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ICT FOR CONTROL SYSTEMS ENGINEERING

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MODEL IDENTIFICATION:

Identification of Transfer Function Models plus Time-delay of a quadrotor helicopter pitch dynamics

PROJECT GOAL

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- Black-box ***continuous-time*** model identification in a bootstrap manner to estimate simultaneously the ***time-delay*** and ***transfer function parameters*** of a quadrotor helicopter pitch dynamics

DATA

- Set of data collected during an experiment on the plant
- Identified model with *tdsrivc* algorithm

TOOLS

- Matlab
- Github: https://github.com/AndreaVRZ/ICTproject

CONTINUOUS-TIME MODEL IDENTIFICATION

A linear time-invariant continuous-time system with input u and output y can always be described by

$$y(t) = G(\rho) \cdot u(t) + \xi(t)$$

where G is the transfer function, ρ the time-domain differential operator and the additive term $\xi(t)$ represents errors and disturbances of all natures.

The ***identification problem*** can be stated as follows: determine a CT model for the original CT system from *N* sampled measurements of the input and output $Z^N = \{u(t_k); y(t_k)\}_{k=1}^{N}$.

There are two fundamentally different time-domain approaches to the problem of obtaining a black-box CT model of a natural CT system from its sampled input-output data: the ***indirect*** and ***direct*** approach. We focus on the second one, in which a CT model is obtained immediately using CT model identification methods. In this direct approach, the model remains in its original CT form.

What are the ***advantages*** of using CT model identification?

There are many advantages in describing a physical system using CT models and also in identifying the CT models directly from sampled data, for example:

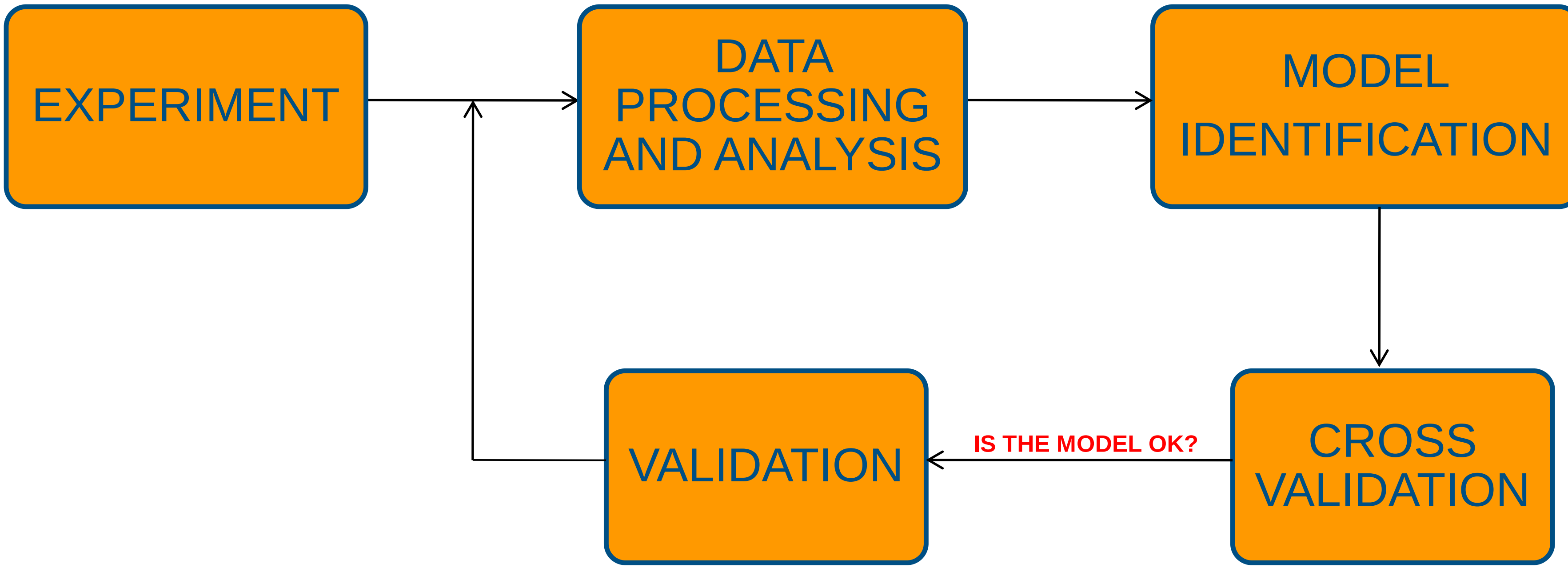
- the estimated parameters are directly linked to the CT model, while the parameters of DT models are a function of the sampling interval and do not normally have any direct ***physical interpretation***.
- the *a* ***priori knowledge of relative degree*** (the difference between the orders of the denominator and numerator) is easy to accommodate in CT models and, therefore, allows for the identification of more parsimonious models than in discrete time.
- with ***non-uniformly sampled data*** systems, the standard DT linear, time-invariant models will not be applicable. On the other hand, the coefficients of CT models are considered to be independent of the sampling period and so they have a built-in capability to cope with the non-uniformly sampled data situation.

