

Exercise 4

$$A = (0, 0, 0, 1)$$

$$B = (0, 5, 0, 0, 1) \quad \text{vertex}$$

$$C = (0, 1/\sqrt{2}, 0, 1)$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & -0,25 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} \text{MODEL-VIEW} \\ \text{PROJECTION MATRIX} \end{array}$$

$$A' = M \times A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = [0, 0, -1/4, 1]$$

$$B' = M \times B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = [1/2, 0, -1/4, 1]$$

$$C' = M \times C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & -1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ 1 \end{bmatrix} = [0, 1/2, 1/4, 1]$$

Now I divide all points by their fourth dimension

$$A' = [0, 0, -1/4] \quad B' = [1/2, 0, -1/4] \quad C' = [0, 1/2, 1/4]$$

1 Remove the z coordinate for each point because we are in 2D

$$A' = (0, 0) \quad B' = \left(\frac{1}{2}, 0\right) \quad C' = \left(0, \frac{1}{2}\right)$$

$$\text{Area of the image} = (1 - (-1)) \cdot (1 - (-1)) = 2 \cdot 2 = 4$$

$$\text{Area of the triangle} = \frac{\|AB\| \cdot \|AC\|}{2} = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$\text{Percentage covered} = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32} = 3,125\%$$