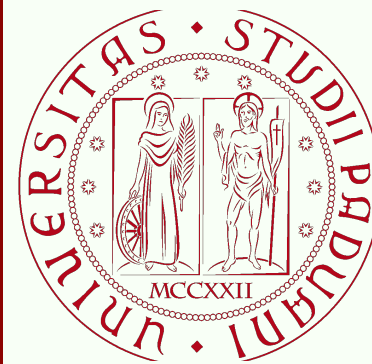




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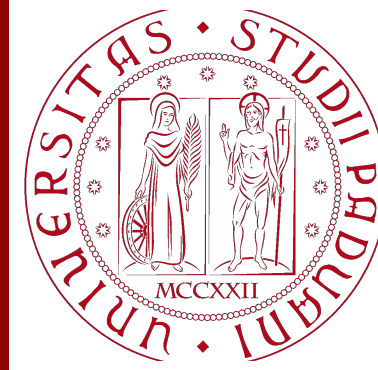
Biathlon analysis with Bayesian inference

Tommaso Amico, Andrea Lazzari 05/07/2022

Advanced Statistics for Physics analysis course 2021/2022, Prof. Alberto Garfagnini



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The Formats

Sprint:

- 3 loops
- One prone shooting range
- One standing shooting range
- A penalty lap for each miss
- The athletes start at equal time intervals

Pursuit:

- 5 loops
- A first prone shooting range
- A second prone shooting range
- A first standing shooting range
- A second standing shooting range
- A penalty lap for each miss
- The athletes start following the sprint's results

Mass start:

- 5 loops
- A first prone shooting range
- A second prone shooting range
- A first standing shooting range
- A second standing shooting range
- A penalty lap for each miss
- The athletes start all together

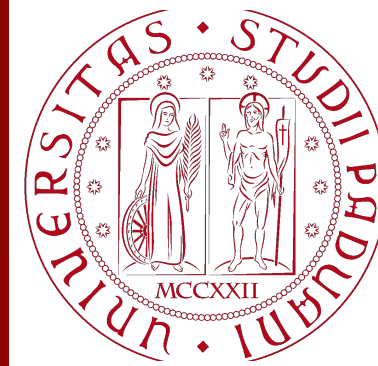
Individual:

- 5 loops
- A first prone shooting range
- A first standing shooting range
- A second prone shooting range
- A second standing shooting range
- 1 additional minute for each miss
- The athletes start at equal time intervals





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The analysis

- Consider only head-to head races
- Separate the data frame, for each athlete, in races where he entered the last shooting range in the top 10 from races where he entered outside of it
- Compute and analyse the efficiency
- Look for differences among men and women using a gaussian approximation and Markov Chain Monte Carlo with the Rjags library

Vaccine analogy

- Placebo shots -> Bullets shot when outside the top 10
- Vaccine shots -> Bullets shot when in the top 10

• Efficiency ->
$$\frac{\text{Placebo} - \text{Vaccine}}{\text{Placebo}} \cdot 100$$

