UNIVERSITÀ DI TRENTO

DEPARTMENT OF

INDUSTRIAL ENGINEERING

Renewable Energy Conversion Systems

Lecture 8

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Lecture 8: Outline

Main topic:

Wind Energy Conversion Systems:

modeling of the permanent magnet synchronous machine (PMSM)

- Scope of the modeling of the PMSM
- Modeling of the PMSM in the natural reference frame (abc)
- Introduction of the synchronous reference frame (dq0)
- Modeling of the PMSM in the synchronous reference frame

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Scope

The goal is to obtain a **mathematical model of the Permanent Magnet Synchronous Machine (PMSM)**, consisting in a set of differential equations, describing the electro-mechanical behaviour of the PMSM

Such model will be the basis for designing the regulators required to control the variables of interest (e.g. current/torque, speed etc.) in the PMSM in both static and dynamic conditions

The mathematical model can be developed in different reference frames, each of which can entail different advantages

The models will be developed considering the **generator sign convention**. Moreover, the circuit is assumed to be linear (e.g. magnetic saturation is excluded)

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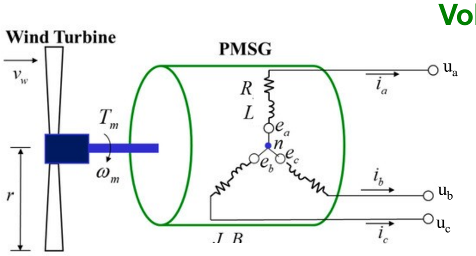
Model of the PMSM in the natural reference frame

We focus on the electrical terminals of the stator windings of the machine, where a set of three phase (*a,b,c*-) emfs is induced by a time varying magnetic flux, whenever the rotor of the machine rotates

Such terminals are considered connected to the three phase grid/power electronic converter, which supplies a set of three phase voltages, *u*



Source: Ming-Fa Tsai et al.

Voltage equations at the stator windings can be written:

$$u_a(t) + Ri_a(t) = \frac{d\lambda_a(t)}{dt}$$

$$u_b(t) + Ri_b(t) = \frac{d\lambda_b(t)}{dt}$$

$$u_c(t) + Ri_c(t) = \frac{d\lambda_c(t)}{dt}$$

R indicates the phase resistance of the stator windings
λ is the magnetic flux linkage
i is the stator phase current

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Model of the PMSM in the natural reference frame

$$u_a(t) + Ri_a(t) = \frac{d\lambda_a(t)}{dt}$$
$$u_b(t) + Ri_b(t) = \frac{d\lambda_b(t)}{dt}$$
$$u_c(t) + Ri_c(t) = \frac{d\lambda_c(t)}{dt}$$

We focus on the flux linkage on the three phases.

Two contributions add to it:

- **The magnetic flux produced by the permanent magnets (PM) in the rotor $\lambda,_{mg}$**
- **The magnetic flux produced by the three phase currents flowing through the stator winding $\lambda,_{i}$**

The main flux produced by the PMs and the additional flux produced by the induced currents act in opposite sense

$$\lambda_a(t) = \lambda_{a,mg}(t) - \lambda_{a,i}(t)$$
$$\lambda_b(t) = \lambda_{b,mg}(t) - \lambda_{b,i}(t)$$
$$\lambda_c(t) = \lambda_{c,mg}(t) - \lambda_{c,i}(t)$$

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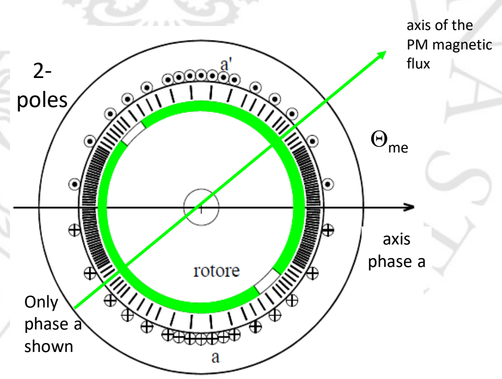
Magnetic flux produced by the PMs

Due to the spatial distribution of the three phase conductors in the stator and pole shaping, flux linkages due to the permanent magnets are approximately sinusoidal

$$\lambda_{a,mg} = \Lambda_{mg} cos(\Theta_{me})$$
$$\lambda_{b,mg} = \Lambda_{mg} cos(\Theta_{me} - 2\pi/3)$$
$$\lambda_{c,mg} = \Lambda_{mg} cos(\Theta_{me} - 4\pi/3)$$