- in CT identification having the ***sampling frequency too high*** with respect to the dominant frequencies of the system is not a problem, while in in DT poles lie too close to the unit circle and the parameter estimation can become ill-defined.
- a ***stiff system*** can be better captured by a CT model estimated from sampled data and the coefficients of this model are independent of the sampling rate. Recall that stiff systems are systems with eigenvalues that are of a different order of magnitude, *i.e.*, the system contains both slow and fast dynamics. Since a DT model is related to a single sampling rate, it is often difficult in such situations to select a sampling rate that captures the complete dynamics of the system without any compromise.

REMARK: unlike the situation in DT identification, where the time-delay is assumed to be an integral number of sampling intervals and is often absorbed into the definition of the numerator polynomial, the time-delay parameter for CT system model is normally associated directly with the input signal and can have a non integral value.

IDENTIFICATION CYCLE

IDENTIFICATION CYCLE



EXPERIMENT

The experiment is done in the following manner:

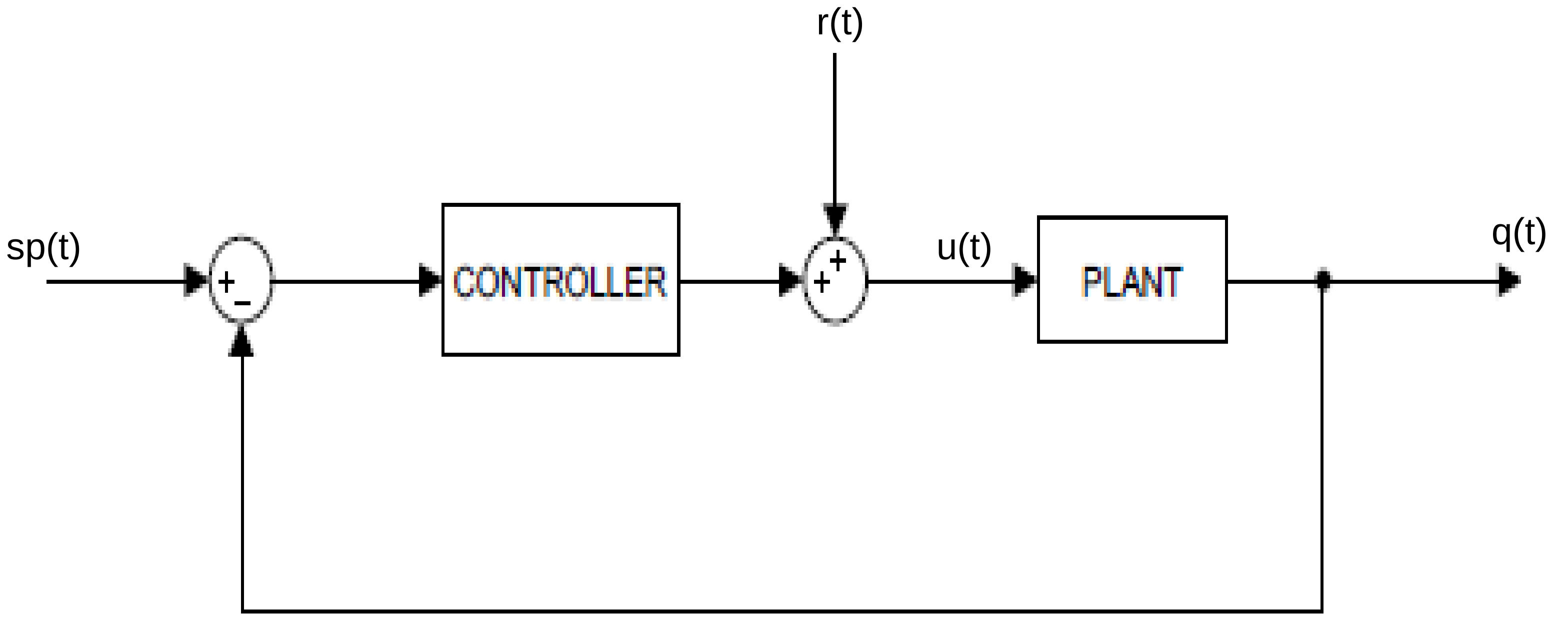
- three PRBS excitation sequences (30s each) in quasi open-loop (pitch attitude controller OFF, supervisor task ON). Pitch control variable amplitude is limited between +/-11% for safety reason. Min-max switching interval are respectively 0.05s and 0.1s.
- final evaluation phase: pitch attitude controller ON and angular set-points variations are imposed.
- at the beginning of the test and between PRBS sequences there are short phases with attitude controller ON (angular set-point=0).