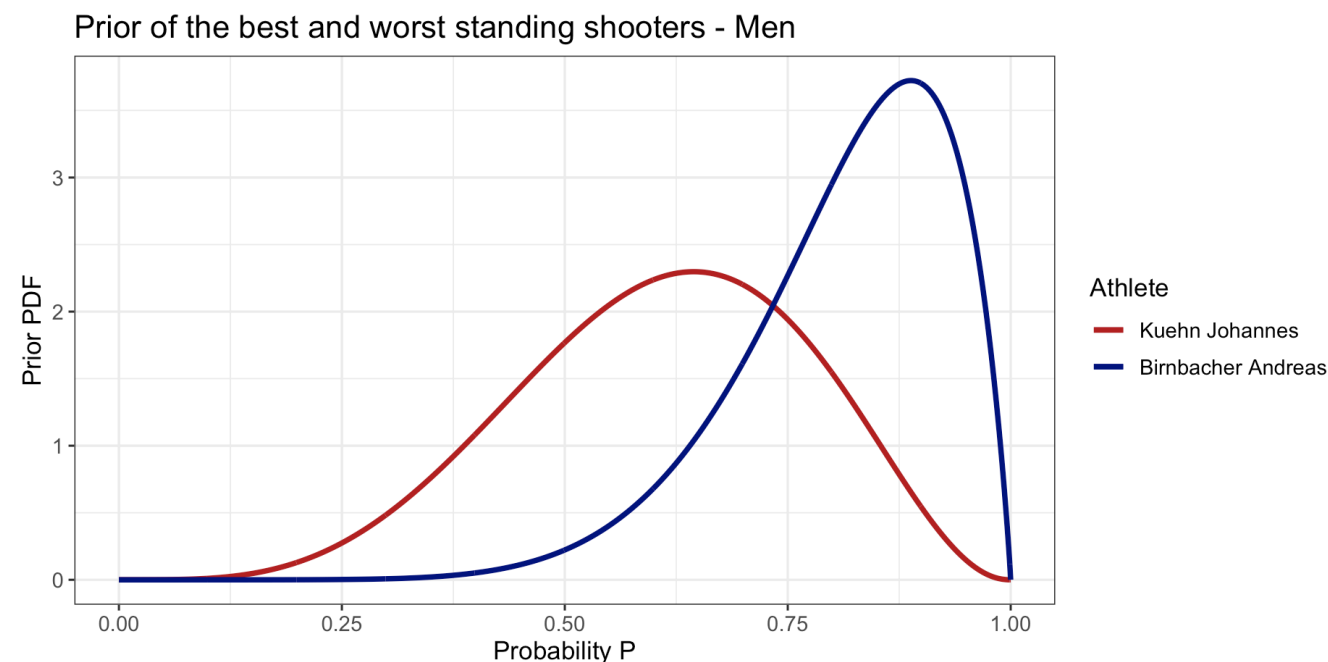


Likelihood

- Binomial distribution

$$PDF(p | n, k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$$

- You either hit the target or you miss it

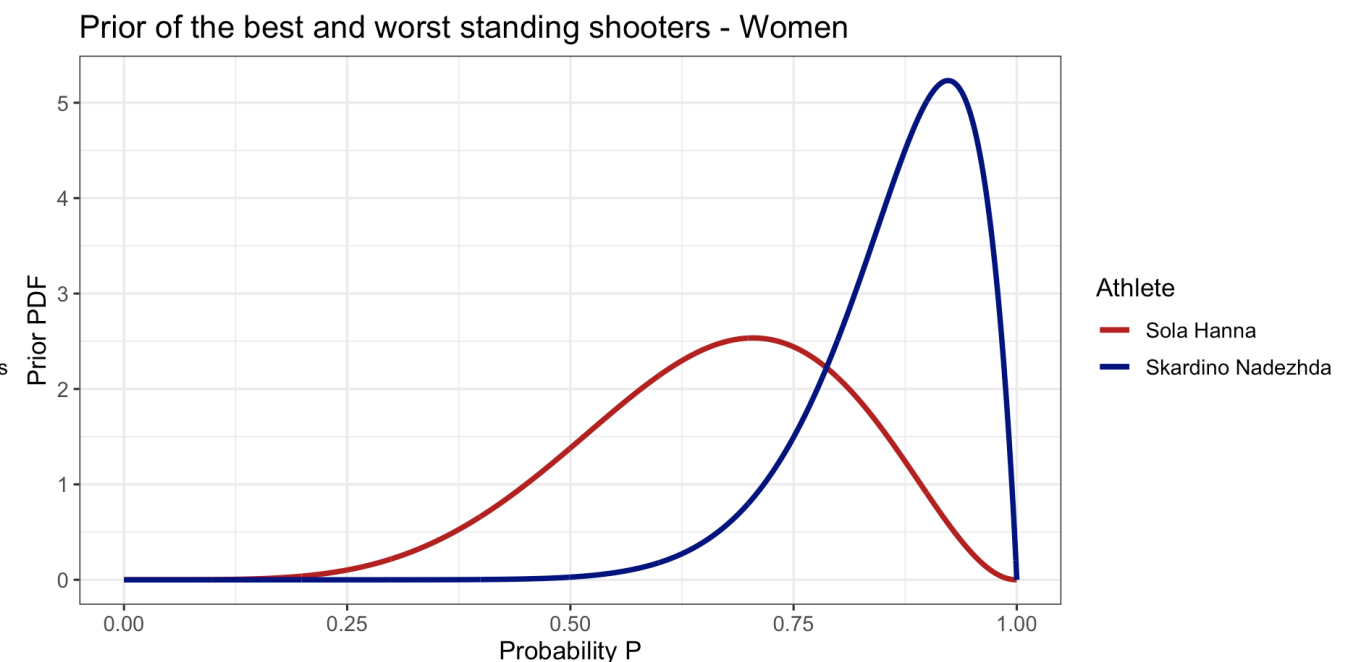


Prior

- Beta distribution

$$PDF(x | \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}$$

- Defined between 0 and 1
- Conjugate properties with the binomial likelihood





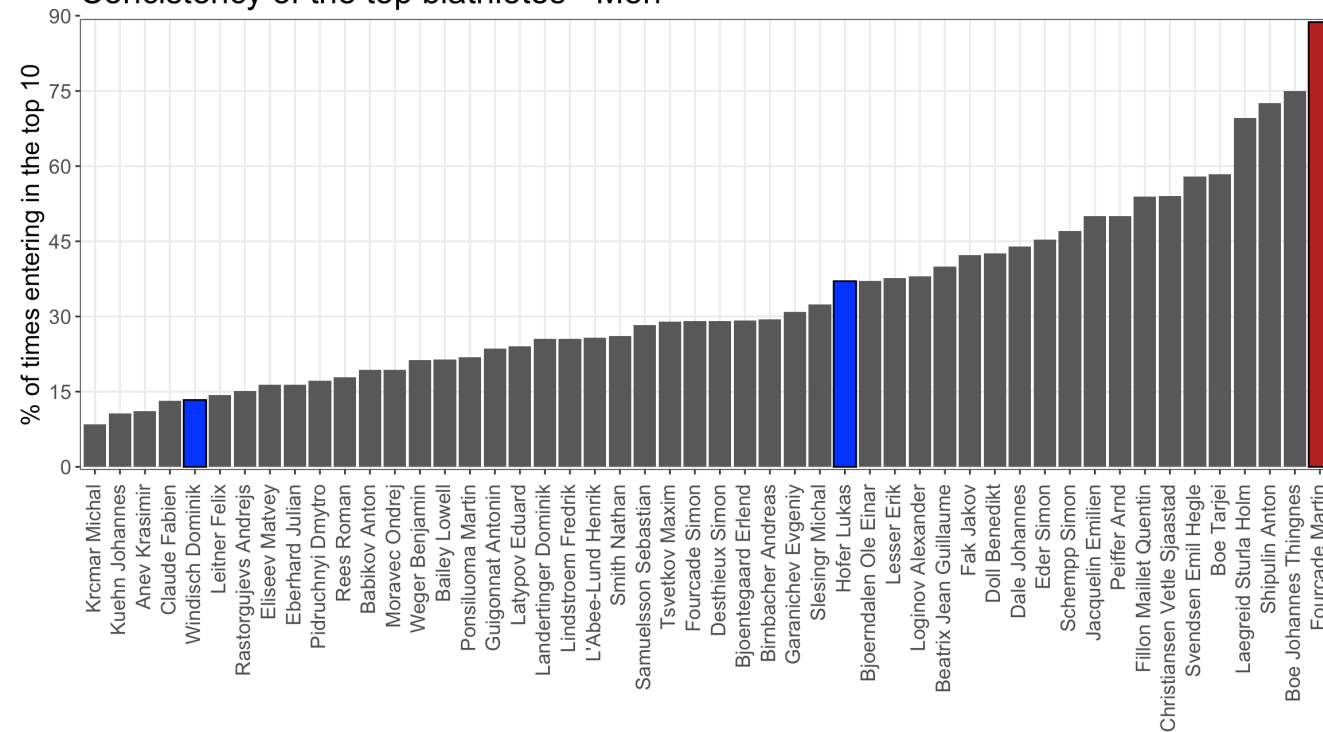
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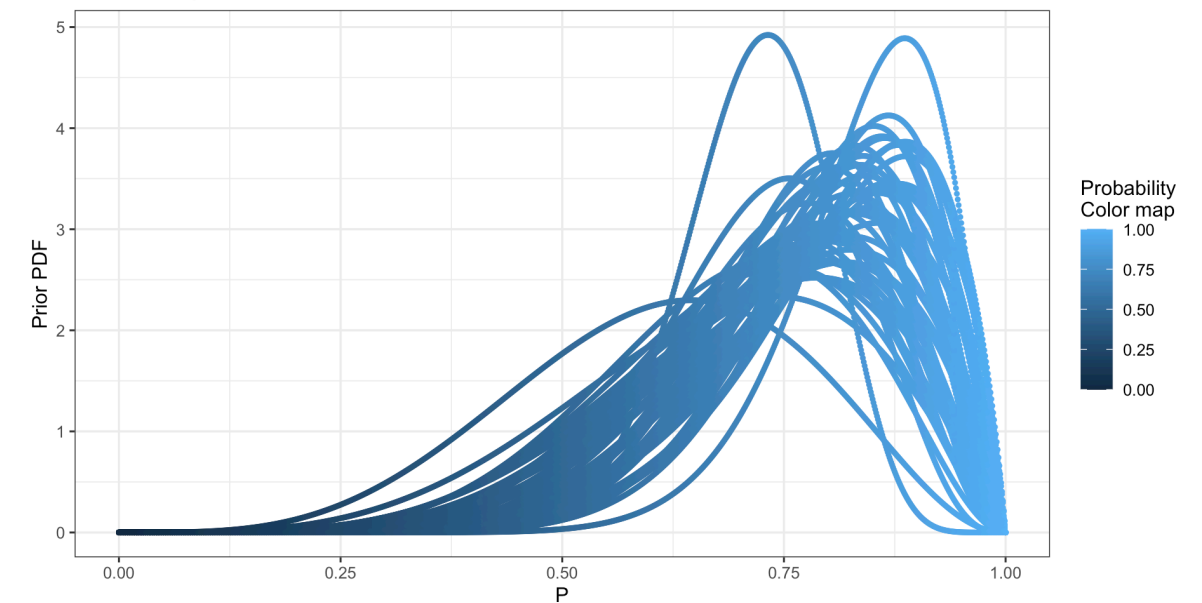
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Preliminary results

