The Economic Journal, **117** (*July*), 883–896. Published by Blackwell Publishing, 9600 Garsington Road, Oxford OX4 2DQ, UK and 350 Main Street, Malden, MA 02148, USA.

EQUILIBRIUM IN THE JUNGLE*

Michele Piccione and Ariel Rubinstein

In the jungle, power and coercion govern the exchange of resources. We study a simple, stylised model of the jungle that mirrors an exchange economy. We define the notion of jungle equilibrium and demonstrate that a number of standard results of competitive markets hold in the jungle.

In the typical analysis of an exchange economy, agents are involved in consumption and exchange goods voluntarily when mutually beneficial. The endowments and the preferences are the primitives of the model. The distribution of consumption in society is determined endogenously through trade.

This article is motivated by a complementary perspective on human interaction. Throughout the history of mankind, it has been quite common (and we suspect that it will remain so in the future) that economic agents, individually or collectively, use power to seize control of assets held by others. The exercise of power is pervasive in every society and takes several forms. Often, power is purely physical. Sometimes, however, power is more gentle. In the male–female 'market', for example, charm and attraction play a key role in obtaining a favourite mate. In an office parking lot, social conventions such as seniority allow control of the preferred parking places. The power of persuasion enables some to convince others to take actions against their own interest.

We introduce and analyse an elementary model of a society, called the *jungle*, in which economic transactions are governed by coercion. The jungle consists of a set of individuals, who have exogenous preferences over a bounded set of consumption bundles (their capacity to consume is finite), and of an exogenous ranking of the agents according to their strength. This ranking is unambiguous and known to all. Power, in our model, has a simple and strict meaning: a stronger agent is able to take resources from a weaker agent.

The jungle model mirrors the standard model of an exchange economy. In both models, the preferences of the agents over commodity bundles and the total endowment of goods are given. The distribution of power in the jungle is the counterpart of the distribution of endowments in the market. As the incentives to produce or collect the goods are ignored in an exchange economy, so are the incentives to build strength in the jungle.

We define a *jungle equilibrium* as a feasible allocation of goods such that no agent would like and is able to take goods held by a weaker agent. We demonstrate several properties that equilibrium allocation of the jungle shares with the equilibrium allocation of an exchange economy. In particular, we will show that under standard assumptions a *jungle equilibrium* exists and is Pareto efficient. In addition, we will show

* We gratefully acknowledge ESRC Grant RES000220394 for financial support. We thank Luca Anderlini, Samuel Bowles, Leonardo Felli, Piero Gottardi, Ben Polak, Heraklis Polemarchakis, Andrea Prat, Tayfun Somnez and Rani Spiegler for their comments.

that there exist prices that 'almost' support the jungle equilibrium as a competitive equilibrium.

The observation that mainstream economic models ignore a variety of human activities that involve power and that are relevant to the distribution and production of wealth is not new. Hirshleifer (1994) remarks that '... the mainline Marshallian tradition has nevertheless almost entirely overlooked what I will call the dark side of the force – to wit, crime, war, and politics' and 'Appropriating, grabbing, confiscating what you want – and, on the flip side, defending, protecting, sequestering what you already have – that's economic activity too'. Grossman (1995) develops a model where agents decide how much effort to put in production and how much to embed in 'extralegal appropriative activities' and studies its equilibrium. Bowles and Gintis (1992) emphasise that 'power is based on the capacity of some agents to influence the behavior of others to their advantage through the threat of imposing sanction' and analysed markets, especially labour markets, where the terms of transactions are determined through a non-Walrasian process where an agent's wealth affects his power. Usher (1989) studies the evolution of despotism and anarchy. Skaperdas (1992) analyses the trade-off between productive and coercive activities when property rights are missing. Moselle and Polak (2001) compare the effects of anarchy and a 'predatory state' on output and welfare.

Our approach is somewhat more abstract. Our main goal is to construct a formal model of involuntary exchange that is similar to the Walrasian model and yields comparable properties. The reason is that our objective is twofold. On the one hand, we have a genuine interest in investigating the effects of power on the distribution of resources. On the other hand, we wish to uncover some of the rhetoric hidden in standard economic theory.

1. The Jungle

1.1. *The Model*

We consider a model with commodities labelled $1, \ldots, K$, and a set of agents, $I = \{1, \ldots, N\}$. An aggregate bundle $w = (w_1, \ldots, w_K) >> 0$ is available for distribution among the agents. Each agent i is characterised by a preference relation $\succsim^i$ on the set of bundles R_+^K and by a consumption set $X^i \subseteq R_+^K$. The preferences of each agent satisfy the standard assumptions of strong monotonicity and continuity. The set X^i is interpreted as the bounds on agent i's ability to consume. We assume that X^i is compact and convex, and satisfies free disposal, that is, $x^i \in X^i$, $y \in R_+^K$ and $y \leq x^i$ implies that $y \in X^i$.

The distribution of resources in the jungle is determined by the relative power of the agents. We choose a particularly simple notion of power. The agents are ordered by a strength relation S. We assume that the binary relation S is a linear ordering (irreflexive, asymmetric, complete, and transitive), and without loss of generality, that $1S2$, $2S3$, ..., $(N-1)SN$. The statement iSj is interpreted as 'i is stronger than j'. If iSj, i can take from j anything that j has. Finally, a *jungle* is defined as a tuple $<\{\succsim^i\}_{i \in I}, \{X^i\}_{i \in I}, w, S>$.

