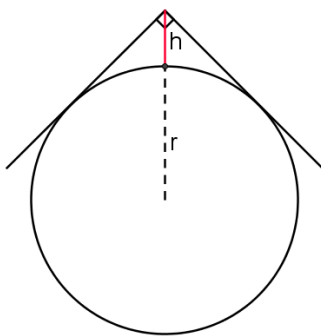


Quadratics

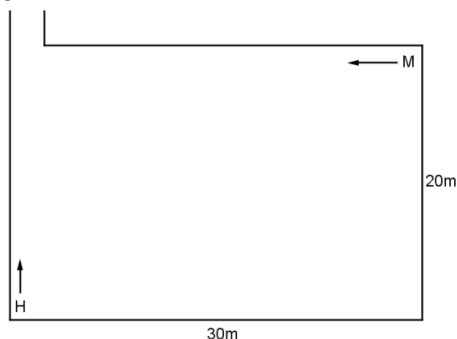
1. Change one number in the equation $(x-1)(x+3) = -9$ so that the roots are equal. There are several ways in which this question can be answered. Can you find all of them?
2. How can you find the radius of the pipe shown if the only measurement you can take is the one marked h ?



3. Unlike many infinitely long expressions $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}}$ has a finite value. What is that value?

4. Show that $\frac{(a+b)^2 + (a-b)^2}{2} = a^2 + b^2$ and use it to evaluate, without a calculator, $102.5^2 + 97.5^2$. What other 'difficult' sums of squares can you add easily in this way?

5. A man and a horse are in a field 30m by 20m respectively from an opening in the corner. They start running towards the opening at the same time. The horse runs 4m for every 3m the man runs.



What is the shortest lasso which will enable the man to catch the horse?

6. Write some digits in a circle. For example

$$\begin{array}{ccc} & 6 & \\ 3 & & 8 \\ & 1 & 5 \end{array}$$

The sum of the squares of the two-digit numbers read clockwise is
 $68^2 + 85^2 + 51^2 + 13^2 + 36^2 = 15915$

The sum of the squares of the two-digit numbers read anticlockwise is
 $63^2 + 31^2 + 15^2 + 58^2 + 86^2 = 15915$

Prove that the two sums will always be equal for any circle of digits.

7. Look at the table of squares of odd numbers below. Either prove that the 10s digit in an odd square number is always even or find a counterexample.

| | | | | |
|--------|---|---|---|---|
| 1^2 | = | 0 | 0 | 1 |
| 3^2 | = | 0 | 0 | 9 |
| 5^2 | = | 0 | 2 | 5 |
| 7^2 | = | 0 | 4 | 9 |
| 9^2 | = | 0 | 8 | 1 |
| 11^2 | = | 1 | 2 | 1 |
| 13^2 | = | 1 | 6 | 9 |
| 15^2 | = | 2 | 2 | 5 |
| 17^2 | = | 2 | 8 | 9 |
| 19^2 | = | 3 | 6 | 1 |
| 21^2 | = | 4 | 4 | 1 |
| 23^2 | = | 5 | 2 | 9 |
| 25^2 | = | 6 | 2 | 5 |

8. Starting with a sum such as $2+7=4+5$

add a constant to every term $5+10=7+8$

and pair off opposite sides: $2+7+7+8=4+5+5+10$

It turns out the sum of the squares are also equal: $2^2 + 7^2 + 7^2 + 8^2 = 4^2 + 5^2 + 5^2 + 10^2$

Investigate!