θ_{me} indicates the the electrical angle between the axis of each phase and the one of the flux produced by the PM

Note that θ_{me} (and consequently the corresponding flux) are time dependent



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Magnetic flux produced by the stator currents

Due to the symmetry of the phase windings and the magnetic isotropy of the structure, phase self-inductances will all be equal, as well as mutual inductances between each winding and the other two.

$$L_a = L_b = L_c = L_s \qquad L_{Mab} = L_{Mbc} = L_{Mca} = -|L_{Ms}|$$

$$\lambda_{a,i} = L_a i_a + L_{Mab} i_b + L_{Mac} i_c = L_s i_a - |L_{Ms}|(i_b + i_c)$$
$$\lambda_{b,i} = L_{Mab} i_a + L_b i_b + L_{Mbc} i_c = L_s i_b - |L_{Ms}|(i_a + i_c)$$
$$\lambda_{c,i} = L_{Mac} i_a + L_{Mbc} i_b + L_c i_c = L_s i_c - |L_{Ms}|(i_a + i_b)$$

and being generally $i_a+i_b+i_c=0$

$$\boxed{\begin{aligned}\lambda_{a,i} &= L\, i_a \\ \lambda_{b,i} &= L\, i_b \\ \lambda_{c,i} &= L\, i_c\end{aligned}}$$

Having set $L=L_s+|L_{Ms}|$

The linkage flux of each phase ***apparently*** depends only on the current circulating in the same phase

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Model of the PMSM in the natural reference frame

Having obtained:

$$\lambda_a = -L\, i_a + \Lambda_{mg} cos(\Theta_{me})$$
$$\lambda_b = -L\, i_b + \Lambda_{mg} cos(\Theta_{me} - 2\pi/3)$$
$$\lambda_c = -L\, i_c + \Lambda_{mg} cos(\Theta_{me} - 4\pi/3)$$

Voltage balance equations can be rewritten as:

$$u_a + Ri_a = -L\frac{di_a(t)}{dt} + e_a$$
$$u_b + Ri_b = -L\frac{di_b(t)}{dt} + e_b$$
$$u_c + Ri_c = -L\frac{di_c(t)}{dt} + e_c$$

having set:

$$e_a = \frac{d\lambda_{a,mg}}{dt} = -\Lambda_{mg}\omega_{me} sin(\Theta_{me}) = \Lambda_{mg}\omega_{me} cos(\Theta_{me} + \pi/2)$$
$$e_b = \frac{d\lambda_{b,mg}}{dt} = \Lambda_{mg}\omega_{me} cos\left(\Theta_{me} + \frac{\pi}{2} - 2\pi/3\right)$$
$$e_c = \frac{d\lambda_{c,mg}}{dt} = \Lambda_{mg}\omega_{me} cos\left(\Theta_{me} + \frac{\pi}{2} - 4\pi/3\right)$$

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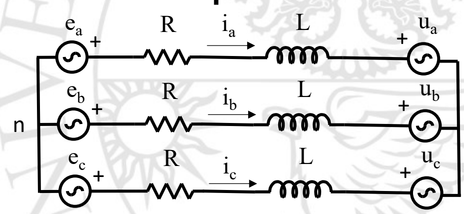
Model of the PMSM in the natural reference frame

Voltage balance equations:

$$u_a + Ri_a + L\frac{di_a(t)}{dt} = e_a$$
$$u_b + Ri_b + L\frac{di_b(t)}{dt} = e_b$$
$$u_c + Ri_c + L\frac{di_c(t)}{dt} = e_c$$

Circuit representation:



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Electro-mechanical energy conversion in PMSM

Electro-mechanical energy conversion in permanent magnets synchronous machines stems from the interaction between two magnetically coupled, independent circuits/subsystems, i.e. the magnetic field (created by the magnets) and the rotating magnetic field created by the stator currents