Throughout the article, we use the standard model of an exchange economy as the benchmark for our results. An exchange economy is defined as a tuple

$<\{\succsim^i\}_{i\in I},\{X^i\}_{i\in I},w,\{w^i\}_{i\in I}>$, where w^i is the initial endowment of agent i and $\sum_{i=1}^N w^i = w$. The jungle differs from the exchange economy in one component. Instead of the distribution of initial endowments, the jungle includes a specification of the power relation.

1.2. *Remarks*

Of course, we are well aware that the model of the jungle under investigation is rudimentary in more than one sense. Our definition of power is crude and extreme and several important issues are deliberately ignored.

(*i*) The model does not indicate the source of power. In our model, the power relation is an exogenous ranking. Analogously, the model of an exchange economy does not indicate the origin of initial endowments. (In a later Section of the article, we discuss production.)

(*ii*) The exercise of power does not involve a loss of resources. The cost of the use of power is not included in jungle model in the same way as the costs of transactions and property right enforcement are not included in the standard exchange model.

(*iii*) As is in standard exchange economies, we do not allow coalitions to be formed. (Some results on coalition formation will be given in a later section.)

These missing elements could be easily embedded into our framework. Our intention, however, is not to achieve generality so much as to devise a model that is comparable to the standard exchange economy and in which the allocation of resources is driven by coercion.

Some readers may object to the assumption that the consumption of agents is bounded. Without such an assumption the jungle equilibrium would be uninteresting as the strongest agent gets all the resources. We disagree with these objections. First, we do not find the presence of bounds on consumption less genuine than the absence of such bounds. Naturally, there are physical limits to what people can consume. Second, we agree that it is not plausible to posit that agents have a bounded desire for wealth. However, we believe that it is not sensible to model the desire for ever increasing wealth in the same fashion as we model the desire to satisfy basic needs. Commonsense reasons for the preference for large wealth are the aspiration to influence the collective allocation, ostentation, or status. It may be fascinating to model these considerations but they are absent here as they are in standard market models.

Note also that one could interpret bounds to consumption as reflecting limitations in an agent's ability to protect his possessions. This interpretation would be satisfactory for some of our analysis but one needs to be careful in interpreting our welfare results since it will undermine the efficiency of the jungle equilibrium.

2. Jungle Equilibrium

Let us introduce the main equilibrium concept. First, some standard definitions. A *feasible allocation* is a vector of non-negative bundles $z = (z^0,z^1,z^2, \ldots, z^N)$ such that $z^0 \in R_+^K$, $z^i \in X^i$ for $i = 1$,.., N, and $\sum_{i=0}^N z^i = w$. The bundle z^0 consists of the goods that are not allocated to any agent. A feasible allocation is *efficient* if there is no other feasible allocation for which at least one agent is strictly better off and none of the other agents is worse off.

A *jungle equilibrium* is an allocation such that no agent can assemble a more preferred bundle by combining his own bundle either with a bundle that is held by one of the agents weaker than him or with the bundle that is freely available. Formally, a *jungle equilibrium* is a feasible allocation z such that there are no agents i and j, iSj , and a bundle $y^i \in X^i$ such that $y^i \leq z^i + z^0$ or $y^i \leq z^i + z^j$ and, in addition, $y^i \succ^i z^i$.

The definition of jungle equilibrium contains a stability condition which appears rather weak in that an agent can appropriate goods belonging to only *one* weaker agent. This condition could be strengthened or generalised by allowing agents to seize commodities assigned to subsets of weaker agents. We will show later that such generalisations are *inconsequential* under conventional assumptions on preferences since the respective jungle equilibria are identical.

We begin with an elementary existence result.

Proposition 1 *A jungle equilibrium exists.*

Proof. Construct an allocation $\hat{z} = (\hat{z}^0, \hat{z}^1, \hat{z}^2, \ldots, \hat{z}^N)$ as follows. Let $\hat{z}^1$ be one of the agent 1's best bundles in the set $\{x^1 \in X^1 \mid x^1 \leq w\}$. Define inductively $\hat{z}^i$ to be one of the agent i's best bundle in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} \hat{z}^j\}$ and $\hat{z}^0 = w - \sum_{j=1}^{N} \hat{z}^j$. The allocation $\hat{z}$ is obviously a jungle equilibrium. QED

Note that the procedure in the proof of Proposition 1 generates a unique allocation $\hat{z}$ when preferences are strictly convex. The allocation $\hat{z}$ results from a process in which each agent maximises his utility in a predetermined order with precedence given to more powerful agents. Other allocations which are stable when an agent takes goods from only one weaker agent can also satisfy the definition of jungle equilibrium. However, as the next Proposition shows, under smoothness assumptions on the preferences and consumption sets of the agents, a jungle equilibrium is unique and hence must have the properties of the allocation constructed in the proof of Proposition 1.

We say that a jungle is *smooth* if, for each agent i,

(i) the preferences are represented by a strictly quasiconcave, and continuously differentiable utility function $u^i : R_+^K \to R$, with $\nabla u^i \gg 0$ and

(ii) there exists a quasiconvex and differentiable function g^i with $\nabla g^i \gg 0$ such that $X^i = \{x^i \in R_+^K \mid g^i(x^i) \leq 0\}$ (at points on the boundary the gradients are defined as limits).

