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Solar power Renewable Energy Conversion Systems

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GENERALITIES



Generalities

Solar power is supplied by the Sun to Earth in the form *radiation*, that travels through the atmosphere and hits the land/sea surface.

This is the primary energy source from which other energy forms derive. E.g.

- *Wind energy* is a consequence of solar energy, as winds are caused by temperature gradients in different regions, which are in turn caused by different exposition to radiation
- Wave energy derives from solar energy too, as waves are generated by the wind/storms (see above)
- Biomasses are grown by solar radiation, and accumulate solar energy in the form of chemical energy, which is used in processes (e.g., combustion).

Solar power uses include:

- *Thermal uses* (hot water production for domestic or industrial uses)
- *Electric power generation*. This is achieved with 2 technologies
 - *Photovoltaic (PV) cells*, which accomplish direct conversion of radiation into electric power
 - *Thermodynamic solar plants*, which use thermoelectric cycles where the input thermal power is provided by the sun



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Solar irradiation maps

SOLAR RESOURCE MAP

GLOBAL HORIZONTAL IRRADIATION



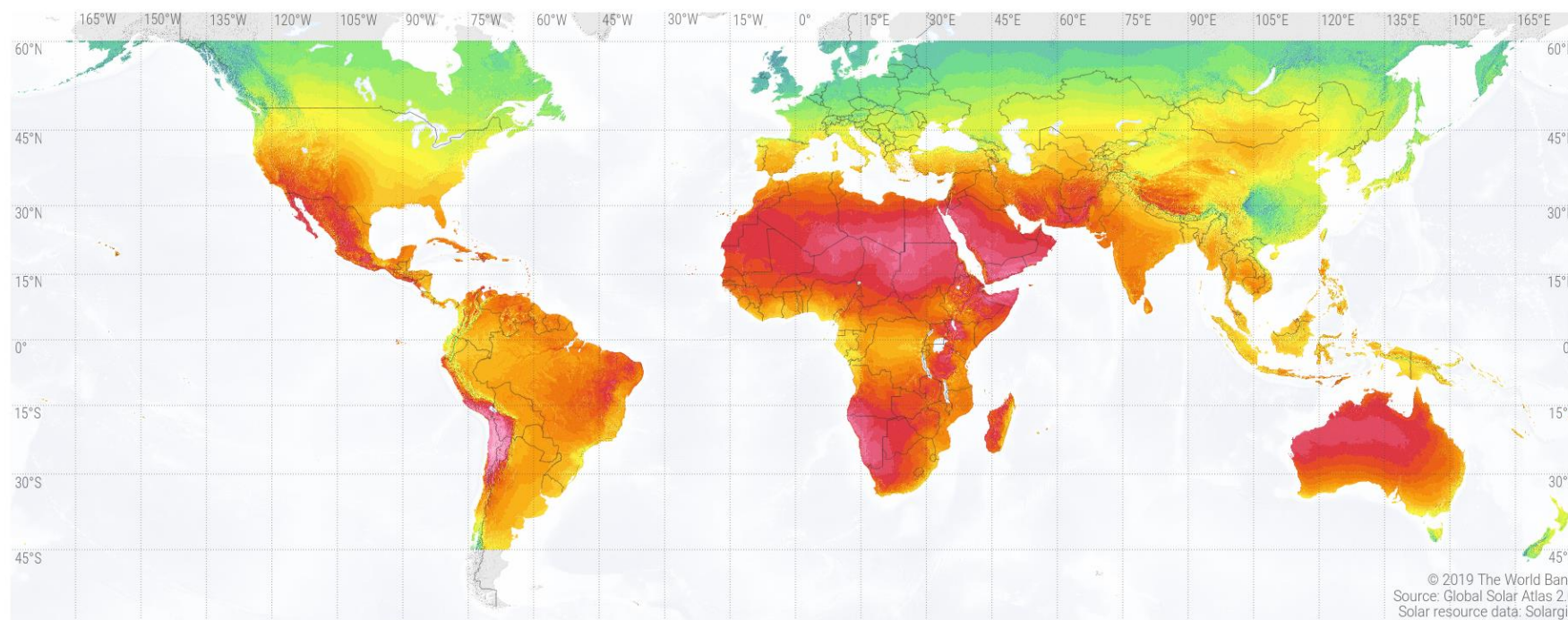
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ESMAP

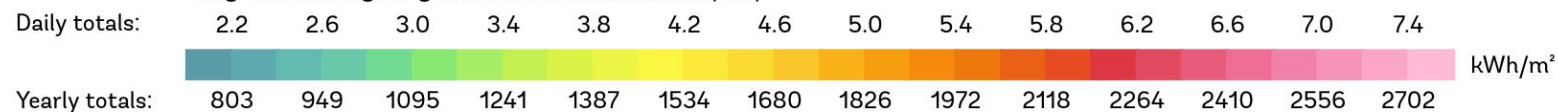


SOLARGIS



© 2019 The World Bank
Source: Global Solar Atlas 2.0
Solar resource data: Solargis

Long-term average of global horizontal irradiation (GHI)



This map is published by the World Bank Group, funded by ESMAP, and prepared by Solargis. For more information and terms of use, please visit <http://globalsolaratlas.info>.



Solar thermal energy

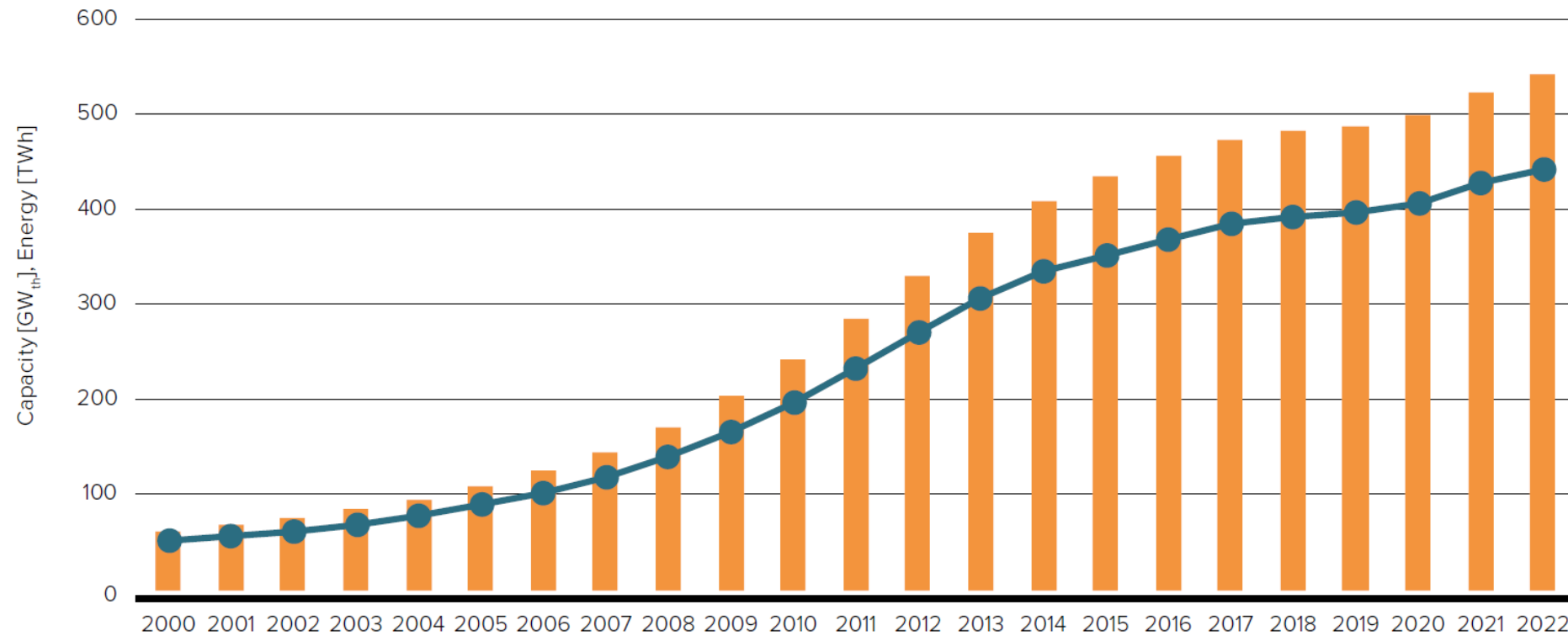


Figure 2: Global solar thermal capacity in operation and annual energy 2000-2022

Global solar thermal capacity in operation [GW_{th}]
Global solar thermal energy yield [TWh]



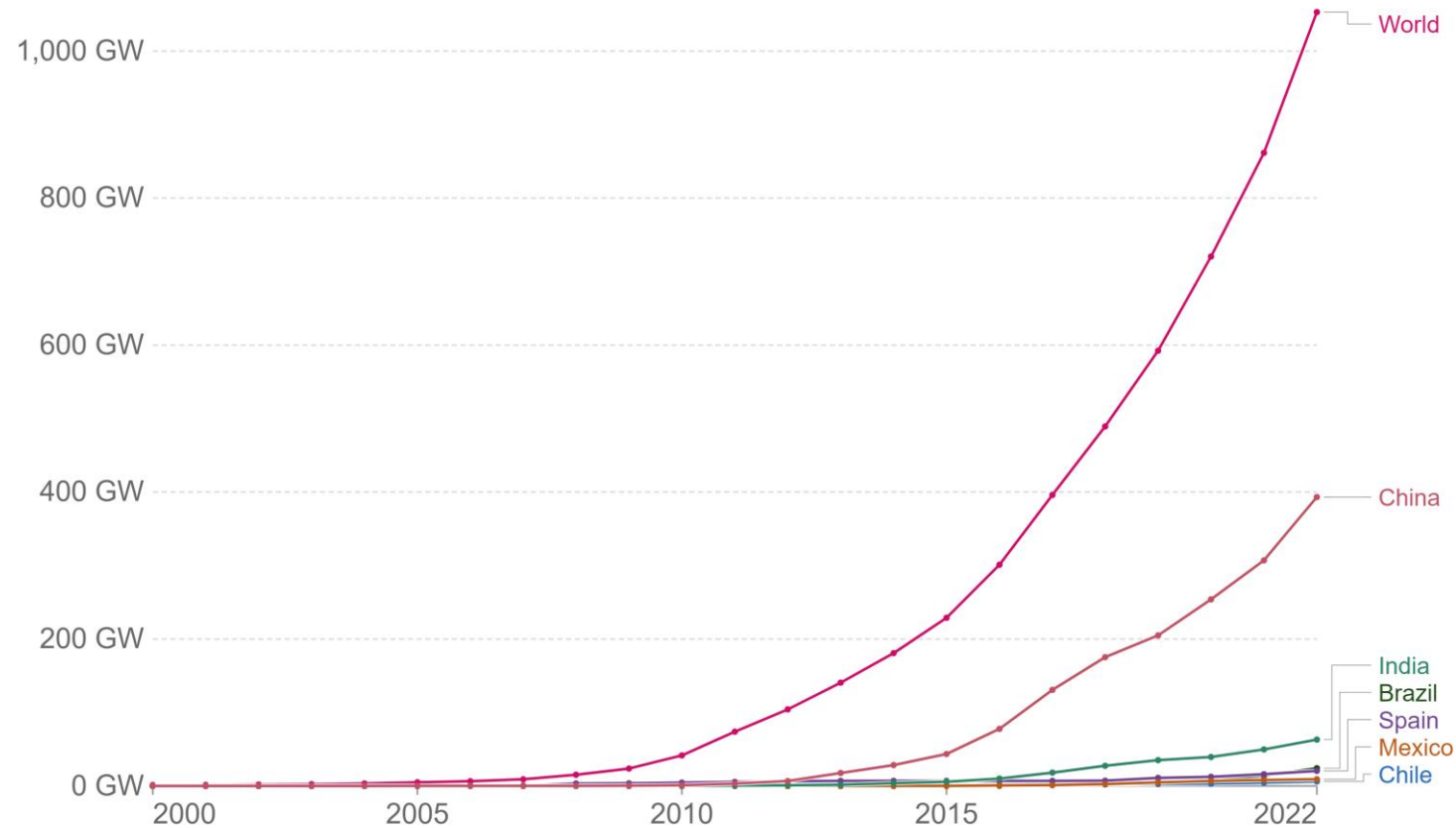


Photovoltaic power

Installed solar energy capacity

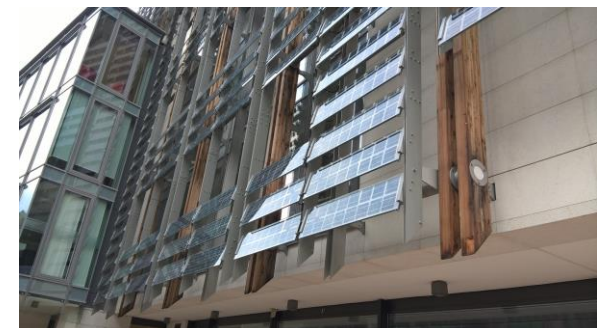
Cumulative installed solar capacity, measured in gigawatts (GW).

