Bananas in a Civilized Jungle

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Contents

1 Introduction

The aim of this dissertation is to extend the setting in which the civilized jungle concept is presented. I will describe the basic features of a civilized jungle as presented by A. Rubinstein and K. Yildiz [5] and show some of their results. Then I formally present a brief review of the welfare theorems, which are a core topic of the essay. Then I enlarge the setting. At first, I analyze a civilized jungle with divisible goods, which is done by introducing consumption bundles for the commodities and consumption set for the agents. This first part has fundamental insights from the paper by M. Piccione and A. Rubinstein [3]. Secondly I introduce production. My ultimate goal is to study an analogous of the second welfare theorem for a civilized jungle with divisible goods and production.

2 Civilized Jungle

Rubinstein and Yildiz [5] present a civilized jungle as a tuple $\langle N, X, (\succeq^i)_{i\in N}, \trianglerighteq, \mathcal{L}\rangle$ where N are the agents, X are the n object, $(\succeq^i)_{i\in N}$ are the preferences of each agents over the objects, $\trianglerighteq$ is a strict and complete ordering over N which represents the power relation between the agents and $\mathcal{L} = \{\geqslant_\lambda\}_{\lambda\in\Lambda}$ is a set of complete and transitive (possibly not antisymmetric) binary relations which stands for different criteria that rank agents.
A distribution of the resources over the agents is represented by an assignment $\mathbf{x}$ that is a map that maps every agent to an object, that is $\mathbf{x} : N \to X : i \mapsto x^i$. We will use the notation $\mathbf{x} = (x^1, \dots, x^2)$.
How do we define an equilibrium? Given an assignment, every agent has to be the strongest among those who envy her and can justify themself. How can someone justify herself among a group of agents? We say that agent $i \in N$ is justified by $\geqslant_\lambda$ over $I \subseteq N$ if $i \geqslant_\lambda j \ \ \forall\, j \in N$, then:

$$i \text{ is justified among } I \text{ if } \exists\, \lambda \in \Lambda \text{ such that she is justified by that criteria over } I.$$

That is, if you are the best in a group for at least one criteria then you are justified in that group. The justification proposed by R-Y is exclusively meritocratic, and therefore far from reality. Nevertheless, that is the concept of a *civilized* jungle. Let $J_{\mathcal{L}}(I)$ be the set of justifiable agents in I. Formally, $J_{\mathcal{L}} : \mathcal{P}(N) \to \mathcal{P}(N)$ such that:

$$\tilde{J}_{\mathcal{L}}(I) = \left\{ i \in N \mid \exists\, \lambda \in \Lambda \text{ s.t. } i \geqslant^\lambda j \ \forall\, j \in I \right\}$$

Actually, the definition of justified agents in a group is given by:

$$J_{\mathcal{L}}(I) = \tilde{J}_{\mathcal{L}}(I) \cap I$$

We restrain the justification only to those who are in the group because we will use this notion just for those who envy or dream about someone, as we will see. Therefore those who are not envious are not interested in being justified.
The last core concept is *envy*. Trivially, an agent i envies another agent j if $x^j \succeq^i x^i$. We define $E(\mathbf{x}, i)$ as the set of agents that envies i, given the assignment $\mathbf{x}$.
We are now able to define the equilibrium concept in a civilized jungle.

Definition 2.1. *An assignment $\boldsymbol{x}$ is a* ***civilized equilibrium*** *if $\forall\, i \in N$ then $i \in J_{\mathcal{L}}(E(\boldsymbol{x}, i))$ and:*

$$i \rhd j \qquad \forall\, j \in J_{\mathcal{L}}(E(\boldsymbol{x}, i))$$

A civilized equilibrium is also called a C equilibrium. Such an equilibrium always exists, although it is not very interesting being just a serial dictatorship according to the power relation.