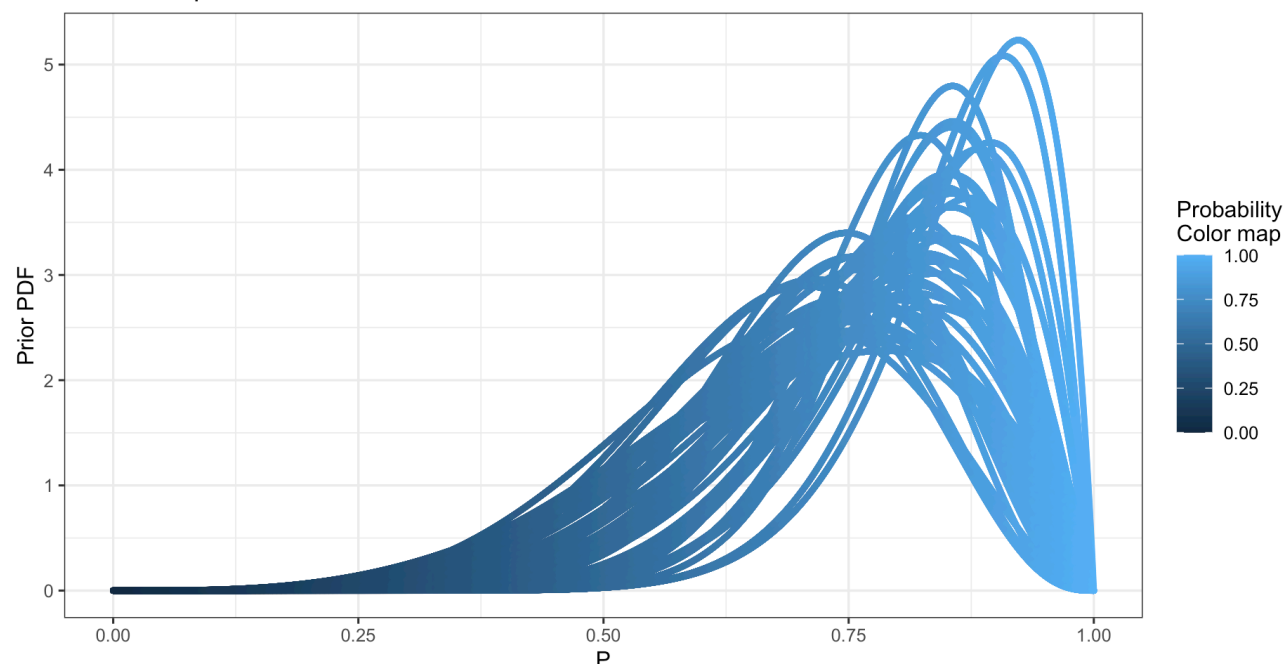
Consistency of the top biathletes - Men



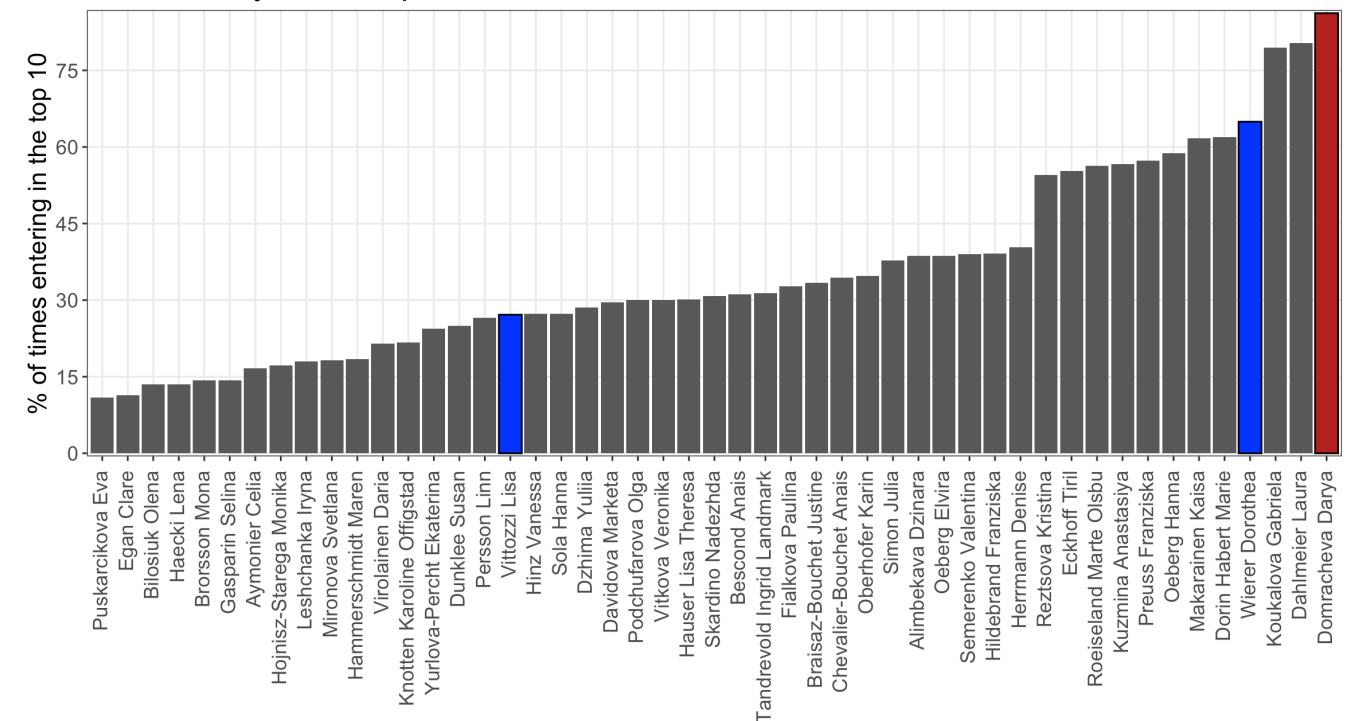
Priors computed - Men



Priors computed - Women

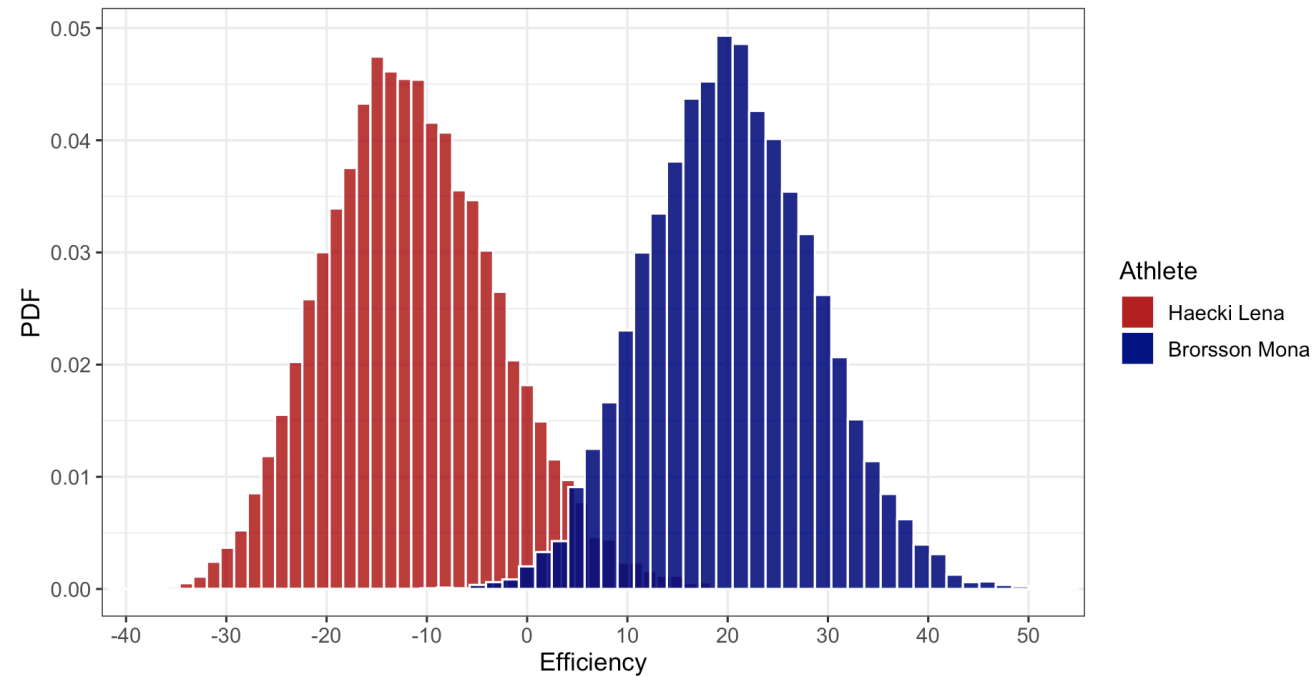


