4. Irradiation Calculations

For many locations in the world the average daily irradiation on a horizontal surface can be looked up. For a more accurate indication of how much energy is falling on a solar panel titled at an angle from the horizontal, a series of calculations must be done. The following sections contain two methods: section 4.1 contains a simplified calculation to find the total daily irradiation falling on a tilted surface that faces towards the equator; section 4.2 describes a more complex calculation that calculates the irradiation on a titled plane hour by hour through the day. Both methods follow the same basic path.

The measured irradiation data that can be referenced is normally in the form of the monthly average daily (MAD) irradiation on a horizontal plane; that is the total irradiation measured in each month divided by the number of days in the month. This figure is made up of the beam and diffused components described in section 2.5 and it is assumed that no reflected radiation is incident on a horizontal surface. To find the MAD irradiation on a tilted surface at the same location the measured data is broken down into its beam and diffused component parts, these are then adjusted for the chosen panel angle and a reflected component added. Because the beam irradiation is a directional and the diffused radiation is not the adjustment for these two values is not the same. Finally the three components are added together to give the total MAD irradiation on the titled panel.

In general:

$$I(\beta) = I_{Bh}\cos\theta_i + I_D R_D + I_{Th} R_A \qquad (W/m^2)(4.1)$$

Where:

$I(\beta)$ = terrestrial insolation intensity on a plane tilted at β degrees to the horizontal surface (W/m^2);
I_{Bh} = terrestrial beam insolation intensity on a horizontal plane (W/m^2);
θ_i = angle of incidence of the radiation on the tilted plane;
I_D = diffused irradiation component on a horizontal plane (W/m^2);
R_D = a convention factor to adjust I_D to a value for a titled plane;
I_{Th} = the total terrestrial insolation intensity on a horizontal plane (W/m^2);
R_A = a convention factor used to find the reflected (or albedo) component for a titled plane.

Note that if $\theta_i > 90°$ the titled plane will be shading itself. Note also that θ_i changes throughout the day as do R_D and R_A. $I(\beta)$ can then be used to find the total irradiation incident on a titled plane ($H(\beta)$) during a whole day by integrating equation 3.14 with respect to time using the sunrise and sunset angles as limits. Thus:

$$H(\beta) = \int_{sunrise}^{sunset} I(\beta)dt \qquad (J/m^2)(4.2)$$

Equation 3.15 can be used to take account of a titled surface shading itself by using values of ω_0 described in section 3.2.

The equations can be used for monthly average values or values for single days.

4.1. Simplified Method For The Surfaces Tilted Towards The Equator

******Work In Radians, Be Careful To Work Only In Wh/m², kWh/m², J/m²***

4.1.1. Calculating The Energy Falling On An Extraterrestrial Horizontal Surface

I_{SC} = solar constant = 1367 W/m^2

Calculate the insolation intensity on a plane perpendicular to sun rays (I_0) on the edge of the atmosphere by correcting I_{SC} for Earth's elliptical orbit.

$$I_0 = I_{SC}\left[1 + 0.034\cos\left(2\pi\frac{n}{365.25}\right)\right] \qquad (W/m^2)(4.3)$$

Calculate the extraterrestrial insolation on a plane horizontal to the Earth's surface (I_{0h}) at the site's latitude.

$$I_{0h} = I_0 \cos\theta_Z$$

Where: $$\cos\theta_Z = \cos\delta\cos\phi\ \cos\omega + \sin\delta\sin\phi \qquad (W/m^2)(4.4)$$

And: $$\delta = 23.45\frac{\pi}{180}\sin\left[2\pi\left(\frac{284+n}{365.25}\right)\right] \qquad (rads)(4.5)$$

Equations 4.3 and 4.5 are now combined to give:

$$I_0 = I_{SC}\left[1 + 0.034\cos\left(2\pi\frac{n}{365.25}\right)\right](\cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta) \qquad (W/m^2)(4.6)$$