As an illustration of the line of argument in the next results, consider an agent i who is allocated a bundle z^i and suppose that a bundle $z^i + \varepsilon^i$ is both feasible and more desirable than z^i. The change ε^i must involve increasing the consumption of some goods, possibly owned by several weaker agents. Using the Lemma below and assuming that his preferences and consumption set satisfy the conditions of a smooth jungle, we will show that there is also a feasible bundle $z^i + \delta^i$ such that

(*i*) it is more desirable than z^i;
(*ii*) it increases the consumption of only one of the goods, one whose quantity was increased in $z^i + \varepsilon^i$;
(*iii*) it decreases the consumption of only one of the goods, one whose quantity was decreased in $z^i + \varepsilon^i$. As $z^i + \delta^i$ increases the consumption of only one good, agent i can improve his utility by taking it from only one agent (obviously ε^i and δ^i can be assumed to be small).

LEMMA 2 *Let $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ be strictly positive vectors, and suppose that $ax > 0$ and $bx < 0$, for some vector $x = (x_1, \ldots, x_n)$. Then, there exists a vector $y = (y_1, \ldots, y_n)$ such that*

(*i*) *$y_k > 0$ for some k for which $x_k > 0$;*
(*ii*) *$y_l < 0$ for some l for which $x_l < 0$;*
(*iii*) *$y_h = 0$ for $h \neq k,l$; and*
(*iv*) *$ay > 0$ and $by < 0$.*

Proof. See Appendix

PROPOSITION 3. *If a jungle is smooth, $\hat{z}$ is the unique jungle equilibrium.*

Proof. Let z be a jungle equilibrium. We will show that z^i is i's best bundle in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} z^j\}$ for any agent i. Strict convexity of preferences will then imply that $z = \hat{z}$, since $z^1 = \hat{z}^1$ and, if $z^j = \hat{z}^j$ for $j < i$, $z^i = \hat{z}^i$.

If $z^i = w - \sum_{j=1}^{i-1} z^j$, the claim is obvious. If not, by strict monotonicity, $g^i(z^i) = 0$. Suppose there exists a bundle $\bar{x}^i$ such that $g^i(\bar{x}^i) \leq 0$ and $u^i(\bar{x}^i) > u^i(z^i)$. Note that, by quasiconvexity of g^i, $\nabla g^i(z^i)(\bar{x}^i - z^i) \leq 0$ and, by strict quasiconcavity and monotonicity $\nabla u^i(z^i)(\bar{x}^i - z^i) > 0$ (Mas Colell *et al.*, 1995, p. 934). Clearly, we can choose $\bar{x}^i$ so that $\nabla g^i(z^i)(\bar{x}^i - z^i) < 0$. Now, applying the Lemma above, there exists a vector y such that

(*i*) $y_k > 0$ for some k for which $(\bar{x}^i_k - z^i_k) > 0$;
(*ii*) $y_l < 0$ for some l for which $(\bar{x}^i_l - z^i_l) < 0$;
(*iii*) $y_h = 0$ for $h \neq k,l$;
(*iv*) $\nabla u^i(z^i)y > 0$ and $\nabla g^i(z^i)y < 0$.

Hence, for small $\epsilon > 0$, adding ϵy_k units of commodity k and subtracting $-\epsilon y_l$ units of commodity l keeps agent i within his consumption set and strictly improves his utility. In particular, if there exists a bundle $\bar{x}^i$ in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} z^j\}$ such that $g^i(\bar{x}^i) \leq 0$ and $u^i(\bar{x}^i) > u^i(z^i)$, then there also exists a bundle $z^i + \epsilon y$ in $\{x^i \in X^i \mid x^i \leq w - \sum_{j=1}^{i-1} z^j\}$, such that $g^i(z^i + \epsilon y) \leq 0$ and $u^i(z^i + \epsilon y) > u^i(z^i)$, obtained taking ϵy_k units of commodity k that are freely available or are held by a weaker agent. QED

One important normative justification for the competitive equilibrium is provided by the 'First Fundamental Welfare Theorem' which states that any competitive allocation is efficient. We will now show that efficiency also holds in the jungle. The proof of the next Proposition contains an argument which also appears in Ghosal and Polemarchakis (1999) Lemma 1.

PROPOSITION 4. *The allocation $\hat{z}$ is efficient.*

Proof. Suppose not and let $(y^0, y^1, \ldots, y^N)$ be a feasible allocation such that $y^i \succsim^i \hat{z}^i$ for every agent i and $y^j \succ^j \hat{z}^j$ for some j. We first show that for every j for whom $y^j \neq \hat{z}^j$ there must be an agent j' stronger than j for whom $y^{j'} \neq \hat{z}^{j'}$. To see this note that, if $y^j \neq \hat{z}^j$ and $y^j \succsim^j \hat{z}^j$, then by the uniqueness of the optimal bundle of j in the convex set $\{x^j \in X^j \mid x^j \leq w - \sum_{h=1}^{j-1} \hat{z}^h\}$ we have that $y^j \notin \{x^j \in X^j \mid x^j \leq w - \sum_{h=1}^{j-1} \hat{z}^h\}$, that is, there is at least one good k for which $y_k^j > w_k - \sum_{j<i} \hat{z}_k^j$. Hence, there must be an agent j' stronger than j for whom $y^{j'} \neq \hat{z}^{j'}$. The Proposition follows noting that, if $y^j \neq \hat{z}^j$ for some agent j, then $y^1 \neq \hat{z}^1$, a contradiction since by hypothesis $y^1 \succsim^i \hat{z}^1$. QED

It follows from Proposition 4 and from the fact that $\hat{z}$ is a jungle equilibrium, as shown in Proposition 1, that if a jungle equilibrium is unique, it is also efficient.

3. The Jungle and the Exchange Economy

In this Section, we compare the jungle to the exchange economy. A *competitive equilibrium* for the exchange economy is a pair (z, p), where z is a feasible allocation and $p = (p_1, \ldots, p_K)$ is a non-negative price vector such that, for every agent i, z^i is i's most preferred bundle in the set $\{x^i \in X^i \mid px^i \leq pw^i\}$.