Our World
in Data



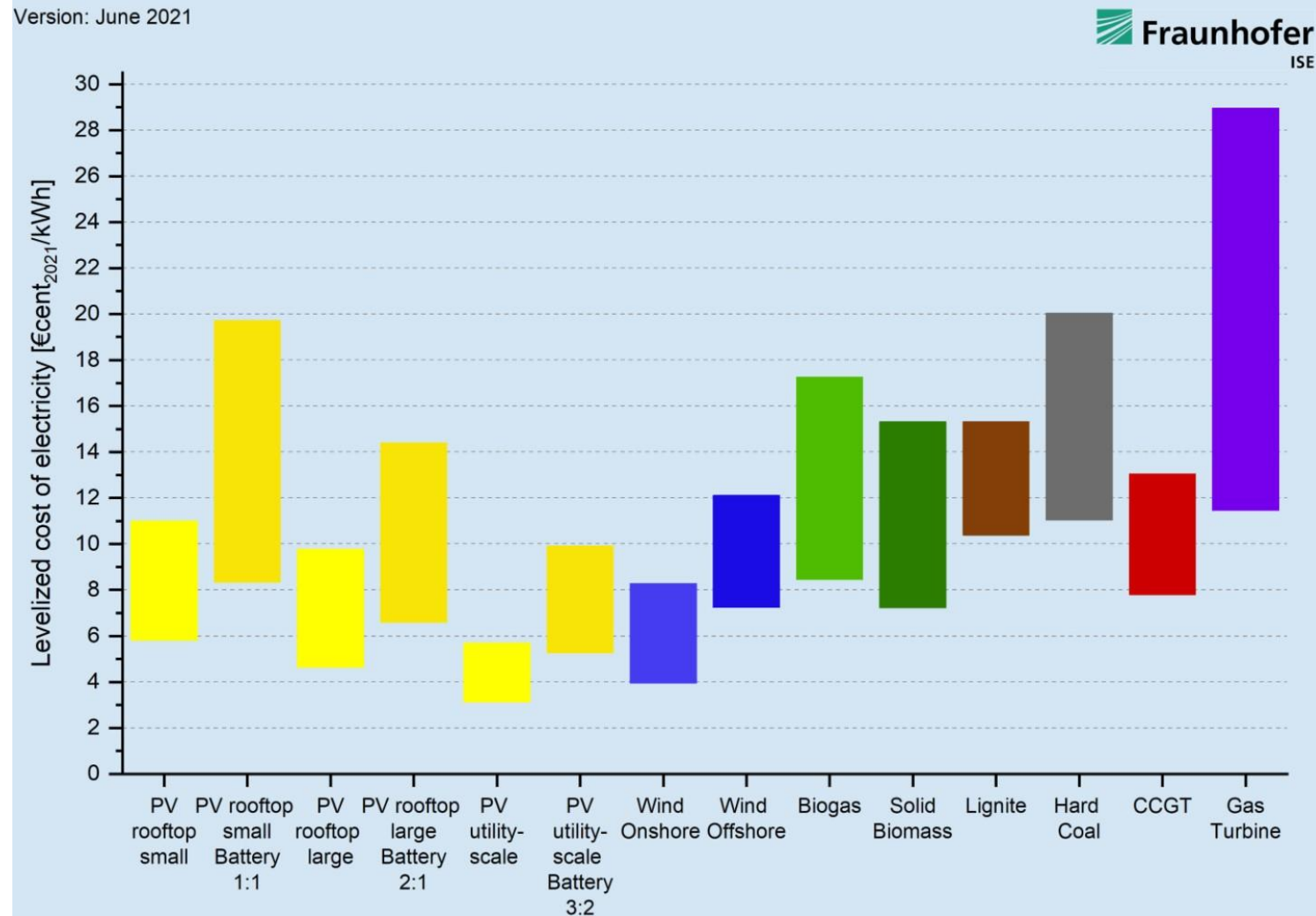
Data source: International Renewable Energy Agency (IRENA)

OurWorldInData.org/renewable-energy | CC BY





LCOE for PV



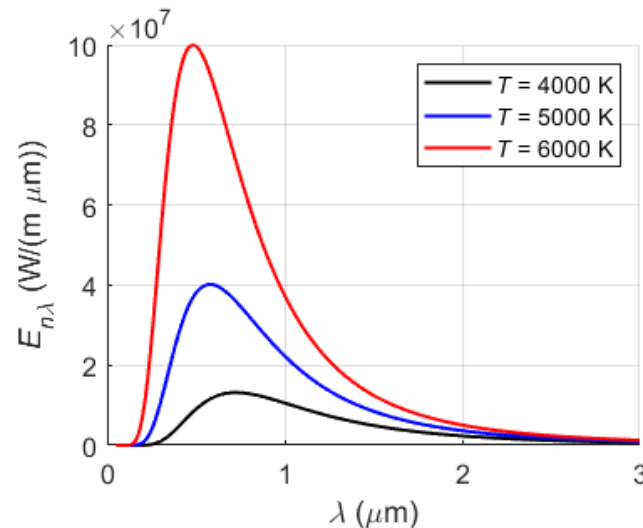
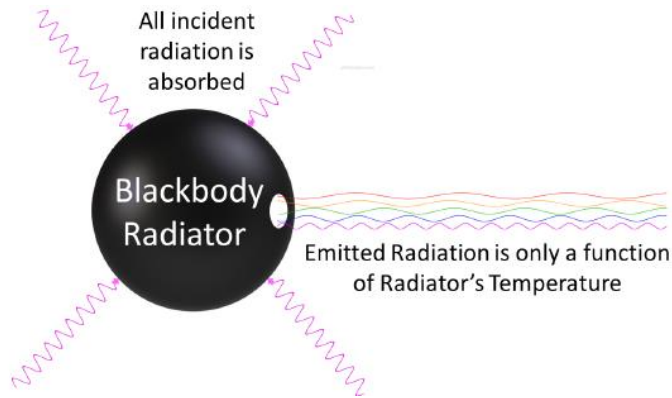


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SOLAR RADIATION

Black body radiation



A *black body* is an object that absorbs all electromagnetic radiation at all wavelengths, and emits *electromagnetic black-body radiation* with a certain spectral distribution that is a function of the body absolute temperature, according to the so-called *Planck law*.

$$E_{n\lambda} = \frac{C_1}{\lambda^5 [\exp(C_2/(\lambda T)) - 1]} \quad \text{W / (m}^2 \cdot \mu\text{m)} \quad \begin{aligned} C_1 &= 3.742 \cdot 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2 \\ C_2 &= 1.439 \cdot 10^4 \mu\text{m} \cdot \text{K} \end{aligned}$$

where T = body temperature [K], λ wavelength [μm]

Total radiated power (per unit surface) over the spectrum:

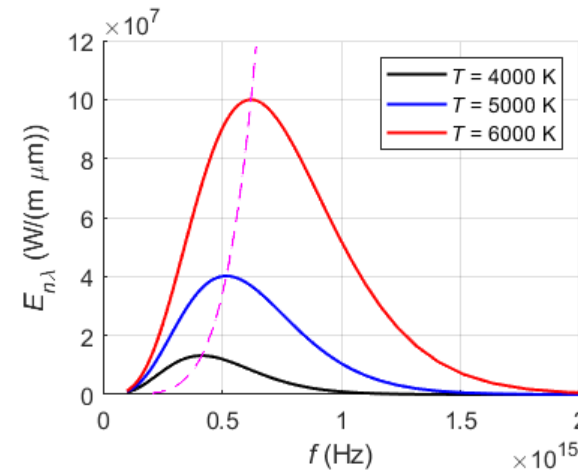
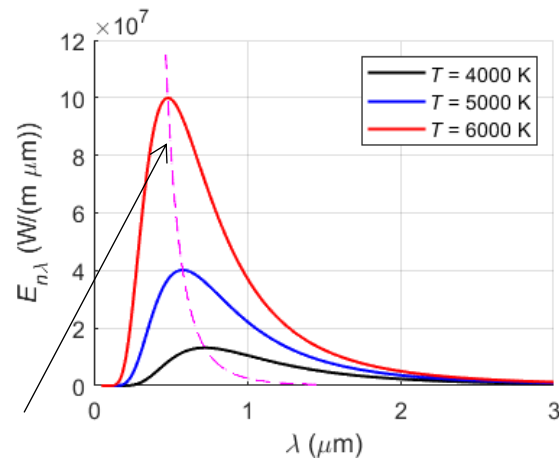
$$I = \int_0^{+\infty} E_{n\lambda}(T, \lambda) d\lambda = \frac{\pi^4}{15} \frac{C_1}{C_2^4} T^4 = \sigma T^4 \quad \begin{aligned} \sigma &= 5.67 \cdot 10^{-8} \text{ W/(m}^2 \text{K}^4) \\ &\text{Stefan-Boltzmann constant} \end{aligned}$$

Black body radiation

Black body radiation is a form of *electromagnetic* radiation, which travels at the speed of light c , and for which frequency and wavelength are related as

$\lambda \cdot f = c = 3 \cdot 10^8 \text{ m/s} \rightarrow$ the higher the wavelength, the smaller the frequency

The higher the temperature, the smaller the wavelengths (i.e., the higher the frequencies) around which the body emits power (Wien law).



Locus of maxima (Wien law):

$$\lambda_{max} T = 2897 \mu\text{m K}$$

Solar radiation

The Sun has a temperature of 5800 K and emits light following a behaviour that closely approximates a black body. The range of *visible light* corresponds to the frequency/wavelength interval where the radiated power is maximum.

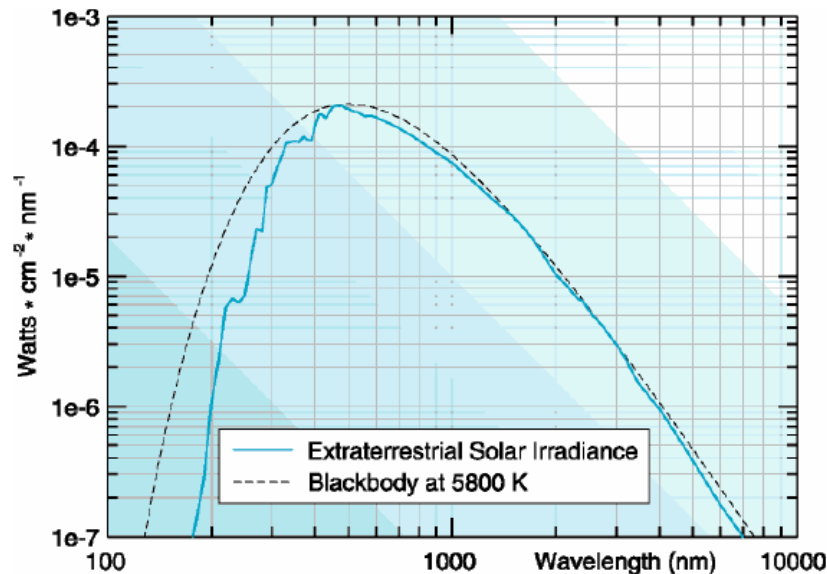
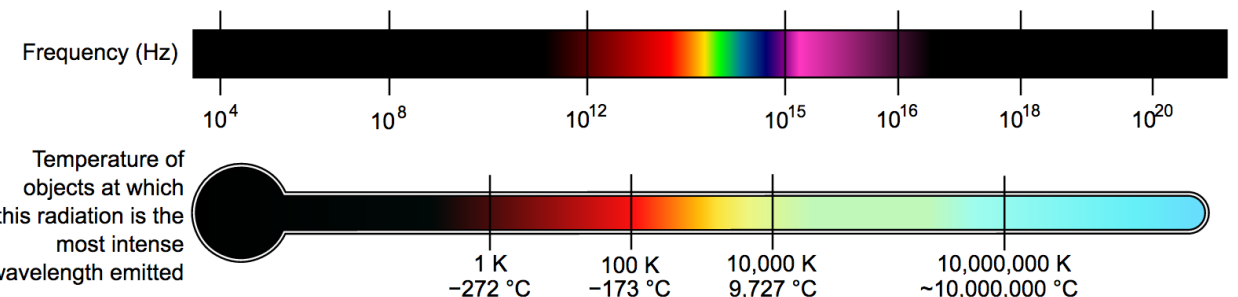
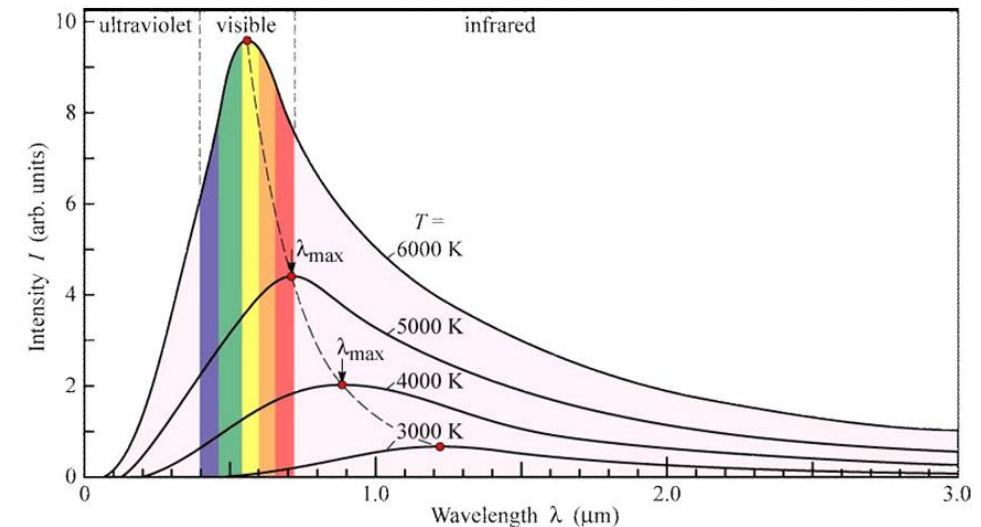