All data are acquired with sampling time:

$$Ts = 0.02s \Rightarrow fs = 50Hz$$

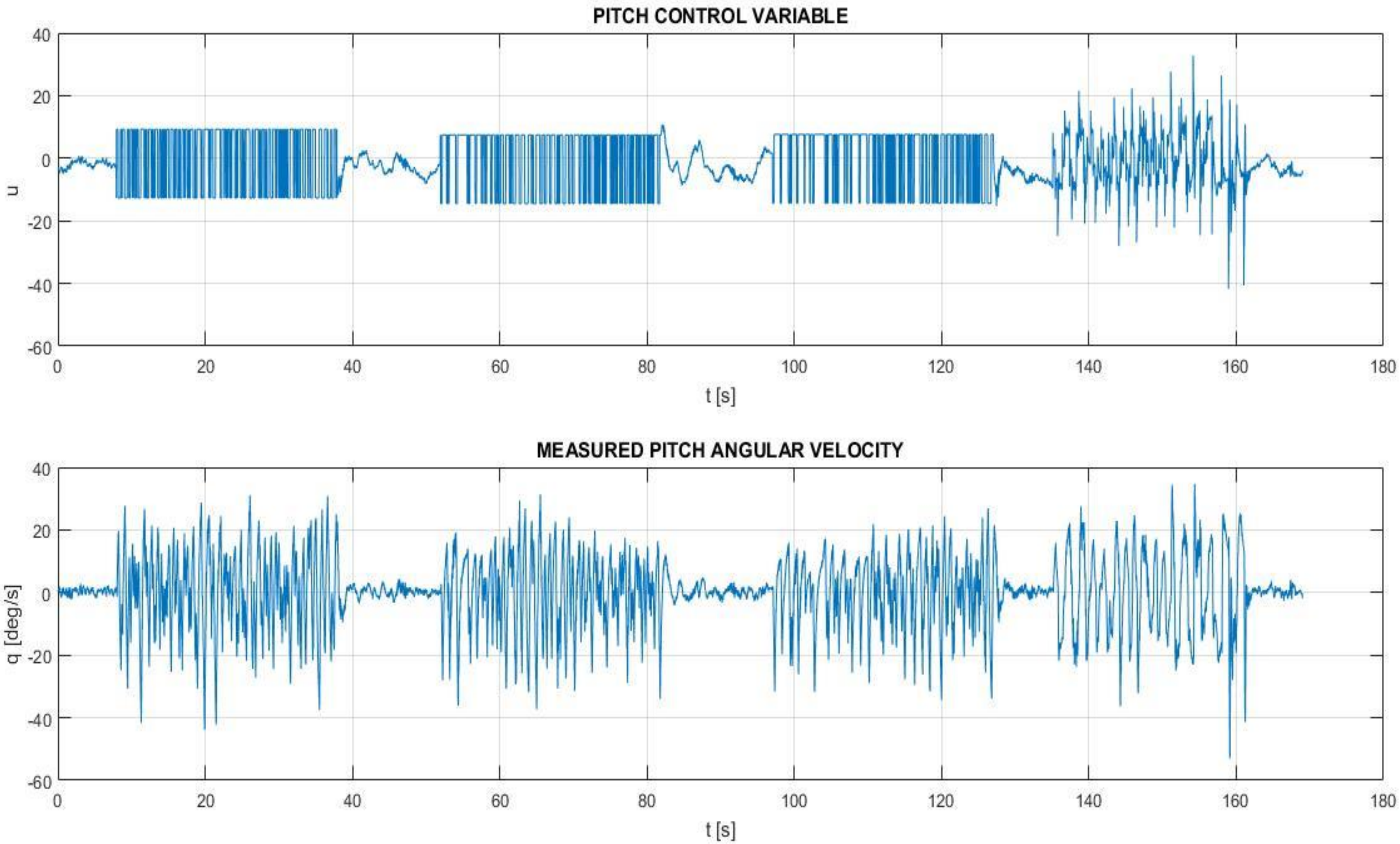
so it is possible to see dynamics up to $25Hz \equiv 157\frac{rad}{s}$.