Example 2.1. *A dichotomous language is a partition into two indifference sets of agents. That is, for each criterion $\lambda \in \Lambda$ if $x \geqslant_\lambda y$ then x and y are respectively in the top and bottom sets of λ. We can therefore partition N by $\sim_\lambda$, where:*

$$x \sim_\lambda y \Leftrightarrow x =_\lambda y$$

And get $N/\sim_\lambda = \{0, 1\}$.
Let the Collegio Carlo Alberto be a Jungle. The agents are all the Professors and the Allievi. The commodities are the different speeds at which an agent can eat at a buffet in the common room. Of course, everyone has the same preference relation: before is better. We summarize by letting N finite, $X \subset \mathbb{N}$ finite, where each number corresponds to a different velocity and $k \succeq^i j$ if and only if $k \leqslant j$, where $\leqslant$ is the usual ordering over the natural numbers for all $i \in N$. We assume that there is a power relation $\trianglerighteq$ over N defined by each agent's capability of winning a buffet race. Nevertheless, one can exercise her power only if she is justified by the one criterion that governs the Collegio: being a scholar. Formally, $\Lambda = \{\lambda\}$ is a singleton and $x \geqslant_\lambda y$ if and only if x is a Professor and y an Allievo.

The only civilized equilibrium is straightforward: the serial dictatorships over the two equivalence classes induced by $\sim_\lambda$.
In general, if $\mathcal{L}$ is a singleton whose element is not a strict ordering, as before, one can define the C equilibrium running serial dictatorships over all the equivalence classes induced by $\sim$.

Example 2.2. *Lower cost of labor in developing countries: agents are firms, commodities are salaries (all negative commodities, preference relation is usual over real numbers), power is money and general power of the firm, language are state laws.*

A language truly brings civilization to a jungle if it has more than one criterion. In fact as R-Y[5] show that if $\mathcal{L}$ consists of just one strict ordering $\geqslant$, then the unique "civilized" equilibrium is a serial dictatorship according to $\geqslant$. Then the C equilibrium does not depend on the power relation over the agents and the civilized jungle is just a normal jungle with another power relation.

Proposition 2.1. *if $\mathcal{L}$ consists of just one strict ordering $\geqslant$, then the unique civilized equilibrium is a serial dictatorship according to $\geqslant$.*

Proof. □

We can think of an uncivilized jungle. Piccione and Rubinstein [3] defined the concept of jungle, as a civilized jungle without a language and a related equilibrium.

Definition 2.2. *An assignment is a* ***jungle equilibrium*** *if:*

$$\forall\ i \in N\ \ \forall\ j \in E(\boldsymbol{x}, i)\ \textit{then}\ i \rhd j$$

Therefore the unique jungle equilibrium is obtained through a serial dictatorship governed by the power relation.

Proposition 2.2. *Let $\langle N, X, (\succeq^i)_{i\in N}, \trianglerighteq\rangle$ be a jungle, a jungle equilibrium is given by the assignment obtained through the serial dictatorship ruled by $\trianglerighteq$.*

Proof. We define a serial dictatorship ruled by $\trianglerighteq$ recursively as $x^1 = \max_{\succeq^1} X$ and:

$$x^i = \max_{\succeq^i} X \backslash \{x^1, \ldots, x^{i-1}\}$$

Where $\max_{\succeq^j} Y$ stands for maximize Y with respect to the preference relation $\succeq^j$. We assume that N is ordered following $\trianglerighteq$, if not, a permutation suffices to extend the definition. This is a jungle equilibrium. Let y □

Which relation do C equilibrium and jungle equilibrium have? Let us state a property of the power relation in order to answer this question.