3.1. *Examples*

Example 1. Allocation of Houses

Consider a jungle with a set of agents $I = \{1, \ldots, N\}$ and a set $H = \{1, \ldots, N\}$ of indivisible commodities, referred to as houses. Each agent i can consume only one house (that is, X^i contains the null and the unit vectors), has strict preferences over X^i, and strictly prefers having any house to having no house.

For any initial endowment that assigns one house to one agent, a competitive equilibrium exists. The following constructive proof is due to David Gale and is cited in Shapley and Scarf (1974). The construction of the equilibrium is useful for the comparison between the competitive and the jungle allocations.

Assume without loss of generality that agent i owns initially house i. First construct a partition $\{I_1, \ldots, I_l, \ldots, I_L\}$ of I as follows. Start with agent $i_0 = 1$ (this choice is arbitrary). Define i_{k+1} as i_k's most preferred house. Continue until $i_{k+1} = i_q$ for some $q \leq k$. Choose $I_1 = \{i_q, \ldots i_k\}$ (the group consists of a 'top trading cycle'). Continue in the same way with all remaining agents and houses until a partition is completed. Choose a sequence of numbers $p_1 > p_2 > \ldots > p_L > 0$ and assign to the houses in I_l the price p_l. Clearly this is a competitive equilibrium price vector. An agent in I_l buys a house in I_l and, if he prefers an house not in I_l, it must be a house in I_t, $t < l$, which he cannot afford.

In this construction, some agents in each round exchange their houses and receive their best house from the set of houses not allocated in earlier rounds. The group of agents who obtain a house in each round has the property that each of its members can obtain his preferred house (among those that have not been allocated in earlier rounds) by reallocating the houses held within the group.

In the construction of the jungle equilibrium, houses are also allocated in rounds. Only one agent makes a choice in each round: he is the strongest agent among those agents who have not made their choices earlier. Define house k_i inductively as i's most preferred house in the set $H - \{k_1, \ldots, k_{i-1}\}$. In any jungle equilibrium agent i obtains house k_i. Suppose not and let agent i be an agent who does not get k_i but any $j < i$ gets k_j. Then, either some agent $t > i$ or no agent gets k_i. In either case, agent i can seize k_i, a contradiction.

Example 2. Allocation of Houses and Gold

This example is a modification of Example 1. Consider a world with a set H of N houses held by an agent (the chief) who is interested only in gold and N merchants who own gold but are interested only in houses. Denote the chief as agent $N + 1$ and gold as good $N + 1$. Gold is divisible but the houses are not. The initial bundle of the chief is $(1,1, \ldots, 1,0)$ containing 0 units of gold and 1 unit of each house. His consumption set contains all bundles of the form $(0,0, \ldots, 0,m)$. Agent $i \neq N+1$ owns an initial amount $m^i > 0$ of gold, can consume only one house, and has a strict preferences over the houses.

In this example, gold is the mirror image of strength. Define house k_i, $i = 1, \ldots, N$, inductively as in Example 1. If $m^1 > m^2 > \ldots > m^N > 0$, then in any competitive equilibrium agent i obtains house k_i. To see this, suppose that in some competitive equilibrium agent i does not obtain k_i but any $j < i$ gets k_j. Then, some agent $t > i$ gets k_i. By definition $m_t < m_i$. The price of k_t cannot exceed m_t and thus k_i is in agent i's budget set, a contradiction. Analogously, consider a jungle in which the chief is weaker than all agents. As in Example 1, in any jungle equilibrium agent i, $i = 1, \ldots, N$, obtains house k_i.

3.2. *The Jungle and the Second Welfare Theorem*

The second fundamental welfare theorem asserts that, under suitable assumptions, any efficient allocation is a competitive equilibrium allocation for some initial endowment. Recall that the difference between the exchange economy and the jungle is that in the former we specify the distribution of initial endowments and in the latter the distribution of strength. Thus, an adaptation of the second welfare theorem to the jungle would state that any efficient allocation is a jungle equilibrium for some power relation. Clearly, such an assertion cannot hold in general since the number of power relations is finite and the number of efficient allocations is infinite.

In Example 1, however, every efficient allocation is a jungle equilibrium for some relation S: if agent j prefers the house owned by i, define iSj. A standard argument by Varian (1974) implies that S does not have cycles and thus can be completed. An analogous result was obtained by Abdulkadiroglu and Sonmez (1998) who show that the set of allocations obtained by serial dictatorship of some order, the set of a competitive allocations, and the set of efficient allocations coincide.

3.3. *Jungle Prices*

In this Section, we attempt to address the question whether, at the jungle equilibrium allocation, a stronger agent is also a wealthier agent. Of course, we first need a criterion for evaluating an agent's holdings. Competitive prices supporting the allocation provide such a measure of value. Here are two cases where the wealth comparison is clear.

In Example 1, the jungle equilibrium allocation is a competitive equilibrium allocation for the exchange economy in which the jungle equilibrium is the initial endowment. Any price vector for which the price of the house assigned to agent i is greater than the price of the house assigned to agent j whenever iSj is an equilibrium price vector for this exchange economy. At these prices, being stronger implies being wealthier. Of course, other equilibrium prices might exist. In particular, if the strongest agent ranks the highest a house that all other agent rank the lowest, there exists a competitive equilibrium price vector in which the strongest agent is the poorest agent.