BLOCK DIAGRAM



COLLECTED DATA

N.B. **u** is expressed as the % difference between front and back rotor command



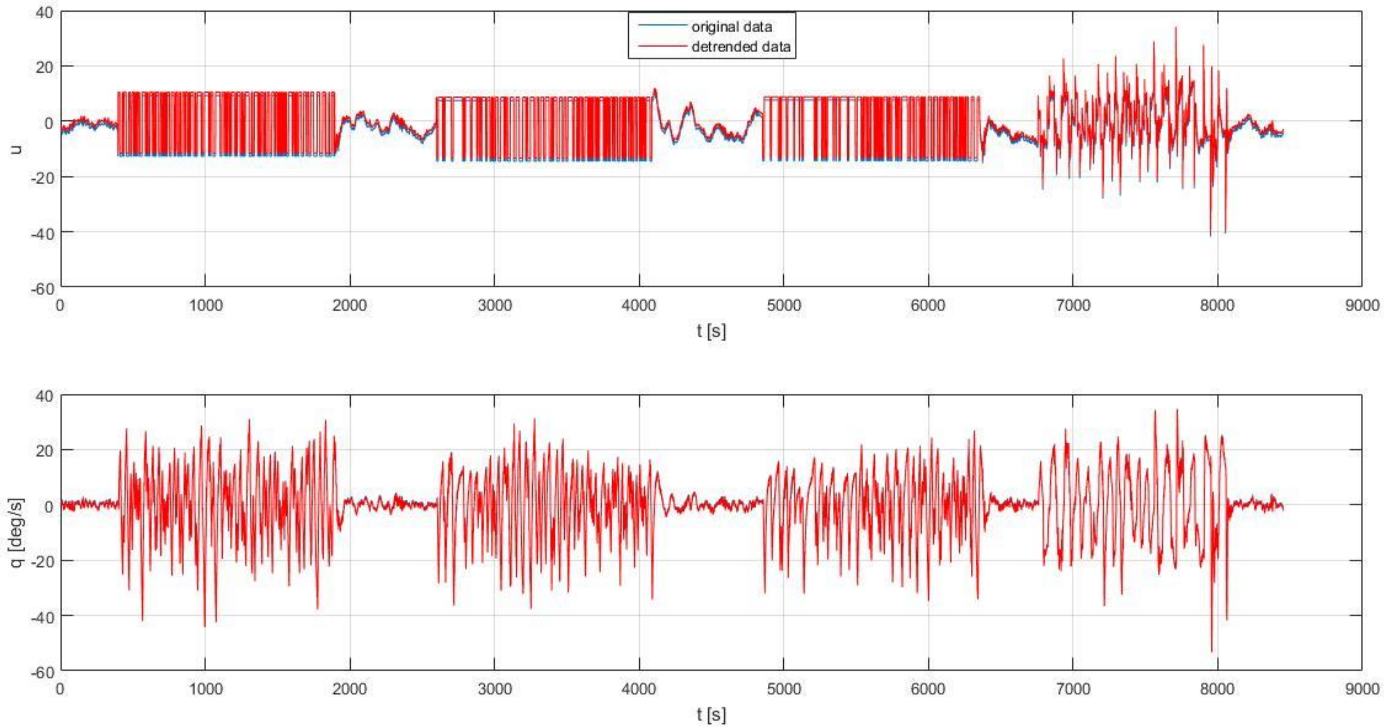
DATA PROCESSING AND ANALYSIS

In black-box modeling some pre-processing operations are needed to obtain satisfying results.

For example:

- **TREND REMOVING**

  Trough the Matlab function *detrend* the mean value (zero order trend) is removed in order to focus on the system dynamics.



- **LEVEL OF INPUT EXCITATION**

  Since the data set is composed by both open-loop and closed-loop data it is not possible to guarantee always the persistent excitation of the system. As consequence it is not advisable to perform any whiteness analysis and the goodness of the model will be considered in the validation.

- **DATA SPLITTING**

  It is important to not use the same data to perform the identification and the validation. This way the validation process is based on an objective criterion to evaluate the quality of the identified model.

  Due to the high number of samples the data set is equally splitted in three *iddata* objects:

  id=data_detrend(1:2817) ⇒ identification data

  cv=data_detrend(2818:5634) ⇒ cross-validation data

  v=data_detrend(5635:8451) ⇒ validation data

MODEL IDENTIFICATION

The full identification process goes through three steps:

- ***Model identification*** based on identification data set.
- ***Cross-validation*** to choose the best model, that has the lowest value of the performance index based on cross-validation data set.
- Final ***validation*** to evaluate and characterize the model got in previous steps.

Our identification problem refers to identification of both transfer function parameters and time-delay in a continuous-time form.

So we rely on the *tdsrivc* algorithm, provided by *CONTSID* Matlab toolbox: based on *srivc* (Simple Refined Instrumental Variable for Continuous-Time OE Model) it computes parameters of continuous-time output-error models with time-delay.

It works under the assumption of white additive noise.

The algorithm is based on a PEM approach with the prediction error as a function of θ (parameters vector) and τ (time-delay):

$$\epsilon(\theta,\tau) = y - G(\rho,\theta) \cdot u(\tau)$$

where ρ denotes the differentiation operator.

A more detailed explanation can be found here.

Starting from the following a priori knowledge about the system singularities:

- number of poles, n_P: 1 – 5
- number of zeros, n_Z: 0 – 4

a model is identified for all the combination of n_P and n_Z, such that $n_P > n_Z$ because a physical system has a strictly proper transfer function.

For example:

M01=tdsrivc(id,[1 1])

it is the identified model using the identification data (id) that has no zeros and one pole. Note that in the square brackets are indicated respectively the number of parameters to be estimated for the numerator (n_b) and the number of parameters to be estimated for the denominator (n_f).

