions (including hedge funds, banks, brokers, and insurers) from 1994 to 2008. Their findings demonstrated that the number of causal connections increased during periods of crisis or market uncertainty, such as the American subprime crisis.

Inspired by Billio et al.'s approach, we analyze the evolution of Granger causality over the last five years using a 60-day rolling-window (approximately three months of data). For each sub-dataset, we test for Granger causality at a lag of 6, which was chosen for computational efficiency. The dataset is advanced forward by 20 days (approximately one month) for each subsequent sub-dataset. For each sub-dataset, we compute the proportion of connections relative to the total possible combinations, enabling us to observe the evolution of Granger causality over the five-year period.

The Granger causality test is performed in both directions, and the results are illustrated in Figure 5.15. The lines in the plot represent the proportion of all the connections analyzed, as detailed in the tables from 5.8 to 5.13. We observe the evolution of Granger causality connections with fixed lags in the VAR models set to p=6 and utilizing a smaller dataset (comprising 60 daily returns).

Figure 5.15 illustrates the ratio of the connections for two sub-datasets and the full range dataset. As illustrated in the literature, the number of connections increases during periods of crisis or market uncertainty, such as the COVID-19 pandemic and the conflict between Ukraine and Russia. The plot indicates peaks during these periods, specifically in early 2020 and late 2022.

Additionally, our analysis indicates that the equity market exhibits a more substantial Granger causality effect compared to the cryptocurrency market. The blue line, representing *equity* *causes cryptocurrency*, generally remains above the red line, indicating that the proportion of connections for *equity causes cryptocurrency* is higher than for *cryptocurrency causes equity*.

