Policy-Conditioned Uncertainty Sets for Robust Markov Decision Processes

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Problem

- Compute a **robust policy** π for an MDP $\langle \mathcal{S}, \mathcal{A}, \tau, R \rangle$ whose transition probabilities $\tau(s_{t+1}|s_t, a_t)$ are *unknown*
- Only a *limited* number of trajectories generated from a **reference policy** $\widetilde{\pi}$ is available
- **Robust optimization** approach:
  - Define uncertainty sets Ξ based on samples such that, with high probability, $\tau \in \Xi$
  - Find the optimal policy against the worst-case dynamics in Ξ:

$$\max_{\pi \in \Pi} \min_{\tau \in \Xi} \rho(\pi, \tau) := \mathbb{E}_{\tau,\pi}\left[\sum_{t=1}^{T-1} R(S_t, A_t, S_{t+1})\right]$$

Motivation

- The majority of the RMDP literature considers **rectangular** uncertainty sets [Wiesemann et al., 2013]:

$$\Xi = \{\tau : \forall s, a \in \mathcal{S}\times\mathcal{A},\ \|\tau(\cdot|s,a) - p(\cdot \mid s,a)\| \leq c\}$$

- **Rectangular** RMDPs:
  - Polynomial-time optimization
  - Robust Bellman optimality equation
  - Very conservative solutions
- **Non-rectangular** RMDPs:
  - **NP-hard** optimization problem in general [e.g., Mannor et al., 2012]

Contributions

1. We propose **policy-conditioned uncertainty sets**:
   - **Non-rectangular** uncertainty sets via *marginal statistics* of the given trajectories
   - **Off-policy robustness**: the impact of the reference policy on the desired control policy is considered in the learning process
   - **Tractable** and **convex** optimization by shifting to parameterized control problems
2. We provide **empirical results** showing the benefits of our approach over rectangular RMDPs

Marginally-Constrained Robust Control Processes

Non-Rectangular Uncertainty Sets via Marginal Features

- We consider **features** $\phi(s_t, a_t, s_{t+1})$ to model the relationships between states and actions
- **Feature expectations** [Abbeel and Ng, 2004] to model the interaction of a policy π with the decision process

$$\boldsymbol{\kappa}_{\phi}(\pi, \tau) = \mathbb{E}_{\tau,\pi}\left[\sum_{t=1}^{T-1} \phi(S_t, A_t, S_{t+1})\right] \quad \simeq \quad \widehat{\boldsymbol{\kappa}}_{\phi} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T-1} \phi\left(s_t^{(i)}, a_t^{(i)}, s_{t+1}^{(i)}\right)$$

- Use feature expectations to define the **uncertainty sets**:

Slack-free: $\Xi_{\widetilde{\pi}}^{\phi} = \left\{\tau : \boldsymbol{\kappa}_{\phi}(\widetilde{\pi}, \tau) = \widehat{\boldsymbol{\kappa}}_{\phi}\right\}$

Slack-based: $\widetilde{\Xi}_{\widetilde{\pi}}^{\phi} = \left\{\tau : \|\boldsymbol{\kappa}_{\phi}(\widetilde{\pi}, \tau) - \widehat{\boldsymbol{\kappa}}_{\phi}\| \leq \epsilon\right\}$

Properties

- Non-rectangularity
- Constrain whole trajectories
- **Generalization** across the state space
- Dependency on the reference policy $\widetilde{\pi}$

Marginally-Constrained Robust MDP

Constrained problem → **Unconstrained problem**

$$\max_{\pi} \min_{\tau \in \Xi_{\widetilde{\pi}}^{\phi}} \left\{\rho(\pi, \tau) - \lambda^{-1} H(\tau)\right\} \quad \to \quad \max_{\boldsymbol{\omega}} \left\{\max_{\pi} \operatorname{softmin}_{\tau} \left(\rho(\pi, \tau) + \boldsymbol{\omega}\cdot\boldsymbol{\kappa}_{\phi}(\widetilde{\pi}, \tau)\right) - \boldsymbol{\omega}\cdot\widehat{\boldsymbol{\kappa}}_{\phi}\right\}$$

