Bayesian inference GW1 data

In[ ]:=
```
ClearAll;
Clear["Global`*"]
```

upload data

```
lcol = 0;
lrow = 0;

Zs = Import[
   "C:\\Users\\mw\\Documents\\dottorato\\branching_mathematica\\GW1\\GW1_data_article
     \\GW1_Y_gen8_num_sim10_θ[0.1 0.2 0.7]zk[120 278 965].txt",
   "Table", Delimiter → ","];
Zs = Zs[[1 ;; Dimensions[Zs][[1]] - lrow, 1 ;; Dimensions[Zs][[2]] - lcol]];
(* select part of the dataset*)
Print["dimension matrix Zt: ", Dimensions[Zs]]
Print[Zs // MatrixForm]
```

```
dimension matrix Zt: {9, 10}
```

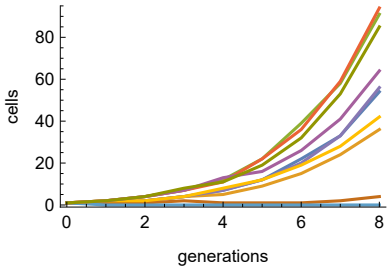
1	1	1	1	1	1	1	1	1	1
1	1	2	2	1	1	0	2	2	2
2	2	4	4	2	1	0	2	4	4
4	4	7	7	4	2	0	4	7	8
7	5	12	11	7	1	0	8	13	11
12	9	22	22	12	1	0	12	16	19
22	15	39	36	20	1	0	19	26	32
33	24	58	59	33	2	0	28	41	53
54	36	91	94	56	4	0	42	64	85

Out[ ]=
```
{120, 278, 965}
```

Out[ ]=
```
{0.1, 0.2, 0.7}
```

```
(*plot trajectories*)
ListLinePlot[Transpose[Zs],
 PlotRange → {-1, Max[Zs] + 1}, Frame → {{True, False}, {True, False}},
 FrameLabel → {{"cells", None}, {"generations", None}},
 DataRange → {0, Length[Zs] - 1}, ImageSize → 200]
```

Out[ ]=



Useful Functions

```
(*------------------Define Definite Polynomial Integration-------
 ---------------------------------------------------------------*)
PolyDInt[y_ + z_, x_, a_, b_] := PolyDInt[y, x, a, b] + PolyDInt[z, x, a, b];
PolyDInt[c_ y_, x_, a_, b_] := c * PolyDInt[y, x, a, b] /; FreeQ[c, x];
PolyDInt[x_^n_., x_ , a_, b_] := b^(n + 1) / (n + 1) - a^(n + 1) / (n + 1);
PolyDInt[c_, x_, a_, b_] := c * b - c * a /; FreeQ[c, x];
PolyDInt[y_, x_, a_, b_] := PolyDInt[Expand[y], x, a, b] /; Not[FreeQ[y, x]];

(*------------------------------------------------------------------------
                                        ---------------------------------
                                        -------*)

(*define conditional likelihood for consecutive counts*)
CondLikelihood[j_, k_] := D[(a0 + a1 * s + a2 * s^2)^j, {s, k}] / k! /. s → 0
(*j_ = start with j cells, k_=end up with k cells*)
```

Analytical method

calculate posteriors

```
organoids = Dimensions[Zs][[2]];                    (*num organoids *)
ngen = Dimensions[Zs][[1]] - 1;                     (*generation*)
ndata = organoids * ngen ;                          (*num of data*)
groupby = 1;                                        (*num of data to use together *)
ndataseq = ndata / groupby ;                        (*num of groups*)

prior = 2;
posteriors = ConstantArray[0, {organoids}];

For[org = 1, org ≤ organoids, org ++,
      likelihood = 1;
      (*create the joint likelihood*)

  For[i = 1, i ≤ ngen, i++, likelihood *= CondLikelihood[Zs[[i, org]], Zs[[i + 1, org]]];
      If[Mod[org, 1] == 0, Print["org: ", org]];
      likelihood = likelihood /. a0 → 1 - a1 - a2; (*apply constraints*)
      (*calculate the bayes' rule numerator*)
      priorlikelihood = MonomialList[prior * likelihood];
  (*divide the numerator in monomials to exploit parallel computation*)
      len = Length[priorlikelihood];
      (*calculate the posterior*)
      posterior = prior * likelihood / Total[ParallelTable[
        PolyDInt[PolyDInt[priorlikelihood[[k]] , a2, 0, 1 - a1], a1, 0, 1], {k, 1, len}]];
      prior = posterior;
      posteriors[[org]] = posterior;

 ] // AbsoluteTiming
```