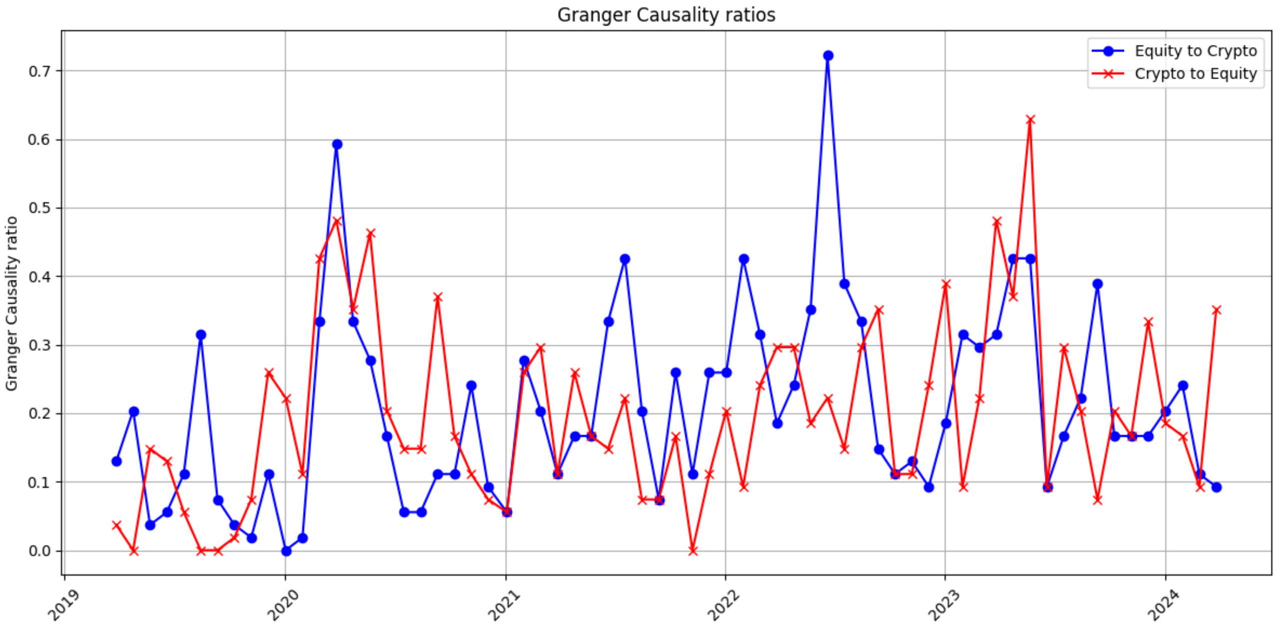


Figure 5.15: Evolution of GC connections - 5% Level

As illustrated in the literature, the number of connections tends to increase during periods of crisis or market uncertainty, such as the COVID-19 pandemic and the conflict between Ukraine and Russia. This phenomenon is evident in the plot, which indicates peaks during these periods, specifically in early 2020 and late 2022.

Asset	Bitcoin	Ethereum	USDT	Ripple	DogeCoin	Cardano	Total
S&P 500	33%	33%	33%	0%	33%	67%	33%
Nikkei 225	45%	64%	36%	45%	36%	82%	62%
Russell 2000	33%	67%	33%	0%	33%	33%	33%
FTSE MIB	0%	17%	50%	0%	17%	17%	17%
NASDAQ	0%	0%	33%	0%	0%	0%	6%
Dow Jones	0%	0%	0%	0%	50%	100%	25%
CAC 40	20%	0%	0%	0%	20%	0%	7%
DAX	33%	33%	17%	17%	0%	33%	22%
FTSE 100	40%	60%	20%	40%	40%	20%	37%
Total	23%	30%	25%	11%	26%	39%	27%

Table 5.14: GC connections ratios for rolling-window - Crypto to equity - 5% Level

Asset	S&P 500	Nikkei 225	Russell 2000	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100	Total
Bitcoin	33%	18%	0%	0%	0%	50%	20%	17%	0%	15%
Ethereum	33%	9%	67%	0%	33%	50%	20%	17%	0%	25%
USDT	33%	9%	67%	0%	0%	50%	20%	17%	40%	26%
Ripple	33%	9%	0%	17%	0%	50%	0%	0%	40%	17%
DogeCoin	0%	45%	33%	33%	0%	50%	60%	17%	40%	31%
Cardano	0%	45%	17%	17%	0%	100%	20%	33%	60%	34%
Total	22%	28%	33%	11%	6%	58%	23%	17%	30%	25%

Table 5.15: GC connections ratios for rolling-window - Equity to crypto - 5% Level

The percentages of Granger causalities connections for the rolling-window sub-datasets are shown, for the *cryptocurrencies to equity* in Table 5.14 and for *equity to cryptocurrencies* in Table 5.15. For both tables, the *Total* in the last rows refers to the ratio of the total amount of links caused by the asset in the same column, while the *Total* in the last column refers to the ratio of the total amount of links by which the asset in the same row was affected, at the 5% level of statistical significance.

Comparing these results with Figure 5.14, it is evident that incorporating the entire dataset (or at least more than three months of data) increases the number of links, as shown in the last row *Total* of Figure 5.14). This indicates that the number of links in the rolling-window analysis decreased significantly. It is important to note that the Granger causality tests are constructed differently: in the rolling-window application, a fixed number of lags were used, whereas in the previous analysis, the lag length was chosen using Akaike's criterion.

In the last row of Table 5.15, we observe that the Russell 2000 and NASDAQ indexes are the least strong in terms of Granger-causing cryptocurrencies, whereas the Dow Jones index caused more than one out of two possible links in the entire rolling-window dataset. An interesting finding is the 100% Granger causality links between the cryptocurrency Cardano and the Dow Jones index. Cardano is one of the most popular cryptocurrencies, with more than four million holders according to the CoinCarp website. However, due to the anonymous nature of cryptocurrencies, information about who holds these addresses is unavailable.

The observed differences in Granger causality connections can be attributed to the varying nature of the financial markets. Equity markets tend to have a more stable and structured environment, which may lead to more predictable and consistent causal relationships. On the other hand, the cryptocurrency market is relatively new and highly volatile, which may result in less stable causality patterns.