Definition 2.3. *A strict ordering $\trianglerighteq$ is* ***weakly $\mathcal{L}$-concave*** *if $\forall\ i, j \in N$ and $\forall\ \lambda \in \Lambda$:*

$$\exists\ i_\lambda \in N\backslash\{j\}\ \textit{s.t.}\ i_\lambda \geqslant_\lambda j\ \textit{and}\ i \rhd i_\lambda \Rightarrow i \rhd j$$

Weak convexity is an extension of the basic notion of convexity. It was proposed by Richter and Rubistein [4] and it allows the defition of the property for any space without the need of an algebraic structure. It is quite intuitive if seen as: if *for all* the criteria that we are using in our language if I (i) can find some one (i_λ) weaker than me and best suited than you (j) then I am stronger than you. Under this condition the jungle equilibrium is a C equilibrium.

Proposition 2.3. *Let $\langle N, X, (\succeq^i)_{i\in N}, \trianglerighteq, \mathcal{L}\rangle$ be a civilized jungle with a weakly $\mathcal{L}$-concave power relation, then the jungle equilibrium is a civilized equilibrium.*

Proof. Let $\mathbf{x}$ be the jungle equlibrium. Of course in a serial dictatorship $E(\mathbf{x}, i) \subseteq \{j \mid i \rhd j\}$, then i is the strongest among $J_{\mathcal{L}}(E(\mathbf{x}, i))$ if i belongs to it. In fact, recall that justified agents in a group are part of that group. If by contradiction i is not the strongest in $J_{\mathcal{L}}(E(\mathbf{x}, i))$ then $i \notin J_{\mathcal{L}}(E(\mathbf{x}, i))$, but then $\forall\ \lambda \in \Lambda$ there exists $i_\lambda \neq i$ such that $i_\lambda \geqslant_\lambda i$ but also $i \rhd i_\lambda$, and so by weak concavity $i \rhd i$. $\square$

Furthermore if the notion of concavity is empowered the jungle equilibrium becomes the only C equilibrium.

Definition 2.4. *A power relation is **strongly $\mathcal{L}$-concave** if* $\forall\ i, j \in N$*:*

$$\exists\ i_\lambda \in N\backslash\{j\}\ \text{s.t.}\ i_\lambda \geqslant_\lambda j\ \text{and}\ i \trianglerighteq i_\lambda \Rightarrow i \rhd j$$

Proposition 2.4. *Let $\langle N, X, (\succeq^i)_{i \in N}, \trianglerighteq, \mathcal{L}\rangle$ be a civilized jungle of strict orderings criteria with a strongly $\mathcal{L}$-concave power relation, then the jungle equilibrium is the unique civilized equilibrium.*

Proof. Let $\mathbf{y}$ be another C equilibrium. Then not being the serial dictatorship implies the existence of $i, j \in N$ such that $i \rhd j$ and $x^j \succeq^i x^i$. Because $\mathbf{y}$ is a C equilibrium $i \notin J_{\mathcal{L}}(E(\mathbf{x}, j))$, then $\forall\ \lambda \in \Lambda\ \exists\ i_\lambda \in N\backslash\{i\}$ such that $i_\lambda \geqslant_\lambda i$. Since $\mathbf{y}$ is a C equilibrium $j \trianglerighteq j_\lambda\ \forall\ \lambda$. Then by strong concavity $j \rhd i$. $\square$