CROSS VALIDATION

Once models with different order have been identified, they are used to obtain the simulated output of the system $\hat{y}$ through the *CONTSID* Matlab function *simc*, that needs as input arguments the identified model and the cross-validation data (cv).

Then it is possible to compute the ***prediction error***

$$\epsilon(t) = y(t) - \hat{y}(t)$$

Finally the goal is to find the best identified model that is the one that minimizes the ***performance index***

$$J = \sum_t (\epsilon(t))^2 = \sum_t (y(t) - \hat{y}(t))^2$$

The results are summarized in the table below:

MODEL	PERFORMANCE INDEX J
M14	68486,31752
M34	98820,99315
M05	107002,3321
M04	108316,0255
M23	109265,1101
M02	115401,7536
M24	118373,2626
M25	121434,0435
M01	125367,2828
M13	130515,7119
M15	142868,3063
M12	147324,3621
M45	147873,0723
M03	152924,7823
M35	176808,8231

VALIDATION

The best model is the model ***M14***, whose continuous-time identified transfer function is:

$$G(s) = e^{-0.0623s} \cdot \frac{1.539 \cdot 10^4 s + 9201}{s^4 + 28.43s^3 + 1818s^2 + 6934s + 2.497 \cdot 10^4}$$

that has ***four*** poles and ***one*** zero, with $\boldsymbol{\tau = 0.0623s}$.

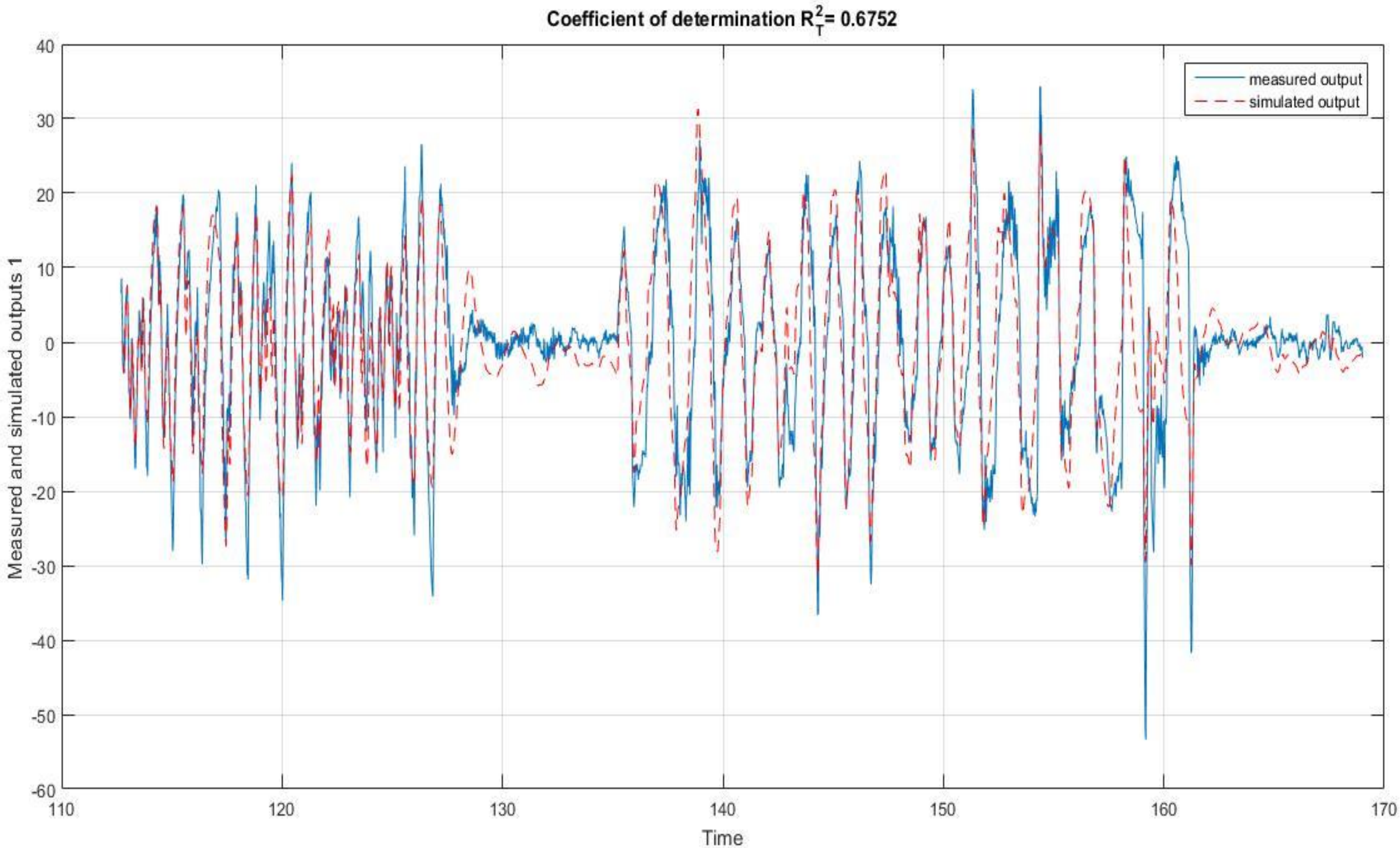
The final step in the identification process is to validate the model through the *CONTSID* Matlab function *comparec*, that compares the simulated output of a continuous-time model with the measured one. It needs as input arguments the validation data (v) and the best identified model.