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Power balance of the PMSM

Power balance can be derived from each phase voltage equation, multiplying it by the corresponding phase current

$$u_a + Ri_a + L\frac{di_a(t)}{dt} = e_a \xrightarrow{\times i_a} p_a$$

$+$

$$u_b + Ri_b + L\frac{di_b(t)}{dt} = e_b \xrightarrow{\times i_b} p_b$$

$+$

$$u_c + Ri_c + L\frac{di_c(t)}{dt} = e_c \xrightarrow{\times i_c} p_c$$

$$\underbrace{u_a i_a + u_b i_b + u_c i_c}_{\text{Instantaneous electrical power injected into the grid}} + \underbrace{R(i_a^2 + i_b^2 + i_c^2)}_{\text{Dissipated power (Joule effect)}} + \frac{d}{dt}\underbrace{[\frac{1}{2}\mathrm{L}(i_a^2 + i_b^2 + i_c^2)]}_{\text{Power used to charge the magnetic energy}} = \underbrace{e_a i_a + e_b i_b + e_c i_c}_{\text{Electro-mechanical power (converted from mechanical to electrical)}}$$

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Power balance of the PMSM

Torque derivation

$$e_a i_a + e_b i_b + e_c i_c = T_m^m \omega_m = \frac{T_m^m \omega_{me}}{p}$$

p: number of pole pairs of the electrical machine

$$T_m^m = p\Lambda_{mg}[i_a cos\left(\Theta_{me} + \frac{\pi}{2}\right) + i_b cos\left(\Theta_{me} + \frac{\pi}{2} - \frac{2\pi}{3}\right) + i_c cos\left(\Theta_{me} + \frac{\pi}{2} - \frac{4\pi}{3}\right)]$$

with

$$\left[\begin{array}{l} i_a(t) = I_M(t)\cos\left(\Theta_{me} + \frac{\pi}{2} - \Psi\right) \\ i_b(t) = I_M(t)\cos\left(\Theta_{me} + \frac{\pi}{2} - \Psi - \frac{2\pi}{3}\right) \\ i_c(t) = I_M(t)\cos\left(\Theta_{me} + \frac{\pi}{2} - \Psi - \frac{4\pi}{3}\right) \end{array}\right]$$

$$\boxed{T_m^m = \frac{3}{2} p\Lambda_{mg} I_M(t)\cos(\Psi)}$$

Ψ Is the phase shift between the currents and the corresponding emfs

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Model of the PMSM in the natural reference frame

The model in the natural reference frame has the advantage of providing immediate interpretation of the physics of the system

However, it is relatively cumbersome due to the presence of equations related to all the three phases

It is also not the most convenient choice to design controllers for the generator

Through the use of new mathematical tools and a different reference framework, a more suitable model for control can be obtained

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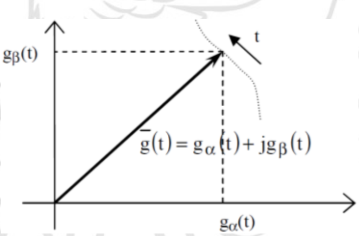
Space vectors

A space vector is a mathematical (symbolic) tool to study the behavior of a three-phase system (also under dynamic conditions) applying a mathematical transformation that reduces the complexity of the equations

It is possible to associate the complex function (i.e. the space vector) g(t), to the set of 3-phase quantities g_a, g_b and g_c

$$\mathbf{g}(t) = \frac{2}{3}\left[g_a(t) + g_b(t)e^{j2\pi/3} + g_c(t)e^{j4\pi/3} \right]$$

This can be represented in the complex plane with α= real axis and β=imaginary axis by a vector with time-variant amplitude and phase



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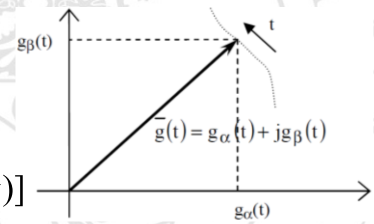
Space vectors in fixed reference frame

$\mathbf{g}(t) = g_\alpha(t) + j\, g_\beta(t)$ **space vector in α, β, corresponding to:**

$$\Re[\mathbf{g}(t)] = g_\alpha(t) = \frac{2}{3}\left[g_a(t) - \frac{g_b(t)}{2} - \frac{g_c(t)}{2}\right]$$

$$\Im[\mathbf{g}(t)] = g_\beta(t) = \frac{2}{3}\left[g_b(t)\frac{\sqrt{3}}{2} - g_c(t)\frac{\sqrt{3}}{2}\right] = \frac{1}{\sqrt{3}}[g_b(t) - g_c(t)]$$



