UNIVERSITÄT OSNABRÜCK

Institute of Cognitive Science
Intuit Data Engineering and Analytics

Bachelor's Thesis

Enhanced A/B Testing

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Enhanced A/B Testing

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Introduction

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Theory

2.1 Basic Idea

A test is set up with two buckets. One is generating actions with a probability $q1$ the other generates them with a probability $q2$. The prior distribution for all q is a Beta distribution, which is a convenient choice because it is a prior distribution for binomial proportions (the Beta distribution is the conjugate family for the binomial likelihood). We also assume no prior knowledge when creating the test case (uninformative prior $Beta(1,1)$). $N1$ customers get assigned to Bucket 1 and $N2$ to Bucket 2. They generate $k1$ and $k2$ actions. The posterior distributions for each individual case is:

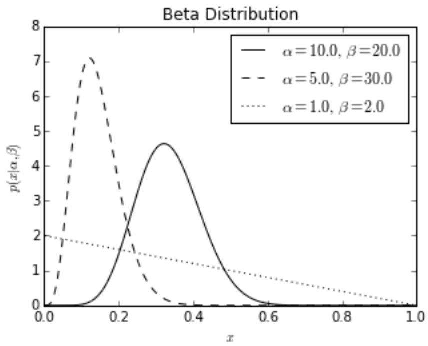
$$
\begin{aligned}
q &= f(k|N) \propto f(N|k)f(k) \\
q1 = Beta(k1+\alpha, N1-k2+\beta) &= \frac{x^{k1+\alpha-1}(1-x)^{N1-k2+\beta-1}}{B(k1+\alpha, N1-k2+\beta)} \\
q2 = Beta(k2+\alpha, N2-k2+\beta) &= \frac{x^{k2+\alpha-1}(1-x)^{N2-k2+\beta-1}}{B(k2+\alpha, N2-k2+\beta)}
\end{aligned}
$$

So each new assignment and each new event would directly alter the distributions parameters. *Questions: What is with returning users? Or users with a high action rate? Is every user a new one?* What is the posterior for $P(q1 > q2|k1, N1, k, 2, N2)$ - that Bucket 1 performs better than Bucket 2 given the collected data? There are in principal two different ways to get an answer.

Analytic

In general for two random variables X, Y with corresponding probability density function f_X, f_Y and cumulative density function F_X, F_Y it holds:

$$
\begin{aligned}
P(X \geq Y) &= \iint_{[x>y]} f_X(x) f_Y(y)\, dy\, dx \\
&= \iint_{-\infty}^{x} f_X(x) f_Y(y)\, dy\, dx \\
&= \int_{-\infty}^{\infty} f_X(x) F_Y(x)\, dx
\end{aligned}
$$



In the paper "Numerical Computation of Stochastic Inequality Probabilities" the author John D. Cook uses symmetries of the distribution to arrive at a set of equations that can be used to calculate the problem recursively.

$$g(k1, N1, k2, N2) = P(X > Y)$$
$$h(k1, N1, k2, N2) = \frac{B(k1 + k2, N1 + N2)}{B(k1, N1)B(k2, N2)}$$

From that one could calculate a base case for a small sample and then continue with:

$$g(k1 + 1, N1, k2, N2) = g(k1, N1, k2, N2) + h(k1, N1, k2, N2)/k1$$
$$g(k1, N1 + 1, k2, N2) = g(k1, N1, k2, N2) - h(k1, N1, k2, N2)/N1$$
$$g(k1, N1, k2 + 1, N2) = g(k1, N1, k2, N2) - h(k1, N1, k2, N2)/k2$$
$$g(k1, N1, k2, N2 + 1) = g(k1, N1, k2, N2) + h(k1, N1, k2, N2)/N2$$

Approximation

For *sufficient large* samples one could again approximate the Beta Distribution with a normal distribution. This means that the following equation should be fulfilled for $B(k, N)$:

$$\frac{k+1}{k-1} \approx 0, \frac{N+1}{N-1} \approx 0$$

The Gaussian Distribution to a given Beta Distribution has the following shape:

$$B(k, N) \approx N\left(\frac{k}{k+N}, \sqrt{\frac{kN}{(k+N)^2(k+N+1)}}\right)$$

The inequality for this case then could be solved by:

$$\begin{aligned} P(X > Y) &= P(0 > Y - X) \\ &= P(0 > \mu_Y - \mu_X + (\sigma_X^2 + \sigma_Y^2)^{\frac{1}{2}} Z) \\ &= P\left(Z < \frac{\mu_X - \mu_Y}{(\sigma_X^2 + \sigma_Y^2)^{\frac{1}{2}}}\right) \\ &= \Phi\left(\frac{\mu_X - \mu_Y}{(\sigma_X^2 + \sigma_Y^2)^{\frac{1}{2}}}\right) \end{aligned}$$

I am not sure if this is really a smart way to do it but what would be maybe interesting is finding out which of the possibilities is the fastest for growing N.

Sampling

One could also sample from the different distributions and take the mean of the of the samples to determine the best distribution. The mean of a Beta distribution $B(k, N)$ is given by:

$$E[X] = \frac{k}{k + N}$$

Open question here: how exactly is the sample size related to the accuracy of the estimate?

2.1.1 n Buckets

The generalization would again depend on the chosen method to evaluate the inequality.

Analytic

I found examples for 3 Beta distributed random variables. But basically any formula would calculate the probability that one variable is greater than the others which results in $n - 1$ equations to solve the problem for n buckets. The same number of equations would occur if we would compute the inequality pairwise with the already presented formula.

Sampling

Additional Buckets would not change the underlying computation but would result in additional draws and new means to compare.

2.1.2 best assignment

The best assignment for a not yet tracked user would be a bucket that minimizes the entropy over the different buckets. The entropy of a Beta Distribution is given by:

$$\begin{aligned} h(q) &= E[-ln(f(x; k, N))] \\ &= \int_0^1 -f(x; k, N) ln(f(x; k, N)) \, dx \\ &= ln(B(k, N)) - (k-1)\psi(k) - (N-1)\psi(N) + (k+N-2)\psi(k+N) \end{aligned}$$

Where ψ is the Digamma function. One could calculate the difference from $h(q_x)$ to $h(q'_x)$ where $x \in 1...n$ number of Buckets and q'_x represents the distribution with the newly assigned user and assign the user the bucket where $max(|h(q_x) - h(q'_x)|)$.

Chapter 3

Methods

This is the first chapter.

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3.1 Data generation

3.2 Data analysis

3.3 Data representation

Chapter 4

Analysis

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4.1 And a section

4.2 another one

Chapter 5

Discussion

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List of Figures

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Osnabrück, May 14, 2015