Fig. 5.7 Extraterrestrial solar irradiance compared to a blackbody.



Incident solar radiation

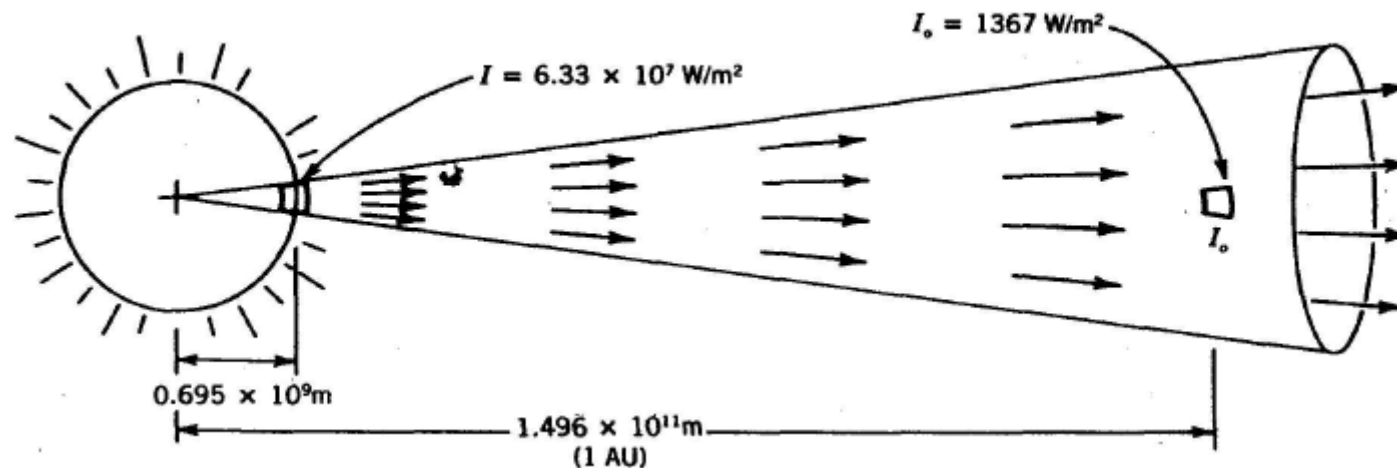
Solar radius: $R_s = 6.95 \cdot 10^5$ km, Solar temperature: $T_s = 5800$ K

Power emitted from the sun (black body model): $P_s = 4\pi R_s^2 \sigma T_s^4 = 3.89 \cdot 10^{26}$ W

Sun-Earth distance: $d_s = 1.496 \cdot 10^8$ km

Radiation reaching Earth (at the boundary of atmosphere): $P_s = I_0 4\pi d_s^2 \rightarrow I_0 = \frac{P_s}{4\pi d_s^2} = 1383$ W/m² (SOLAR CONSTANT)

Slightly more accurate measurements of the solar constant led to a value (used in engineering practice) of $I_0 = I_{SC} = 1367$ W/m²

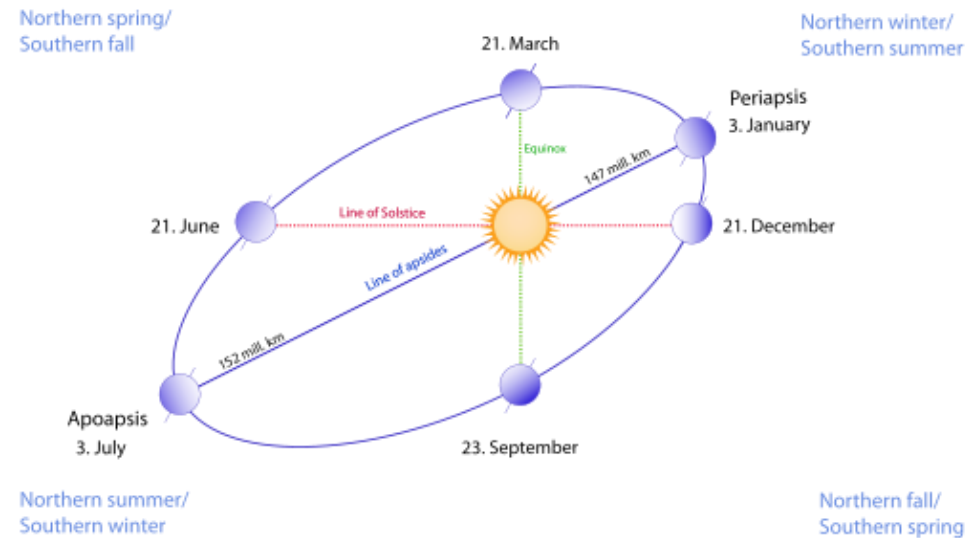


Incident solar radiation

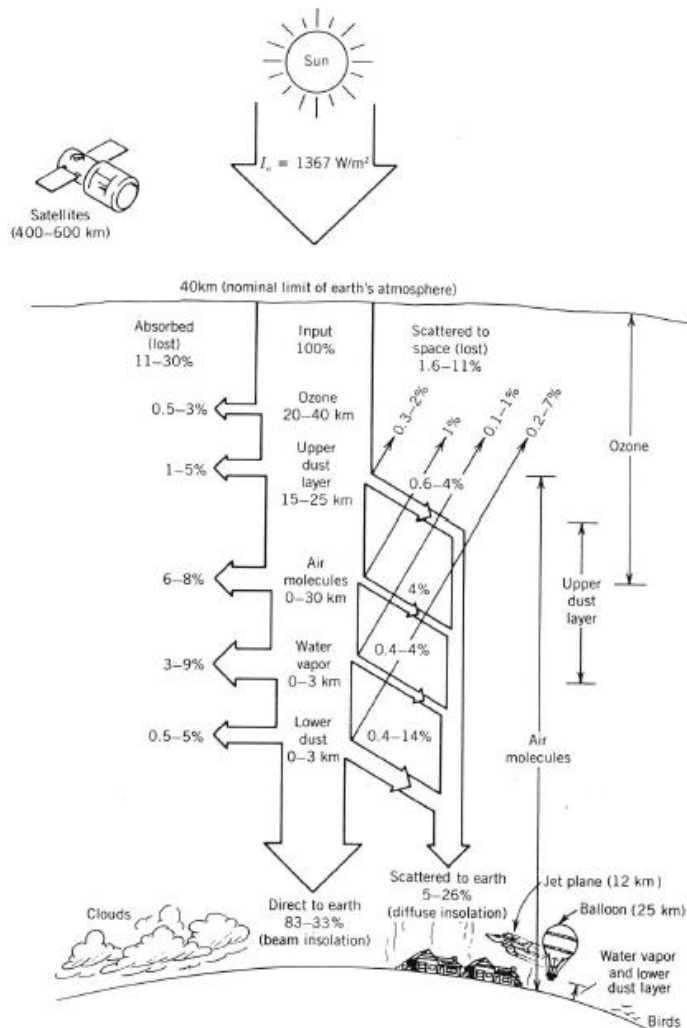
The value of the solar constant further needs to be corrected, as a function of the day of year, to account for the elliptic shape of the Earth orbit (the Sun-Earth distance is not constant)

$$I_0 = I_{sc} \left[1 + 0.034 \cos \left(2\pi \frac{n}{365} \right) \right]$$

where n is the day of the year ($n=1 = 1^{\text{st}}$ Jan, $n = 365 = 31$ Dec).



Incident solar radiation



Only a portion of the extraterrestrial radiation reaches ground.

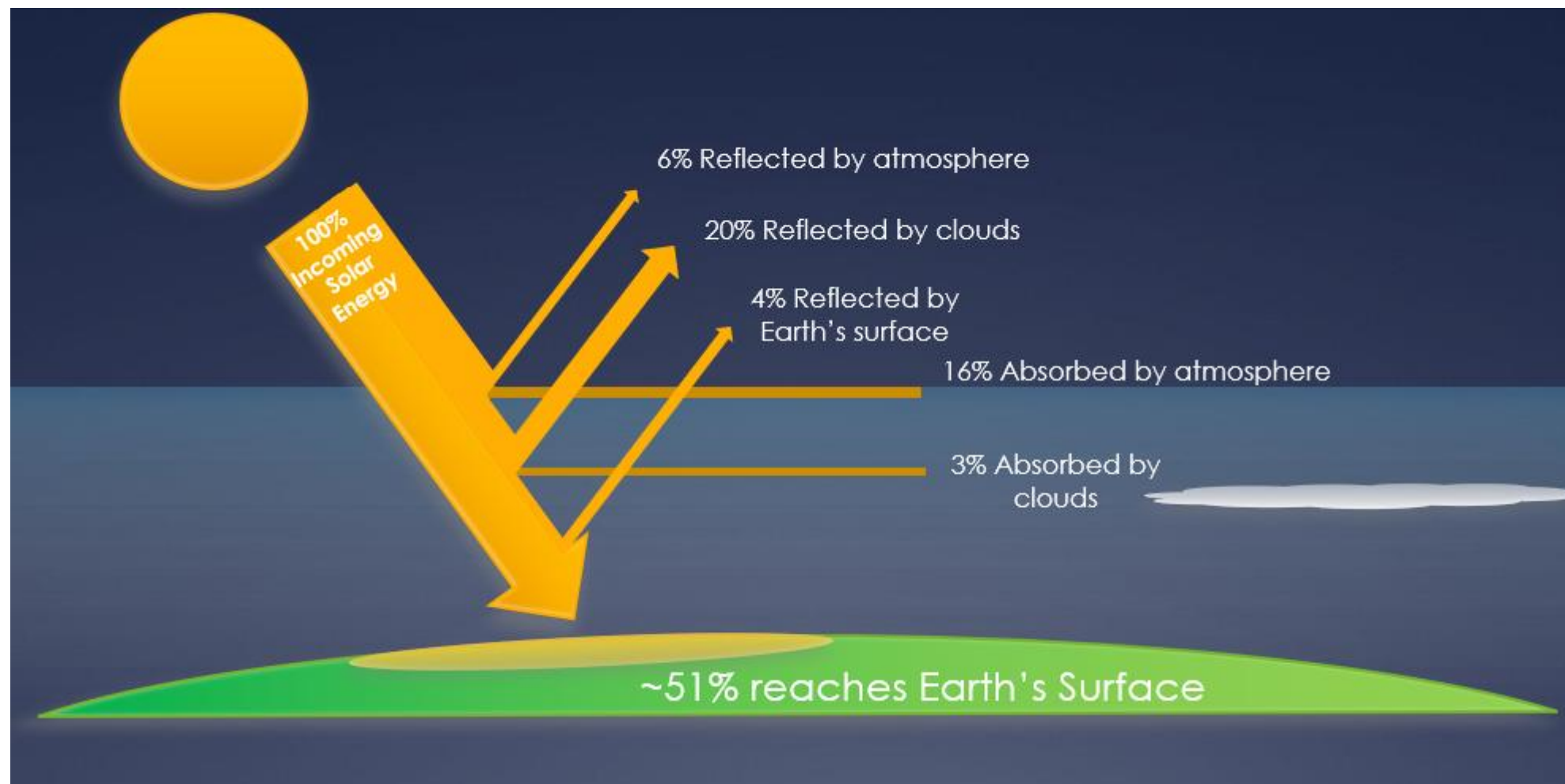
Part of the extraterrestrial radiation is reflected at the boundary of the atmosphere. Part is then absorbed or reflected within the atmosphere.