3 Welfare theorems

Considered this simple economic model we want to establish whether or not the welfare theorems hold. Let us recall them heuristically: the first one states that under pretty weak hypotheses any competitive equilibrium is Pareto optimal while the second one states that under more stringent assumptions any Pareto optimal allocation can be attained as a competitive equilibrium through a suitable price vector (and share allocation). Let's talk math. We will present the welfare theorems in (almost) their most general formulation, following D. Acemoglu [1]. At first we need to define the building block of this theory, i.e. a stylized economy. We set $N \in \mathbb{N}^* = \mathbb{N} \cup \{+\infty\}$ as the number of agents, $K \in \mathbb{N}^*$ as the number of commodities and $\mathcal{F}$ as the set of firms. Commodities can be taken in a continuum, but this setting is sufficient for our purposes. Each agent $i \leqslant N$ has a *preference relation* $\succeq^i$ on the set of bundles $\mathbb{R}_+^K$ and a *consumption set* $X^i \subseteq \mathbb{R}_+^K$. We assume the preference relation to be strongly monotone and continuous (WHY??) and the consumption set to be compact, convex and that it satisfies free disposal (???). We won't allow consumption to negative, the extension is straightforward. We define $\mathbf{X} = \prod_{i=1}^N X^i$ the aggregate consumption set as the cartesian product of all the consumption sets. We then denote $Y^f \subseteq \mathbb{R}^K$ (???) for each $f \in \mathcal{F}$ as the production set of firm f. We assume each production set to be a cone and denote $\mathbf{Y} = \prod_{f \in \mathcal{F}} Y^f$. At last we define the *profit share* deriving from the aggregate production as $\theta = (\theta^f)_{f \in \mathcal{F}}$ where $\theta^f \in \mathbb{R}_+^N$ represents the redistribution of firm f's production to each agent. We normalize to 1 for each firm, that is $\sum_{i=1}^N \theta_f^i = 1 \ \forall \ f \in \mathcal{F}$.

Definition 3.1. *The tuple* $\langle N, \mathcal{F}, \boldsymbol{X}, \boldsymbol{Y}, \omega, \theta \rangle$ ...

4 Civilized Jungle with divisible commodities

I now extend the setting of a civilized jungle and the relative equilibrium concept for divisible commodities. Relying on the paper by P and R [3] I define for each agent $i \in N$ a consumption set $X^i \subseteq \mathbb{R}^K_+$ and $X = (X^1, \ldots, X^N)$. I also have to specify an aggregate bundle $w = (w_1, \ldots, w_K)$, where $w_k \in \mathbb{R}_+ \ \forall\ k$. Therefore a **civilized jungle** is a tuple:

$$\langle N, K, (X^i)_{i \in N}, w, (\succeq^i)_{i \in N}, \trianglerighteq, \mathcal{L} \rangle$$

Because agents can consume more than just one commodity in different quantities, economic scenarios cannot be represented by an assignment, instead they are modeled through allocations. An allocation is a vector $\mathbf{z} \in \mathbb{R}^K_+ \times X$, where the first coordinate stands for the unused goods, therefore $\sum_{i=0}^{N} z_i = w$. I now adapt the civilized equilibrium concept by redefining the notion of "envy". In this context, agent are not envious, because their economic situation can be, and realistically is always, composed of divisible commodities. Given an allocation, an agent can "dream" about another agent if she sees in the dreamed one's property a more preferred allocation. To keep it simpler, I do not allow coercion on multiple agents. I define formally the concept of a dreamer.

Definition 4.1. *Given an allocation* $\boldsymbol{z}$ *and* $i, j \in N$ *we say that* i ***dreams*** *about* j *if:*

$$\exists\ y^i \in X^i \textit{ such that } y^i \succeq^i z^i \textit{ and } y^i \leqslant z^i + z^j$$

We define the new set of those who dream someone in a given allocation:

$$D(\mathbf{z}, i) = \left\{ j \in N \mid \exists\ y^i \in X^i \text{ s.t. } y^i \succeq^i z^i,\ y^i \leqslant z^i + z^j \right\}$$

In a jungle equilibrium, every agent has to be the strongest among all those who dream of her. In a civilized jungle, agents have to justify themself through at least one criteria in order to "fight" for commodities. Once agents are justified, jungle law prevails.

Definition 4.2. *An allocation* $\boldsymbol{z}$ *is a* ***civilized equilibrium*** *if* $\forall\ i \in N$ *the following hold:*

1. $i \in J_{\mathcal{L}}(D(\boldsymbol{z}, i))$
2. $i \rhd j \ \ \forall\ j \in J_{\mathcal{L}}(D(\boldsymbol{z}, i))$
3. $D(\boldsymbol{z}, 0) = \varnothing$

The first two conditions were previously explained, while the third one assures that the unused goods aren't useful for anyone.
Does the relation between jungle and civilized equilibrium still hold? Let us briefly recall the jungle equilibrium concept.