Consistency of the top biathletes - Women

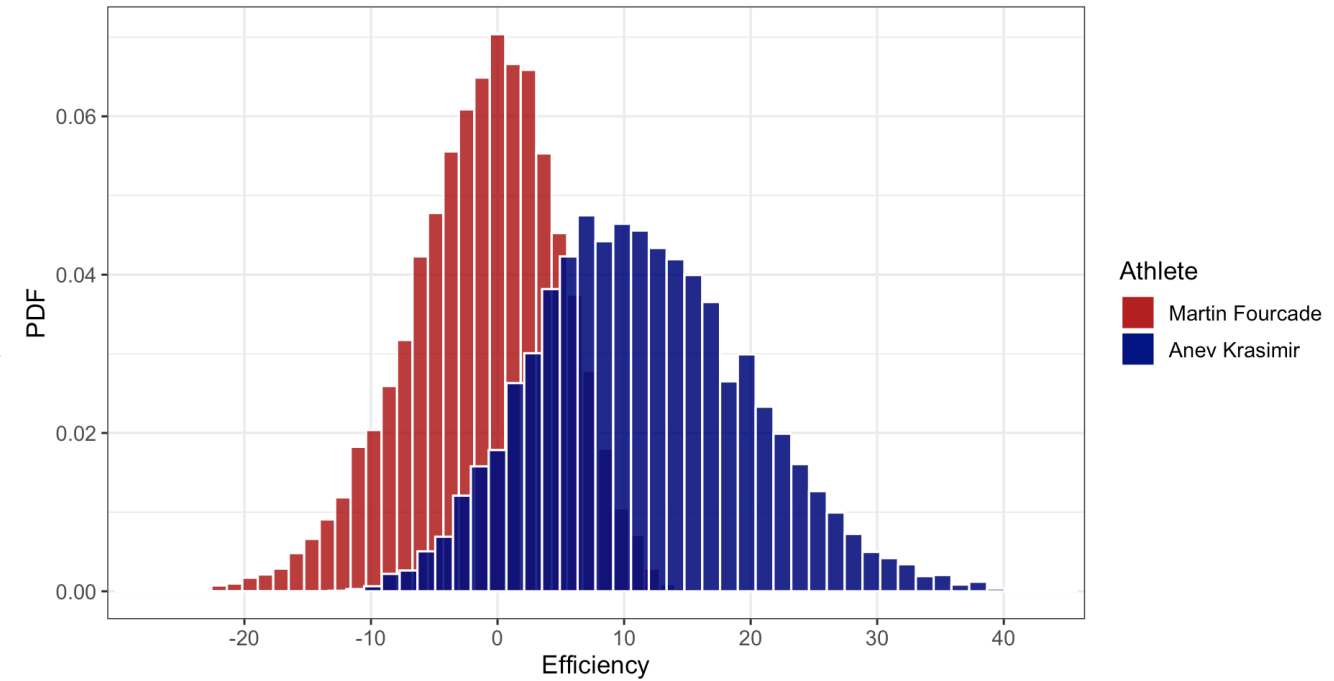


Results

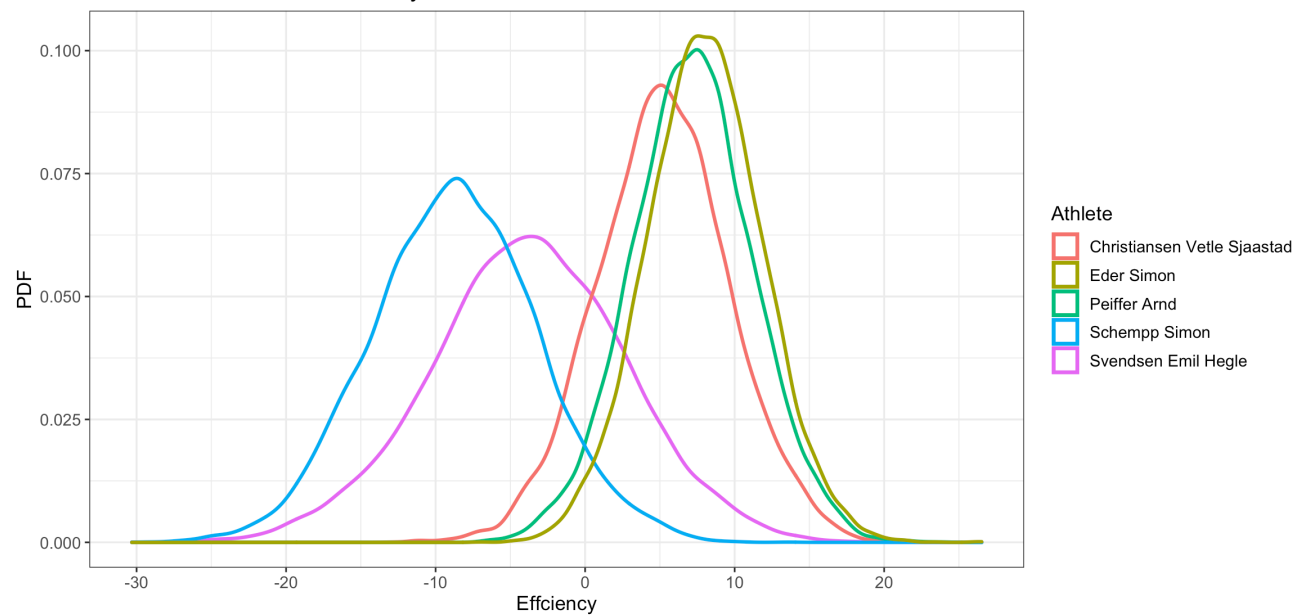
Athletes with the best and worst efficiency - Women