When agents have the same preferences and consumption sets, and prices supporting a jungle equilibrium as a competitive equilibrium exist, the relationship between power and wealth is unambiguous: the value of an agent's jungle equilibrium bundle increases with his strength. To see this, consider a jungle equilibrium z and suppose that (z,p) is a competitive equilibrium. If iSj then $z^i \succsim^i z^j$ and thus also $z^i \succsim^j z^j$. Since j chooses z^j it must be that $pz^i \geq pz^j$ since, if $pz^i < pz^j$, then $(z^i + z^j)/2 \in X^j$, $p(z^i + z^j)/2 \leq pz^j$ and, by strict convexity, $(z^i + z^j)/2 \succ^j z^j$.

The evaluation of wealth in the general model is subtle. The existence of competitive prices supporting the jungle equilibrium is a major problem since consumption sets are bounded and exhibit a satiation point. Under our assumptions about preferences and consumption sets, an exchange economy might not have a competitive equilibrium.

In the absence of conditions which guarantee the existence of competitive prices, we follow a less straightforward route. Instead of one price vector which clears each market, we consider a sequence of price vectors such that, for each agent, the induced sequence of demands (given his jungle equilibrium bundle as endowment) converges to the jungle equilibrium bundle. Thus, far enough in the sequence, the price vector 'almost' clears the market. The Appendix contains a detailed construction of such a sequence for an example in which competitive equilibrium prices do not exist as well as the proof for the following proposition:

PROPOSITION 5. *Suppose that the jungle is smooth. In the exchange economy in which $w^i = \hat{z}^i$, $i = 1, \ldots, N$, there exists a sequence of price vectors p^n such that, for every agent i, the sequence of demands of agent i given p^n converges to $\hat{z}^i$.*

Proof. See Appendix.

The price sequence in the proof of Proposition 5 has the limit property that if iSj then the prices of the goods exhausted by agent i are in relative terms 'infinitely larger' than the prices of the goods exhausted by agent j, thus making i infinitely wealthier than j.

A related approach which provides a solution concept for economies in which competitive equilibria may not exist is found in Florig (2001) (see also references therein). One possible interpretation of his concept is that the economy contains several currencies. An equilibrium specifies the rates of exchange between each commodity and each currency. Trade between currencies is prohibited and currencies ordered by a strict hierarchy: a low-ranked currency can only be used in a smaller set of markets than a high-ranked currency.

4. Production

In the previous Sections, the jungle has been analysed within a context that is parallel to a pure exchange economy, namely, a given aggregate bundle is distributed among agents. A natural question as to our construction and the analogy with the exchange economy is whether a similar analysis can be conducted in the context of an economy with production. In this Section, we suggest and discuss briefly such an extension.

The *jungle with production* is a tuple $<\{\succsim^i\}_{i\in I},\{X^i\}_{i\in I},Y,S>$. It is the same as before, with the exception that the aggregate bundle w is replaced by $Y \subseteq R_+^K$, the set of bundles which can be produced in the economy. The model without production is a special case of the new model with $Y = \{x \in R_+^K : x \leq w\}$. We assume that the set Y is compact and convex, satisfies free disposal, and has an additional property: given any production vector y, if it is possible to increase the production of some commodity k, it is possible to do it at the expense of any single commodity which is produced in y, without changing the production level of other commodities. Formally, if y, $y' \in Y$, where $y_k < y'_k$ and $y_l > 0$, there exists $y'' \in Y$ such that $y_k < y''_k$, $y_l > y''_l$, and $y_h = y''_h$ for any $h \neq l,k$. This condition is satisfied when all commodities produced require a positive amount of the same input and all production functions are strictly increasing. However, it will fail if, for example, input a is used to produce good 1, input b is used to produce 2, and both inputs are needed to produce good 3.

We now adapt the notion of jungle equilibrium to this model. We begin with the formal definition: a *feasible activity* is a vector of non-negative bundles $z = (z^1,z^2, \ldots, z^N)$ such that $z^i \in X^i$ for $i = 1, \ldots, N$, and $\sum_{i=1}^N z^i \in Y$. A *jungle equilibrium* is a feasible activity z such that there are no agents i and j, iSj, and a feasible activity y such that $y^h = z^h$ for any $h \notin \{i,j\}$ and $y^i \succ^i z^i$.

This definition requires some discussion. The bundle z^i is interpreted as a claim of agent i for goods. In equilibrium, the total claims are feasible in that there exists a production plan (implicit in the set Y) which generates $\sum_{i=1}^N z^i \in Y$. To 'break' an equilibrium, agent i must propose a production and distribution plan derived by reducing the claims of at most one weaker agent. It is important to notice that the new activity can involve reshuffling of the production plans. In some cases, this reshuffling might be quite problematic. Take for example the case where the set of goods contains the labour of each agent. The reshuffling proposed by an agent can include a reallocation of the use of labour of stronger agents across production processes as long as their final consumption is not affected.

PROPOSITION 6. *(i) A jungle equilibrium exists.*
(ii) If a jungle is smooth, the jungle equilibrium is unique and efficient.

Proof. (*i*) Construct the allocation $\hat{z} = (\hat{z}^1, \hat{z}^2, \ldots, \hat{z}^N)$ as follows. Let $\hat{z}^1$ be agent 1's best bundle in the set $\{x^1 \in X^1 \mid x^1 \in Y\}$. Define inductively $\hat{z}^i$ to be agent i's best bundle in $\{x^i \in X^i \mid x^i \leq y - \sum_{j=1}^{i-1} \hat{z}^j$ for some $y \in Y\}$. The allocation $\hat{z}$ is obviously a jungle equilibrium.