The zero-sequence (i.e. homopolar) component needs to be added, if present

$$g_0(t) = \frac{g_a(t) + g_b(t) + g_c(t)}{3} \neq 0$$

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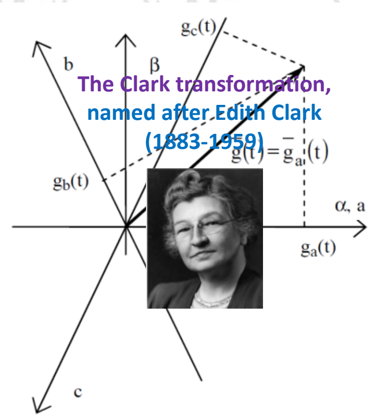
Space vectors in fixed reference frame

Any generic set of 3-phase quantities is fully described by the following components

$$\begin{bmatrix} g_\alpha(t) \\ g_\beta(t) \\ g_0(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} g_a(t) \\ g_b(t) \\ g_c(t) \end{bmatrix}$$

In this way, a linear transformation from the three-phase natural reference frame a-b-c to a new reference frame α-β-0 in space having three fixed axes shifted by 90° with respect to each other



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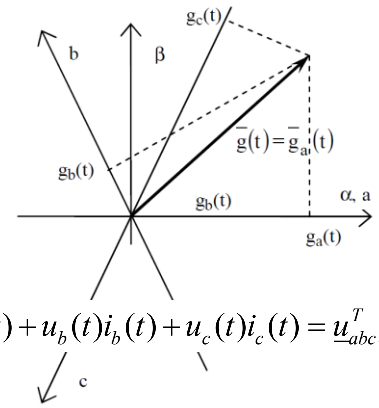
Space vectors in fixed reference frame

The corresponding inverse transform is:

$$\begin{bmatrix} g_a(t) \\ g_b(t) \\ g_c(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{bmatrix} \begin{bmatrix} g_\alpha(t) \\ g_\beta(t) \\ g_0(t) \end{bmatrix}$$



As defined, the α–β-0 transformation is amplitude invariant.

It is, however, not power invariant $p(t) = u_a(t)i_a(t) + u_b(t)i_b(t) + u_c(t)i_c(t) = \underline{u}_{abc}^T \underline{i}_{abc}$

$$p(t) = \frac{3}{2}\left[u_\alpha(t)i_\alpha(t) + u_\beta(t)i_\beta(t)\right] + 3u_0(t)i_0(t)$$

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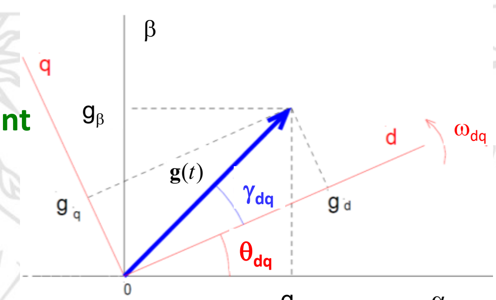
Space vectors in rotating reference frame

In electrical machines applications it is convenient to express the space vector in an orthogonal reference frame (d-q-0) that is rotating with angular frequency ω_{dq} with respect to the (α–β–0) frame $\boldsymbol{g}_{\alpha\beta}(t) = g_\alpha + jg_\beta = |\boldsymbol{g}|e^{j\gamma_{\alpha\beta}}$

This is obtained by another (non-linear) transformation $\boldsymbol{g}_{dq}(t) = g_d + jg_q = |\boldsymbol{g}|e^{j\gamma_{dq}}$



The d-q-0 transformation is amplitude invariant

$|\boldsymbol{g}_{\alpha\beta}| = |\boldsymbol{g}_{dq}| = |\boldsymbol{g}|$

Being: $\gamma_{\alpha\beta} = \gamma_{dq} + \theta_{dq}$

$\boldsymbol{g}_{\alpha\beta} = |\boldsymbol{g}|e^{j(\gamma_{dq}+\theta_{dq})} = |\boldsymbol{g}|\, e^{j\gamma_{dq}} e^{j\theta_{dq}} = \boldsymbol{g}_{dq}\, e^{j\theta_{dq}}$