To find the extraterrestrial irradiation energy falling on a plane horizontal to the Earth's surface throughout a whole day, integrate equation 4.6 with respect to time between sunrise ($\omega = -\omega_S$) and sunset ($\omega = \omega_S$). The resulting equation is:

$$H_{0h} = \frac{86400}{\pi} I_{SC}\left[1 + 0.034\cos\left(2\pi\frac{n}{365.25}\right)\right](\cos\phi\cos\delta\sin\omega_S + \omega_S\sin\phi\sin\delta) \qquad (J/m^2)(4.7)$$

Note that 86 400 = number of seconds in 24 hours and equation 4.7 gives H_{0h} in J/m^2. Since there are 3600 seconds in an hour and 86 000/3 600 = 24, the following equation gives H_{0h} in Wh/m^2.

$$H_{0h} = \frac{24}{\pi} I_{SC}\left[1 + 0.034\cos\left(2\pi\frac{n}{365.25}\right)\right](\cos\phi\cos\delta\sin\omega_S + \omega_S\sin\phi\sin\delta) \qquad (Wh/m^2)(4.8)$$

Where: $$\cos\omega_S = -\tan\phi\tan\delta \qquad (4.9)$$

If n is set to the 15th of each month (i.e. n = 15 for January and n = 46 for February etc), equation 4.8 can be used to calculate monthly average daily H_{0h} values. When MAD irradiation values are used the symbol $\overline{H}_{0h}$ replaces H_{0h} to indicate average values.

4.1.2. The Clearness Index

Next the clearness index (K_T) is defined as the ratio of the total irradiation reaching a horizontal plane at the location on the Earth's surface and the extraterrestrial irradiation on a horizontal plane above the location. Thus K_T is an indication of how much of the Sun's radiation is lost to scattering and absorption in the atmosphere. If MAD values are used:

$$\overline{K}_T = \frac{\overline{H}}{\overline{H}_{0h}} \tag{4.10}$$

$\overline{H}$ = the monthly average daily irradiation on a horizontal plane at the Earth's surface. This is the value that can be referenced from weather station data.
$\overline{H}_{0h}$ the monthly average daily value of extraterrestrial radiation energy falling on a horizontal plane. Calculated from equation 4.7 or 4.8.

K_T values calculated in this way are valid in the range $0.3 \leq K_T \leq 0.8$ between $\phi = \pm 55°$. K_T can be used to find the ratio of diffused irradiation to total irradiation for a horizontal surface:

$$\frac{\overline{D}}{\overline{H}} = 1 + 1.13\overline{K}_T$$

Therefore: $$\overline{D} = \overline{H}\left(1 - 1.13\overline{K}_T\right) \quad (\text{J/m}^2 \text{ or Wh/m}^2)(4.11)$$

Where $\overline{D}$ is the monthly average daily value for the diffused irradiation incident on a horizontal surface.

4.1.3. Calculation Of The Beam Irradiation On A Tilted Surface

It is assumed that no irradiation reflected from the ground makes it onto a horizontal surface. Therefore:

$$\overline{H} = \overline{B} + \overline{D} \tag{4.12}$$

Where $\overline{B}$ is the monthly average daily value for the beam irradiation on a horizontal plane. Since $\overline{H}$ is measured and $\overline{D}$ has been calculated from equation 4.10, the beam irradiation component incident on a horizontal surface on the average day for each month is given by:

$$\overline{B} = \overline{H} - \overline{D} \tag{4.13}$$

For a titled plane at angle β to horizontal we can define the ratio R_B:

$$R_B = \frac{\overline{B}(\beta)}{\overline{B}} = \frac{\text{MAD beam irradiation on a tilted surface}}{\text{MAD beam irradiation on a horizontal surface}} \tag{4.14}$$

It is assumed that the ratio of extraterrestrial irradiation on a surface titled at angle β to the extraterrestrial irradiation on a horizontal plane also equals R_B so that:

$$R_B = \frac{\overline{B}(\beta)}{\overline{B}} = \frac{\overline{H}_0(\beta)}{\overline{H}_{0h}} \tag{4.15}$$