Solar radiation reaching ground has a wavelength almost entirely comprised in the range 0.3–2.5 μm .

Part of the global radiation reaching ground is reflected because of the ground reflectivity, called *albedo*. Albedo coefficients (expressing the percentage of reflected incident radiation) are highly variable for different ground types:

Terrain	Albedo, ρ
Snow	95%
Desert	30%
Brush	25%
Dark soil	10%
Water	5%

Incident solar radiation





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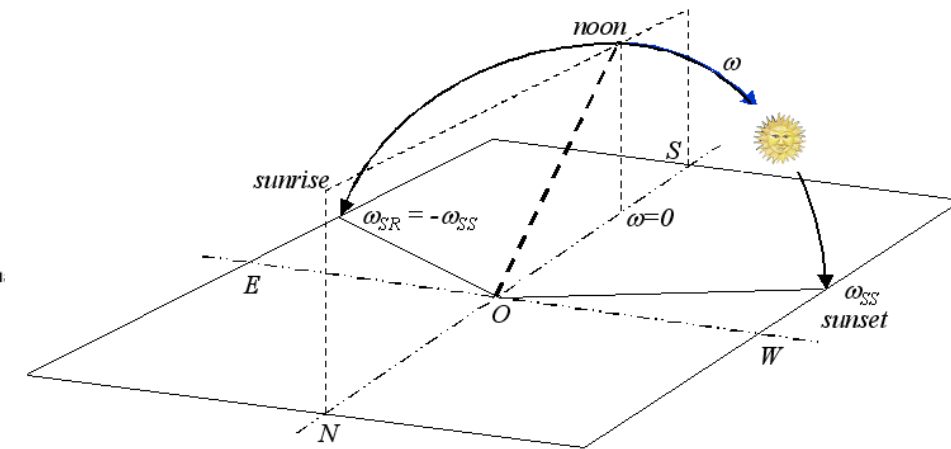
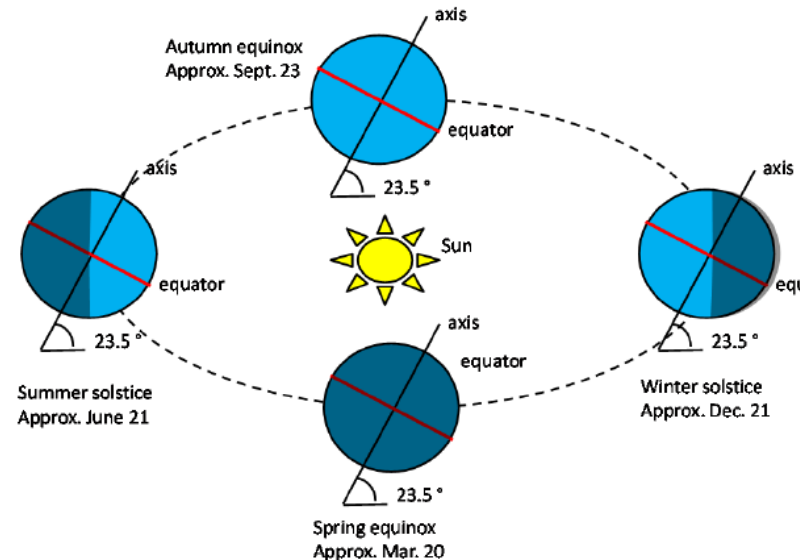
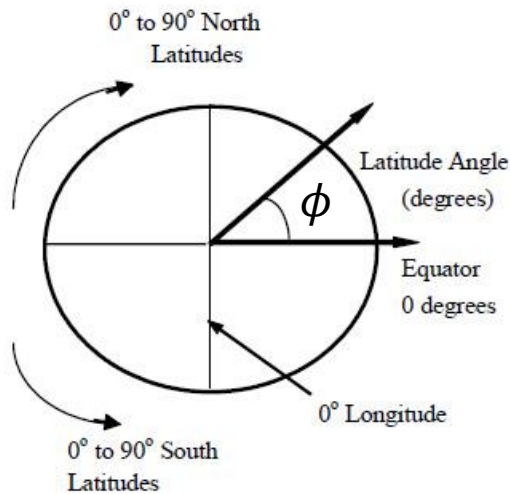
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SOLAR GEOMETRY

Solar angles

The amount of solar radiation that reaches a certain location, a certain day of the year at a given time, is influenced by:

- location latitude ϕ ($\phi = 0$ at the equator, north positive)
- solar declination (i.e. the angular position of the sun at solar noon at the equator) δ , which depends on the day of the year
- hour of the day, expressed through the hour angle of sun ω ($\omega = 0$ at noon, afternoon positive)



Solar angles

Solar declination δ :

$$\delta = 23.45^\circ \sin \left(360^\circ \frac{284+n}{365} \right)$$

where n is the day of the year ($n = 1$: 1 Jan., $n = 365$: 31 Dec.)

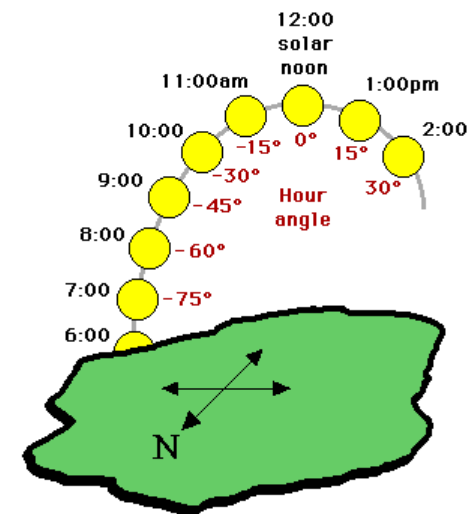
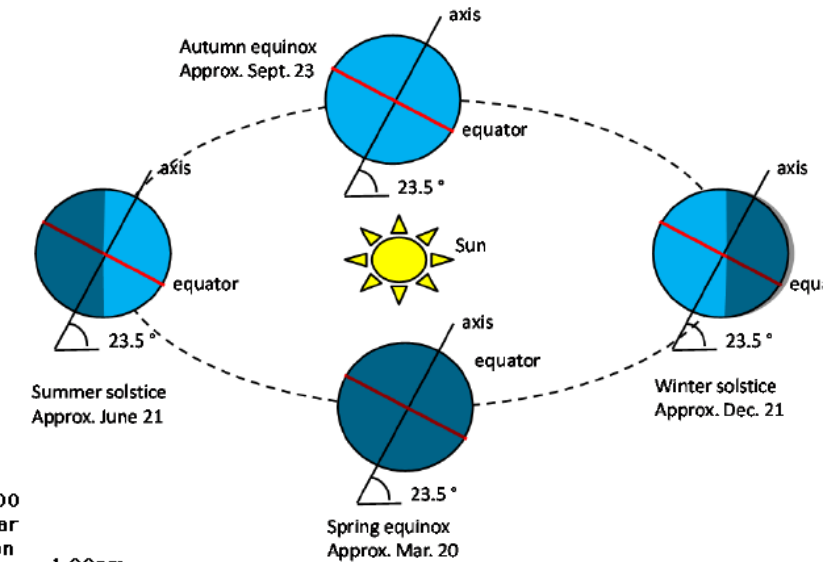
$\delta = \pm 23.45^\circ$ approx. at the solstices ($n = 172$, 21 Jun., $n = 355$, 21 Dec.)

$\delta = 0^\circ$ approx. at the equinoxes ($n = 81$, 22 Mar., $n = 264$, 21 Sep.)

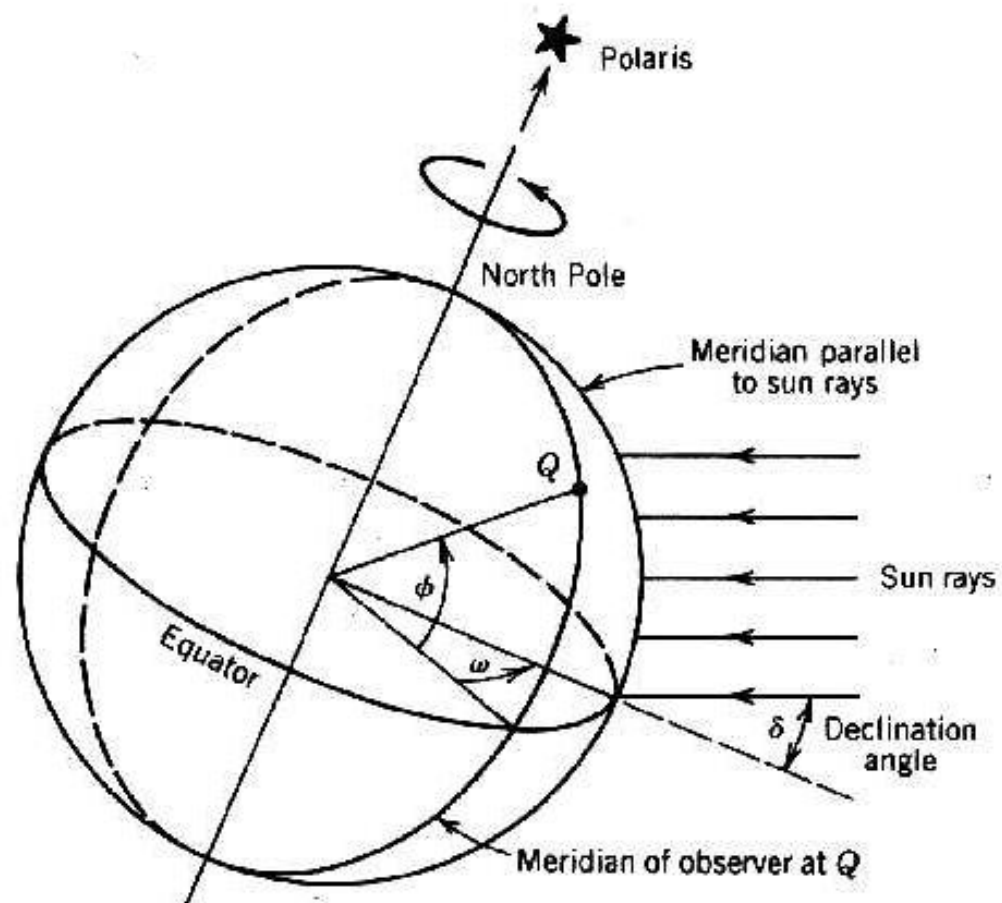
Hour angle of sun ω

$$\omega = 15^\circ (t - 12)$$

where $t \in [0,24)$ is the hour of the day



Solar angles



Incidence angles

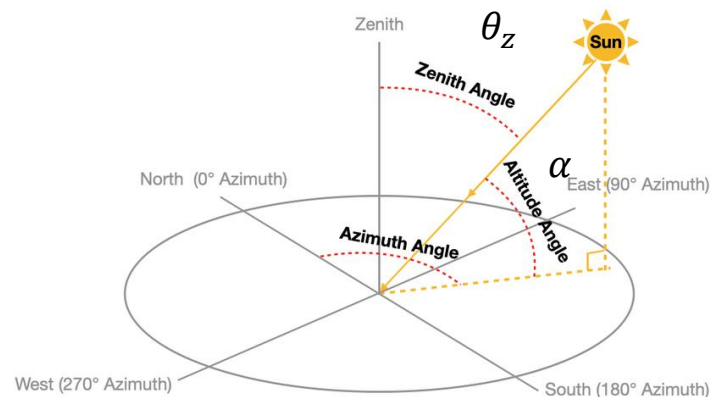
Zenith angle θ_z , angle between the beam direction and the direction orthogonal to ground

$$\cos \theta_z = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega$$

At sunrise/sunset, the sun hits the ground tangentially $\rightarrow \cos \theta_z = 0$

