Master thesis project:

Modeling and identification of Limit Cycle Oscillations in thermoacoustic instabilities

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Thermoacoustics studies phenomena where the interaction between acoustic wave and heat transfer effects plays a crucial role [1]. A broad class of problems that is described within this framework is represented by special types of heat engines and refrigerators where the conversion heat-work is performed with no moving parts by exploiting pressure waves. These machines have a high potential for the development of sustainable energy systems, and this has stimulated a large body of research on the topic [2].

Thermoacoustic instabilities are another very important class of phenomena [3]. They arise in systems where acoustic waves adversely couple with unsteady heat release, determining loss of stability in the form of self-sustained oscillations, or Limit Cycle Oscillations (LCO). This problem is particulary relevant in jet engines, where a complex combustion dynamics take place, and therefore research in this domain has largely focused on suppressing these behaviours to improve efficiency [4]. However, from an energy harvesting perspective, thermoacoustic self-sustained oscillations have a great, yet unexplored, potential. Indeed, one can think of extracting the mechanical power from the heat-excited waves and convert it into electricity or directly use it to drive other thermoacoustic machines (e.g. the aforementioned heat pumps). Before attempting to actively control these oscillations in order to achieve the desired performance, it is of paramount importance the development of reliable models which allow the response to be predicted and the design parameters' effect on the system's behaviour to be quantified.

Most of the literature for control-oriented models has considered a simplified representation consisting of a linear (single harmonic) acoustic field in feedback loop with the heat release modelled as a static delayed saturation-like nonlinearity [5]. While this schematization is sufficient to justify the presence of LCO and, in some cases, to capture amplitude and frequency of the oscillations, research has suggested that it might not be an exhaustive description of the problem. For example, in [6] non-harmonic oscillators and the coexistence of multiple attractors were found experimentally. Modeling, and its closely related task of identification, of thermoacoustic instabilities is thus still an open problem and an active area of research.

1 Project research directions

The project is articulated around two main objectives.

1.1 Derivation of a model from first principles

First, a numerical model (based on first principles) to quantitatively describe thermoacoustic instabilities will be derived. On one hand, this model will overcome limitations and/or simplifications of most of the present control-oriented approaches. On the other, it will provide a thorough understanding of the most important physical mechanisms featuring the problem. This understanding will be key to inform the following system identification process. Moreover, it will provide guidelines for the well-known trade-off between complexity and reliability needed for models used for active control.

In this step the student will be aided by the vast literature available on acoustics and thermodynamic modelling of combustion instabilities. In addition, several open-source software (e.g. *DeltaEC* from Los Alamos National Laboratory, *Oscilos* from Imperial College London) are available to take inspiration or directly model specific sub-components of the system.

The developed model will be tested against literature results. In particular, its ability to reproduce the dynamics of the Rijke tube [7], a known benchmark for thermoacoustic instabilities, will be assessed. Simulation of the models can be performed via different approaches, e.g. time marching, harmonic balance, numerical continuation.

1.2 System identification for oscillating systems

The second objective is the investigation and development of system identification strategies tailored for nonlinear systems experiencing LCOs. Nonlinear system identification is a far less developed topic than its linear counterpart, and approaches able to encode the existence of an LCO as qualitative constraint of the identification process are still lacking despite a few important attempts [8–10]. Therefore, there is a great potential for research that looks at this very challenging problem -considering also that its outcomes could be transferred to other engineering domains where LCOs are relevant (e.g. power systems, aerospace, biology).
A starting point for this step could be represented by classic nonlinear system identification schemes (e.g. Hammerstein–Wiener models; basis functions-based black box, semi-physical modelling grey box), moving then to approaches which makes direct use of the information of periodicity of the response (e.g. flame-describing function [11]) . In view of the known problem of identifiability of oscillating systems (due to the limited amount of information gathered in the steady-state response), it is envisaged that specific approaches will be explored (e.g. harmonic regressors [12]) and aspects such as experiment design will deserve a special focus.

While typically system identification methods are ultimately tested against experiments, due to the substantial work expected for the investigation of new algorithms and approaches, this will not be asked in the project. Instead, the identification algorithms will be numerically evaluated on a data bank of test systems generated by the model developed in the first step.

2 Work flow and possible timeline

The project main steps are briefly recapped in the following:

- literature review on thermoacoustic instabilities and system identification (with focus on nonlinear systems experiencing LCOs) - 0.5/1 month ca.
- development of a model which is able to capture instabilities in different thermoacoustic systems (e.g. jet engines, Rijke tube) and is amenable to simulation with different techniques (time marching, harmonic balance, bifurcation) - 1/2 month ca.
- investigation of system identification algorithms tailored for nonlinear systems exhibiting self-sustained oscillations - 2/3 month ca.
- writing-up dissertation - 1 month ca.

Beneficial skills (or interests) of the prospective applicant: modelling physical systems using thermodynamics and acoustic principles; studying nonlinear dynamics behaviours such as bi-stability and LCOs within rigorous mathematical framework (e.g. bifurcation); investigating control theory-based system identification approaches and their application to the thermoacoustic problem.

Earliest start: September 2019.

References

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