In equations 4.6 and 4.7 ω_S is used for a horizontal plane, however for titled plane ω_0 must be used as described in section 3.2, also for a tilted plane ϕ is replaced by (ϕ - β) (from equation 3.13). So that:

$$\omega_0 = \min\{\omega_S, \omega_s'\} \tag{4.16}$$

where ω_0 is either ω_S or ω_S', depending on which is the smaller value and:

$$\cos\omega_S' = -\tan(\phi - \beta)\tan\delta \tag{4.17}$$

Thus, from figure 4.6 and 4.7:

$$R_B = \frac{\cos(\phi - \beta)\cos\delta\sin\omega_0 + \omega_0\sin(\phi - \beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_S + \omega_S\phi\sin\delta} \tag{4.18}$$

$$\overline{B}(\beta) = \overline{B}R_B = \overline{B}\left[\frac{\cos(\phi - \beta)\cos\delta\sin\omega_0 + \omega_0\sin(\phi - \beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_S + \omega_S\phi\sin\delta}\right] \quad (J/m^2, Wh/m^2) \tag{4.19}$$

Values of ω_S and ω_0 should be kept in radians in equation 4.19.

4.1.4. Calculation Of The Diffused Component On A Tilted Surface

The diffused irradiation on a horizontal surface was calculated with equation 4.11, this value is in turn used to calculate the diffused irradiation on a surface tilted from the horizontal. To this end a convention factor is defined as the ratio:

$$R_D = \frac{\overline{D}(\beta)}{\overline{D}} = \frac{1}{2}(1 + \cos\beta) \tag{4.20}$$

Thus: $$\overline{D}(\beta) = R_D\overline{D} = \frac{1}{2}(1 + \cos\beta)\overline{D} \quad (J/m^2, Wh/m^2) \tag{4.21}$$

4.1.5. Calculation Of The Reflected Component On A Tilted Surface

It is assumed that no reflected (albedo) radiation is incident on a horizontal plane so that to calculate the monthly average daily reflected radiation on a tilted surface ($\overline{A}$) is found by multiplying $\overline{H}$ by a conversion factor:

$$R_A = \frac{1}{2}(1 - \cos\beta)\rho \tag{4.22}$$

$$\overline{A}(\beta) = R_A \overline{H} = \frac{1}{2}(1-\cos\beta)\rho\overline{H} \qquad (J/m^2\ Wh/m^2)(4.33)$$

where ρ is the reflectivity of the surrounding ground. Values of ρ are given in table 4.1.

Ground Cover	***Reflectivity***	***Ground Cover***	***Reflectivity***
Dry bare ground	0.2	Pale soil	0.3
Dry grassland	0.3	Dark soil	0.1
Desert sand	0.4	Water	0.1
Snow	0.5-0.8	Vegetation	0.2

Table 4.1: Reflectivity Values.

4.1.6. Calculation Of The Total Irradiation On A Tilted Surface

The beam, diffused and reflected components are now added together to give the monthly average daily total irradiation on the titled surface. Thus:

$$\overline{H}(\beta) = \overline{B}(\beta) + \overline{D}(\beta) + \overline{A}(\beta) \qquad (J/m^2\ Wh/m^2)(4.34)$$

Often written as:

$$\overline{H}(\beta) = R_B\overline{B} + \frac{1}{2}(1+\cos\beta)\overline{D} + \frac{1}{2}(1-\cos\beta)\rho\overline{H} \qquad (J/m^2\ Wh/m^2)(4.35)$$

Note that if β = 0 (surface horizontal) then cos β = 1 and reflected irradiation does not contribute to the total irradiation.

The method detailed in this section can be used for an average day for each month and for any value of β. A spreadsheet can be used to calculate values for all possible angles and therefore the optimum angle can be found for each month.

