Benchmarking Transfer Entropy Methods for the Study of Linear and Nonlinear Cardio-Respiratory Interactions

Supplementary Material

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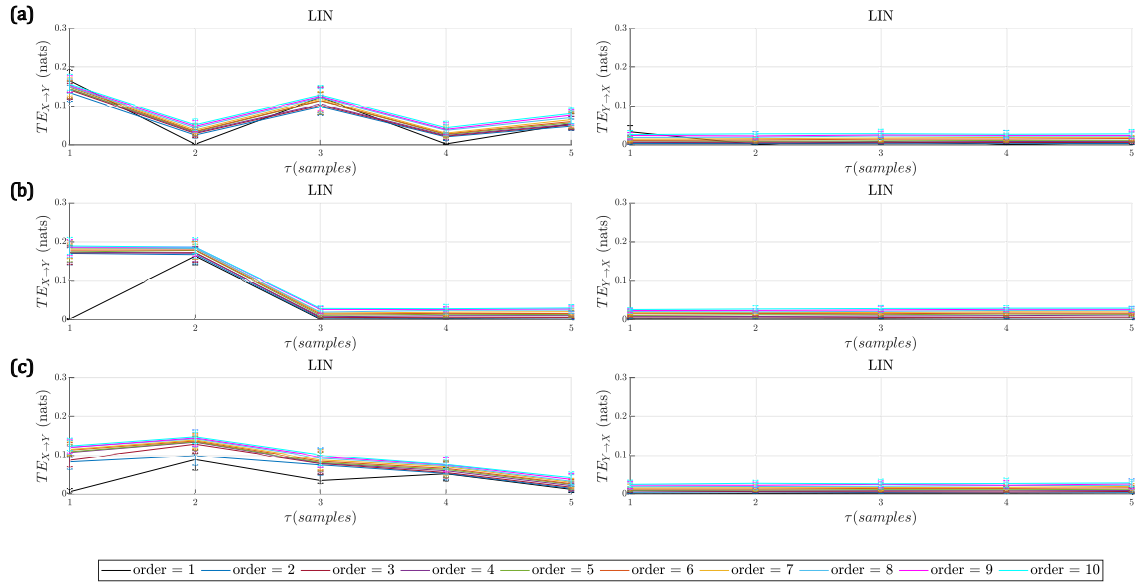
1 Results

1.1 Simulation study

Figure 1: Results of changing the lag for the embedding vectors generation and the model order of the LIN method, for (**a**) linear model (interaction at $\tau = 1$), (**b**) non-linear model (interaction at $\tau = 2$), and (**c**) linear + non-linear model (interaction at $\tau = 2$ and $\tau = 4$). Each plot shows the median TE vs. lag (τ) in samples. The error bars indicate the interquartile range. Each line in the plot corresponds to a model order varying from 1 to 10.

Table 1: Scores of TEE for the assessment of the best model order for LIN for each simulation model.

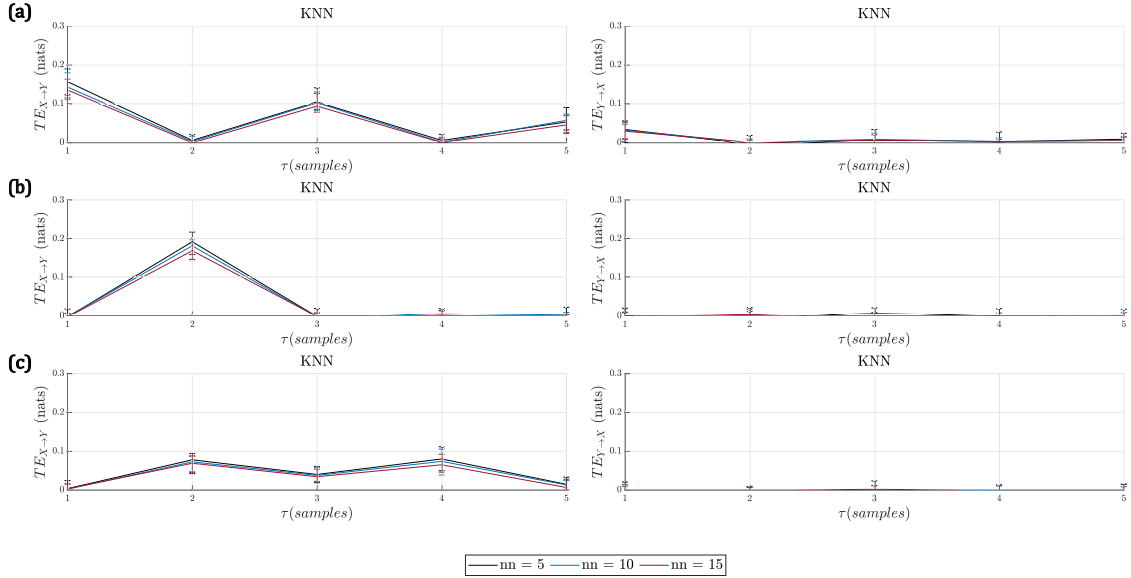| model order	Linear model TEE	Non-linear model TEE	Linear + Non-linear model TEE
1	**2.09**	**1.03**	**1.37**
2	2.44	2.07	2.13
3	2.45	2.10	2.00
4	2.48	2.18	2.07
5	2.59	2.19	2.08
6	2.59	2.26	2.10
7	2.66	2.30	2.11
8	2.82	2.38	2.14
9	2.89	2.44	2.15
10	2.96	2.49	2.19

