Introduction to Numerical Methods for Macroeconomics

Macroeconomics 3: TA class #1

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About Myself

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About the TA Classes for Macro 3

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- Now it's time for fun: empirical tools!

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- Two options
  - Perturbation methods
  - Projection methods

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Objective: **get familiar with projection methods and related applications**

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- Many options to work with Python: VSCode, Spyder, PyCharm, Jupyter Notebooks, etc.
- These classes: Jupyter Notebooks (VSCode behind the scenes)

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Perturbation methods

- Rely on Taylor expansion
- Require differentiability of the model
- Low computational costs

Projection methods

- Rely on Bellman equations
- Allow for heterogeneity, discontinuities
- High computational costs

Intro to Numerical Methods for Economists: Hands-on Example

Example: **Neoclassical Stochastic Growth Model**

$$\max_{C_t, K_{t+1}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$\text{s.t.} \begin{cases} C_t + K_{t+1} = Z_t K_t^{\alpha} + (1+\delta) K_{t-1} & \forall\ t \\ \log(Z_{t+1}) = (1-\rho)\mu + \rho \log(Z_t) + \log(\varepsilon_{t+1}) & \forall\ t \\ \varepsilon_{t+1} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) & \forall\ t \\ C_t, K_{t+1} > 0 & \forall\ t \\ K_0, Z_0 \text{ given} & \end{cases}$$

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Variables

- Endogenously predetermined: Z_t, K_t
- Exogenous shocks: ε_{t+1}
- Controls: C_t, K_{t+1}
- Forward looking: C_{t+1}

Equations for the equilibrium

$$\begin{cases} u'(C_t) = \beta \cdot \mathbf{E}_t \left(u'(C_{t+1}) \left[\alpha K_{t+1}^{\alpha-1} + 1 - \delta \right] \right) \\ C_t + K_{t+1} = Z_t K_t^{\alpha} + (1+\delta)K_t \\ \log(Z_{t+1}) = (1-\rho)\mu + \rho \log(Z_t) + \log(\varepsilon_{t+1}) \end{cases}$$

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Focus on the equations that characterize the equilibrium (w/ CRRA utility)

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- A (deterministic) steady state
- Derivatives of each equation

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Log-linear representation of the model (1^{st} order Taylor expansion around the steady state)

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Can solve this system of linear equations with linear algebra (e.g., Schur decomposition)

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Focus on the optimization problem (write the associated Bellman equation)

$$V(K,Z) = \max_{C,K'} u(C) + \beta \mathbf{E}\left(V(K',Z')|Z\right)$$

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- A unique fixed point $V(K,Z)$

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In a computer

- Define the domains for K and Z
- Define a function that maximizes $u(C) + \beta\mathbf{E}(\ldots)$ s.t. …
- Iterate the function until convergence

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Can crack this by letting the computer loop the contraction mapping **T**

Intro to Numerical Methods for Economists: Projection Methods (cont'd)

For more complicated supply-demand models (let the "real" equilibrium price be P^*)

1. Guess an equilibrium price $P^{(h)}$
2. Obtain the policy functions associated to the Bellman equation, for the given price $P^{(h)}$
3. Define the excess demand function $D(P)$
4. Observe that $D(P)$ is decreasing in P
   - If $D(P^{(h)}) > 0$, then $P^{(h)} < P^*$
   - If $D(P^{(h)}) < 0$, then $P^{(h)} > P^*$
5. Propose a new guess $P^{(h+1)}$ accordingly
6. Repeat steps 1–5 until $|P^{(h+1)} - P^{(h)}| < \epsilon$

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Why do projection methods matter?

- Models that suffer from derivatives (i.e., where higher order moments matter)
- Models with heterogeneous agents
- Models with binding constraits

Intro to Python

Moving to a Jupyter Notebook