$\boldsymbol{g}_{dq} = \boldsymbol{g}_{\alpha\beta}\, e^{-j\theta_{dq}}$

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Space vectors in rotating reference frame

$\boldsymbol{g}_{dq} = \boldsymbol{g}_{\alpha\beta}\, e^{-j\theta_{dq}}$

The previous can be decomposed on the d-q axes:

$$\boldsymbol{g}_{dq} = g_d + jg_q = \boldsymbol{g}_{\alpha\beta}\, e^{-j\theta_{dq}} = (g_\alpha + jg_\beta)(\cos(-\theta_{dq}) + j\sin(-\theta_{dq}))$$

$$\begin{bmatrix} g_d \\ g_q \end{bmatrix} = \begin{bmatrix} \cos(\theta_{dq}) & \sin(\theta_{dq}) \\ -\sin(\theta_{dq}) & \cos(\theta_{dq}) \end{bmatrix} \begin{bmatrix} g_\alpha \\ g_\beta \end{bmatrix}$$

orthonormal matrix

The homopolar component is unaffacted by the transformation

$$\theta_{dq} = \int_0^t \omega_{dq}(t)dt$$

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Space vectors in rotating reference frame

The complete transform is:

$$\begin{bmatrix} g_d \\ g_q \\ g_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_{dq}) & \sin(\theta_{dq}) & 0 \\ -\sin(\theta_{dq}) & \cos(\theta_{dq}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_\alpha \\ g_\beta \\ g_0 \end{bmatrix}$$

The Park transformation, named after Robert Park (1902-1994)

The corresponding inverse transform is:

$$\begin{bmatrix} g_\alpha \\ g_\beta \\ g_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_{dq}) & -\sin(\theta_{dq}) & 0 \\ \sin(\theta_{dq}) & \cos(\theta_{dq}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_d \\ g_q \\ g_0 \end{bmatrix}$$

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Space vectors in rotating reference frame

The Clark and Park transforms can be applied in a sequence to move from the natural reference frame to the rotating reference frame…

$$\underline{T}_{abc\rightarrow dq0} = \frac{3}{2}\begin{bmatrix} \cos(\theta_{dq}) & \cos(\theta_{dq}-2\pi/3) & \cos(\theta_{dq}-4\pi/3) \\ -\sin(\theta_{dq}) & -\sin(\theta_{dq}-2\pi/3) & -\sin(\theta_{dq}-4\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

…or viceversa:

$$\underline{T}_{dq0\rightarrow abc} = \begin{bmatrix} \cos(\theta_{dq}) & -\sin(\theta_{dq}) & 1 \\ \cos(\theta_{dq}-2\pi/3) & -\sin(\theta_{dq}-2\pi/3) & 1 \\ \cos(\theta_{dq}-4\pi/3) & -\sin(\theta_{dq}-4\pi/3) & 1 \end{bmatrix}$$

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Coordinates transformation example

Let's consider a set of three-phase balanced voltages (of positive sequence):

$$\theta_{dq} = \int \omega(t)\,dt$$

$$u_a = \cos(\omega t)$$

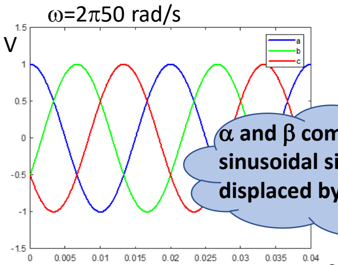
$$u_b = \cos(\omega t - 120°)$$

$$u_c = \cos(\omega t + 120°)$$

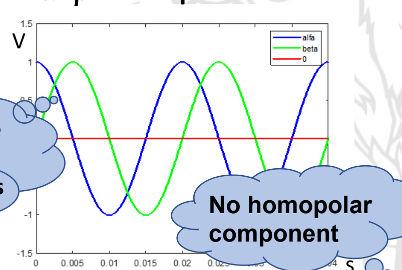
$$\begin{bmatrix} u_\alpha(t) \\ u_\beta(t) \\ u_0(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} u_a(t) \\ u_b(t) \\ u_c(t) \end{bmatrix}$$

$$\begin{bmatrix} u_d(t) \\ u_q(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta_{dq}) & \sin(\theta_{dq}) & 0 \\ -\sin(\theta_{dq}) & \cos(\theta_{dq}) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} u_\alpha(t) \\ u_\beta(t) \\ u_0(t) \end{bmatrix}$$