Figure 2: Results of changing the lag for the embedding vectors generation and the number of neighbors (nn) of the KNN method, for (**a**) linear model (interaction at $\tau = 1$), (**b**) non-linear model (interaction at $\tau = 2$), and (**c**) linear + non-linear model (interaction at $\tau = 2$ and $\tau = 4$). Each plot shows the median TE vs. lag (τ) in samples. The error bars indicate the interquartile range. Each line in the plot corresponds to the number of neighbors varying from 5 to 15.

Table 2: Scores of TEE for the assessment of the best number of neighbors (nn) for KNN for each simulation model.

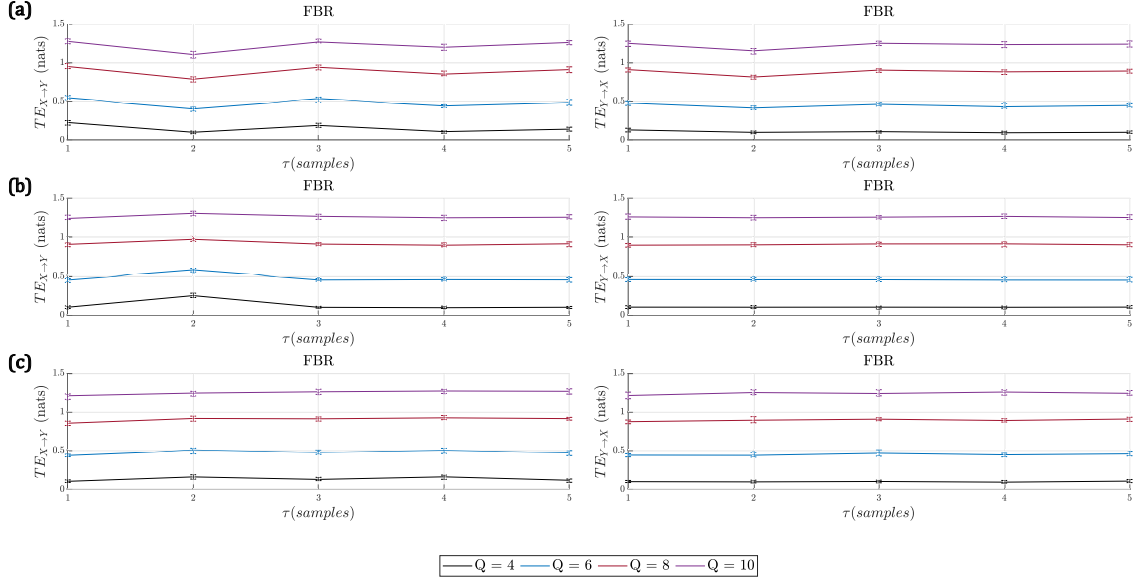| nn	Linear model TEE	Non-linear model TEE	Linear + Non-linear model TEE
5	2.07	**1.055**	1.37
10	2.13	1.061	1.36
15	**2.04**	1.056	**1.33**

Figure 3: Results of changing the lag for the embedding vectors generation and the quantization levels (Q) of the FBR method, for (**a**) linear model (interaction at $\tau = 1$), (**b**) non-linear model (interaction at $\tau = 2$), and (**c**) linear + non-linear model (interaction at $\tau = 2$ and $\tau = 4$). Each plot shows the median TE vs. lag (τ) in samples. The error bars indicate the interquartile range. Each line in the plot corresponds to a number of levels varying from 4 to 10.

Table 3: Scores of TEE for the assessment of the best number of quantization levels (Q) for FBR for each simulation model.

