Macroeconomics 3

TA session 2, Exercise set

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This is a set of exercises to get you started with Value Function Iteration. **No hand-in required**. For every exercise, there might be more than one way to obtain the intended result, so experimenting is encouraged. If you have questions, feel free to ask me.

1 Getting artistic with PFI and convergence

Consider the Neoclassical Growth Model we saw in class

$$V(k) = \max_{c,k'} \frac{c^{(1-\sigma)}}{1-\sigma} + \beta V(k')$$
$$\text{s.t.} \quad \begin{cases} c + k' \leq k^\alpha + (1-\delta)k \\ c \geq 0 \end{cases}$$

Run again the Policy Function Iteration exercise (you already have most of the code). This time however, keep track of the policy functions $c_i(k)$ and $k'_i(k)$ visited by the iterative path and plot them along with the actual results. The graphical result should amount to multiple lines for each policy function, in increasing order of color intensity as the sequence $(c_i(k), k'_i(k))_{i=0}^N$ converges to the result (i.e., almost invisible for the initial guess, pitch black for the actual results).

2 Understanding how sensitive the projection method is (and how to fix it)

Consider again the Neoclassical Growth Model and look at the code that implements the projection method.

1. Try different initial guesses for the policy function on consumption $c_0(k)$ and see how the results change depending on that. *[Hint: be creative to maximize the benefit out of this exercise. Think of several functional forms for the initial guess].*

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2. Find a way to force the algorithm to give a policy function $k'(k)$ that passes through the real value of the steady state. *[Hint: you might want to change the number of equations that are given as input to* `scipy.optimize.fsolve()`*]*

3 Solving globally (part of) the RBC model

Consider the (slightly modified) building block of the RBC model: a consumer that has to decide how much to consume, save and work, taking prices (wage and interest rate) as given.

$$\max_{(C_t, S_{t+1}, N_t)_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$
$$\text{s.t.} \quad \begin{cases} C_t + S_{t+1} \leq w_t N_t + (1+r_t) S_t & \forall\ t \in \{0, 1, \ldots\} \\ N_t \in [0,1] & \forall\ t \in \{0, 1, \ldots\} \\ C_t \geq 0 & \forall\ t \in \{0, 1, \ldots\} \\ S_0 \text{ given.} \end{cases}$$

Solve the problem of this guy:

1. Identify the state variables and the control variables.

2. Write the problem in recursive formulation.

3. Assuming that $w_t = w$ and $r_t = r$ at all dates t, write some code that implements VFI in this case. *[Yes, you get to choose reasonable values for all the parameters! It's fun!]*

4. Run again the exercise by using PFI.

5. Using the policy functions for the control variables, simulate a path of optimal consumption, savings and labor, given appropriate initial conditions (but avoid starting from the steady state!). Comment on how interesting these (deterministic) simulations are.