Kinematic and Dynamic Parameters of the UR5 Robot Manipulator

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1 Introduction

Serial robots are widely used in manufacturing, handling material, and tele-operation. In the recent years, Universal Robots have developed a series of robotic manipulators that is now widely used by many universities and industries. This robot is claimed to be fast, easy to program, flexible, safe and offers low level programming access of the robot controller with high cycle time [1]. Among the UR products the family of UR3, UR5 and UR10 have received a great attention within the robotics community and industries specifically by the robotic research community. In this document, the main information for deriving the mathematical model for kinematics and dynamics of the UR5 robot (see Fig. 1) is presented. The

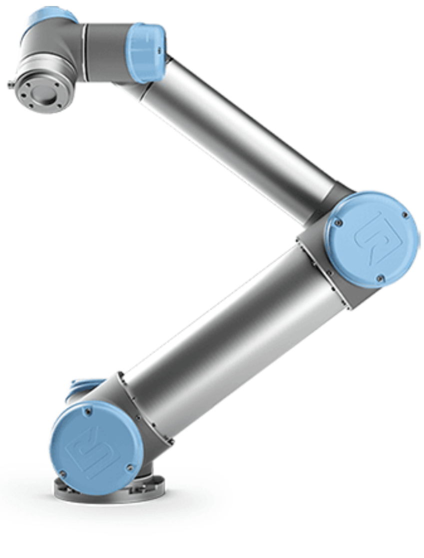

Figure 1: The UR5 manipulator.

kinematic model has to include a full mathematical development for the forward and inverse kinematic equations of the robot, while the dynamic model has to take into account the mass inertial matrix, centrifugal and Coriolis matrix and gravity force vector. These models are implemented in Matlab environment, preferably in the Simscape Multibody environment.

The UR5 is a collaborative robot, i.e. it is *safe* because it stops as soon as an object sensed by a force sensor in one of the joints is hitted. In Table 1, the specifications given by Universal Robots are stated. One important statement from the specifications is the repeatability of 0.1 mm. The robotic arm consists of six revolute joints. In this document, these joints are

weight	18.4 kg
payload	5 kg
reach	850.0 mm
joint ranges	± 360 °
joint max speed	180 °/s
TCP max speed	1 m/s
degrees of freedom	6 rotational joints
repeatability	± 0.1 mm
I/O power supply	12 V/24 V, 600 mA
communication	TCP/IP, Ethernet socket & Modbus TCP
programming	Polyscope graphical user interface
IP classification	IP54
power consumption	150 W
power supply	10-240 VAC, 50-60 Hz
materials	aluminium, ABS and plastic
temperature	Working range of 0-50 °C
operating life	35000 h

Table 1: Technical specifications of the UR5 robotic arm.

referred to as *Base, Shoulder, Elbow, Wrist1, Wrist2* and *Wrist3*. The *Shoulder* and *Elbow* joint are rotating perpendicular to the *Base* joint. These three joints are connected with long links. The wrist joints control the Tool Center Point (TCP) in the right orientation. The main source of information is [2].

2 Parameters for the kinematic model

A Universal Robots robot has a not-mono-centric wrist which means that there is not a wrist center point defined as the common point of all the three rotations axes of joints 4, 5 and 6. This solution is due to the fact that each joint has its own motor. In fact, there are not any transmission organs such as belts or gearboxes. A not-mono-centric wrist configuration does not allow to split arm and wrist singularities since the wrist rotations q_4, q_5 and q_6 affect the End Effector (EE) linear velocity. In fact, as reported in Fig. 2, wrist joints axes do not converge in a single point, but axes of joints 4 and 6 are parallel. The kinematic structure of the UR5 manipulator is described in terms of the corresponding Denavit-Hartenberg parameters. Universal Robots relies on the standard convention, i.e. the i-th joint connects links i and $i+1$, and the z_i axis is aligned with the axis of the joint $i+1$. The following four parameters are defined for each link i:

- *offset distance a_i:* the distance between z_i and z_{i-1} measured along x_i;
- *translation distance d_i:* the distance between the x_i and x_{i-1} axes measured along the positive direction of z_{i-1};
- *twist angle α_i:* the angle between the z_{i-1} and z_i axes. It is the angle required to align the z_{i-1} axis with the z_i axis in the right-hand direction around the x_i axis;
- *joint angle θ_i:* the angle between the x_{i-1} and x_i axes. It is the angle required to align the x_{i-1} axis with the x_i axis in the right-hand direction around the z_{i-1} axis.