Definition 4.3. *An allocation* $\boldsymbol{z}$ *is a* ***jungle equilibrium*** *if* $\nexists i, j \in N$ *s.t.* $i \rhd j$ *and* $\exists\ y^i \in X^i$ *s.t.* $y^i \succeq^i z^i$ *and:*

$$y^i \leqslant z^i + z^j \textit{ or } y^i \leqslant z^i + z^0$$

We can reformulate definition 4.3 as follows, in terms of dreamers.

Definition 4.4. *An allocation* $\boldsymbol{z}$ *is a* ***jungle equilibrium*** *if:*

- $\forall\ i \in N\ : i \rhd j \ \ \forall\ j \in D(\boldsymbol{z}, i)$
- $D(\boldsymbol{z}, 0) = \varnothing$

Through this *equivalent* definition it is clear the further step required by civilization, in equilibrium agents have to be the strongest among the justified dreamers, not among those who just dream. We recall that justified among a group are defined to be part of that group. The definition could be also given without this condition, letting agents be justified in groups of which they are not part, but it seems unreasonable in our setting, given that only dreamers are interested in being justified.

Example 4.1. *New*

Example 4.2. *From PR*

Proposition 4.1. *In a weakly concave jungle, a jungle equilibrium is civilized.*

Proof. The second condition is fulfilled. $J_{\mathcal{L}}(D(\mathbf{z}, \cdot)) \subseteq D(\mathbf{z}, \cdot)$, therefore being the strongest in the latter implies being stronger in the justified envious, if part of it. Then by contradiction as before we prove the belonging. $\square$

Of course, the uniqueness under strict concavity does not hold. The proof by R-Y[5] breaks instantly when the existence of another C equilibrium implies that there is at least one powerful envious, meaning that she is stronger than the envied one. It is not true when goods are more than just units, possibly there are multiple best bundles for each agent. Clearly, under smoothness and strict concavity uniqueness is attained.

5 Civilized Welfare Theorems

I now investigate whether or not the welfare theorems hold in a Civilized Jungle. As a first step, we have to adapt the statement of the theorems to this setting. Evidently, a competitive allocation is a civilized equilibrium while the concept of *Pareto efficiency* is intended as follows.

Definition 5.1. *An allocation* $\boldsymbol{z}$ *is* ***Pareto efficient*** *if does not exist another* $\boldsymbol{y}$ *allocation s.t:*

$$y^i \succeq^i z^i \ \forall\, i \in N \text{ and } \exists\, j \in N : y^j \succ^j z^j$$

That is, an allocation is Pareto efficient if no one can improve her situation without making anyone else worse off.
In further sections, production will be introduced in the jungle setting, until then the welfare theorems are stated in their simplified versions.

5.1 First civilized theorem

The first theorem can be interpreted as:

A civilized equilibrium is Pareto efficient.

We know from P-R that unique jungle equilibria are indeed Pareto efficient and, from now on just efficient. R-Y show that a strong result holds for civilized jungles.

Proposition 5.1. *Given a civilized jungle with a language of strict orderings such that there exists no agent* i, j *where one is ranked right above the other and the opposite relation holds for the language. If the power relation is not weakly concave then there exists a preference profile such that there exists no pareto efficient C equilibrium.*

As we already lose the first welfare theorem power for indivisible commodities we shift our attention to the second one, which has a civilized version in P-R.

5.2 Second civilized theorem

The second welfare theorem, in its standard formulation, guarantees for every Pareto efficient allocation, under suitable assumption, the existence of a price vector and an endowment that sustains that allocation as a competitive one. In a jungle, even if civilized, the only currency is brute force, then prices and personal endowments are substituted by a power relation. Therefore we can restate the theorem as:

For every Pareto efficient allocation there exists a power relation which sustains the allocation as a civilized equilibrium.