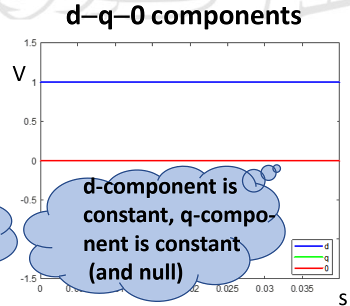
$\omega=2\pi 50$ rad/s



α–β–0 components



d–q–0 components



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Space vector notation in the PMSM model

The space vector notation can be used to represent the model of the PMSM in a more compact form, lumping the information related to the three phases

Due to the absence of neutral point accessibility in the machine, and assuming balanced three phase operating conditions, no homopolar component is present, and α–β components are sufficient to capture the full dynamics of the machine

Flux linkages due to permanent magnets become

$$\lambda_{a,mg} = \Lambda_{mg} cos(\Theta_{me})$$
$$\lambda_{b,mg} = \Lambda_{mg} cos(\Theta_{me} - 2\pi/3)$$
$$\lambda_{c,mg} = \Lambda_{mg} cos(\Theta_{me} - 4\pi/3)$$

$$\Rightarrow \quad \boldsymbol{\lambda}^s_{mg} = \Lambda_{mg}\, e^{j\theta_{me}}$$

The superscript «s» indicates a fixed reference frame, linked to the stator of the machine (i.e. α–β ref frame)

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Space vector notation in the PMSM model

The space vector notation used for the electromotive forces gives:

$$\boldsymbol{e}^s = \frac{d\lambda^s_{mg}}{dt} = \frac{d(\Lambda_{mg} e^{j\theta_{me}})}{dt} = \mathrm{j}\, \Lambda_{mg}\omega_{me}\, e^{j\theta_{me}} = \mathrm{j}\omega_{me}\, \boldsymbol{\lambda}^s_{mg}$$

And finally, the stator voltage electrical equations can be obtained as:

$$\boldsymbol{e}^s = R\boldsymbol{i}^s + L\frac{d\boldsymbol{i}^s(t)}{dt} + \boldsymbol{u}^s = \mathrm{j}\omega_{me}\, \boldsymbol{\lambda}^s_{mg}$$

space vector formulation

that can be rewritten along the real and imaginary axes as:

$$u_\alpha = -Ri_\alpha - L\frac{di_\alpha}{dt} - \omega_{me}\lambda_{\beta,mg} \qquad u_\beta = -Ri_\beta - L\frac{di_\beta}{dt} + \omega_{me}\lambda_{\alpha,mg}$$

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Model of the PMSM in a rotating reference frame

Goal: we want to rewrite the previous equation in a reference frame (*d-q*) that is rotating at the synchronous speed ω_{me} and with the real axis, *d*, aligned with the rotor polar axis

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Stator electrical equations in the stationary reference frame (d-q)

The space vector notation in the fixed ref frame is:

$$\boldsymbol{e}^s = R\boldsymbol{i}^s + L\frac{d\boldsymbol{i}^s(t)}{dt} + \boldsymbol{u}^s = \mathrm{j}\omega_{me}\,\boldsymbol{\lambda}_{mg}^s$$

Choosing a new, rotating reference frame, that is synchronous with the rotor and has the d-axis aligned with the polar axis of the rotor implies that that λ_{mg}^s has only the real component and this simplifies the formulas:

For the generic space vector, the transition from stationary (s) to rotating (r) frame:

$$\boldsymbol{g}^r = \boldsymbol{g}^s\, e^{-j\theta_{me}}$$
$$\boldsymbol{g}^s = \boldsymbol{g}^r\, e^{j\theta_{me}}$$

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Stator electrical equations in the stationary reference frame (d-q)

From:

$$\boldsymbol{e}^s = R\boldsymbol{i}^s + L\frac{d\boldsymbol{i}^s(t)}{dt} + \boldsymbol{u}^s = \mathrm{j}\omega_{me}\,\boldsymbol{\lambda}_{mg}^s$$

Having: $\boldsymbol{\lambda}_{mg}^r = \Lambda_{mg} + \mathrm{j}0$

$$R\boldsymbol{i}^r + L\frac{d\boldsymbol{i}^r(t)}{dt} + \mathrm{j}\omega_{me}L\boldsymbol{i}^r + \boldsymbol{u}^r = \mathrm{j}\omega_{me}\Lambda_{mg}$$