Out[•]=

```
{528.854, Null}
```

In[•]:=

Find Marginals

In[•]:=
```
pa0s = ConstantArray[0, {organoids}];
pa1s = ConstantArray[0, {organoids}]; pa2s = ConstantArray[0, {organoids}];
For[org = 1, org ≤ organoids, org ++,
    If[Mod[org, 1] == 0, Print["org: ", org]];
   posterior = MonomialList[posteriors[[org]]];
   len = Length[posterior];
   (*Exploiting parallel computing*)
   pa1s[[org]] = Total[ParallelTable[PolyDInt[posterior[[k]], a2, 0, 1 - a1], {k, 1, len}]];

  pa2s[[org]] = Total[ParallelTable[PolyDInt[posterior[[k]], a1, 0, 1 - a2], {k, 1, len}]];

   posterior = MonomialList[posteriors[[org]] /. a2 → 1 - a0 - a1];
   len = Length[posterior];
   pa0s[[org]] = Total[ParallelTable[PolyDInt[posterior[[k]], a1, 0, 1 - a0], {k, 1, len}]];

 ] // AbsoluteTiming
```

Out[•]=
```
{1335.6, Null}
```

Find Estimators

In[•]:=
```
Ea0s = ConstantArray[0, {organoids}]; sa0s = ConstantArray[0, {organoids}];
Ea1s = ConstantArray[0, {organoids}]; sa1s = ConstantArray[0, {organoids}];
Ea2s = ConstantArray[0, {organoids}]; sa2s = ConstantArray[0, {organoids}];
For[org = 1, org ≤ organoids, org++,
     If[Mod[org, 1] == 0, Print["org: ", org]];
      (*find estimates*)
      a0pa0s = MonomialList[a0 * Simplify[pa0s[[org]]]];
      Ea0s[[org]] =
   Total[ParallelTable[PolyDInt[a0pa0s[[k]], a0, 0, 1], {k, 1, Length[a0pa0s]}]];
      a1pa1s = MonomialList[a1 * Simplify[pa1s[[org]]]];
      Ea1s[[org]] =
   Total[ParallelTable[PolyDInt[a1pa1s[[k]], a1, 0, 1], {k, 1, Length[a1pa1s]}]];
      a2pa2s = MonomialList[a2 * Simplify[pa2s[[org]]]];
      Ea2s[[org]] =
   Total[ParallelTable[PolyDInt[a2pa2s[[k]], a2, 0, 1], {k, 1, Length[a2pa2s]}]];


      a0pa0s = a0 * a0pa0s;
      sa0s[[org]] =
   Sqrt[Total[ParallelTable[PolyDInt[a0pa0s[[k]], a0, 0, 1], {k, 1, Length[a0pa0s]}]] -
     Ea0s[[org]]^2];
      a1pa1s = a1 * a1pa1s;
      sa1s[[org]] =
   Sqrt[Total[ParallelTable[PolyDInt[a1pa1s[[k]], a1, 0, 1], {k, 1, Length[a1pa1s]}]] -
     Ea1s[[org]]^2];
      a2pa2s = a2 * a2pa2s;
      sa2s[[org]] =
   Sqrt[Total[ParallelTable[PolyDInt[a2pa2s[[k]], a2, 0, 1], {k, 1, Length[a2pa2s]}]] -
     Ea2s[[org]]^2];

 ] // AbsoluteTiming
```

Out[•]=
```
{3597.17, Null}
```

Plot mean and standard deviations of marginal distributions

```
plta0 = ListLinePlot[Table[Around[Ea0s[[org]], sa0s[[org]]],  {org, 1, organoids}],
    PlotLegends → Placed[{"mean"}, {Right, Top}],
    PlotStyle → Directive[Opacity[0.5], Red], PlotRange → {0, 1},
    PlotLabel → TextString[a0], Frame → {{True, False}, {True, False}},
    FrameLabel → {{None, None}, {"data  (x" <> ToString[ngen] <> " gen)", None}}];
plta1 = ListLinePlot[Table[Around[Ea1s[[org]], sa1s[[org]]],  {org, 1, organoids}],
    PlotLegends → Placed[{"mean"}, {Right, Top}],
    PlotStyle → Directive[Opacity[0.5], Green], PlotRange → {0, 1},
    PlotLabel → TextString[a1], Frame → {{True, False}, {True, False}},
    FrameLabel → {{None, None}, {"data  (x" <> ToString[ngen] <> " gen)", None}}];
plta2 = ListLinePlot[Table[Around[Ea2s[[org]], sa2s[[org]]],  {org, 1, organoids}],
    PlotLegends → Placed[{"mean"}, {Right, Bottom}],
    PlotStyle → Directive[Opacity[0.5], Blue], PlotRange → {0, 1},
    PlotLabel → TextString[a2], Frame → {{True, False}, {True, False}},
    FrameLabel → {{None, None}, {"data  (x" <> ToString[ngen] <> " gen)", None}}];

GraphicsRow[{Show[{plta0} ], Show[{plta1}], Show[{plta2}]},
 Spacings → 5, ImageSize → 600]
```

Out[•]=