If dealing with non-divisible goods, R-Y [5] have proved an analogous version of the second welfare theorem. The two key hypotheses are strict orderings as languages and restraining efficient allocation to *J-costrained* efficient allocation. The first one guarantees a clear power relation. The latter is an important and necessary constraint: in a civilized equilibrium each agent has to justify herself among those who envy her, otherwise she cannot use her force against them. This assumption is not necessary for an efficient assignment. It is therefore necessary to introduce the following class of assignments.

Definition 5.2. *An allocation* $\boldsymbol{z}$ *is J-constrained if* $i \in J_{\mathcal{L}}(E(\boldsymbol{z}, i)) \ \forall\, i \in N$.

Once we constrain an efficient assignment to a language we can inquire under which conditions on $\mathcal{L}$ each efficient assignment is an equilibrium. Turns out that for divisible commodities the adapted proof of *Proposition 3* from R-Y[5] breaks immediately. The idea is to build the power relation as a completion of a non-cyclic binary relation over a subset of N. In particular, the subset over which the (possibly incomplete) binary relation is defined is such that it guarantees the assignment to be C equilibrium. I now present the theorem from R-Y and then show why it does not hold in our more general setting.

Theorem 5.2. *Let $\langle N, (X^i)_{i \in N}, (\succeq^i)_{i \in N}, \mathcal{L} \rangle$ be a tuple as above. Then, for every J-constrained efficient assignment $\boldsymbol{x}$ there exists a power relation $\trianglerighteq$ such that $\boldsymbol{x}$ is a C equilibrium for the corresponding civilized jungle.*

Proof. Let $\mathbf{x}$ be a J-constrained efficient assignment. Let P be a binary relation over N such that for each $i, j \in N$ then jPi if i envies j and she is justifiable in $E(\mathbf{x}, j)$. If we show P to be non-cyclic, then it is a pre-order over N. This comes from a standard result in order theory[1], I will talk more about it later in this section. By completing P, we'd get a power relation $\trianglerighteq$, which sustains $\mathbf{x}$ as a C equilibrium. Indeed:

- $i \in J_{\mathcal{L}}(E(\mathbf{z}, i))$ for all $i \in N$ because $\mathbf{x}$ i J-constrained
- If i is justifiable in $E(\mathbf{z}, j)$, then jPi, then $j \rhd i$

Let us show that P is non-cyclic. Suppose by contradiction that for some $I = \{1, 2, \ldots, m\}$ we have $1P2P \ldots PmP1$. Let us define the allocation $\mathbf{y}$ as $y^i = x^{i-1}$ for $i \in I$ and $y^i = x^i$ for $i \notin I$. The assignment $\mathbf{y}$ is justifiable and pareto dominates $\mathbf{x}$. The latter is obvious by construction. By recalling that the operator $L_{\mathcal{L}}$ is monotone decreasign[2] we prove $\mathbf{y}$ to be justified, indeed for $i \in N$ then:

- If $i \in I$ then $E(\mathbf{y}, i) \subseteq E(\mathbf{x}, i-1)$
- If $i \notin I$ then $E(\mathbf{y}, i) \subseteq E(\mathbf{x}, i)$

But i is justified in both $E(\mathbf{x}, i)$ and $E(\mathbf{x}, i-1)$, because $\mathbf{x}$ is justified and by definition of P. □