4.2. Hourly Method For Surfaces Tilted In Any Direction

4.2.1. Calculation Of Hourly Irradiation Values

Firstly, equation 4.7 is used to calculate $\overline{H}_{0h}$ for each month. Then $\overline{K}_T$ is calculated from equation 4.10 ($0.3 \le K_T \le 0.8$ still applies). For this method equation 4.11 is replaced with the following relationships:

$$\frac{\overline{D}}{\overline{H}} = 1.391 - 3.560\overline{K}_T + 4.189\overline{K}_T^{\,2} - 2.137\overline{K}_T^{\,3} \qquad \text{when } \omega_S < 81.4° \ (4.36)$$

$$\frac{\overline{D}}{\overline{H}} = 1.311 - 3.022\overline{K}_T + 3.427\overline{K}_T^{\,2} - 1.821\overline{K}_T^{\,3} \qquad \text{when } \omega_S > 81.4° \ (4.37)$$

Where ω_S is calculated from equation 3.13.

The average daily irradiation values for each month are broken down into hourly values using the ratio r_t:

$$r_t = \frac{\text{hourly total irradiation}}{\text{daily total irradiation}}$$

Note that to calculate the hour angle (ω) the hours should be centred on the half hour (e.g. 11:00-12:00).

$$r_t = \frac{\pi}{24}(a + b\cos\omega)\frac{\cos\omega - \cos\omega_S}{\sin\omega_S - \omega_S\cos\omega_S} \tag{4.38}$$

where: $$a = 0.409 + 0.5016\sin\left(\omega_S - \frac{\pi}{3}\right)$$

$$a = 0.6609 - 0.4767\sin\left(\omega_S - \frac{\pi}{3}\right)$$

Where ω is the hour angle in the middle of each hour and is quoted in radians. ω_S is calculated from equation 3.13 and is also in radians. ω is negative in the morning, zero at solar noon and positive in the afternoon.

The total irradiation for each hour ($\dot{H}$) of the 'average' day of each month is then calculated:

$$\dot{H} = r_t\overline{H} \tag{4.39}$$

The diffused component of the total irradiation for each hour is now calculated using the ratio:

$$r_d = \frac{\text{hourly diffused irradition}}{\text{daily total diffused irradiation}}$$

$$r_d = \frac{\pi}{24}\frac{\cos\omega - \cos\omega_S}{\sin\omega_S - \omega_S\cos\omega_S} \tag{4.40}$$

So that the hourly diffused irradiation for the 'average' day of each month:

$$\dot{D} = r_d\overline{D} \tag{4.41}$$

The hourly beam component ($\dot{B}$) is calculated from the relationship stated in equations 4.12 and 4.13:

$$\dot{B} = \dot{H} - \dot{D} \tag{4.42}$$

4.2.2. Calculation Of The Total Hourly Irradiation On A Titled Surface

The hourly irradiation on a tilted surface ($\dot{H}(\beta)$) is calculated using an equation similar to equation 4.35:

$$\dot{H}(\beta) = \dot{B}\dot{R}_b + \dot{D}\left(\frac{1+\cos\beta}{2}\right) + \left(\frac{1-\cos\beta}{2}\right)\rho\dot{H} \qquad (4.43)$$

In this equation: $\dot{R}_b = \dfrac{\cos\theta_i}{\cos\theta_Z}$

Where θ_i is the angle of incidence of the Sun's rays on the tilted plane (figure 3.2) calculated from equation 3.11, and θ_Z is the solar zenith angle (figure 1.7)calculated from equation 3.12.

Using this method, values of ω_S' are not used (section 3.2) and since hourly figures of θ_i and θ_Z are calculated, self-shading of the tilted plane is apparent when θ_i is greater than 90° and $\dot{B}\dot{R}_b$ becomes negative – and therefore is taken as zero. Note the $\dot{R}_b$ is an hourly figure.

The daily totals for the average day of each month are then found by adding the hourly figures together.