(*ii*) Let z be a jungle equilibrium. We need to show that z^i is i's best bundle in $\{x^i \in X^i | x^i \leq y - \sum_{j=1}^{i-1} z^j$ for some $y \in Y\}$. Suppose not. Then there exists a feasible activity v such that $v^j = z^j$ for all $j < i$ and $v^i \succ^i z^i$. Because of monotonicity, $v^i_k > z^i_k$ for some commodity k. Since z is a jungle equilibrium, $z^j_l > 0$ for at least one commodity l and agent $j > i$ as otherwise player i can suggest to produce $\sum_{j=1}^n v^j$ and allocate it so that he gets v^i and any other agent j gets z^j (that is, weaker agents get the zero vector). By the same argument as in Proposition 3, there exists a bundle $\hat{v}^i \in X^i$ such that $\hat{v}^i_k > z^i_k$ for only one commodity k, $\hat{v}^i_h < z^i_h$ for only one commodity h, and $u^i(\hat{v}^i) > u^i(z^i)$. We will now show that there exists an activity in Y that can be redistributed so that any $j < i$ gets z^j and agent i's bundle consists of z^i_m for $m \neq k$ and $z^i_k + \varepsilon$ for good k when ε is sufficiently small. This is obvious if $k = l$. If $k \neq l$, by the assumption about Y and convexity of X_i, we can take ε to be sufficiently small so that only j's bundle needs to be changed to increase z^i_k. The claim then follows as agent i can freely dispose of good h to keep within X^i. Finally, efficiency follows from the fact that $\hat{z}$ is efficient.

4.1. *Concluding Comments by MP*

The aim of this article is to investigate theoretically an environment in which transactions are governed by coercion. Its main goal is to demonstrate at an abstract level the richness of analysis when the allocation rule is driven by agents using power to appropriate resources.

We have emphasised the analogy between the initial endowments in the exchange economy and the initial distribution of power in the jungle for the determination of the final distribution of commodities among the agents. I wish to add a few simple remarks.

In an exchange economy, the interior efficient allocations can be supported as a competitive equilibrium allocation for some redistribution of the endowments. In so far as the jungle precludes 'redistribution' of power, it also precludes redistribution of resources. One possible avenue for a more flexible definition of power is to interpret the consumption sets as restrictions not only on the ability to consume but also on the ability to seize resources held by others. Such an interpretation, however, will in general undermine the efficiency of the jungle equilibrium.

As we have seen, the inclusion of production processes in this model is problematic. First, it is not clear if and how firms should operate in the jungle as ownership is not defined. Second, the jungle mechanism might not generate production efficiency. In our model, we evade this problem by using the production possibility set. If, however, stronger agents determine which particular combinations of inputs are used in production, efficiency may not ensue. Suppose, for example, that $N = 2$ and that there are several ways of producing the satiation bundle of agent 1. The inputs mix commandeered by agent 1 for the production of his satiation bundle does not necessarily maximise agent 2's utility subject to agent 1 consuming his satiation bundle. An efficient jungle equilibrium exists but it may not be achieved if agent 1 does not include in his considerations the utility level of agent 2. A similar problem can arise in the case of no production when the utility of agent 1 is linear. Production, however, seems to make inefficiency more generic.

In future research, we plan to extend our model to include the possibility of agents forming coalitions. Preliminary results suggest that coalition stability is a subtle issue.

4.2. *Concluding Comments by AR*

Economic theory is the study of mechanisms by which society organises its economic activities and, in particular, distributes its resources. Most of economic theory deals with variants of the market system in which goods enter the world with attached ownership rights and are allocated by means of prices. One way in which to view our article is as a study of an additional mechanism which is not uncommon in real life, even outside the jungle.

However, in my view, this article is not purely a study of the jungle economy. When I present the model in public lectures, I ask the audience to imagine that they are attending the first lecture of a course at the University of the Jungle, designed to introduce the principles of economics. I use this rhetorical device to emphasise that the article is only a rhetorical exercise aimed at shedding light on the implicit message that Microeconomics' students receive from us.

Being faithful to the classical economic tradition, we constructed a model which is close to an exchange economy. We used terminology that is familiar to any economics student. After having defined the notion of jungle equilibrium, we conducted the same type of analysis which can be found in any microeconomics textbook on competitive equilibrium. We showed existence and then discussed the first and second fundamental welfare theorems. We emphasised the analogy between the initial endowments of an exchange economy and the initial distribution of power in the jungle as determining the distribution of commodities among the agents. Were I teaching this model, I would also add the standard comments regarding externalities and the place for government intervention.

There are arguments which attempt to dismiss the comparison between markets and jungles. One can argue that the market has the virtue of providing incentives to 'produce' and to enlarge the size of the 'pie' to be distributed among the agents. One can also argue, however, that the jungle provides incentives to develop power (physical, intellectual or mental) which is an important social asset. Agents make efforts to produce more goods. Agents who wish to be stronger are an asset for a society which can then defend itself against invaders or evade others in order to accumulate resources.

One might argue that market mechanisms save the resources that would have been wasted in conflicts. Note, however, that under complete information a stronger agent can persuade a weaker agent to part with his goods with no resistance. Societies often create rituals which aid people in recognising relative power and thereby avoid the costs of conflict. Under incomplete information, the market also wastes resources. And finally, I have not mentioned the obvious trade costs which are also associated with market institutions.

One might argue that labour is a good which should be treated differently. The long history of slavery shows this to be inaccurate.