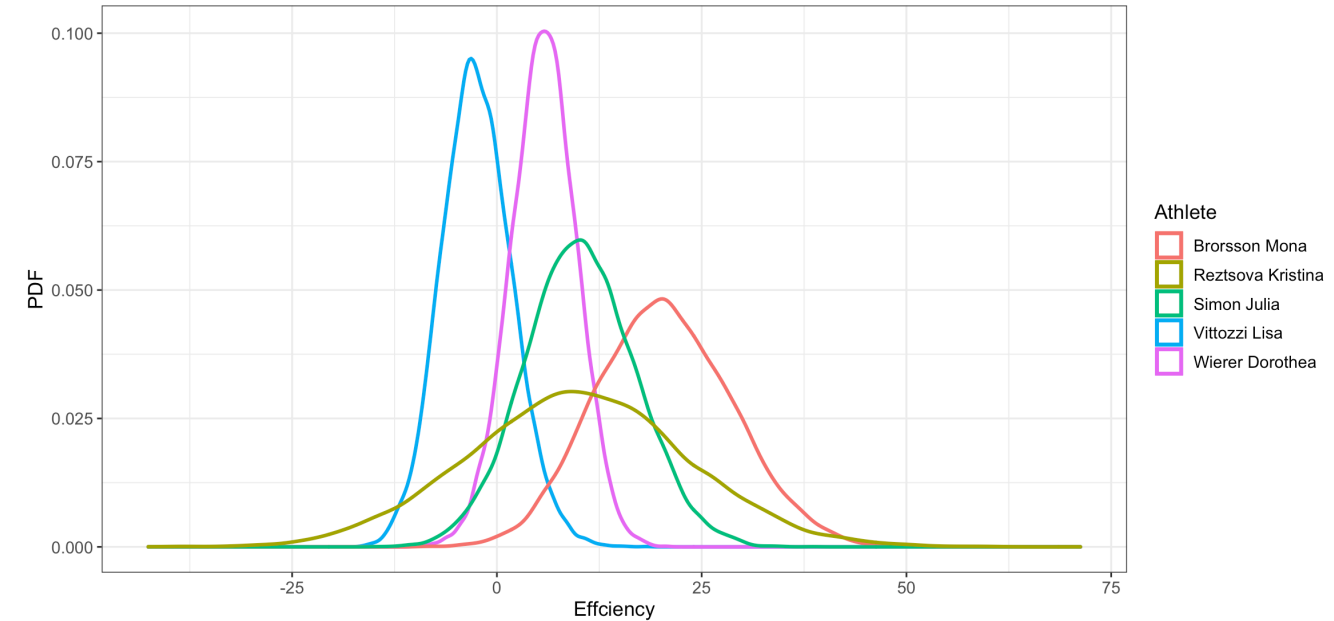
Efficiency of the most and least consistent biathlete - Men

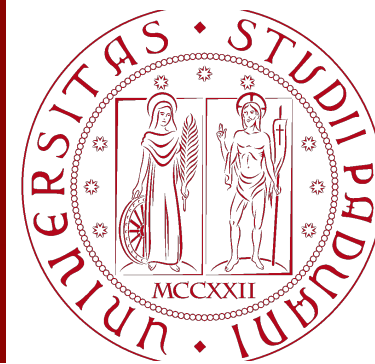


Athletes with related efficiency distributions - Men



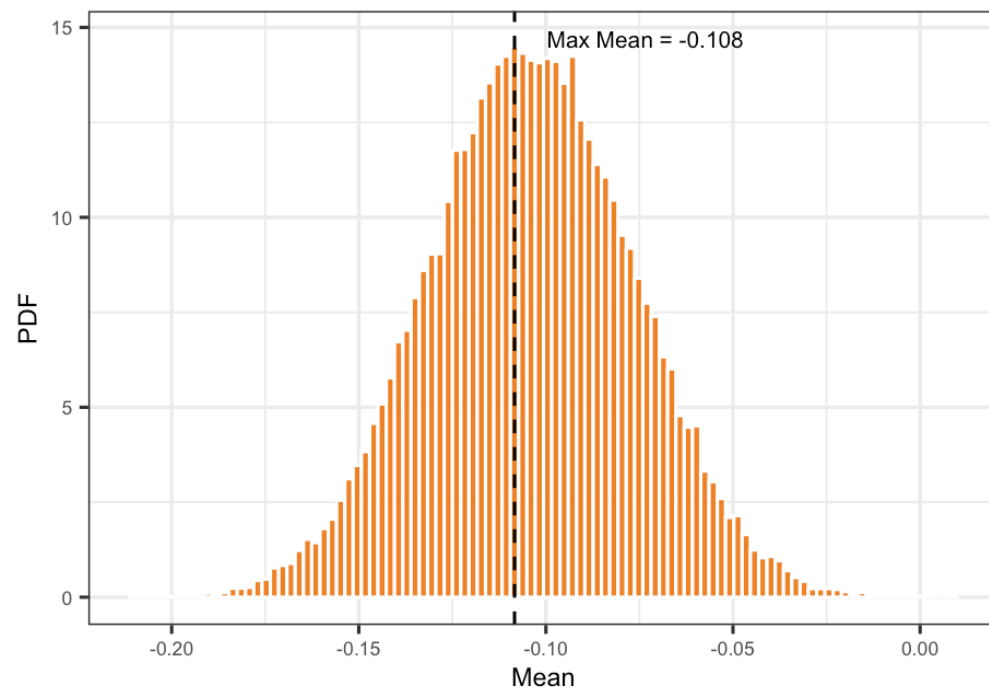
Athletes with related efficiency distributions - Women