Hint: $\frac{d\mathbf{i}^s}{dt} = \frac{d(\mathbf{i}^r e^{j\theta_{me}})}{dt} = e^{j\theta_{me}}\frac{d\mathbf{i}^r}{dt} + \mathbf{i}^r\omega_{me}je^{j\theta_{me}}$

Which can be further decomposed on the d and q axes:

$$u_d = -Ri_d - L\frac{di_d}{dt} + \omega_{me}Li_q$$

$$u_q = -Ri_q - L\frac{di_q}{dt} - \omega_{me}Li_d + \omega_{me}\Lambda_{mg}$$

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Stator power balance in the synchronous reference frame (d-q)

Power balance can be derived from each phase voltage equation, multiplying by the corresponding phase current

$$u_d = -Ri_d - L\frac{di_d}{dt} + \omega_{me}Li_q \xrightarrow{\times\, i_d} p\sim_d$$

$$+$$

$$u_q = -Ri_q - L\frac{di_q}{dt} - \omega_{me}Li_d + \omega_{me}\Lambda_{mg} \xrightarrow{\times\, i_q} p\sim_q$$

Proportional to:

$$\underbrace{u_d i_d + u_q i_q}_{\text{Instantaneous electrical power injected into the grid}} + \underbrace{R\left(i_d^2 + i_q^2\right)}_{\text{Dissipated power (Joule effect)}} + \underbrace{\left[L\left(i_d\frac{di_d}{dt} + i_q\frac{di_q}{dt}\right)\right]}_{\text{Power used to charge the magnetic energy}} = \underbrace{\omega_{me}\Lambda_{mg}i_q}_{\text{Electro-mechanical power (converted from mechanical to electrical)}}$$

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Torque in the synchronous reference frame (d-q)

Remembering that the transformation from abc to dq is not power invariant

$$T_m^m \omega_m = \frac{3}{2}\omega_{me}\Lambda_{mg} i_q \quad \Rightarrow \quad T_m^m = \frac{3}{2} p \Lambda_{mg} i_q$$

Observations:

- The torque only depends on the q-component of the current (i.e. the component in quadrature with respect to the flux linkage from the permanent magnet, i.e. the current component in phase with the emf)

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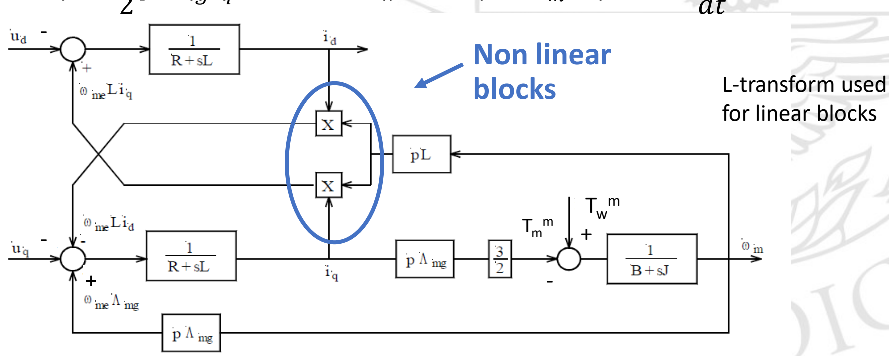
Block diagram of the PMSM with isotropic rotor in d-q

From:

$$u_d = -Ri_d - L\frac{di_d}{dt} + \omega_{me} L i_q \qquad \omega_{me} = p\omega_m$$

$$u_q = -Ri_q - L\frac{di_q}{dt} - \omega_{me} L i_d + \omega_{me}\Lambda_{mg}$$

$$T_m^m = \frac{3}{2} p \Lambda_{mg} i_q \quad \textbf{and:} \quad T_w^m = T_m^m + B_m\omega_m + Jm\frac{d\omega_m}{dt}$$



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Next step(s)

Goal: Having obtained the model of the PMSM in the synchronous reference frame d-q-0 is instrumental to designing the control loop(s) of the generator in the same reference frame, which brings advantages in terms of simplicity of design and tuning

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Lecture 8: Reference material

Lecture slides

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