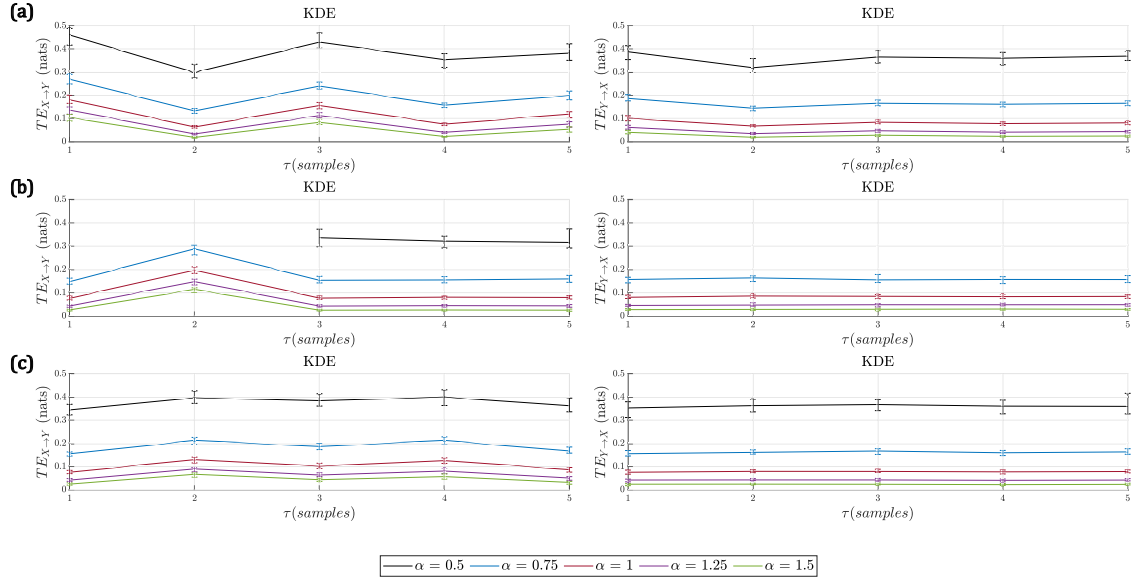| Q	Linear model TEE	Non-linear model TEE	Linear + Non-linear model TEE
4	**3.39**	**2.59**	**2.09**
6	4.44	4.14	2.39
8	4.67	4.73	2.46
10	4.79	4.84	2.49

Figure 4: Results of changing the lag for the embedding vectors generation and the multiplier of the kernel bandwidth (α) of the KDE method, for (**a**) linear model (interaction at $\tau = 1$), (**b**) non-linear model (interaction at $\tau = 2$), and (**c**) linear + non-linear model (interaction at $\tau = 2$ and $\tau = 4$). Each plot shows the median TE vs. lag (τ) in samples. The error bars indicate the interquartile range. Each line in the plot corresponds to a value of α varying from 0.5 to 1.5.

Table 4: Scores of TEE for the assessment of the best multiplier of the kernel bandwidth (α) for KDE for each simulation model.

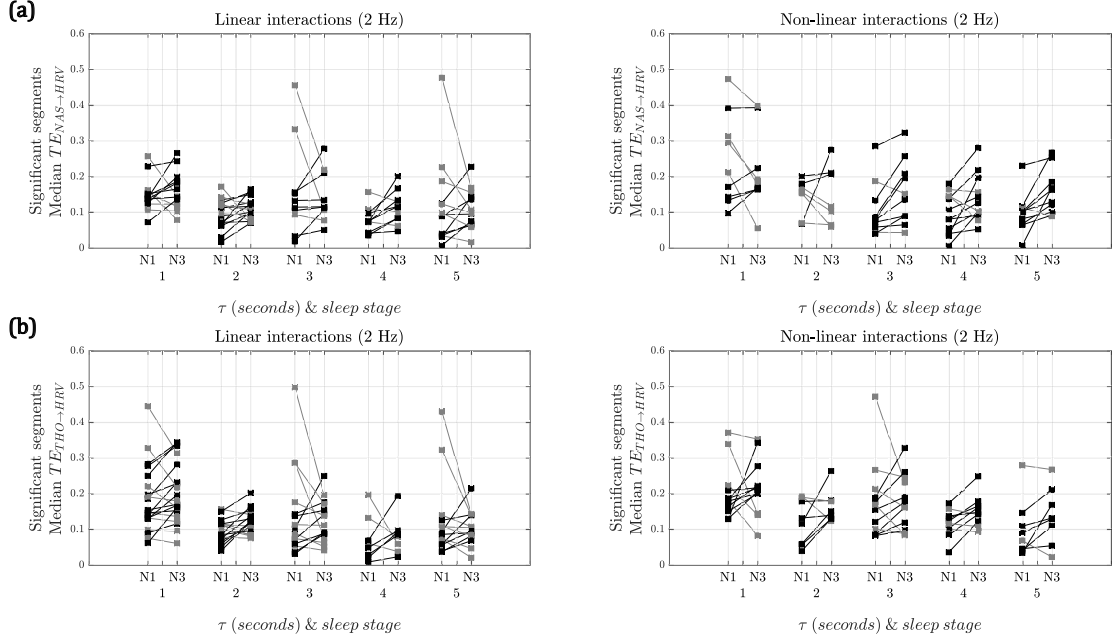| α	Linear model TEE	Non-linear model TEE	Linear + Non-linear model TEE
0.5	4.18	-	2.37
0.75	3.71	3.13	2.20
1	3.29	2.59	2.03
1.25	2.93	2.19	1.91
1.5	**2.71**	**1.91**	**1.81**

1.2 Application to Real Data

Figure 5: Median $TE_{RESP \to HRV}$ of significant segments for each kind of interaction at 2 Hz vs. lag (τ) in seconds. **(a)** $RESP = NAS$, **(b)** $RESP = THO$. Gray lines represent higher interactions in NREM1, while black lines correspond to higher interactions in NREM3. Linear interactions are on the left, for which significant differences using NAS are found at $\tau = 2$ and $\tau = 4$ seconds. Non-linear interactions are on the right, for which significant differences using NAS are found at $\tau = 1$ and $\tau = 3$ seconds. No significant differences are found using THO.