Sunrise/sunset angles (they are opposite to one another) ω_s : $\cos \omega_s = -\tan \delta \tan \phi$

Derivation @ the blackboard



The complementary of θ_z is called *altitude angle* (usually referred to as α)

Incidence angles

Let us now consider a surface which is inclined with respect to ground. We call:

γ the surface azimuth angle ($\gamma=0$ if the surface is facing south, positive towards west)

β the slope angle between the plane of the surface and the horizontal

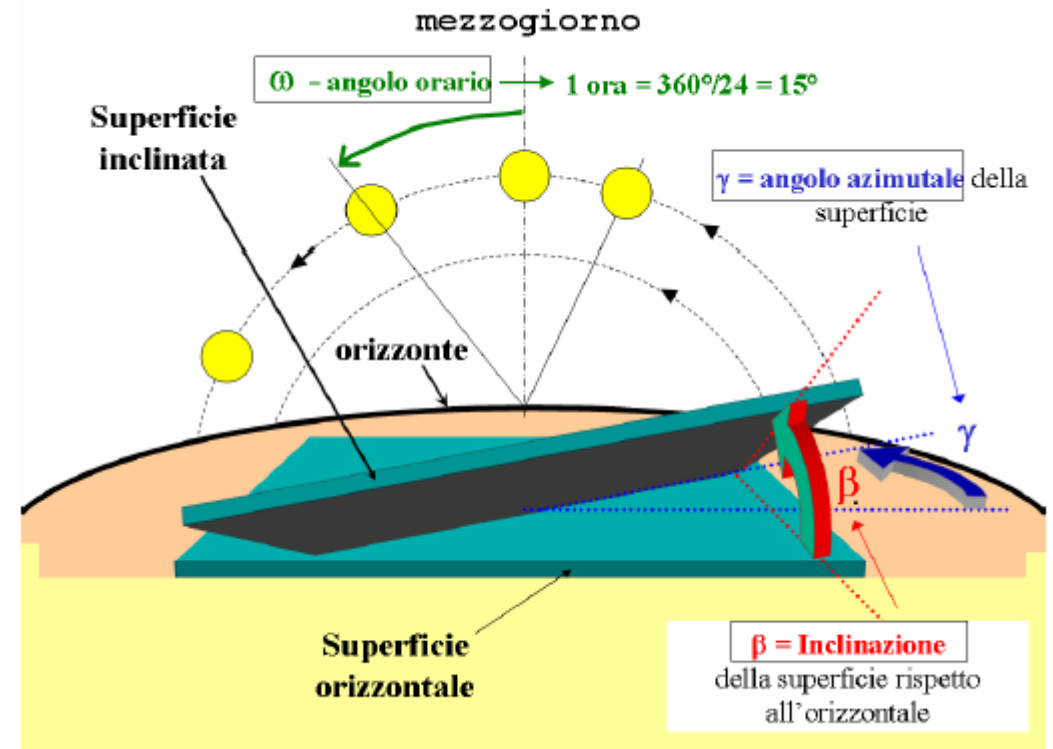
The angle θ between the normal vector to the surface and the sun beams (called angle of incidence) is then given by:

$$\begin{aligned}\cos \theta = & \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma \\ & + \cos \delta \cos \phi \cos \beta \cos \omega + \cos \delta \sin \phi \sin \beta \cos \gamma \cos \omega \\ & + \cos \delta \sin \beta \sin \gamma \sin \omega\end{aligned}$$

In the case of a surface facing South ($\gamma = 0$) this becomes:

$$\cos \theta = \cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta$$

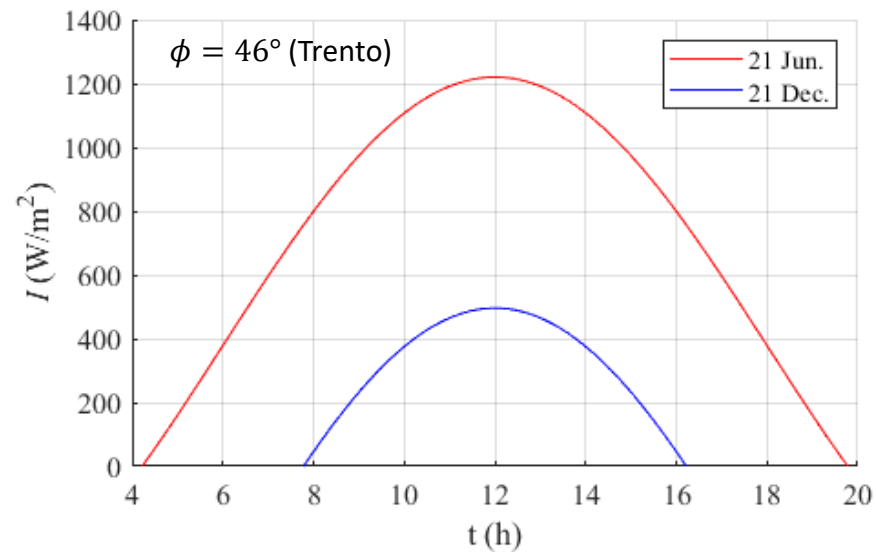
Derivation @ the blackboard



Extraterrestrial radiation

The incident *extraterrestrial radiation* on a *horizontal plane* at the border of atmosphere is

$$I = I_0 \cos \theta_z = I_{sc} \left[1 + 0.034 \cos \left(2\pi \frac{n}{365} \right) \right] (\sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega) \quad [\text{W/m}^2]$$





Extraterrestrial and incident irradiation

To find the *extraterrestrial irradiation* (i.e., the radiated energy) falling on a horizontal plane (tangent to the Earth's surface) throughout a *whole day*, we can integrate the previous equation over time, remembering that $\omega = \frac{\pi}{12}(t - 12)$, between sunrise and sunset ($\omega = \pm \omega_s$), leading to

$$\bar{H}_{h0} = \frac{86400[s]}{\pi} \left[1 + 0.034 \cos \left(2\pi \frac{n}{365} \right) \right] (\omega_s \sin \delta \sin \phi + \cos \delta \cos \phi \sin \omega_s) \quad [\text{J/m}^2]$$

The *daily irradiation* that really reaches ground (a horizontal surface) will be reduced by a factor called *clearness index*

$$K_T = \frac{\bar{H}_h}{\bar{H}_{h0}}$$

which typically varies in the range 0.3-0.8

[Libraries](#) of measurements of the daily irradiation \bar{H}_h on horizontal surfaces on ground are available from observatories, typically in the form of *monthly average values*.

This data, combined with the formulation seen above, allow constructing datasets of monthly average values of K_T



Beam vs diffused irradiation

The incident radiation on a surface holds two components:

- *beam radiation*, which is the solar radiation received from the Sun not scattered by the atmosphere;
- *diffuse radiation*, received from the Sun after its direction has been changed by scattering by the atmosphere

With reference to the irradiation on a horizontal surface: $\bar{H}_h = \bar{B}_h + \bar{D}_h$

Some datasets explicitly report average values for the beam and diffuse irradiation (\bar{B}_h , \bar{D}_h respectively) at given locations. If such data is not available, empirical correlations exist, which allow estimating realistic values for the two components (given \bar{H}_h) based on the clearness index:

$$\bar{D}_h = \bar{H}_h(1.390 - 4.027 K_T + 5.553 K_T^2 - 3.108 K_T^3)$$

$$\bar{B}_h = \bar{H}_h - \bar{D}_h$$

Irradiation on a tilted surface

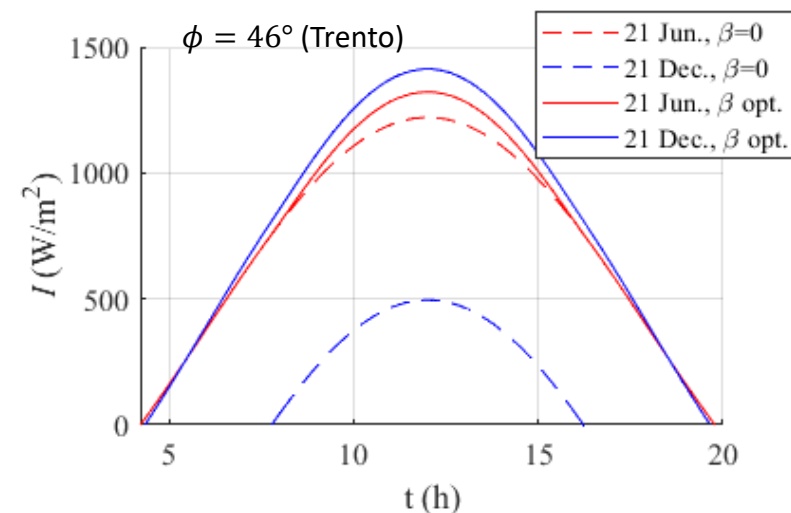
In the case of a tilted surface, the considerations seen so far for the extraterrestrial radiation/irradiation can be repeated, using $\cos \theta$ (instead of $\cos \theta_z$) in the expression of the incident extraterrestrial radiation.

Let us consider the case of a surface facing South ($\gamma = 0$):

$$I(\beta) = I_0 \cos \theta = I_{sc} \left[1 + 0.034 \cos \left(2\pi \frac{n}{365} \right) \right] (\sin \delta \sin(\phi - \beta) + \cos \delta \cos \omega \cos(\phi - \beta)) \quad [\text{W/m}^2]$$

$$\bar{H}_0(\beta) = \frac{86400[s]}{\pi} \left[1 + 0.034 \cos \left(2\pi \frac{n}{365} \right) \right] (\omega_s \sin \delta \sin(\phi - \beta) + \cos \delta \sin \omega_s \cos(\phi - \beta)) \quad [\text{J/m}^2]$$

Changing β (e.g. using a tracking system on a solar panel) allows increasing the amount of average radiation hitting the surface





Irradiation on a tilted surface

Knowing the daily irradiation components on a horizontal surface on the ground, $(\bar{H}_h, \bar{B}_h, \bar{D}_h)$, one can estimate the *direct* and *diffuse* components on the *titled surface*:

$$\bar{B}(\beta) = R_b \bar{B}_h, \quad \text{with } R_b = \frac{\bar{H}_0(\beta)}{\bar{H}_{h0}} = \frac{\omega_s \sin \delta \sin(\phi - \beta) + \cos \delta \sin \omega_s \cos(\phi - \beta)}{\omega_s \sin \delta \sin \phi + \cos \delta \cos \phi \sin \omega_s}$$

i.e., we assume that the ratio $R_b = \bar{B}(\beta)/\bar{B}_h$ equals the ratio of extraterrestrial irradiation on a tilted surface over the extraterrestrial irradiation on a horizontal surface.

As for the diffuse radiation, it is assumed that

$$\bar{D}(\beta) = \bar{D}_h \frac{1 + \cos \beta}{2}$$

which makes use of a convention factor such that:

- when $\beta = 0$, the surface gets the same diffused radiation as the horizontal
- when $\beta = \pi/2$, the surface gets $\frac{1}{2}$ the diffused radiation of the horizontal (as the other half goes to the back of the surface)
- when $\beta = \pi$, the surface gets no diffused radiation (as this entirely goes to the back of the surface)



Irradiation on a tilted surface

The global irradiation on the tilted surface can be finally estimated as

$$\bar{H}(\beta) = \bar{B}(\beta) + \bar{D}(\beta) + \bar{A}(\beta)$$

where $\bar{A}(\beta)$ is an additional component due to the beams reflected towards the surface by the ground (albedo), estimated as

$$\bar{A}(\beta) = \rho \frac{1 - \cos \beta}{2} \bar{H}_h$$

where ρ is the albedo coefficient



Irradiation on a tilted surface

Recap - calculation of the daily irradiation on a tilted surface – beam, diffuse and global

- 1) Calculate \bar{H}_{h0} and $\bar{H}_0(\beta)$ from location data and equations
- 2) Get the global horizontal irradiation \bar{H}_h from a database
- 3) (if daily data for beam/diffuse irradiation are available -> go to 4). Calculate K_T , and use it to obtain \bar{D}_h from correlations ($\bar{D}_h = \bar{D}_h(\bar{H}_h, K_T)$) and \bar{B}_h by difference ($\bar{B}_h = \bar{H}_h - \bar{D}_h$)
- 4) Calculate $R_b = \frac{\bar{H}_0(\beta)}{\bar{H}_{h0}}$
- 5) Calculate $\bar{B}(\beta) = R_b \bar{B}_h$, $\bar{D}(\beta) = \bar{D}_h \frac{1+\cos\beta}{2}$ and $\bar{A}(\beta) = \rho \frac{1-\cos\beta}{2} \bar{H}_h$
- 6) Calculate the global irradiation $\bar{H}(\beta) = \bar{B}(\beta) + \bar{D}(\beta) + \bar{A}(\beta)$

Note: daily irradiation values can be used to generate *hourly irradiation profiles* (useful in the evaluation of solar panels performance), based on correlations available on technical standards



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THERMAL SOLAR



Solar collectors

A solar collector is a heat exchanging device used to convert solar energy absorbed from incident solar radiation into thermal energy. Energy harvested from the radiation is used to heat a fluid that flows into a piping system within the collector.