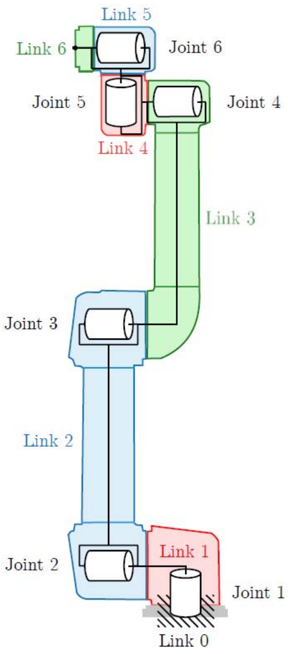

Figure 2: Sketch of the UR5 manipulator.

For each link of the UR5, the DH parameters of the robot that are reported in Table 2[1]. With q_i, $i = 1, \ldots, 6$, we denote the degrees of freedom of the robot arm. Besides, these

joint	q_i [°]	d_i [m]	a_i [m]	α_i [°]	$\tilde{q}_i$ [°]
Base	q_1	0.089159	0	90	0
Shoulder	q_2	0	-0.425	0	0
Elbow	q_3	0	-0.39225	0	0
Wrist1	q_4	0.10915	0	90	0
Wrist2	q_5	0.09465	0	-90	0
Wrist3	q_6	0.0823	0	0	0

Table 2: UR5 Denavit-Hartenberg parameters.

quantities have to be summed to the *offset angles* $\tilde{q}_i$ specified by the manufacturer, to obtain the joint angles θ_i:

$$\theta_i = q_i + \tilde{q}_i, \qquad i = 1, \ldots, 6$$

Based on the DH parameters, the homogeneous transformation H_i^{i-1} that describes position and orientation of the i-th frame with respect to the $(i-1)$-th one is immediately obtained as

$$\begin{aligned} H_i^{i-1} &= \mathrm{Trans}(z_{i-1}, d_i) \cdot \mathrm{Rot}(z_{i-1}, \theta_i) \cdot \mathrm{Trans}(x_i, a_i) \cdot \mathrm{Rot}(x_i, \alpha_i) \\ &= \begin{pmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

[1] `https://www.universal-robots.com/articles/ur/application-installation/dh-parameters-for-calculations-of-kinematics-and-dynamics/`

3 Parameters for the dynamic model

To implement the dynamical model of the UR5 manipulator, the inertial parameters are required. In Table 3, the masses of each link and the positions of the corresponding centers of gravity are reported. The latter are expressed in the reference system of the i-th link identified by the DH parameters. Finally, link inertia matrices are reported as follows; they are

link	m_i [kg]	$r_{G,i}$ [m]
1	3.7	$(0.0\,,-0.02561\,,0.00193)^{\mathrm{T}}$
2	8.393	$(0.2125\,,0.0\,,0.11336)^{\mathrm{T}}$
3	2.33	$(0.15\,,0.0\,,0.0265)^{\mathrm{T}}$
4	1.1219	$(0.0\,,-0.0018\,,0.01634)^{\mathrm{T}}$
5	1.1219	$(0.0\,,0.0018\,,0.01634)^{\mathrm{T}}$
6	0.1879	$(0.0\,,0.0\,,-0.001159)^{\mathrm{T}}$

Table 3: Mass and position of the center of gravity of each link.

expressed in the reference system of the i-th link identified by the DH parameters, and the unit of measure is kg·m^2. The idea has been to approximate each link with cylinders with different densities, [3].

$$I_1 = \begin{pmatrix} 0.0067 & 0 & 0 \\ 0 & 0.00640 & \\ 0 & 0 & 0.0067 \end{pmatrix} \quad I_2 = \begin{pmatrix} 0.0149 & 0 & 0 \\ 0 & 0.3564 & 0 \\ 0 & 0 & 0.3553 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 0.0025 & 0 & 0.0034 \\ 0 & 0.0551 & 0 \\ 0.0034 & 0 & 0.0546 \end{pmatrix} \quad I_4 = \begin{pmatrix} 0.0012 & 0 & 0 \\ 0 & 0.0012 & 0 \\ 0 & 0 & 0.0009 \end{pmatrix}$$

$$I_5 = \begin{pmatrix} 0.0012 & 0 & 0 \\ 0 & 0.0012 & 0 \\ 0 & 0 & 0.0009 \end{pmatrix} \quad I_6 = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix}$$

References

[1] `www.universal-robots.com`

[2] M. Sorli, *Dynamic parameters identification of a UR5 robot manipulator.* Master Thesis in Mechanical Engineering, Politicnico di Torino, 2020.

[3] K. Kufieta, *Force Estimation in Robotic Manipulators: Modeling, Simulation and Experiments, UR5 as a case study.* Thesis from Norwegian University of Science and Technology, Department of Engineering Cybernetics, Jan 29, 2014.