The function gives us the ***coefficient of determination*** calculated as

$$R_T^2 = 1 - \frac{\sigma(y - \hat{y})}{\sigma(y)}$$

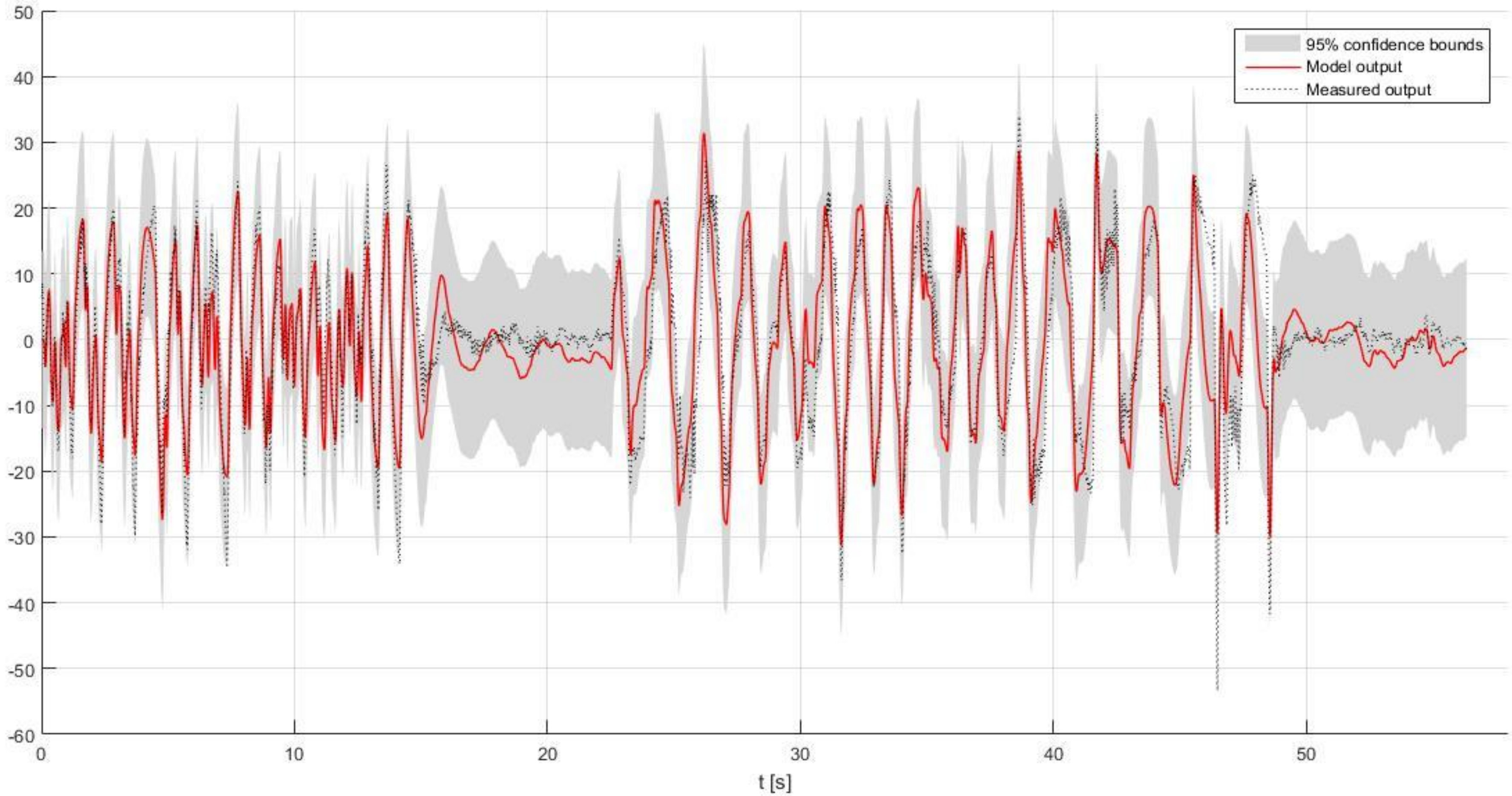
The closer to 1 the coefficient, the better the identified model.