5.5 Conclusions

This thesis has undertaken an in-depth analysis of the interconnections between equity markets and cryptocurrencies using econometric models, specifically the VAR model and Granger causality tests. The research explained the intricate dynamics and predictive relationships that exist between these financial markets, highlighting both opportunities and challenges for investors and policymakers.

The introduction of cryptocurrencies, led by Bitcoin, has revolutionized the financial landscape by offering decentralized, transparent, and efficient alternatives to traditional financial systems. This study explored the foundational aspects of the cryptocurrency market, emphasizing the advantages brought by blockchain technology. The transformation of Bitcoin from a speculative asset to a legitimate component of the global financial system underscores the critical role of regulatory frameworks in this evolution. The unique characteristics of cryptocurrencies, such as their decentralized nature, advanced security, transparency, and potential for financial inclusivity, were highlighted.

A comprehensive literature review provided the context for understanding the existing body of knowledge on the relationship between equity and cryptocurrency markets, revealing significant gaps. This thesis aimed to test the bidirectional causality and interactions between these markets. The empirical evidence underscored the necessity of a robust methodological approach to accurately capture dynamic interactions.

The methodology section detailed the application of the VAR model to examine interdependencies between equity and cryptocurrency markets, ensuring reliability through careful selection of lags, stationarity tests, and stability conditions. Granger causality tests further clarified predictive relationships, providing a comprehensive understanding of how shocks in one market can influence the other.

The results revealed significant findings. The VAR model indicates strong bidirectional causality between major cryptocurrencies and equity indices, particularly during periods of high market activity. The impulse response functions provided deeper insights into the magnitude and duration of impacts, demonstrating that shocks in the cryptocurrency market could substantially affect equity markets, and vice versa. Key observations included the increased interconnections during the COVID-19 pandemic and the dynamic nature of these relationships over different time frames.

From Chapter 5, several important observations emerged. The period from 2019 to 2020, characterized by high market volatility and the advent of the COVID-19 pandemic, showed stronger interconnections between markets. The VAR model often selected higher lags for this period, indicating stronger short-term connections. In contrast, the 2021 to 2024 period, although still exhibiting significant interactions, showed a tendency for the model to select smaller lags, reflecting a relatively less interconnected market compared to the pandemic period. This variability in lag selection underscores the importance of considering different time horizons in understanding market dynamics.

The Granger causality tests revealed that cryptocurrencies such as Bitcoin and Ethereum had a significant influence on equity markets, particularly during periods of heightened market activity. Conversely, equity indexes like the S&P 500 and Nikkei 225 exhibited strong predictive power over cryptocurrency prices, indicating a mutual influence. The findings from the rolling-window analysis further highlighted the dynamic nature of these relationships, with varying degrees of causality observed over different time frames. This dynamic analysis provided a nuanced view of how the strength and direction of causal relationships evolved over time, reflecting changes in market conditions and external shocks.

The growing involvement of institutional investors in the cryptocurrency market is identified as a crucial factor driving these interconnections. This institutional presence not only brings substantial capital but also introduces professional trading strategies and risk management practices that increase market stability. The role of macroeconomic factors, such as central bank policies and economic indicators, was also highlighted as significant drivers of these intermarket dynamics. The analysis emphasized how macroeconomic stability, interest rate policies, and economic growth indicators influenced both equity and cryptocurrency markets, reinforcing their interconnected nature.

Regulatory developments emerged as a pivotal element influencing the relationship between equity and cryptocurrency markets. The evolution of regulatory frameworks, from initial neglect to comprehensive oversight, has profound implications for market behavior and investor confidence. The thesis emphasized the necessity of coordinated international regulatory efforts to address the challenges posed by the global nature of cryptocurrencies and to ensure financial stability. The discussion covered various regulatory approaches, from strict licensing requirements to outright bans, and their differential impacts on market integration and investor behavior.