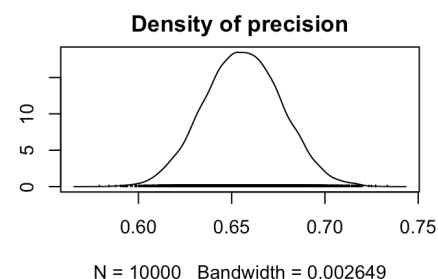
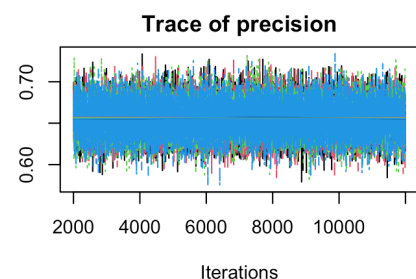
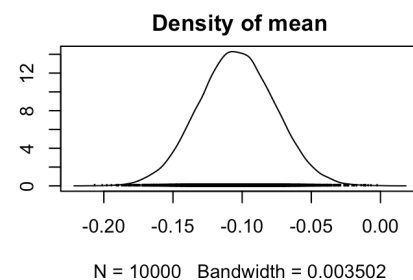
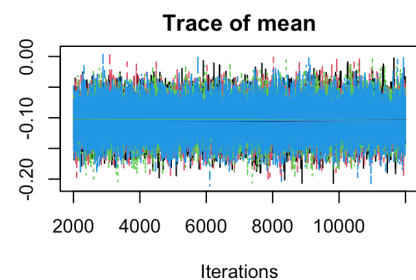
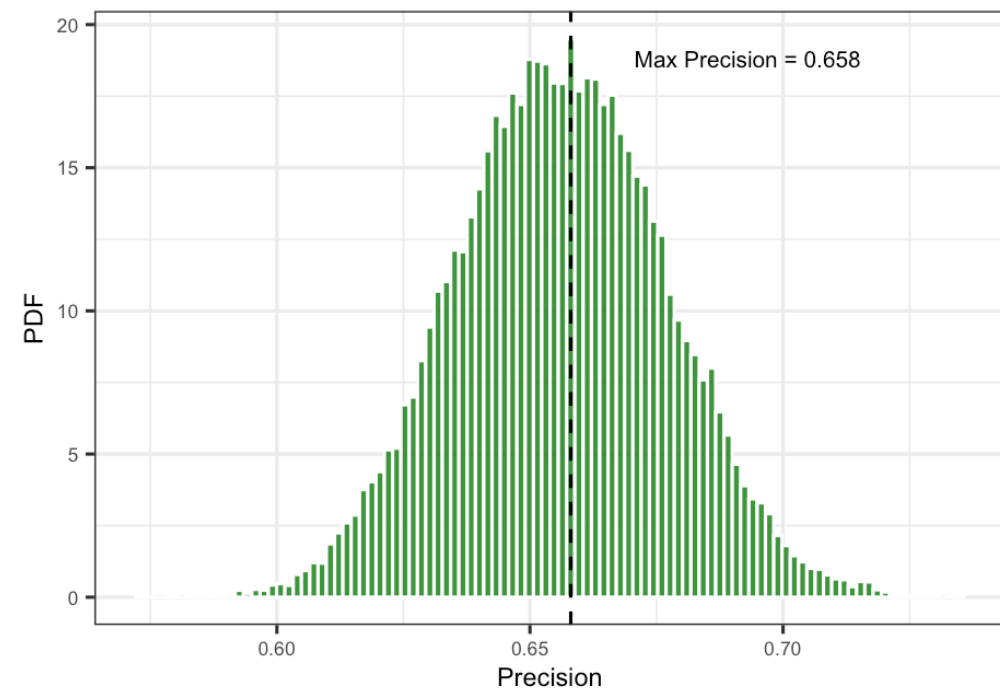


Markov Chain Monte Carlo

Posterior Distribution for the Mean of the Normal



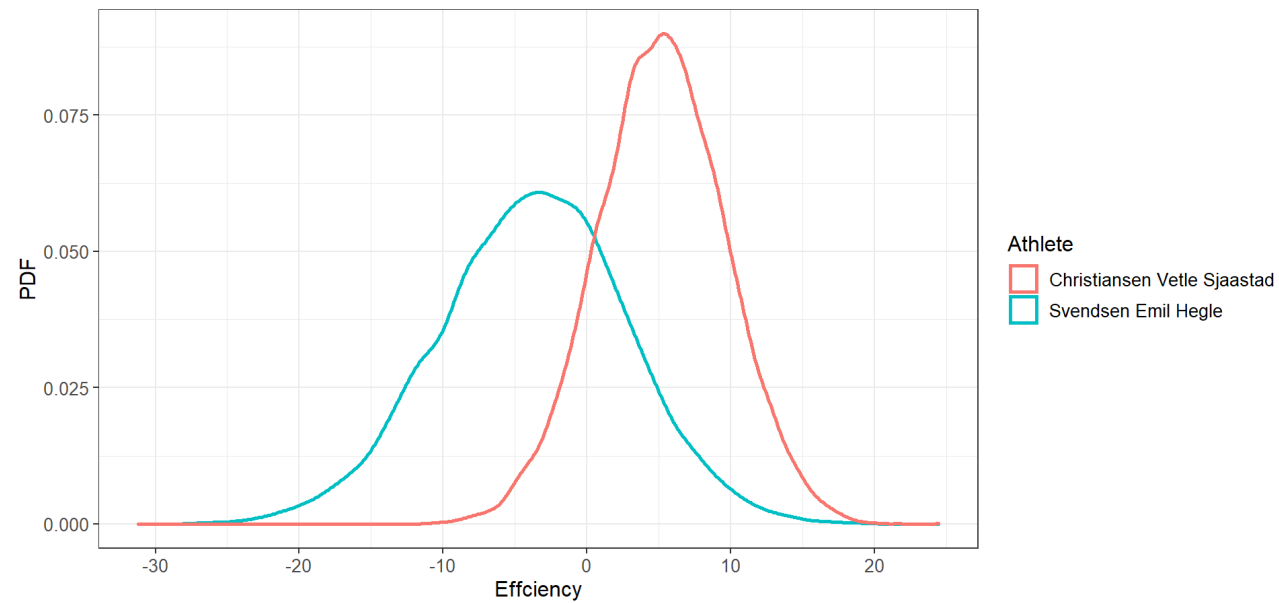
Posterior Distribution for the Precision of the Normal



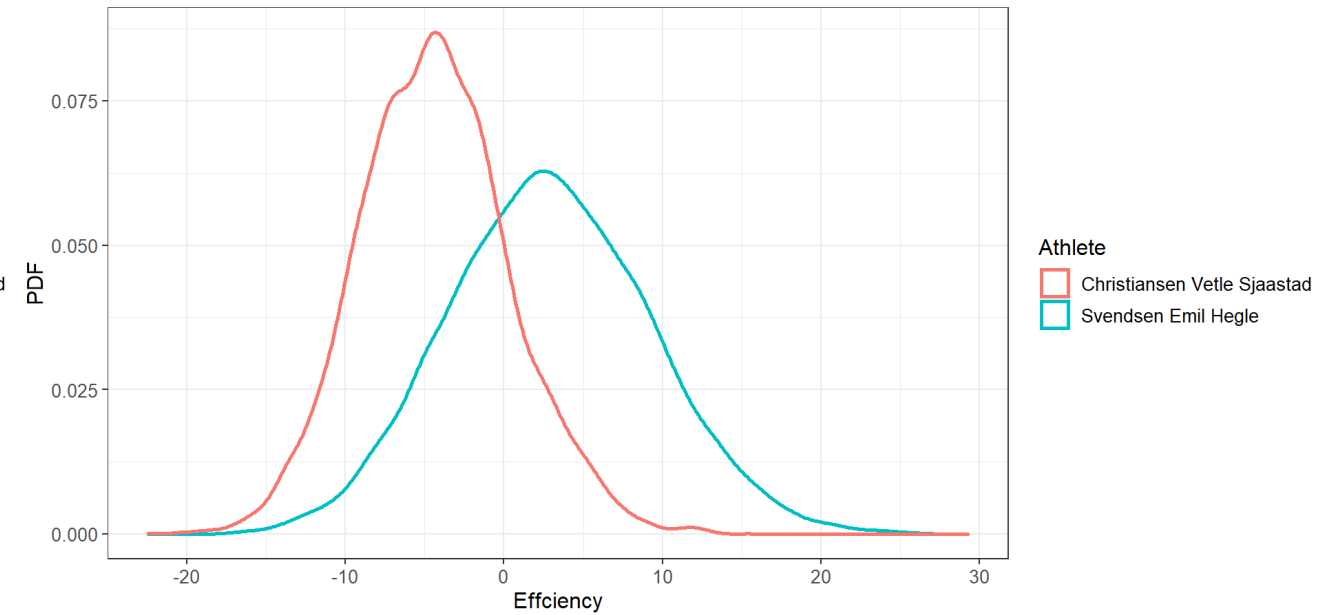
- Posterior distribution of the mean
- Posterior distribution of the precision:
$$\tau = \frac{1}{\sigma^2}$$
- Chains of the MCMC
- The slight difference between male and female athletes can be explained by chance alone