One might also argue that the virtue of the market system is that it utilises people's natural desire to acquire wealth. On the other hand, the jungle uses the people's natural willingness to exercise power and to dominate without employing central government.

Obviously, I am not arguing in favour of adopting the jungle system. Overall, the relative comparison of the jungle and market mechanisms depends on our assessment of the characteristics with which agents enter the model. If the distribution of the initial holdings in the market reflects social values which we wish to promote, we might regard the market outcome as acceptable. However, if the initial wealth is allocated unfairly, dishonestly or arbitrarily, then we may not favour the market system. Similarly, if power is desirable we might accept the jungle system but if the distribution of power reflects brutal force which threatens our lives we would clearly not be in favour.

Appendix

Proof of Lemma 2: First note that $x_l < 0$ for some l and $x_k > 0$ for some k. Second note that, if $a_l/b_l < a_k/b_k$ for some k for which $x_k > 0$ and some l for which $x_l < 0$, the claim follows by choosing $y_k > 0$ and $y_l < 0$ such that $b_k/b_l < -y_l/y_k < a_k/a_l$. Finally, we show that it is impossible that $a_l/b_l \geq a_k/b_k$ whenever $x_l < 0$ and $x_k > 0$. If so, let a_h/b_h be the lowest a_l/b_l associated with $x_l < 0$. Then $ax = \sum_{i=1}^{n}(a_i/b_i)x_i b_i \leq \sum_{i=1}^{n}(a_h/b_h)x_i b_i = (a_h/b_h)bx$, a contradiction as $ax > 0$ and $bx < 0$. QED

Example (Jungle Prices): Consider a jungle with $N = 2$, $K = 3$, where agent i's utility is $3x_1^i + 2\sqrt{3 + x_2^i} + 2\sqrt{3 + x_3^i}$, his consumption set is defined by $x_1^i + x_2^i + x_3^i - 3 \leq 0$ and the aggregate initial endowment bundle is $(1, \frac{5}{2}, 2)$.

It is easy to verify that (1,1,1) is the (unique) optimal bundle of agent 1 among all feasible bundles. Then, if we assign $(0, \frac{3}{2}, 1)$ to agent 2, we have the jungle equilibrium $\hat{z}$.

Suppose that a competitive equilibrium price vector p exists when $\hat{z}$ is the initial endowment. The allocation $\hat{z}$ must also be a competitive equilibrium allocation as it is efficient. Agent 1's first order conditions are

$$3 - \theta^1 - \lambda^1 p_1 = 0$$

$$\frac{1}{\sqrt{\hat{z}_2^1 + 3}} - \theta^1 - \lambda^1 p_2 = 0$$

$$\frac{1}{\sqrt{\hat{z}_3^1 + 3}} - \theta^1 - \lambda^1 p_3 = 0.$$

Note that $\lambda^1 > 0$. Now, since $\hat{z}_2^1 = \hat{z}_3^1$, we must have $p_2 = p_3$. However, agent 2's consumption set is not binding at $\hat{z}^2$ and only the budget constraint is binding. Thus, the demands of agent 2 for goods 2 and 3 must be identical whereas $\hat{z}_2^2 \neq \hat{z}_3^2$.

Despite non existence of equilibrium prices, one can find a sequence of price vectors p^n such that the sequence of demands given p^n converges to $\hat{z}^i$ for $i = 1{,}2$. Take $p_1^n \equiv 1$, $p_2^n/p_3^n \equiv 2\sqrt{2}/3$, the marginal rate of substitution of agent 2 at $\hat{z}^2$, and let p_2^n and p_3^n converge to zero to make any consumption of good 1 suboptimal for agent 2.

Proof of Proposition 5: Without loss of generality, we label the commodities so that all commodities except for 1, …, t^i are exhausted by agents stronger than i, that is, $w_j - \sum_{h=1}^{i-1} \hat{z}_j^h = 0$ if and only if $j > t^i$. The Kuhn-Tucker conditions for agent i are

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} = 0 \text{ for } j = 1, \ldots, t^{i+1}$$

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \gamma_j^i = 0 \text{ for } j = t^{i+1} + 1, \ldots, t^i$$

with non negative multipliers λ^i for the consumption set and γ_j^i for the binding resource constraints.

The first set of equations refers to the commodities that are not exhausted by agent i. The second set of equations refers to the commodities that are exhausted by agent i.

The constraints are necessary for agent i's maximisation. If agent i exhausts all the goods, one can set $\lambda^i = 0$ and $\gamma_j^i = \partial u^i(\hat{z}^i)/\partial x_j$. If not, $g^i(\hat{z}^i) = 0$ because of strict monotonicity of preferences and the 'constraint qualification' condition is satisfied: the gradients (the derivatives with respect to the commodities 1, …, t^i) of the constraints ($\hat{z}_j^i - w + \sum_{h=1}^{i-1} \hat{z}_j^h \leq 0$, $j = 1, \ldots, t^i$ and $g^i(\hat{z}^i) \leq 0$) that are satisfied with equality are linearly independent. In fact, the gradient of g^i (recall that $g^i(\hat{z}^i) = 0$) is a vector of t^i positive numbers and the gradients of the remaining constraint holding with equality are at most $t^i - 1$ unit vectors.

