

Exercise 1

Write a class similar to

`binomialmodel.optionValuation.europeanOption,`

where you value this time a so called Knock-out barrier option for a call option, i.e., an option that gives the usual payoff of a call option with strike K but only conditional to the fact that the value of the underlying S remains within a given interval (B_d, B_u) for all times until maturity, with

$$B_d < K, S_0 < B_u.$$

Consider to implement it in such a way that both the barriers or only one of them can be active. The option is called:

- Up-and-out if $B_d = 0$;
- Down-and-out if $B_u = \infty$.

Exercise 2

Consider the market where you have zero interest rate $\rho = 0$, and where the risky asset $\bar{S} = (\bar{S}_t)_{t=0,1,\dots,T}$ is defined by

$$\bar{S}_t = \log(S_t),$$

where $S = (S_t)_{t=0,1,\dots,T}$ is the binomial model described in the script, i.e.,

$$S_t = S_0 \cdot Y_1 \cdot \dots \cdot Y_t, \quad t = 1, \dots, T,$$

where Y_t can take the two values d, u with $0 < d < 1 < u$, for any $t = 1, \dots, T$, and $(Y_t)_{t=1,\dots,T}$ are i.i.d. and such that Y_{t+1} is independent of \mathcal{F}_t .

Find the risk neutral probability Q of such a market, i.e., compute the probability

$$q = Q(Y_t = u)$$

such that \bar{S} is a martingale under Q . Check that $0 < q < 1$.