VALIDATION



FPE=21.71 MSE=21.60

It is also possible to evaluate the goodness of the best identified model using the *CONTSID* Matlab function *shadedplot* that adds to previous data the 95% confidence bounds.



ADDITIONAL IDENTIFICATION

As additional attempt the identification process is repeated using only ***open-loop data*** to see whether there was any improvement with respect to the previous case.

The ***same*** procedure is adopted:

- ***data trend removing***
- ***data splitting***

  id=data_detrend(401:1900) ⇒ *identification data*

  cv=data_detrend(2601:4100) ⇒ *cross-validation data*

  v=data_detrend(4851:6350) ⇒ *validation data*
- ***level of input excitation***: in this case it is meaningful doing this analysis since only open-loop data are considered. The Matlab function *pexcit* gives us 50, that is the degree of persistent excitation. The intuitive interpretation of the degree of excitation in an input is the order of a model that the input is capable of estimating in an unambiguous way.
- ***model identification*** using the *CONTSID* Matlab function *tdsrivc*.
- ***cross-validation***: the results are in the next slide.

ADDITIONAL IDENTIFICATION

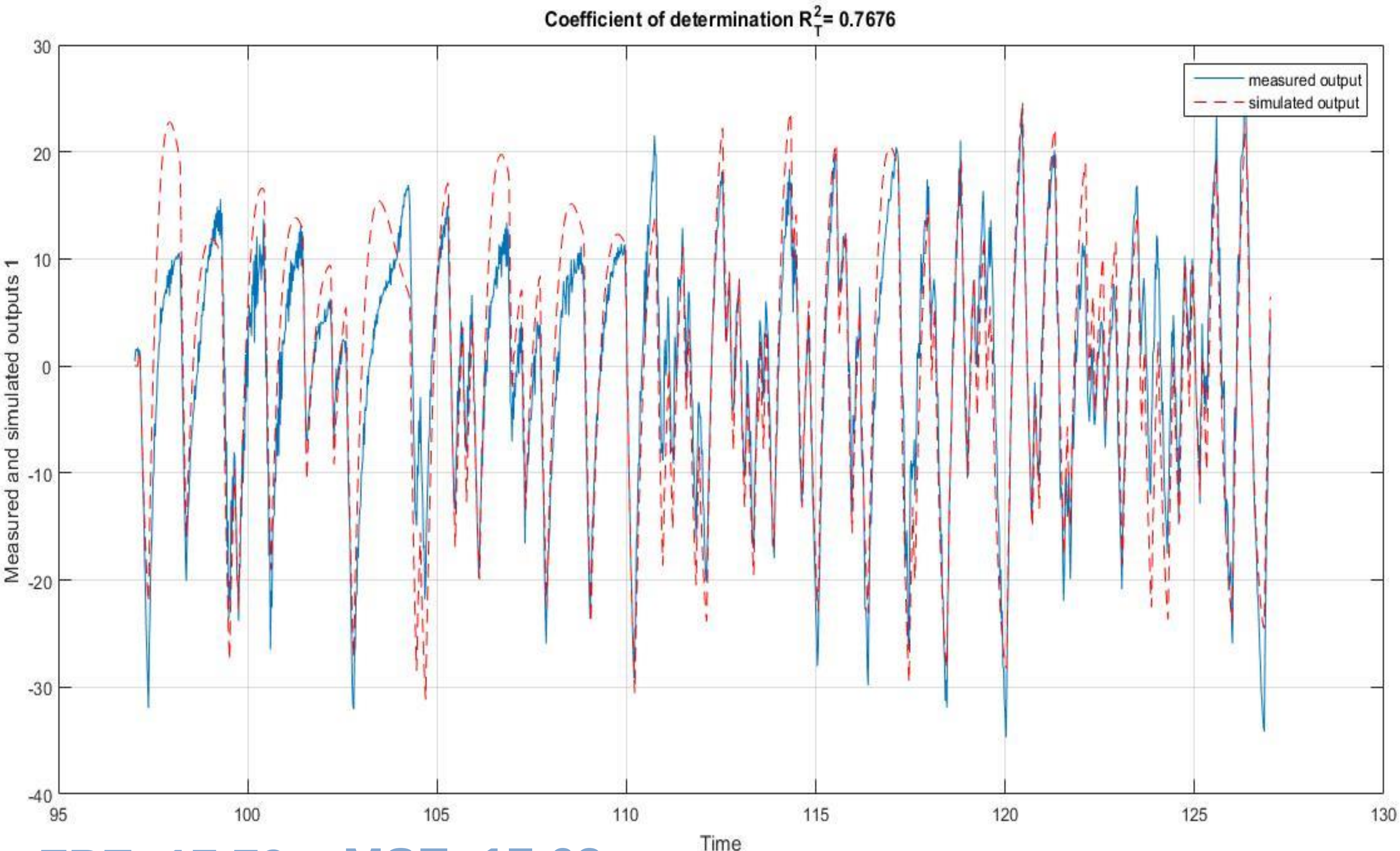
MODEL	PERFORMANCE INDEX J
M14	33197,97264
M34	33515,15025
M15	33550,50839
M25	44185,20299
M23	50057,54879
M24	52618,81977
M04	53703,25829
M45	55340,70165
M02	55967,26254
M13	56314,92714
M01	56867,03833
M05	60415,18003
M35	61102,73758
M12	62348,4976
M03	63493,62889