For later use, we add a set of residual conditions which is trivially satisfied

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \lambda^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \gamma_j^i \leq 0 \text{ for } j = t^i + 1, \ldots, K$$

Consider now an exchange economy with $w^i = \hat{z}^i$, $i = 1, \ldots, N$. For $\hat{z}^i$ to be i's optimal choice given a price vector $(p_1, \ldots, p_K)$, the Kuhn-Tucker sufficient conditions are

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j = 0 \text{ for } j = 1, \ldots, t^{i+1}$$

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j = 0 \text{ for } j = t^{i+1} + 1, \ldots, t^i$$

$$\frac{\partial u^i(\hat{z}^i)}{\partial x_j} - \mu^i \frac{\partial g^i(\hat{z}^i)}{\partial x_j} - \theta^i p_j \leq 0 \text{ for } j = t^i + 1, \ldots, K.$$

Hence, $\hat{z}^i$ is an optimal choice given any price vector p^i such that $p_j^i = 0$, $j = 1, \ldots, t^{i+1}$, and $p_j^i = a\gamma_j^i$, $j = t^{i+1}+1, \ldots, t^i$, $p_j^i \geq a\gamma_j^i$, $j = t^i+1, \ldots, K$, where $a > 0$.

In general, one cannot construct a price vector for which the demands of every agent i are equal to $\hat{z}^i$. However, we construct a sequence (p^ε) of price vectors such that the demands of every agent i converge to $\hat{z}^i$ as $\varepsilon \to 0$.

Let $d(j)$ be the first player i for whom $\sum_{h=1}^{i} \hat{z}_j^h = w_j$ (agent $d(j)$ exhausts commodity j) and define $p_j^\varepsilon = \varepsilon^{d(j)}\gamma_j^{d(j)}$ if $\gamma_j^{d(j)} > 0$ and $p_j^\varepsilon = \varepsilon^{d(j)+\frac{1}{2}}$ if $\gamma_j^{d(j)} = 0$. If commodity j is never exhausted then set $p_j^\varepsilon = \varepsilon^{N+1}$. This price vector is such that the price of a good which is exhausted by player i is by an order of magnitude larger than the price of a good which is exhausted by agent $i + 1$, and the price ratio of any two goods j and k which are exhausted by the same agent i is γ_j^i/γ_k^i if both $\gamma_j^i > 0$ and $\gamma_k^i > 0$.

We will show that, as $\varepsilon \to 0$, the demand of agent i given the price vector p^ε converges to $\hat{z}^i$ for each i. Note that a direct application of the continuity of i's demand is not sufficient as p^ε converges to the null vector. Consider the price vector $\hat{p}^{\varepsilon i}$ obtained from p^ε by replacing p_j^ε, $j = t^i+1, \ldots, K$, with $\varepsilon^i \gamma_j^i$. Clearly i's demands given $(1/\varepsilon^i)\,\hat{p}^{\varepsilon i}$ converge to $\hat{z}^i$ by continuity. Suppose that the vector of demands given the price vector $(1/\varepsilon^i)p^\varepsilon$ converges along some subsequence to $\tilde{z}^i \neq \hat{z}^i$. Note that $(1/\varepsilon^i)\hat{p}^{\varepsilon i}\,\tilde{z}^i = (1/\varepsilon^i)p^\varepsilon\,\tilde{z}^i$ (since $\hat{p}^{\varepsilon i}$ differ from p^ε only in goods which were exhausted by stronger players) and $(1/\varepsilon^i)\hat{p}^{\varepsilon i} \leq (1/\varepsilon^i)p^\varepsilon$ for small ε. Since $\tilde{z}^i$ is in the budget constraint of agent i when the price vector is equal to $\lim_{\varepsilon\to 0}(1/\varepsilon^i)\hat{p}^{\varepsilon i}$, strict convexity implies that $u^i(\tilde{z}^i) < u^i(\hat{z}^i)$. A contradiction is thus obtained noting that $\hat{z}^i$ is in the budget constraint given $(1/\varepsilon^i)p^\varepsilon$ for any ε.

London School of Economics
Tel Aviv University and New York University

Submitted: 16 January 2006
Accepted: 3 June 2006

References

Abdulkadiroglu, A. and Sonmez, T. (1998). 'Random serial dictatorship and the core from random endowments in house allocations problems', *Econometrica*, vol. 66, pp. 689–701.
Bowles, S. and Gintis, H. (1992). 'Power and wealth in a competitive capitalist economy', *Philosophy and Public Affairs*, vol. 21, pp. 324–53.
Florig, M. (2001). 'Hierarchic competitive equilibria', *Journal of Mathematical Economics*, vol. 35, pp. 515–46.
Ghosal, S. and Polemarchakis, H. M. (1999). 'Exchange and optimality', *Economic Theory*, vol. 13, pp. 629–42.
Grossman, H. (1995). 'Robin Hood and the redistribution of property income', *European Journal of Political Economy*, vol. 11, pp. 399–410.
Hirshleifer, J. (1994). 'The dark side of the force', *Economic Inquiry*, vol. 32, pp. 1–10.
Mas Colell, A., Whinston, M.D. and Green, J.R. (1995). *Microeconomic Theory*, Oxford: Oxford University Press.
Moselle, B. and Polak, B. (2001). 'A model of predatory state', *Journal of Law, Economics, and Organization*, vol. 17, pp. 1–33.
Shapley, L. and Scarf, H. (1974). 'On cores and indivisibility', *Journal of Mathematical Economics*, vol. 1, pp. 23–37.
Skaperdas, S. (1992). 'Cooperation, conflict and power in the absence of property rights', *American Economic Review*, vol. 82, pp. 720–39.
Usher, D. (1989). 'The dynastic cycle and the stationary state', *American Economic Review*, vol. 79, pp. 1031–44.
Varian, H. (1974). 'Equity, envy and efficiency', *Journal of Economic Theory*, vol. 9, pp. 63–91.