Top 10 versus Top 5

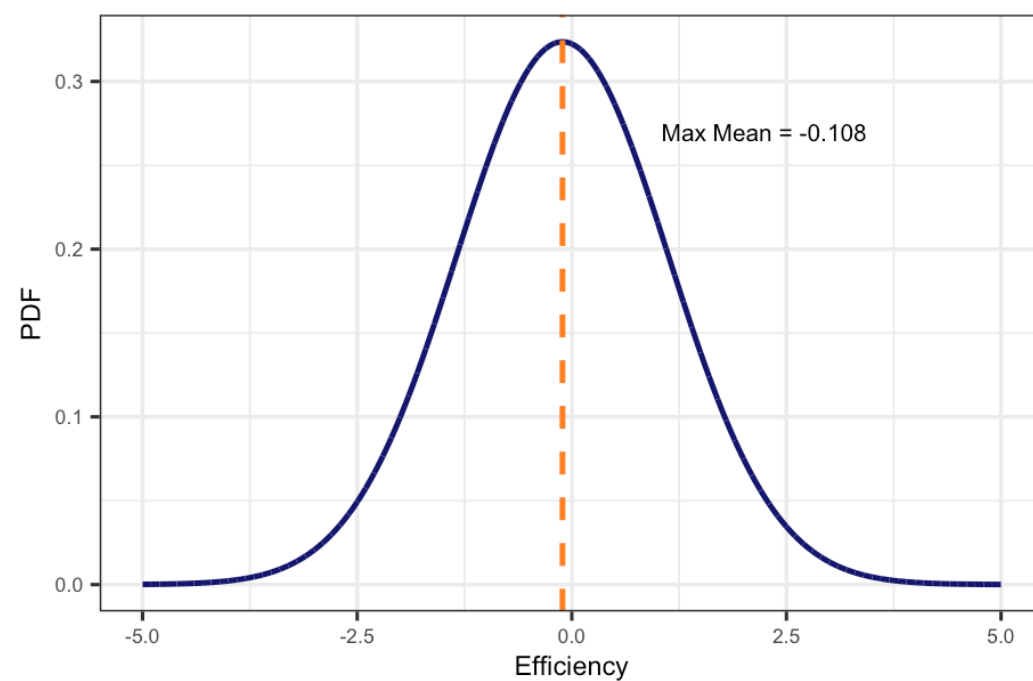
Athletes with related efficiency distributions - Top10 - Men



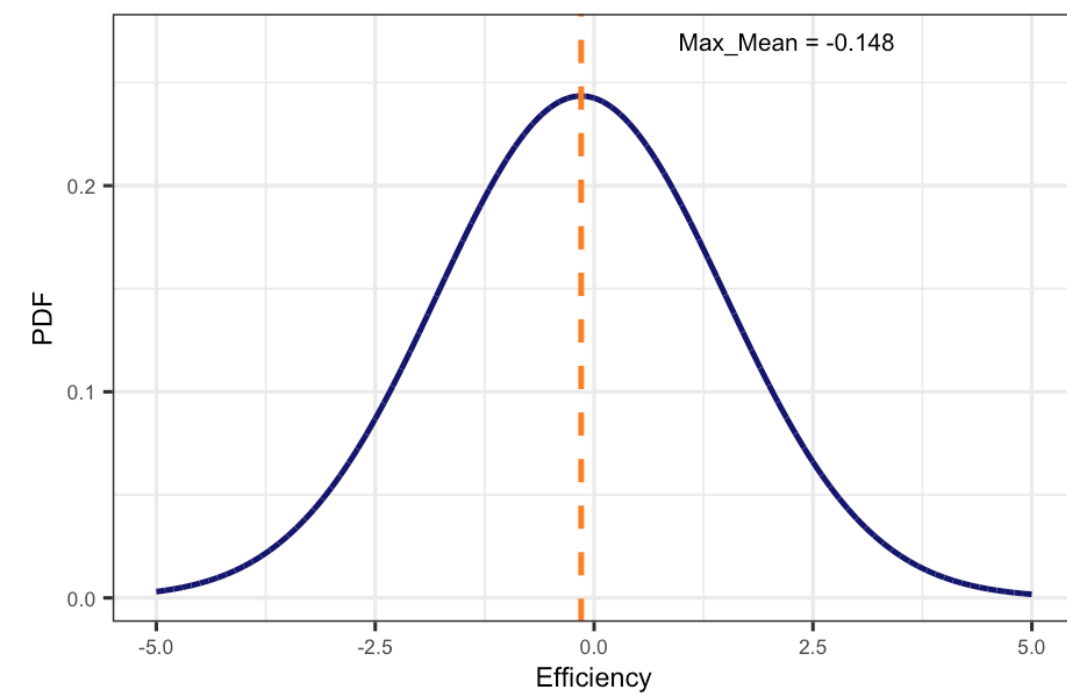
Athletes with related efficiency distributions - Top5 - Men



Retrieved Normal Distribution for the Efficiency
Difference between Male and Female athletes

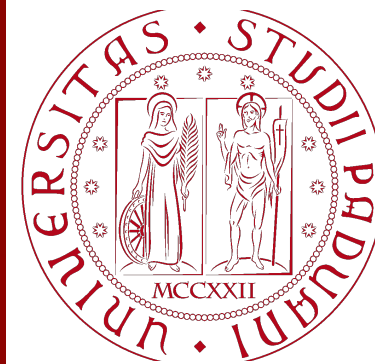


Retrieved Normal Distribution for the Efficiency
Difference between Male and Female athletes





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Individual's analysis

- Divide the data frame between individual races and the other 4-shooting-ranges race formats
- Take into accounts only records where the athlete was a perfect 19 hits after the first 19 shots
- Compute the percentage of times the last shot was good in pursuits and mass starts
- Set up the null hypothesis H_0 : the probability of hitting the last shot in individuals is greater and equal then in pursuits and mass starts
- Perform a one sided hypothesis test trying to prove the alternative hypothesis H_1 : the last shot of the an individual is the toughest in biathlon