In conclusion, this thesis provides a detailed and rigorous analysis of the interconnections between equity and cryptocurrency markets. The findings highlight the importance of understanding these relationships for effective portfolio management and policy formulation. The integration of cryptocurrencies into the mainstream financial system presents both opportunities for diversification and risks that require careful regulatory oversight. Future research should continue exploring the long-term impacts of technological innovations and regulatory changes on these evolving markets, ensuring that the benefits of financial innovation are realized while mitigating potential systemic risks. This ongoing research is essential to adapt to the rapid evolution of digital finance and to develop strategies that balance innovation with financial stability. Understanding these complex interrelations will be crucial for investors and policymakers aiming to navigate the increasingly interconnected global financial landscape effectively.

Appendix

Additional Tables

Asset	S&P 500	Nikkei 225	Russell 2000	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	1.61	0.62	1.31	1.00	1.36	1.55	1.07	0.50	1.06
Ethereum	2.90	2.00	2.18	1.32	2.29	3.02	2.42	0.66	2.75
USDT	0.05	0.01	0.02	0.02	0.02	0.07	0.03	-0.00	0.04
Ripple	1.50	0.90	1.17	1.11	1.27	1.86	1.45	0.46	1.96
DogeCoin	0.18	-0.39	-0.14	-0.30	-0.04	0.66	1.37	-0.03	-0.58
Cardano	2.26	1.44	1.81	1.52	1.93	1.90	1.93	1.08	1.76

Table 5.16: Covariance between cryptocurrency and equity returns from January 2019 to December 2020

Asset	S&P 500	Nikkei 225	Russell 2000	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.94	0.85	0.88	0.82	0.87	1.47	1.51	0.57	1.20
Ethereum	1.37	1.14	1.25	1.18	1.22	1.83	1.98	0.84	1.60
USDT	-0.00	0.00	-0.00	-0.00	-0.00	0.00	0.01	0.00	0.01
Ripple	1.11	0.89	0.87	1.05	1.03	1.65	1.86	0.60	1.40
DogeCoin	1.78	-0.04	1.41	1.16	1.57	1.59	0.64	-0.46	2.15
Cardano	1.34	1.21	1.16	1.14	1.13	2.03	2.13	0.75	2.02

Table 5.17: Covariance between cryptocurrency and equity returns from January 2021 to April 2024

	Crypto						**Equity Index**								
Asset	**BTC**	**ETH**	**USDT**	**XRP**	**DOGE**	**ADA**	**S&P**	**Nikkei**	**Russell**	**FTSE MIB**	**NASDAQ**	**Dow Jones**	**CAC**	**DAX**	**FTSE 100**
BTC	16.44	15.73	0.16	13.09	17.88	15.77	1.19	1.30	1.04	0.79	1.06	1.50	1.95	0.29	2.07
ETH	15.73	27.31	0.14	17.55	22.11	20.47	1.94	2.05	1.60	1.21	1.62	2.28	2.82	0.35	2.95
USDT	0.16	0.14	0.03	0.10	0.07	0.15	0.01	0.03	0.01	0.01	0.01	0.03	0.03	0.00	0.03
XRP	13.09	17.55	0.10	40.64	20.27	21.92	1.26	1.44	0.98	0.88	1.12	1.73	2.26	0.41	2.35
DOGE	17.88	22.11	0.07	20.27	251.75	25.97	1.18	1.04	0.83	0.63	0.96	1.24	1.54	-0.14	1.62
ADA	15.77	20.47	0.15	21.92	25.97	33.51	1.68	1.68	1.40	1.12	1.43	1.98	2.56	0.59	2.79
S&P	1.19	1.94	0.01	1.26	1.18	1.68	1.88	1.05	1.55	1.18	1.57	1.04	1.07	0.43	1.33
Nikkei	1.30	2.05	0.03	1.44	1.04	1.68	1.05	1.59	0.99	0.82	1.00	1.56	1.62	0.34	1.82
Russell	1.04	1.60	0.01	0.98	0.83	1.40	1.55	0.99	1.57	1.16	1.50	0.97	0.97	0.49	1.25
FTSE MIB	0.79	1.21	0.01	0.88	0.63	1.12	1.18	0.82	1.16	1.15	1.13	0.77	0.73	0.43	1.00
NASDAQ	1.06	1.62	0.01	1.12	0.96	1.43	1.57	1.00	1.50	1.13	1.63	1.00	1.04	0.50	1.28
Dow Jones	1.50	2.28	0.03	1.73	1.24	1.98	1.04	1.56	0.97	0.77	1.00	1.67	1.87	0.33	1.89
CAC	1.95	2.82	0.03	2.26	1.54	2.56	1.07	1.62	0.97	0.73	1.04	1.87	2.34	0.34	2.11
DAX	0.29	0.35	0.00	0.41	-0.14	0.59	0.43	0.34	0.49	0.43	0.50	0.33	0.34	1.39	0.41
FTSE 100	2.07	2.95	0.03	2.35	1.62	2.79	1.33	1.82	1.25	1.00	1.28	1.89	2.11	0.41	2.79

