**Computational Finance and its implementation in Python with applications to option pricing, Green finance and Climate risk.**
**Exercise Handout 3**

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### Exercise 1

Consider again the market where you have zero interest rate $\rho = 0$, and where the risky asset $\bar{S} = (\bar{S}_t)_{t=0,1,\dots,T}$ is defined by

$$\bar{S}_t = \log(S_t),$$

where $S = (S_t)_{t=0,1,\dots,T}$ is the binomial model described in the script, i.e.,

$$S_t = S_0 \cdot Y_1 \cdot \dots \cdot Y_t, \quad t = 1, \dots, T,$$

where $Y_t$ can take the two values $d$, $u$ with $0 < d < 1 < u$, for any $t = 1, \dots, T$, and $(Y_t)_{t=1,\dots,T}$ are i.i.d. and such that $Y_{t+1}$ is independent of $\mathcal{F}_t$.
In Exercise 2 of Handout 1 you had to find the risk neutral probability $Q$ of such a market, i.e., the probability

$$q = Q(Y_t = u)$$

such that $\bar{S}$ is a martingale under $Q$, and in Exercise 2 of Handout 2 you had to implement such a model in Python, similarly to what has been done in the classes of

`binomialmodel.creationandcalibration`.

- Valuate now a call option for given parameters under this new model.
- Consider the model above with $N$ time steps, for $N$ large enough, and parameters

$$u = e^{\sigma\sqrt{T/N}}, \qquad d = 1/u. \tag{1}$$

  Find the dynamics

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t, \quad t \geq 0,$$

  of a process $X$ which is approximated by $\bar{S}$ under this setting.

  Is it possible to compute the analytic value of a call option under these dynamics for $X$? If yes, check that the price obtained under $\bar{S}$ with parameters (1) for large $N$ approximate this price.