The whole proof relies on the possibility of constructing an assignment $\mathbf{y}$ that pareto dominates $\mathbf{x}$. For divisible commodities, two main problems arise. We first think about non-civilized jungles. We call the first one *reciprocal dreaming*, that is the instance in which two agents dream each other. This situation could clearly occur even in the non-divisible goods setting, but only in non-efficient assignments. Indeed, if two monkeys envy each other they can simply switch their bananas and get a more efficient assignment. When bananas are divisible, reciprocal dreaming does not imply inefficiency. But if in an efficient allocation two agents are reciprocal dreamers, then no power relation will sustain the allocation as an equilibrium. Indeed, every power relation would not be strict as for the reciprocal dreamers i, j the equilibrium condition would imply $i \rhd j$ and $j \rhd i$. We found a necessary condition. If we impose non-reciprocal dreaming can we prove the theorem? Not yet, the very essence of the more general setting prevents the allocation $\mathbf{y}$ from being constructed. While for non-divisible commodities monkeys could just switch different bananas, in this setting what is dreamed by two monkeys could be unfeasible together. We therefore should follow a different path, a straightforward extension of the proof is impossible.
Actually, a general result holds: the II welfare theorem does not hold for uncivilized jungles with divisible commodities. Given an allocation, we can sustain it as a C equilibrium only if in the subset of dreamers there are no reciprocal dreamers (we can impose this condition) and those who dream are weaker than the dreamed one. We can easily construct a Pareto efficient allocation where the two previous conditions are fulfilled, but the binary relation resulting doesn't satisfy OWC, which will be studied later, and therefore cannot be extended to a complete order (because of 6.1).

[1]Varian (1974) for non cyclic implies completable
[2]Meaning that $A \subseteq B \Rightarrow J_{\mathcal{L}}(A) \supseteq J_{\mathcal{L}}(B)$

Example 5.1. *Let three monkeys fight for one banana, one nut, and one stick. Monkey a prefers one banana and the stick over everything, monkey b prefers the banana and the nut over everything else, and monkey c prefers the nut and the stick. The preference relations are:*

$$\begin{cases} (1,0,1) \succ (1,0,0) \text{ strictly better than all others, over which she is indifferent.} \\ (1,1,0) \succ (0,1,0) \text{ strictly better than all others, over which she is indifferent.} \\ (0,1,1) \succ (0,0,1) \text{ strictly better than all others, over which she is indifferent.} \end{cases}$$

Consider the allocation $z = ((1,0,0),(0,1,0),(0,0,1))$. It is Pareto efficient (the only allocations where someone is strictly better off are obtained if b takes the banana from a, or if c takes the nut from b or a takes the stick, in all instances, the robbed are worse off). There is no reciprocal dreaming, b dreams about a and is dreamed by c. To have a power relation $\rhd$ sustaining z as a C equilibrium it has to be the case that:

$$a \rhd b \text{ and } b \rhd c$$

But then it must be the case that $a \rhd c$, which is absurd because a dreams about c's stick.

6 Jungle policies

In this section, we present how jungle institutions can implement civilized equilibria. We think of a jungle institution as a policy maker who can implement power relations and languages. Such an entity will be able, under certain circumstances, to impose an allocation as an equilibrium. The main concept is **civilized compatibility** (*C-compatibility*) of a language with respect to an allocation. We will prove that for every allocation which admits a C-compatible language a C equilibrium can be attained.

I will add an example and maybe some more economic description.

Let us take a little detour on the basics of order theory, which will help us precisely spot where we can ask for sufficient conditions to be matched for a completion to a preorder.
I will add a formal introduction to orders and some basic definitions.
We'll make use of the following definition.

Definition 6.1. *Let $\geq$ be a binary relation on a non-empty set X, $>$ be the asymmetric part of $\geq$ and $T(\geq)$. We say that $\geq$ satisfies* ***only weak cycles*** *(OWC) if:*

$$xT(\geq)y \Rightarrow \neg(y > x)$$

As its name suggests, the OWC condition is a weaker condition than acyclicity. Recall that for any binary relation R its transitive closure is denoted by $T(R)$. The transitive closure of R is the smallest binary relation that contains R (in the sense that xRy implies $xT(R)y$ for all x, y) is transitive and any transitive binary relation containing R also contains $T(R)$.
We'll need the following proposition.