Table 5.18: Variance-Covariance matrix of returns for the full range sample

Asset	S&P 500	Nikkei 225	Russell 2000	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.22	0.09	0.20	0.17	0.20	0.22	0.15	0.10	0.13
Ethereum	0.31	0.26	0.25	0.18	0.26	0.33	0.28	0.10	0.27
USDT	0.11	0.01	0.05	0.06	0.04	0.17	0.07	-0.01	0.09
Ripple	0.14	0.09	0.12	0.13	0.13	0.18	0.14	0.06	0.15
DogeCoin	0.01	-0.02	-0.01	-0.02	-0.00	0.03	0.07	-0.00	-0.03
Cardano	0.23	0.14	0.20	0.17	0.20	0.20	0.18	0.15	0.15

Table 5.19: Correlations of returns from January 2019 to December 2020

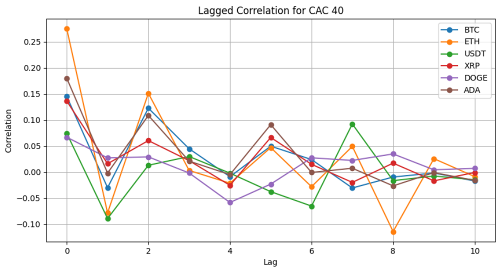
Asset	S&P 500	Nikkei 225	Russell 2000	FTSE MIB	NASDAQ	Dow Jones	CAC 40	DAX	FTSE 100
Bitcoin	0.21	0.21	0.22	0.25	0.21	0.36	0.27	0.12	0.20
Ethereum	0.24	0.21	0.24	0.28	0.23	0.35	0.28	0.15	0.22
USDT	-0.06	0.05	-0.06	-0.04	-0.05	0.09	0.08	0.01	0.10
Ripple	0.15	0.12	0.13	0.20	0.15	0.24	0.21	0.09	0.16
DogeCoin	0.09	-0.00	0.08	0.08	0.09	0.09	0.03	-0.03	0.10
Cardano	0.20	0.18	0.20	0.24	0.19	0.34	0.26	0.11	0.24

Table 5.20: Correlations of returns from January 2021 to April 2024

Asset	Test Statistic	p-value	Lags used	# obs used	Critical Value (5%)
Bitcoin	-37.300	0	0	1384	-2.864
Ethereum	-25.638	0	1	1383	-2.864
USDT	-14.011	0	10	1374	-2.864
Ripple	-27.405	0	1	1383	-2.864
DogeCoin	-11.718	0	7	1377	-2.864
Cardano	-24.851	0	1	1383	-2.864
S&P 500	-11.752	0	8	1376	-2.864
Nikkei 225	-11.024	0	9	1375	-2.864
Russell 2000	-13.198	0	7	1377	-2.864
FTSE MIB	-13.528	0	7	1377	-2.864
NASDAQ	-11.678	0	8	1376	-2.864
Dow Jones	-10.955	0	8	1376	-2.864
CAC 40	-11.348	0	8	1376	-2.864
DAX	-12.338	0	8	1376	-2.864
FTSE 100	-11.555	0	8	1376	-2.864