There exist two types of collectors:

- *non-concentrating collectors*, also called *flat-plate collectors*
- *concentrating collectors*, which make use of concentration surfaces (mirrors, parabolic troughs or solar dishes)

Flat-plate collectors are the simplest solar energy harvesters. They capture both beam and diffuse radiation and convert it into thermal energy. They are typically used for applications requiring energy delivery at modest temperatures (below 100 °C).



Flat-plate collectors - structure

Flat-plate collectors components include:

Absorber Plate: a flat dark-colored surface responsible for absorbing sunlight. It is made from materials with high thermal conductivity to efficiently transfer heat to the piping/fluid system.

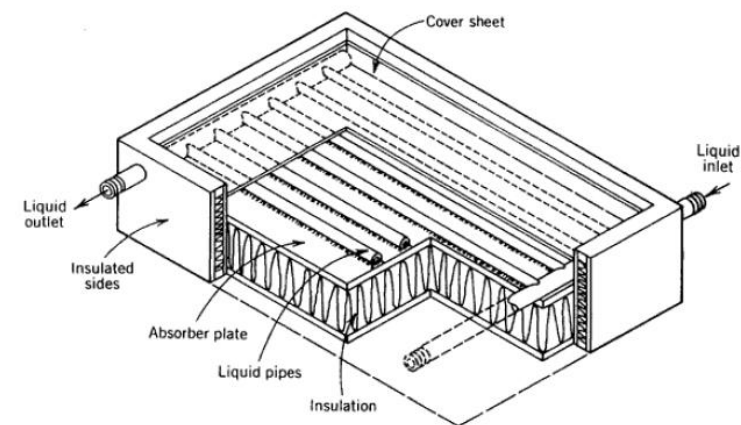
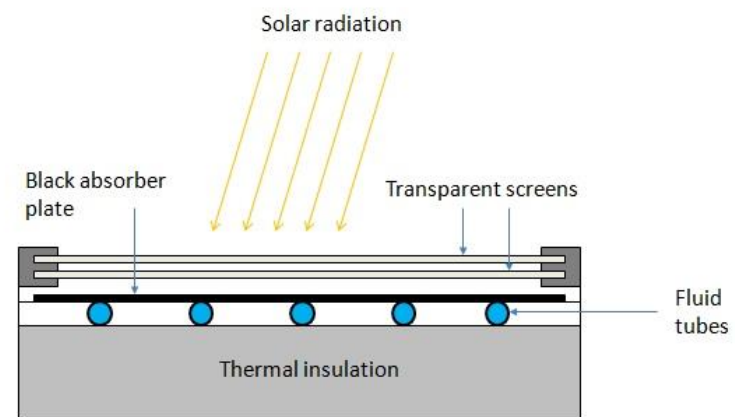
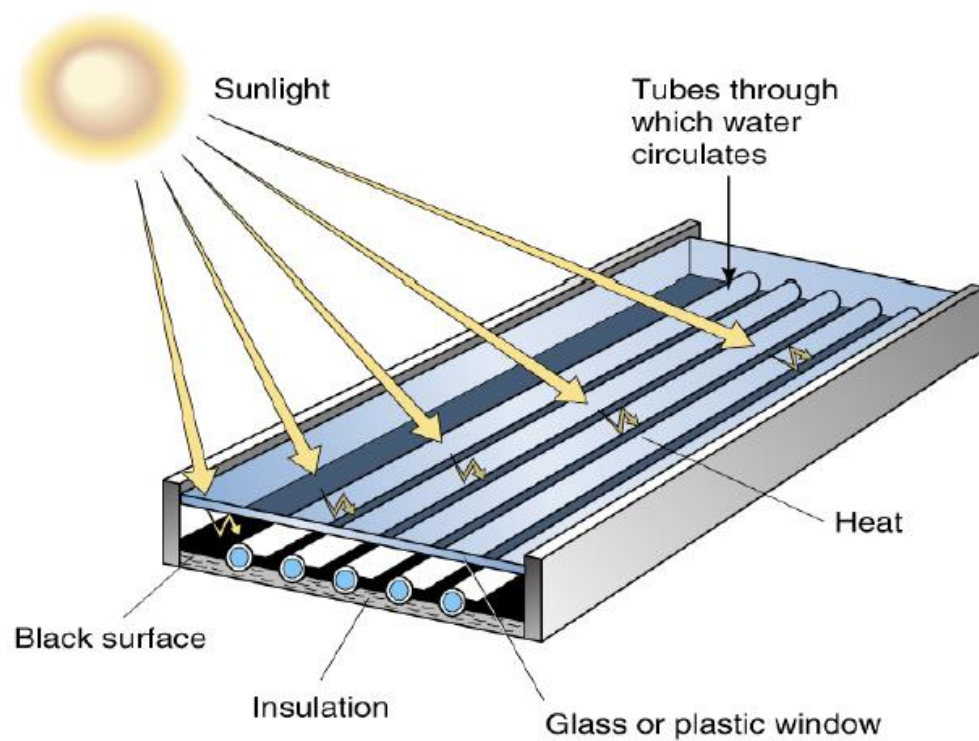
Transparent Cover: Positioned above the absorber plate, usually made of glass or plastic. It allows sunlight to pass through while minimizing heat loss through convection and conduction (creating a “greenhouse effect”), while also sheltering the absorber plate.

Insulation: The sides and bottom of the collector are insulated (with fiberglass or foam) to reduce heat losses.

Heat Transfer Fluid : A heat transfer fluid (usually water or antifreeze liquids such as glycol) flows through tubes (made of highly conducting materials, e.g. Cu or Al), hence absorbing heat from the absorber plate and carrying it to a heat exchanger for further use.

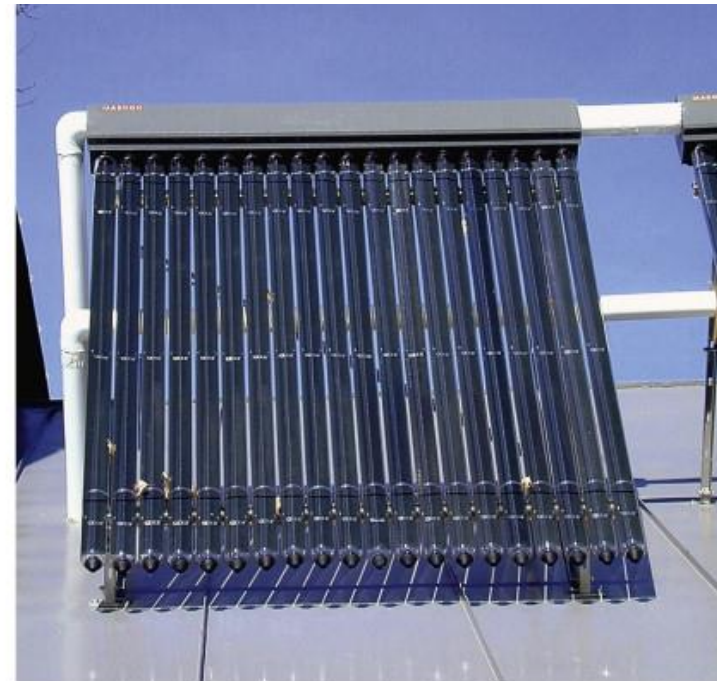
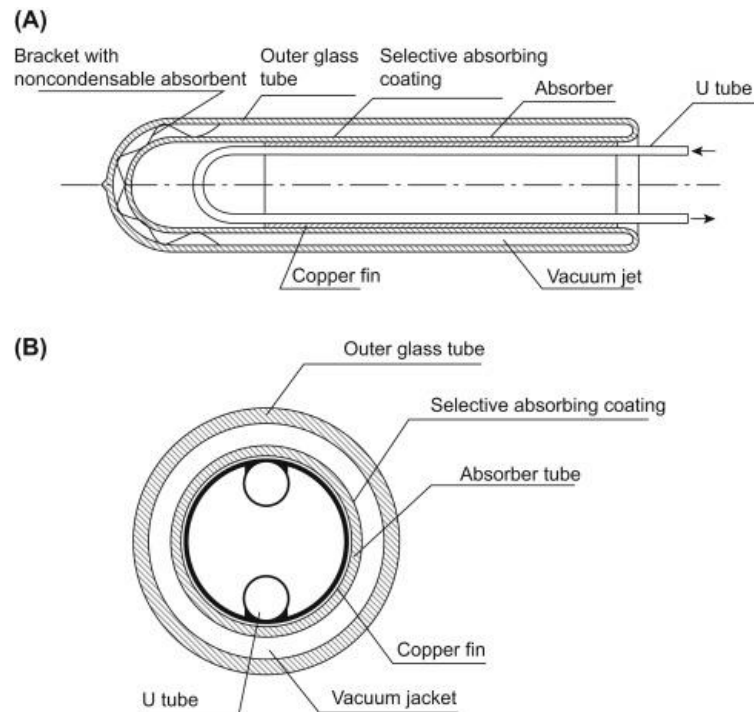
Heat Exchanger: The heat exchanger transfers heat from the transfer fluid to another fluid (e.g. domestic hot water) for final application.