Proposition 6.1. *Let R be a binary relation on a non-empty set X. Then R can be extended to a complete preorder if and only if it satisfies OWC.*

Proof. I will write it. Via Sziplrajn extension theorem and a good reference I found.[2] □

Having acquired the necessary mathematical tools, we now turn our attention to C compatibility and why it plays a crucial role in defining a C equilibrium. Formally, what do we mean by C compatibility of a language? Let z be an allocation and let us introduce a collection of pairs of agents:

$$\mathcal{R}_z = \Big\{\{a, b\} \,|\, aT(\trianglerighteq)b \text{ and } bT(\trianglerighteq)a\Big\}$$

Where $\vartriangleright$ is defined by:

$$a \vartriangleright b \text{ if } b \in D(z, a)$$

That is, $\mathcal{R}_z$ is the collection of all pairs of agents that are "equally powerful" under the transitive closure of $\vartriangleright$, where this binary relation is defined to satisfy the C equilibrium condition.

Definition 6.2. *Let z be an allocation. A language $\mathcal{L}$ is C-compatible if $\forall\ \{a, b\} \in \mathcal{R}_z$ then:*

$$\forall\ \lambda \in \Lambda\ \exists\ c_\lambda \in D(z, a)\ :\ c_\lambda >_\lambda b$$

C-compatibility is a strong hypothesis on the relation between an allocation and a language. We require the language to break the power relation between every equally powerful agent, under the C equilibrium condition. The policy value of a language comes out for C-compatible languages, as it decides who is going to prevail between potentially equally strong agents. A C-compatible language is a sufficient tool to sustain an allocation as a C equilibrium.

Theorem 6.2. *Let $\langle N, K, (X^i)_{i\in N}, w, (\geq^i)_{i\in N}\rangle$ be a tuple as above. Then, for every $(\mathcal{L}, z)$ C compatible allocation there exists a power relation $\trianglerighteq$ such that* ***z*** *is a C equilibrium for the corresponding civilized jungle.*

Proof. Let us consider the civilized jungle given by the tuple and the C-compatible language. Let us define $\rhd$ over a subset of N pairs as follows:

$$i \rhd j \text{ if } j \in J_{\mathcal{L}}(D(z,i))$$

If we prove $\rhd$ to satisfy only weak circles we can extend it to a complete preorder. Let us suppose by absurd that:

$$iT(\unrhd)j \text{ and } j \rhd i$$

By definition of the binary relation $i \in J_{\mathcal{L}}(D(z,j))$. Furthermore $jT(\unrhd)i$. Therefore $\{i,j\} \in \mathcal{R}_z$, which implies that i is not justifiable in $D(z,j)$. $\square$

In the proof we implicitly use the fact that the binary relation defined is the restriction to the justified dreamers of the one defined in definition 6.2. That is why the last implication holds.

The interpretation of this theorem is straightforward. We suppose that a jungle institution can distribute power and decide a language. If a jungle institution wants to attain a specific allocation as a civilized equilibrium it has just to check whether the allocation admits a C-compatible language. If so, the C-compatible language and the power relation derived from the C equilibrium definition itself will define the civilized jungle under which the allocation is a civilized equilibrium.

As a consequence, it would be interesting to study sufficient conditions on the existence of a C-compatible language for a given allocation. We could also investigate if the existence of a C-compatible language to an allocation implies some necessary conditions on the allocation itself. Those conditions would, ideally, help jungle institutions understand whether a certain allocation is actually supportable as an equilibrium or not.

7 Working monkeys

I now implement *production* in a civilized jungle with divisible commodities, from now on just a civilized jungle.
P-R propose that the endowment ω be replaced by a set Y which is a collection of endowments. Y stands for the production component of the jungle meaning that it shapes the agents' behavior by defining the goods' quantity. In this representation of the production side of the jungle, the endowments are defined a priori, as the power relation. My goal is to describe an endogenous production in a civilized jungle.
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