Table 5.21: Statistics referring to the ADF test for stationarity

Additional Graphs



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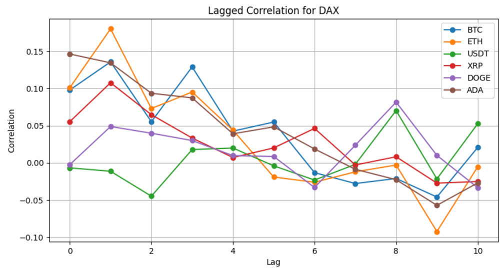
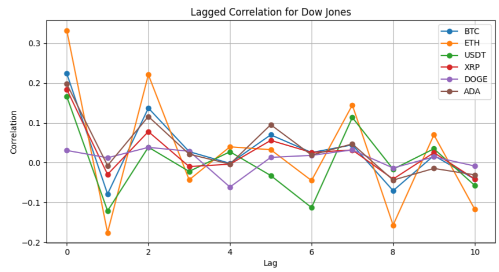
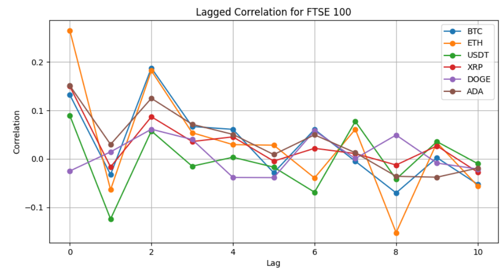


