Bewley-type Models: Huggett & Aiyagari

Macroeconomics 3: TA class #5

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Plan for Today

Objective: **Replicate the results in [Huggett, 1993] and [Aiyagari, 1994]**

Sub-goals:

- Understand the papers
- Learn about “binning” (a.k.a., non-stochastic simulations)
- Learn about transition dynamics (a.k.a., MIT shocks)

Overview

[Huggett, 1993]

- Tries to address the Equity Premium Puzzle
- Uses a simple exchange economy with incomplete insurance
- Solid numerical approach: guaranteed to converge
- Finds that incomplete cannot account for the EPP

[Aiyagari, 1994]

- Tries to see if precautionary savings explain aggregate savings
- Uses an RBC economy with incomplete insurance
- Tentative numerical approach: converges, maybe, nobody knows why
- Finds that precautionary savings cannot account for aggregate savings

Huggett: Overview

Question	How to address the Equity Premium Puzzle (EPP) in macro models?
Approach	A heterogeneous-agents model with incomplete insurance
Challenge	Need to nail mechanism and quantification
Findings	Preventing over-saving does it qualitatively, but not quantitatively

Huggett: Methodology

Equity Premium Puzzle: spread between risk-free rate and risky rate too small in models relative to the data

Two avenues

- Risky rate too small in models
- Risk-free rate too large in models

Huggett

- Risk-free rate is too large in models (at the time, innovative!)
- Can reduce the risk-free rate by reducing the demand for the risk-free asset
- Two alternatives
  - Prevent lenders from saving too much (unrealistic assumption)
  - Prevent borrowers from taking too much debt (more realistic)
- Representative-agent models cannot cut it: requires a zero net financial position in GE

Huggett: Model

Ex-ante identical consumers solve the following

$$\max_{C_t, A_{t+1}} \mathbf{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right)$$

$$\text{s.t.} \begin{cases} C_t + A_{t+1} \leq Y_t + (1+r_t)A_t & \forall\ t \\ A_{t+1} \geq \underline{A} & \forall\ t \\ \log(Y_{t+1}) = (1-\rho)\mu + \rho \log(Y_t) + \varepsilon_{t+1} & \forall\ t \\ \varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) & \forall\ t \end{cases}$$

The borrowing constraint $\underline{A}$ is such that it may bind for some consumers (i.e., $\underline{A}$ is higher than the natural debt limit)

Huggett: Numerical Approach

1. At iteration j, guess an equilibrium interest rate $r^{(j)}$

2. Solve for the policy function $A'(A, Y)$

3. Combine $A'(A, Y)$ with Π to obtain the endogenous transition matrix Q

4. Compute the ergodic distribution $\lambda(A, Y)$ by iterating Q enough times

5. Compute the net excess demand $E^d(r) = \sum_A \sum_Y \lambda(A, Y)\, A'(A, Y)$

6. Use a root-solver to bring the LHS to the RHS (i.e., zero)
   - If $E^d\left(r^{(j)}\right) > 0$, then $r^{(j)} < r^*$
   - If $E^d\left(r^{(j)}\right) < 0$, then $r^{(j)} > r^*$
   - Set $r^{(j+1)}$ accordingly and repeat 2–6

Aiyagari: Overview

Question What are the determinants of aggregate savings in the US?

Approach A heterogeneous-agents RBC model

Challenge Lucas' diversification argument: idiosyncrasies average out in the aggregate

Findings
- Precautionary savings do not matter for aggregate savings, quantitatively
- Can generate precautionary savings without prudence
- Model matches the US cross-sectional distribution of household wealth

Aiyagari: Methodology

Fix aggregate (capital) savings K

Look at composition of the aggregate

- Thourough exploration of endogenous distribution of agents
- Focus on left tail of wealth distribution (constrained, or almost, agents)

Features of the model

- Relatability: just an RBC model
- Source of heterogeneity: labor endowments
- No aggregate uncertainty: prices r and w fixed in equilibrium

Aiyagari: Model

Households (demand side)

$$\max_{c_t, k_{t+1}} \mathbf{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right)$$

$$\text{s.t.} \begin{cases} c_t + k_{t+1} \leq w l_t + (1+r) k_t & \forall\, t \\ c_t, k_{t+1} \geq 0 & \forall\, t \\ l_{t+1} = (1-\rho)\mu + \rho l_t + \varepsilon_{t+1} & \forall\, t \\ \varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) & \forall\, t \end{cases}$$

Firms (supply side)
(note: no time subscripts)

$$\max_{K,L} A K^{\alpha} L^{1-\alpha} - rK - wL$$

Market clearing (recursive notation, ergodic equilibrium)

$$K = \int_k \int_l \lambda(k, l)\, k'(k, l) \mathrm{d}l\, \mathrm{d}k$$

Aiyagari: Numerical Approach

1. At iteration j, guess an aggregate level of capital holdings $K^{(j)}$
2. Use the FOCs of the firm to compute r and w
3. Solve for the households' policy function $k'(k, l)$
4. Combine Π with $k'(k, l)$ to obtain the endogenous transition matrix Q
5. Compute the ergodic distribution $\lambda(k, l)$ by iterating Q enough times
6. Compute aggregate savings $\hat{K}$ as

$$\hat{K} \equiv \sum_k \sum_l \lambda(k, l)\, k'(k, l)$$

7. Check if $\hat{K}$ is consistent with $K^{(j)}$
   - If $\hat{K} \neq K^{(j)}$, then set $K^{(j+1)}$ using the dampening scheme for $\theta \in [0, 1]$

$$K^{(j+1)} \equiv \theta \hat{K} + (1 - \theta) K^{(j)}$$

   - Repeat steps 2–7

Are These Bewley-Type Models? Yes!

- Consumers are ex-ante identical
  - One maximization problem describes everybody (ex-ante!)
  - Idiosyncratic uncertainty and policy functions place consumers differently on the distribution of asset holdings

- In equilibrium, consumers are heterogeneous
  - At the “dawn of time,” there is a distribution of agents, depending on endowments
  - Based on (different) income, consumers choose (different) savings

- At the *ergodic* equilibrium
  - Consumers are distributed according to an endogenous ergodic distribution
  - The *ergodic* distr. does **not** mean that there is no dynamics (i.e., not a deterministic steady s.)
  - Across periods, consumers are “reshuffled” such that the ergodic distribution is maintained

- Technical note (nothing to do with Bewley type-ness)
  - There is idiosyncratic uncertainty
  - There is **no** aggregate uncertainty

Comparing Huggett & Aiyagari

They look like the same model… They are, almost

	Huggett	**Aiyagari**
Supply side	Exogenous	Endogenous
Financial market	Asset	Capital
Research question	Equity Premium Puzzle	Composition of aggr. savings
Numerical solution	Net excess demand	Consistency w/ guess

Fundamental difference in numerical algorithm:

- Aiyagari assumes that K is a sufficient statistic for $\lambda(k, l)$
- Assumption is baked-in the optimization problem of firms (i.e., FOC wrt K)
- This assumption makes the algorithm unreliable, from a maths/theory point of view
- "It converges, but nobody knows why"
  - Is the convergence point an equilibrium?
  - Is the equilibrium unique?
  - Is this equilibrium cherry-picked?

Practice Time

Moving to a Jupyter Notebook

References

Aiyagari, S. R. (1994).
Uninsured Idiosyncratic Risk and Aggregate Saving.
The Quarterly Journal of Economics, 109(3):659–684.

Huggett, M. (1993).
The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies.
Journal of Economic Dynamics and Control, 17(5-6):953–969.