Flat-plate collectors

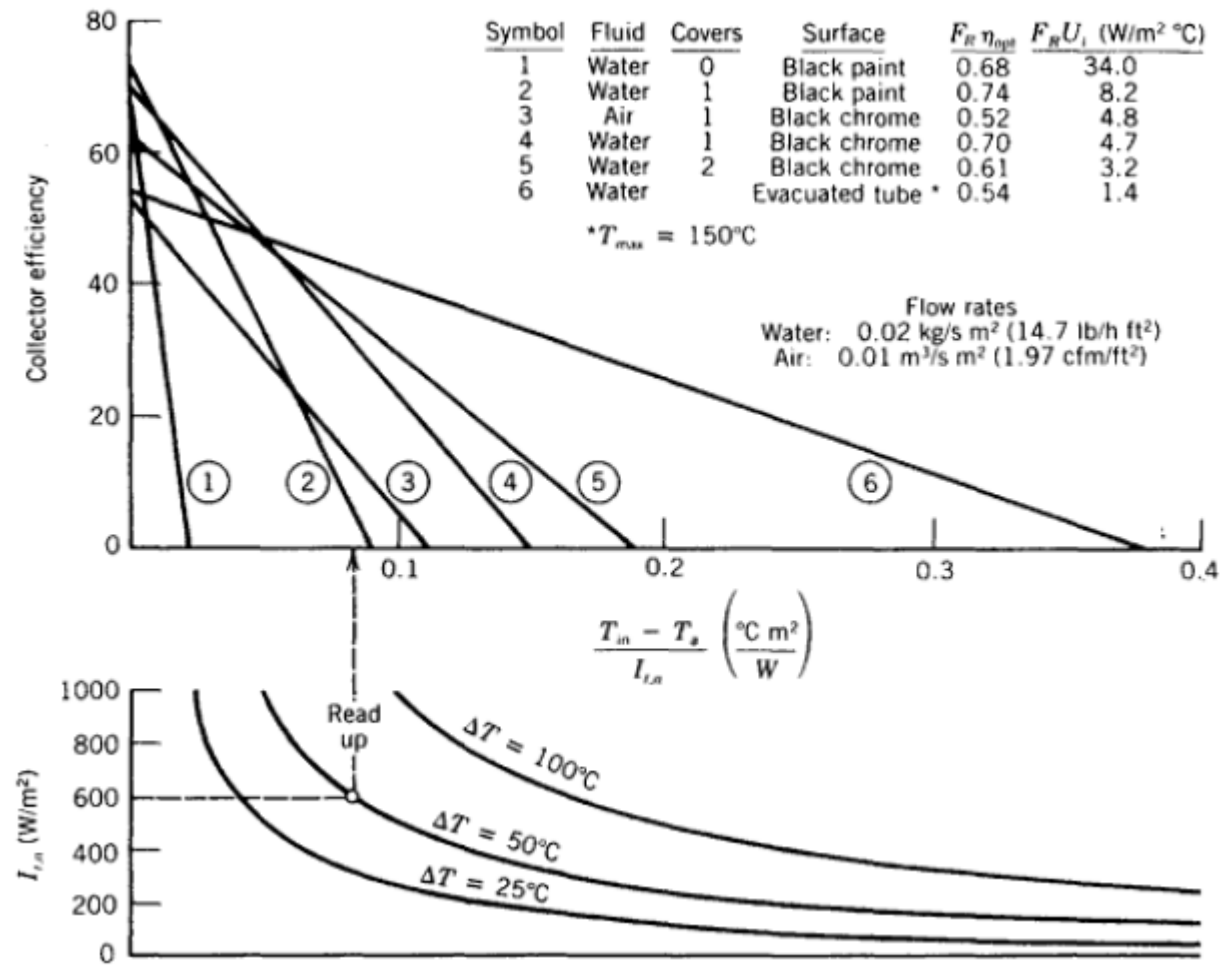


Flat-plate vacuum tube collectors

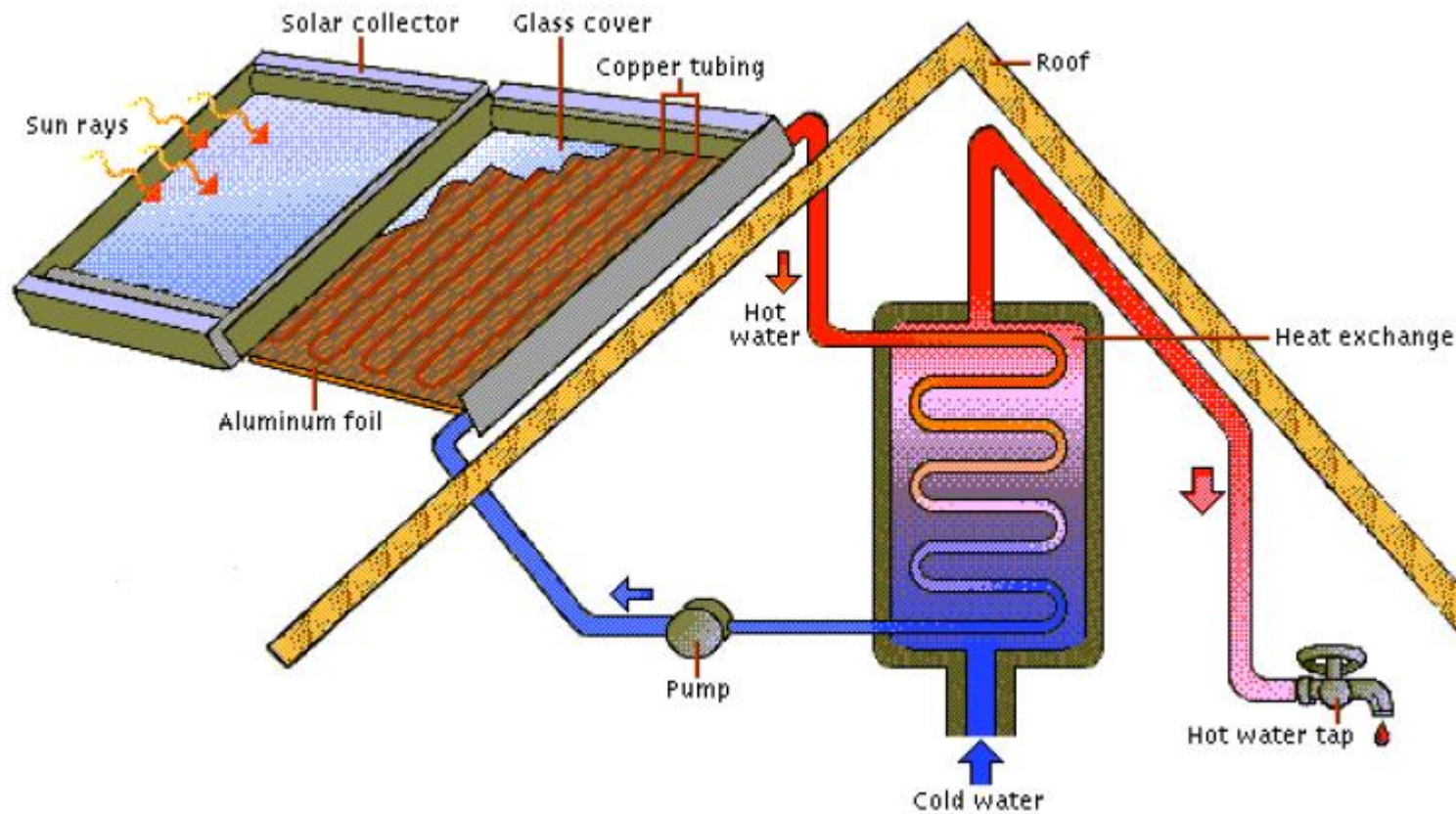
A special class of flat-plate collectors are the so-called *vacuum tube collectors*. These are made of absorber tubes bounded to a heat pipe, covered with selective coating and suspended in vacuum (which improves thermal insulation). These are typically used for moderate temperatures (above 90 °C) or very cold climates.



Flat-plate collectors efficiency

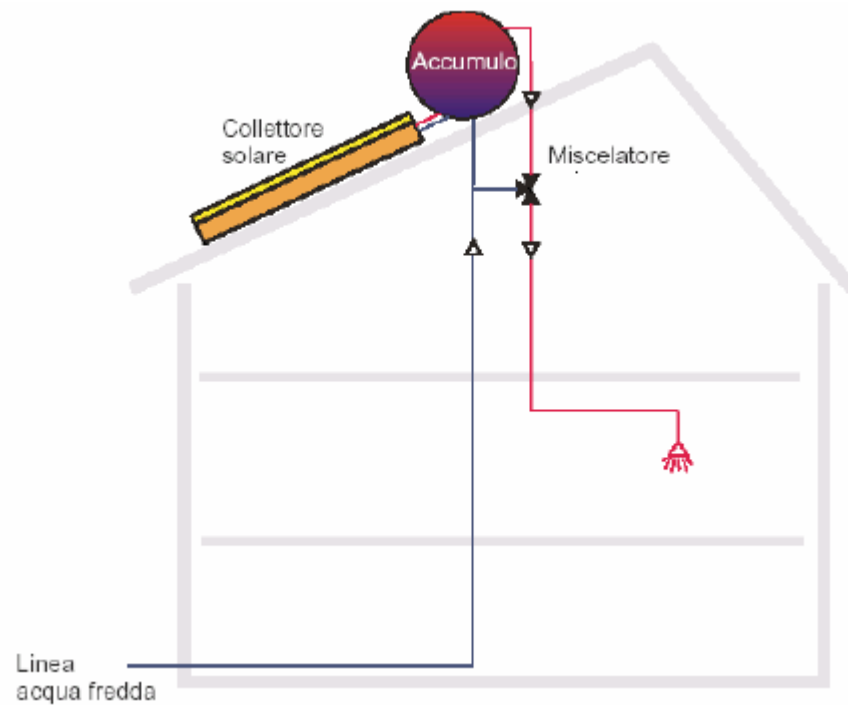


Flat-plate collectors in applications

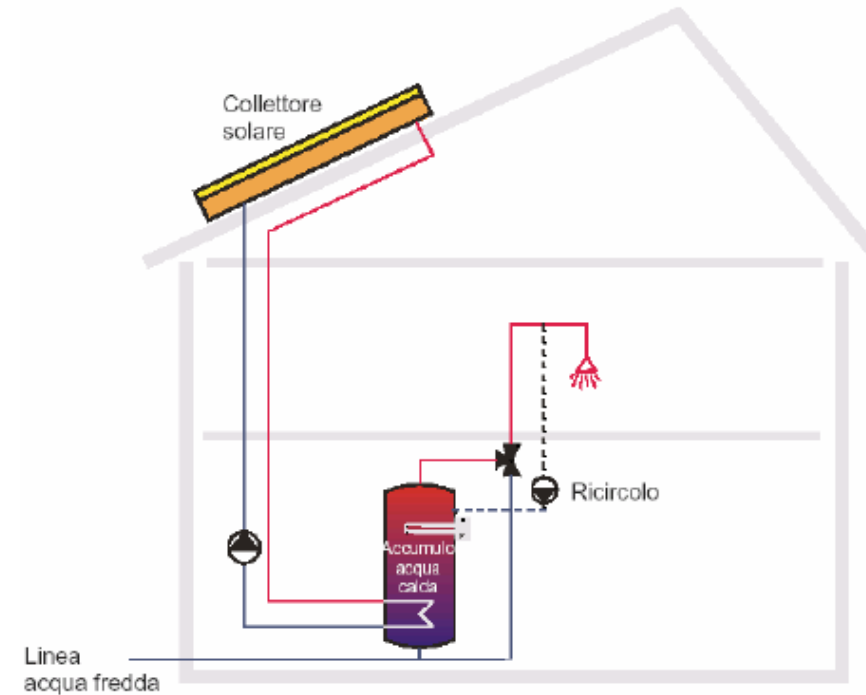


Flat-plate collectors in applications

Integrated storage



External storage





Consumptions, sizing

STIMA CONSUMO ACQUA CALDA 1

Modalità di utilizzo	Litri (al giorno per persona)
Comfort basso	30 (20 - 40)
Comfort medio	50 (40 - 60)
Comfort alto	70 (60 - 80)

STIMA CONSUMO ACQUA CALDA 2

Tipologia di utilizzo	Litri al giorno	Note
Ospedale	60	Per posto letto
Case di riposo	40	-
Scuole	5	-
Caserme	30	-
Industrie	20	-
Uffici	5	-
Campeggi	30	Per persona
Hotel cat. superiore	160	Per stanza
Hotel cat. inferiore	100	Per stanza
Palestre	35	Per utilizzatore
Lavanderie	6	Per kg lavati
Ristoranti	10	Per pasto
Bar	2	Per consumazione

Dalle seguenti tabelle è possibile determinare un dimensionamento medio della dimensione dei pannelli solari e del volume di accumulo in funzione del numero di persone servite dall'impianto:

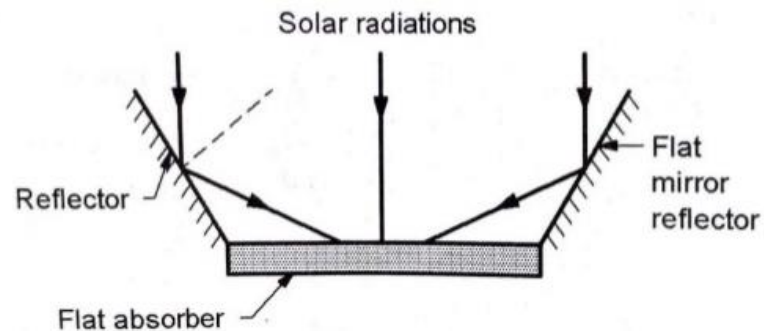
DIM. COLLETTORI PIANI

Numero di persone	2/4	4/6	6/8
Consumo medio giornaliero (litri)	150	250	350
Mq collettore piano	2,4	4	6
Volume accumolo (litri)	200	300	400
Grado di copertura medio (35°)	60%	66%	66%

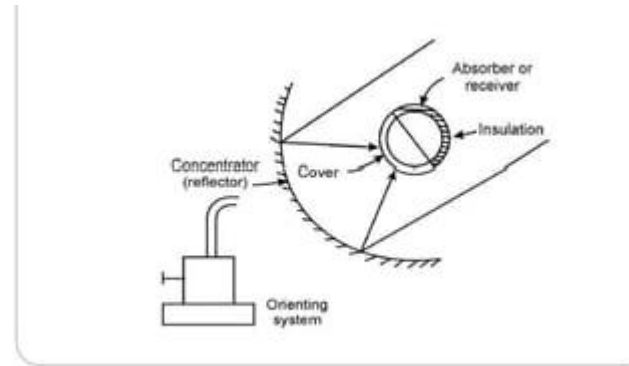
DIM. COLLETTORI TUBI SOTTOV.