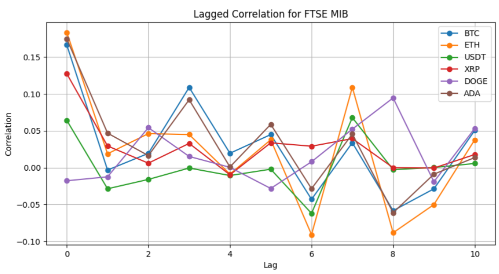
(b) DAX



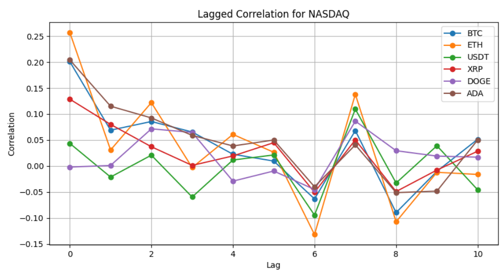
(c) Dow Jones



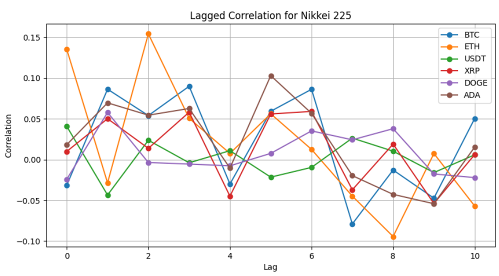
(d) FTSE 100



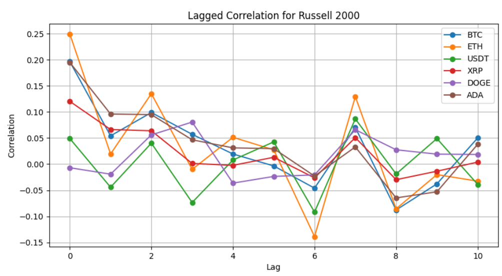
(e) FTSE MIB



(f) NASDAQ

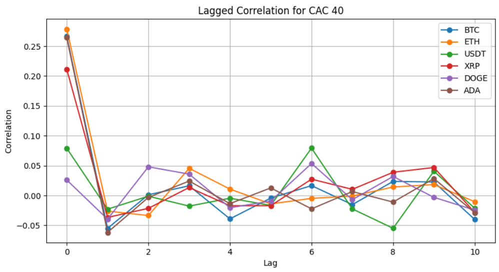


(g) Nikkei 225

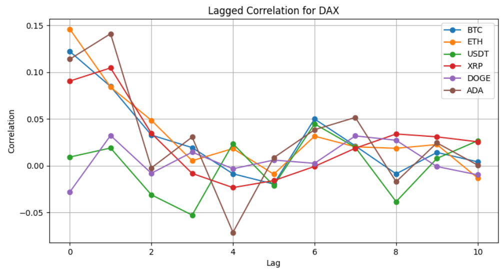


(h) Russell 2000

Figure 5.16: Lagged correlations for equity indexes from 2019 to 2020



(a) CAC 40



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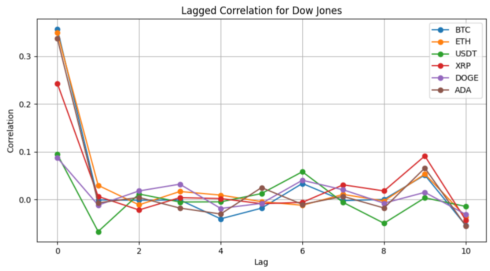
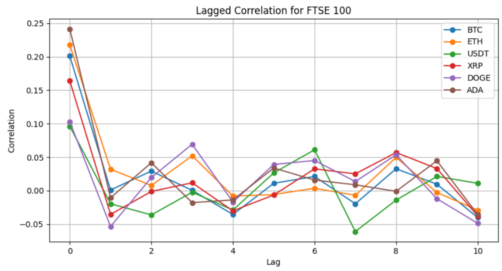


(c) Dow Jones



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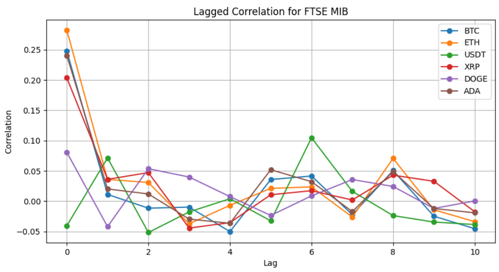
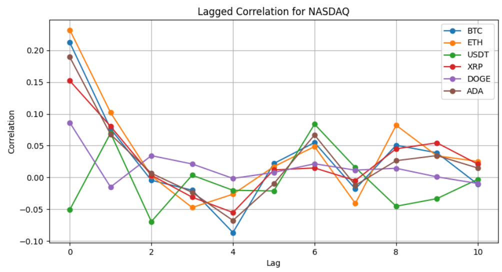
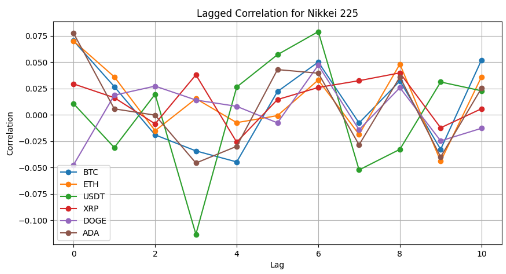