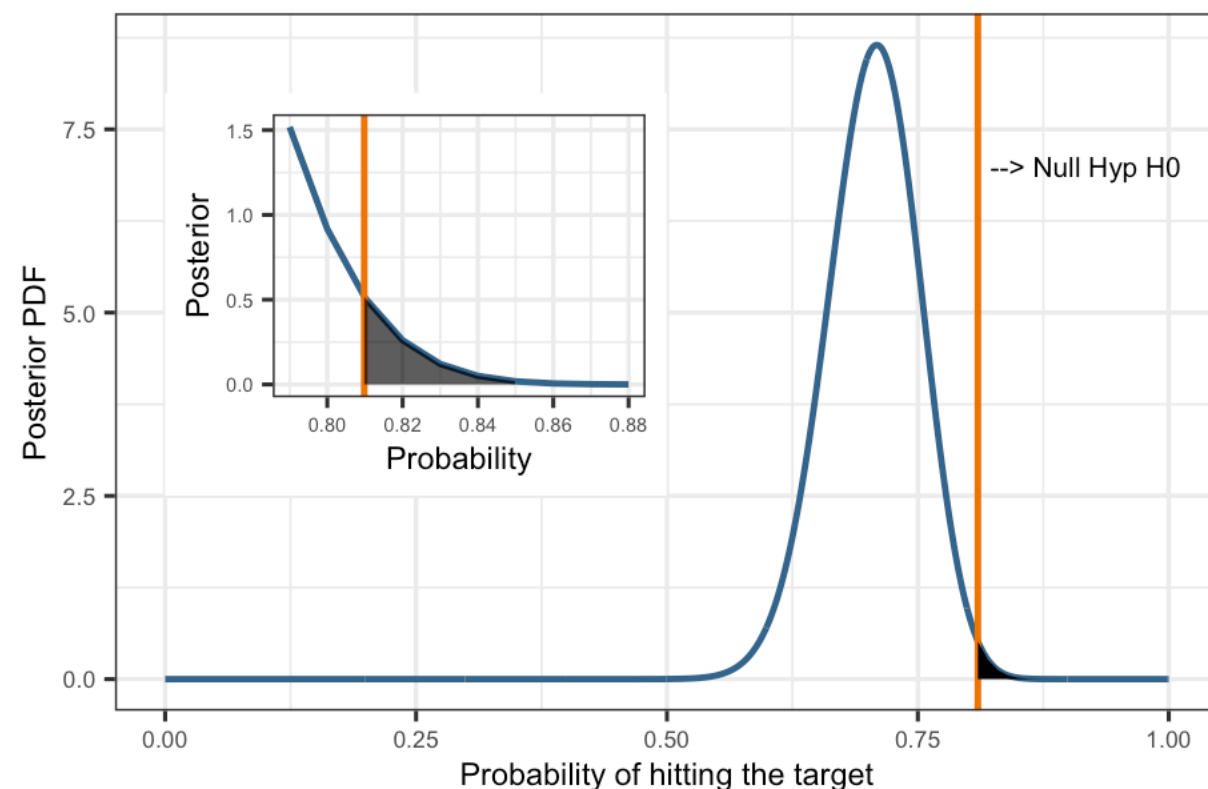




Results

Posterior Distribution Men

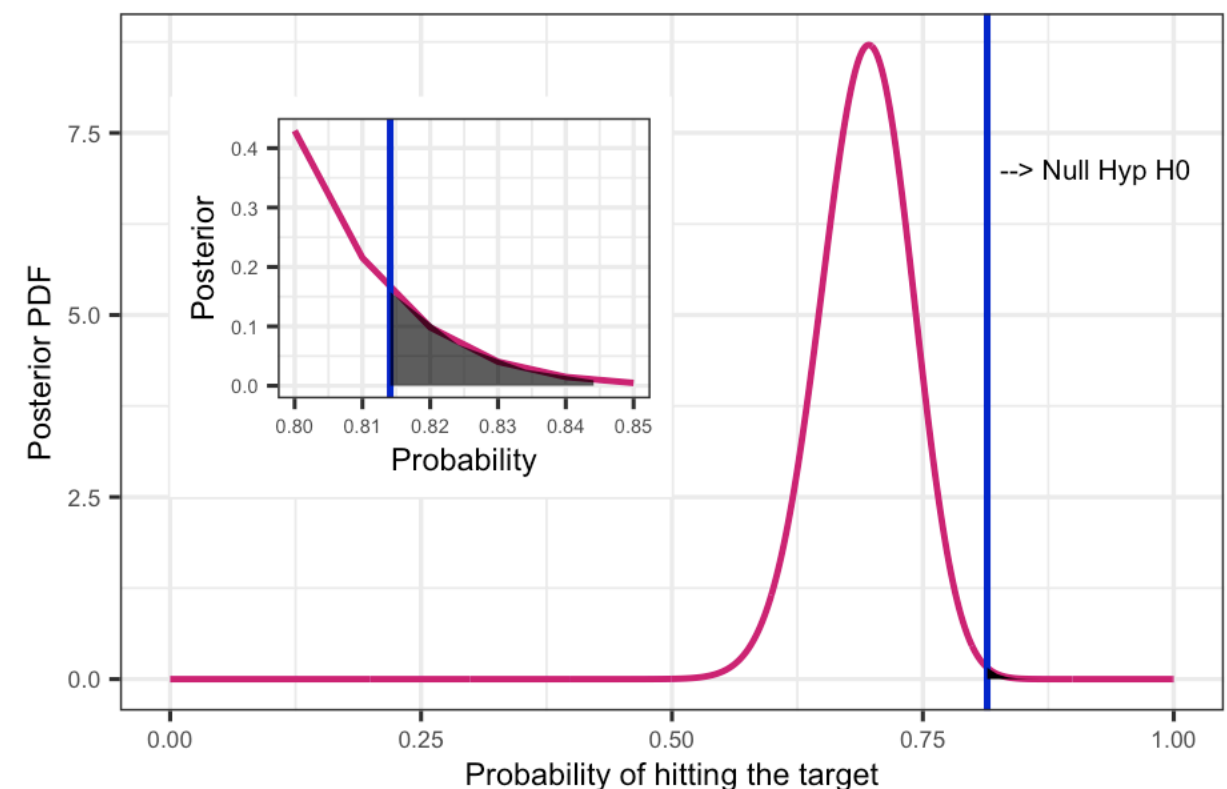
Bayesian Hypothesis Testing



- The value of the integral is 0.007
- We reject the Null hypothesis for men with 99% probability

Posterior Distribution - Women

Bayesian Hypothesis Testing



- The value of the integral is 0.002
- We reject the Null hypothesis for women with 99% probability

Conclusions

Efficiency

- We show how being in contention for a meaningful position influences some biathletes: we have cases where the performance is improved and, more often, where the performance worsens
- Both athletes with great experience and sharp-shooters can be influenced
- We find confirmation on the reputation of biathletes that are known for crumbling under pressure
- The behaviour is confirmed when looking at the top 5 with a few interesting outliers

Hypothesis test

- We determined, for both men and women, with 99% probability that the last shot in individuals is more difficult than in other 4-range races, if you have hit the first 19 bullets