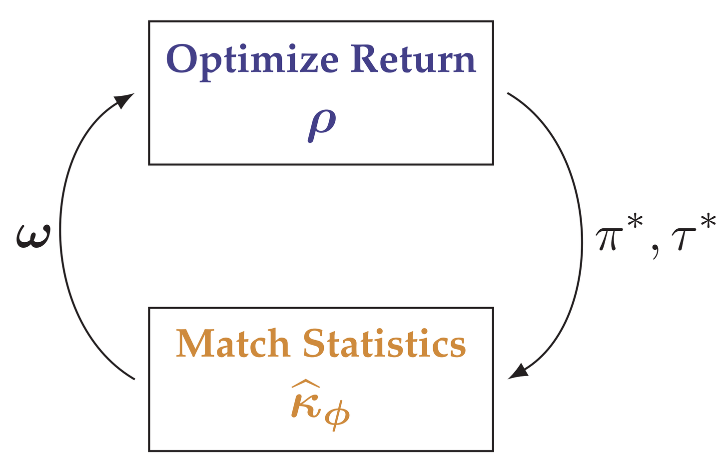
Alternated Optimization

1. **Optimize return ρ**. Find the equilibrium (π^*, τ^*) of the inner zero-sum game using *min-max dynamic programming*:

$$(\pi^*, \tau^*) \leftarrow \max_{\pi} \operatorname{softmin}_{\tau} \left\{\rho(\pi, \tau) + \boldsymbol{\omega}\cdot\boldsymbol{\kappa}_{\phi}(\widetilde{\pi}, \tau)\right\}$$

2. **Match Statistics $\widehat{\boldsymbol{\kappa}}_{\phi}$**. Update parameters $\boldsymbol{\omega}$ so that τ^* matches the sample statistics with respect to the reference policy $\widetilde{\pi}$:

$$\boldsymbol{\omega} \leftarrow \boldsymbol{\omega} + \eta\left(\boldsymbol{\kappa}_{\phi}(\widetilde{\pi}, \tau^*) - \widehat{\boldsymbol{\kappa}}_{\phi}\right)$$



Mixed-Objective Minimax Optimal Control

- **Issue**. Solving the zero-sum game at step 1 requires finding dynamics τ that minimize the sum of two expected returns under different policies
  - **NP-hard** problem [Petrik et al., 2016] → *Non-Markovian* solution
- **Main result**. A Markovian solution exists when augmenting the state space with a continuous **belief state** which keeps track of the relative importance of the two policies:

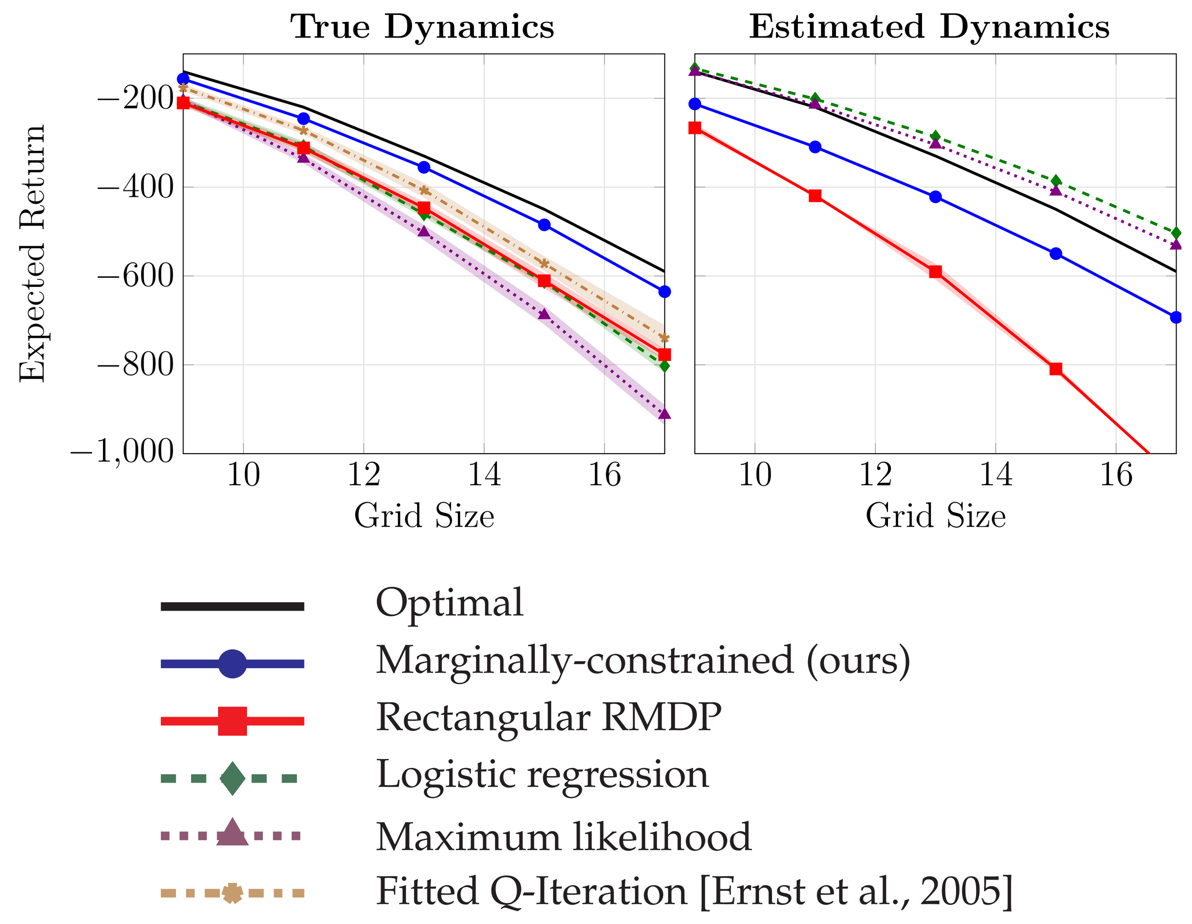
$$b_t = \frac{\prod_{i=1}^{t} \pi(a_i|h_i)}{\prod_{i=1}^{t} \pi(a_i|h_i) + \prod_{i=1}^{t} \widetilde{\pi}(a_i|s_i)} \quad \to \quad b_{t+1} = \frac{b_t \pi(a_{t+1}|h_{t+1})}{b_t \pi(a_{t+1}|h_{t+1}) + (1-b_t)\widetilde{\pi}(a_{t+1}|s_{t+1})}$$

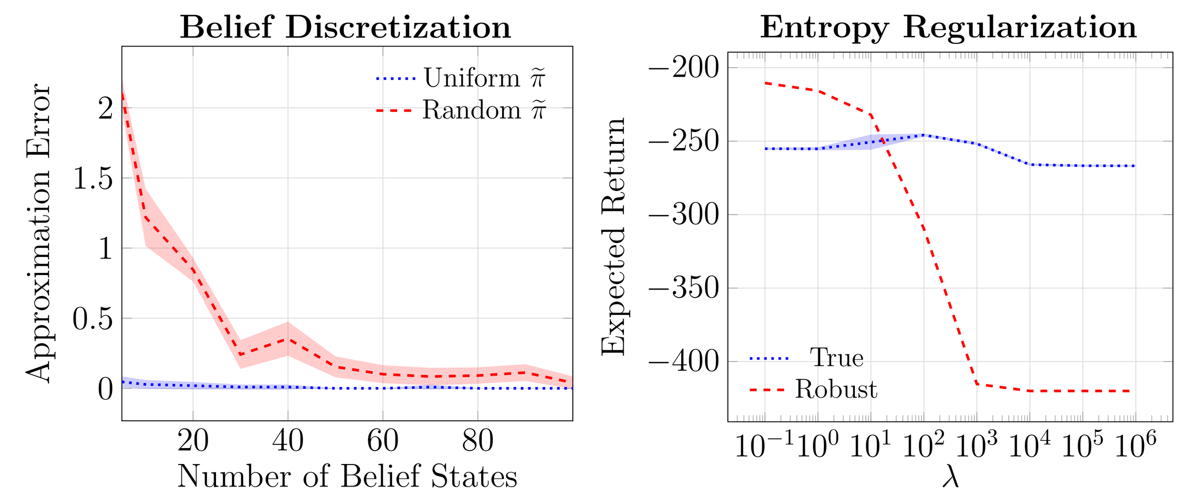
- The equilibrium can be found by solving a **min-max dynamic program** using discretized belief states:

$$\tau^*(s_{t+1}|s_t, a_t, b_t) = \frac{e^{-\lambda Q(s_t, a_t, b_t, s_{t+1})}}{\sum_{s'_{t+1}} e^{-\lambda Q(s_t, a_t, b_t, s'_{t+1})}} \qquad \pi^*(s_t, b_{t-1}) = \arg\max_{a_t} Q_R(s_t, a_t, b_t)$$

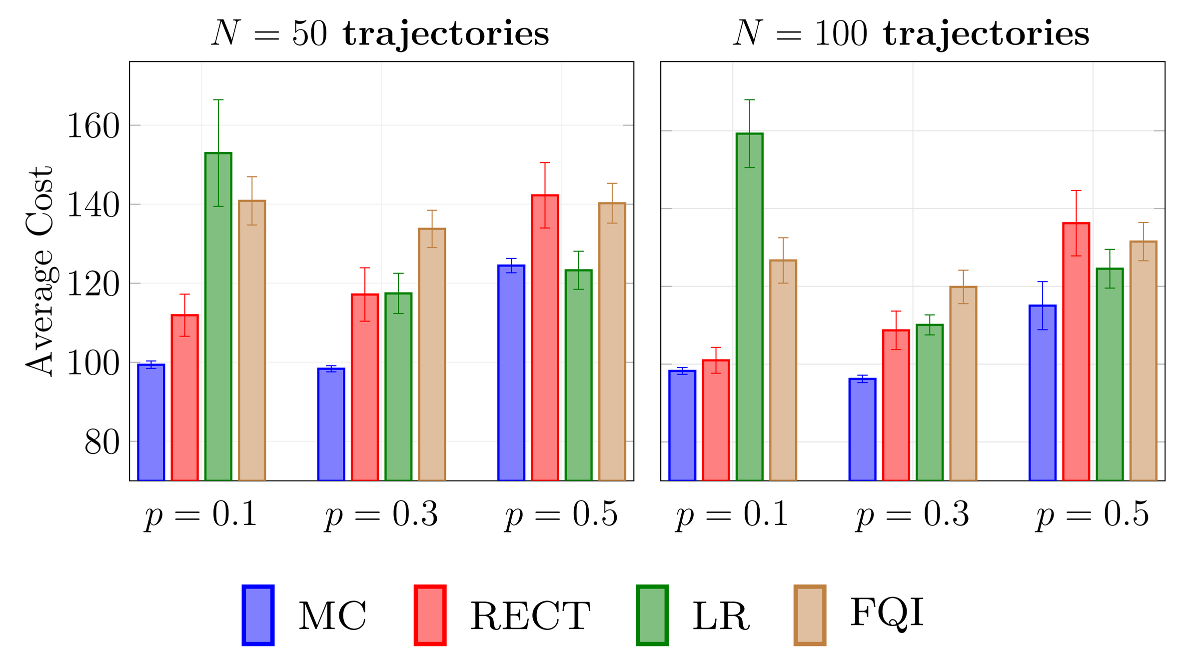
Results

Grid World





Invasive Species



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