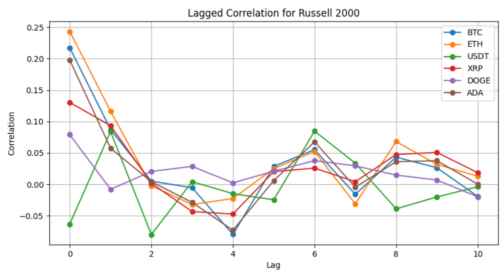
(e) FTSE MIB



(f) NASDAQ

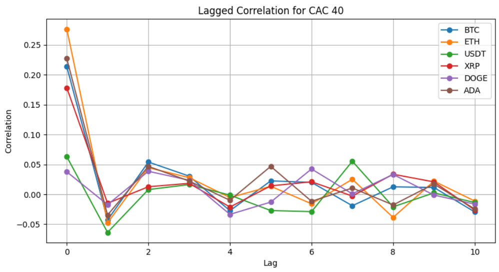


(g) Nikkei 225

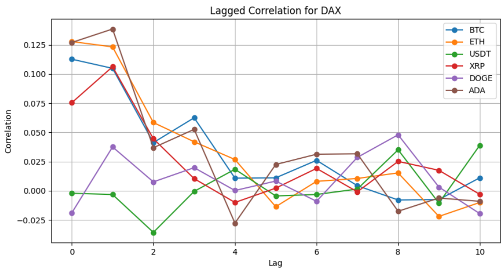


(h) Russell 2000

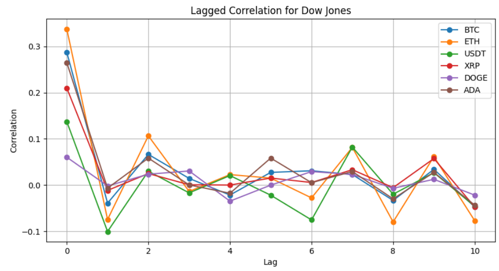
Figure 5.17: Lagged correlations for equity indexes from 2021 to 2024



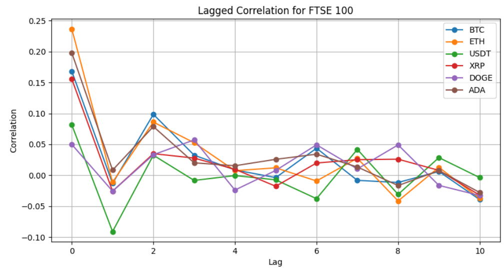
(a) CAC 40



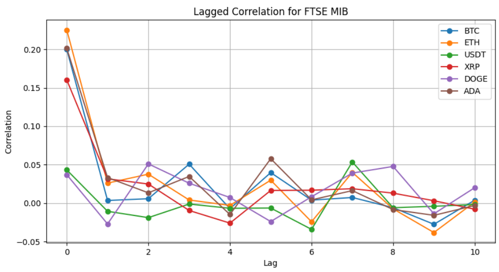
(b) DAX



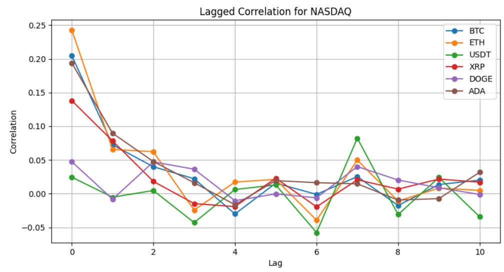
(c) Dow Jones



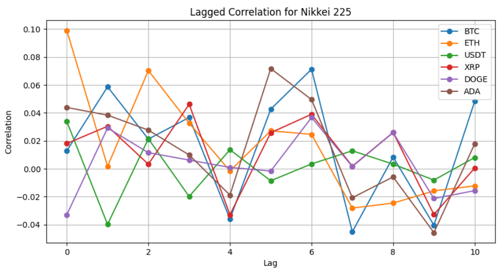
(d) FTSE 100



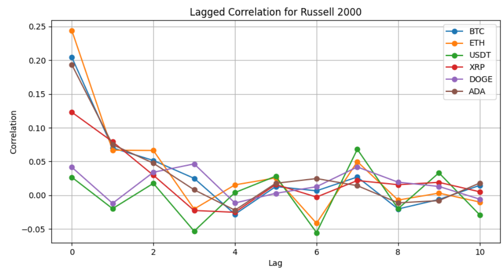
(e) FTSE MIB



(f) NASDAQ



(g) Nikkei 225



(h) Russell 2000

Figure 5.18: Lagged correlations for equity indexes for the full sample (from 2019 to 2024)

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