Numero di persone	2/4	4/6	6/8
Consumo medio giornaliero (litri)	150	250	350
Mq collettore tubi sottov.	2	3	4
Volume accumolo (litri)	200	300	400
Grado di copertura medio (35°)	59%	65%	65%

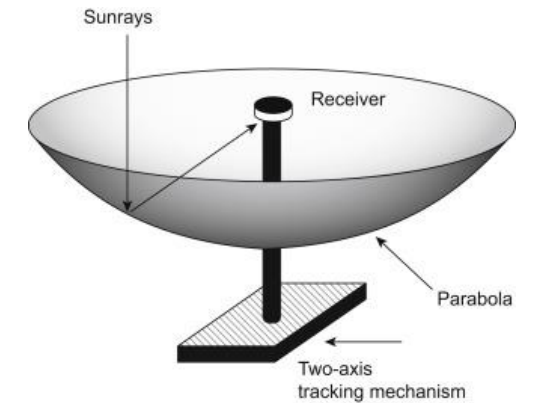
Concentrating collectors



Fixed concentrator



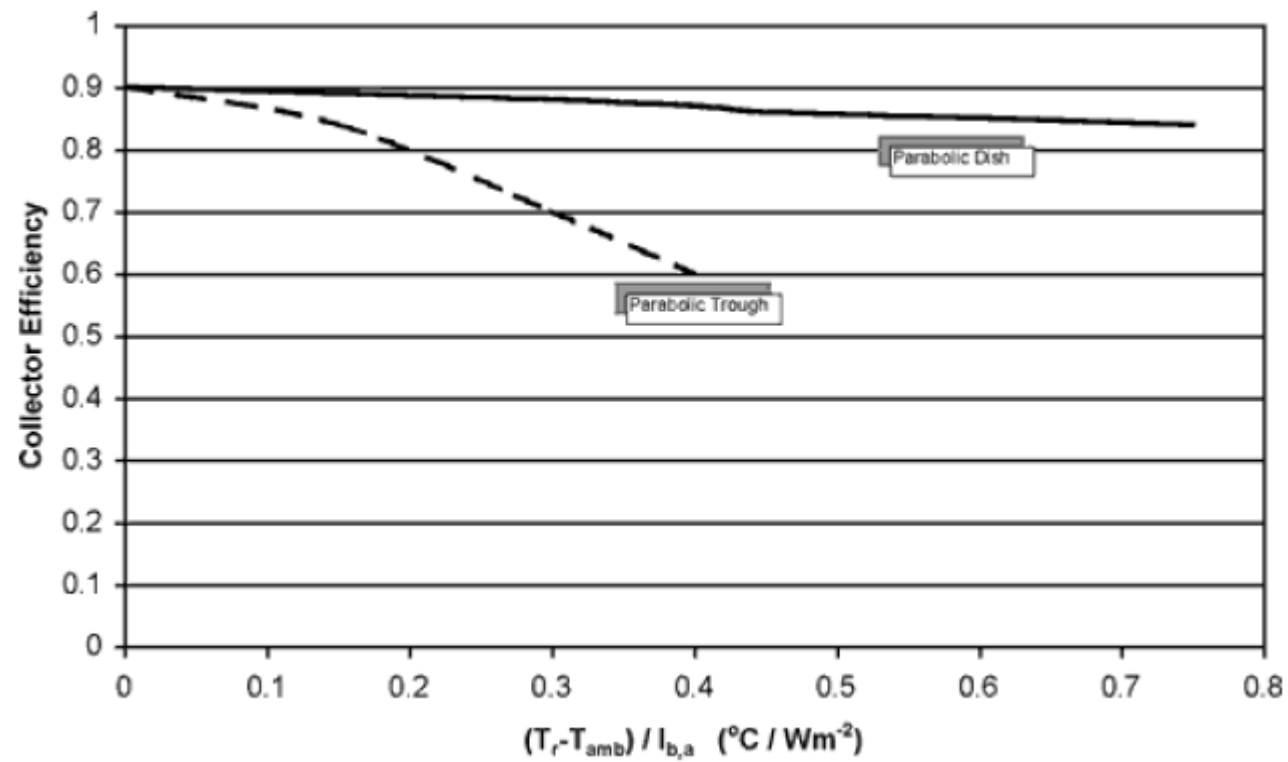
Parabolic trough



Parabolic dish

Type of Collector	Concentration Ratio C	Typical Working Temperature Range (°C)
Flat plate collector	1	≤ 70
High-efficiency flat plate collector	1	60–120
Fixed concentrator	2–5	100–150
Parabolic trough collector	10–50	150–350
Parabolic dish collector	200–2000	250–700
Central receiver tower	200–2000	400–1000

Concentrating collectors





Water storage

In general, the rate at which thermal energy is produced (by solar collectors) and the rate at which it is consumed do not match. To make harvested energy (here, hot water) usable in an asynchronous manner, *energy storage* is necessary.

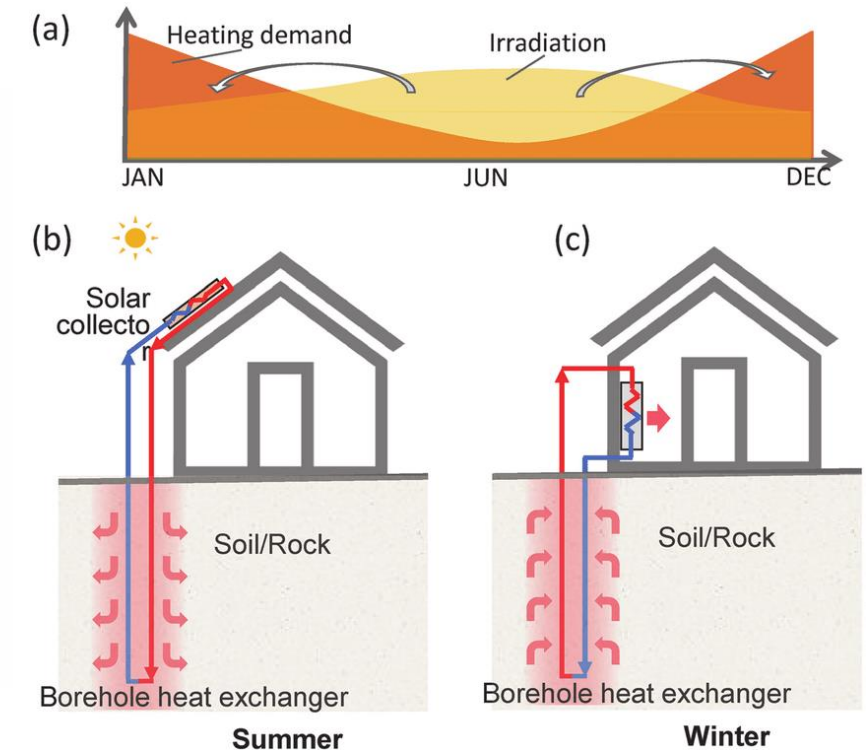
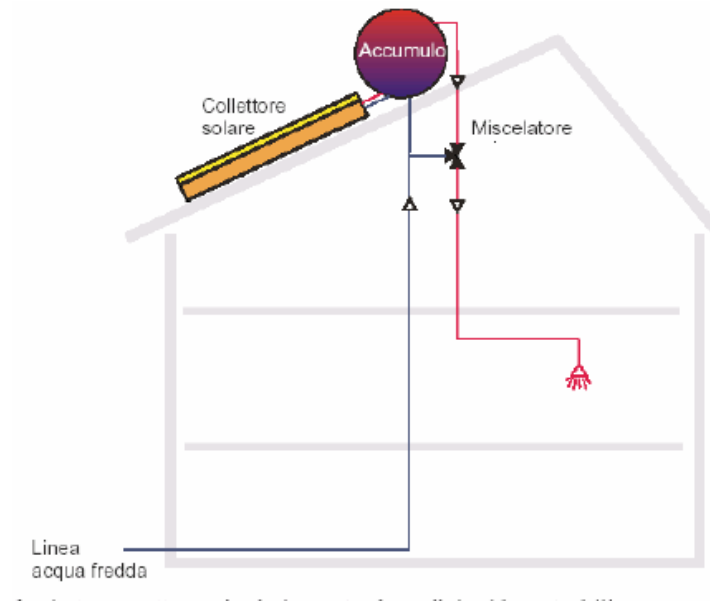
In the case of domestic utilities, thermal energy is stored within tanks housing large volumes of hot water. The larger the tank capacity (i.e., volume), the higher the buffering capacity of the system (i.e., the ability to temporally decouple production and consumption).

The optimum capacity of an energy storage system depends on the expected time dependence of solar radiation availability, the nature of loads, the manner in which auxiliary energy is provided (e.g., auxiliary heaters), and the costs (e.g. the costs of large storage systems).

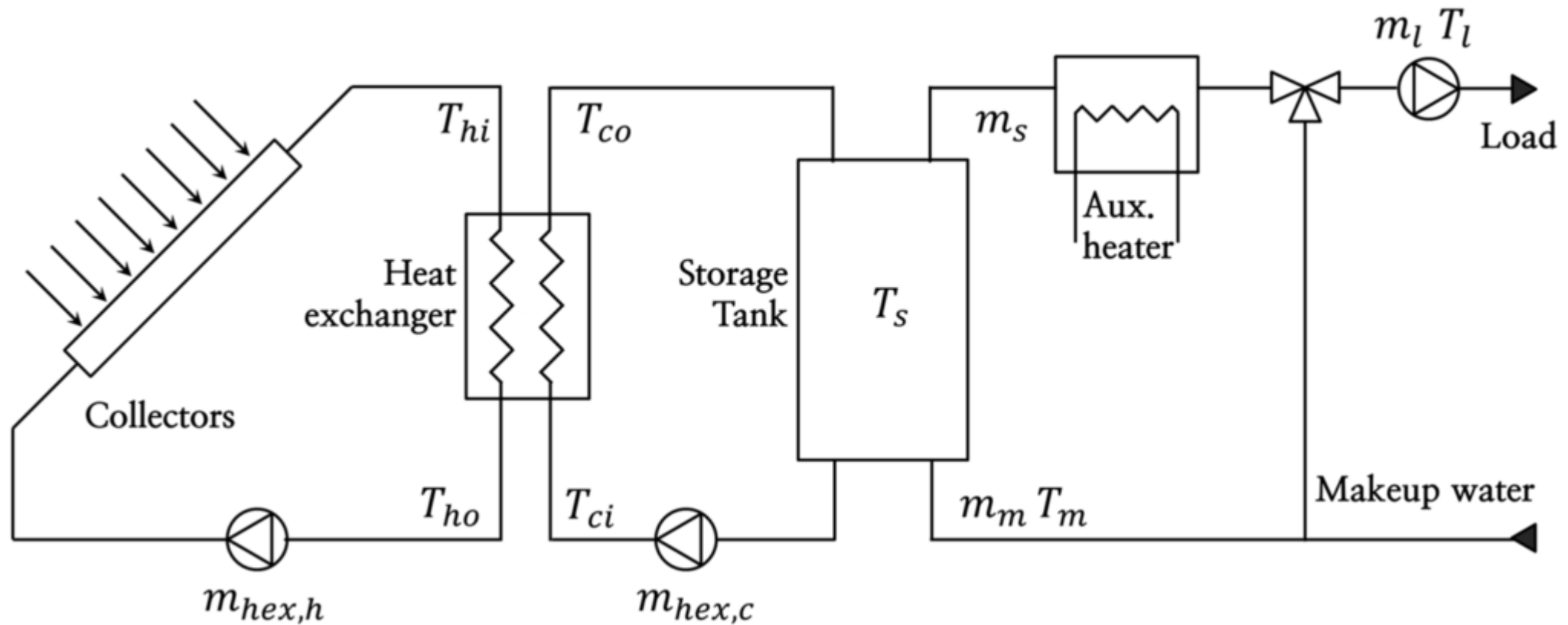
Water storage

Thermal energy storage for solar collectors can encompass different time scale. We distinguish between

- short term (hourly/daily) storage systems. They decouple production/consumption on a daily basis, by relying of water tanks with size of 100-500 l (for households)
- long term storage systems. These allow decoupling production/consumption of a seasonal basis. They typically make use of underground tanks, or use the ground itself (e.g., water-filled caves) to store thermal energy.



Water storage (short-term)





Sizing criteria

People in Household	1	2	3	4	5	6
Size of Storage Tank	40 gal	40 gal	60 gal	80 gal	90 gal	100 gal
	Square Feet of Solar Collector					
Sunbelt	20-26	20-26	30-40	40-53	46-60	50-67
Mountain & South	23-32	23-32	34-48	46-64	51-72	57-80
Midwest & Atlantic	32-40	32-40	48-60	64-80	72-90	80-100
North West & New England	40-53	40-53	60-80	80-107	90-120	100-133

1 gal = 3.78 l

1 sq foot = 0.0929 m²