- ***validation***: the best model is the model ***M14***, whose continuous-time identified transfer function is:

$$G(s) = e^{-0.0715s} \cdot \frac{1.614 \cdot 10^5 s + 5.064 \cdot 10^4}{s^4 + 69.56s^3 + 1.369 \cdot 10^4 s^2 + 5.453 \cdot 10^4 s + 1.047 \cdot 10^5}$$

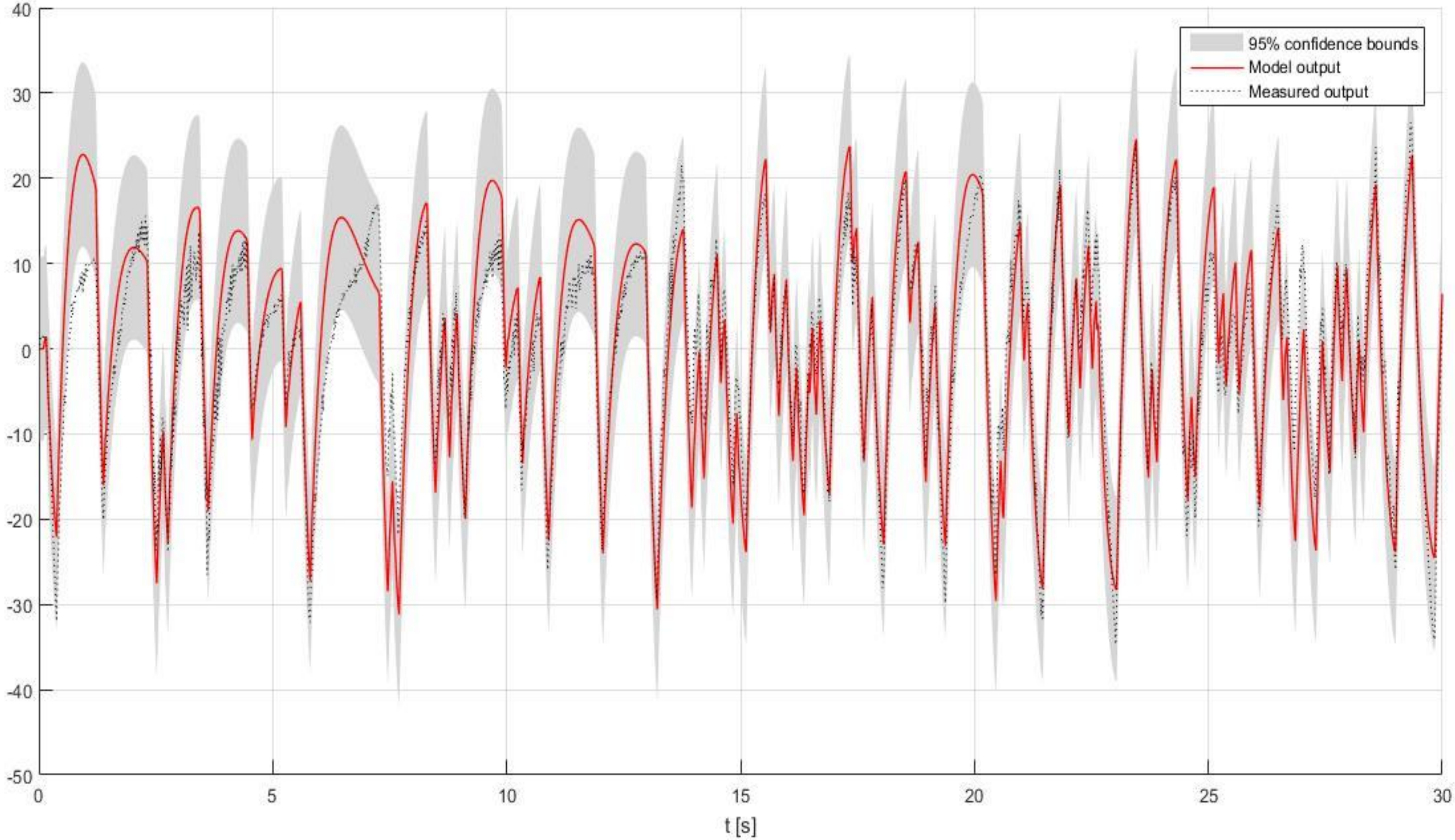
that has ***four*** poles and ***one*** zero, with $\boldsymbol{\tau = 0.0715s}$.

The goodness of the model is explained in the following slides.



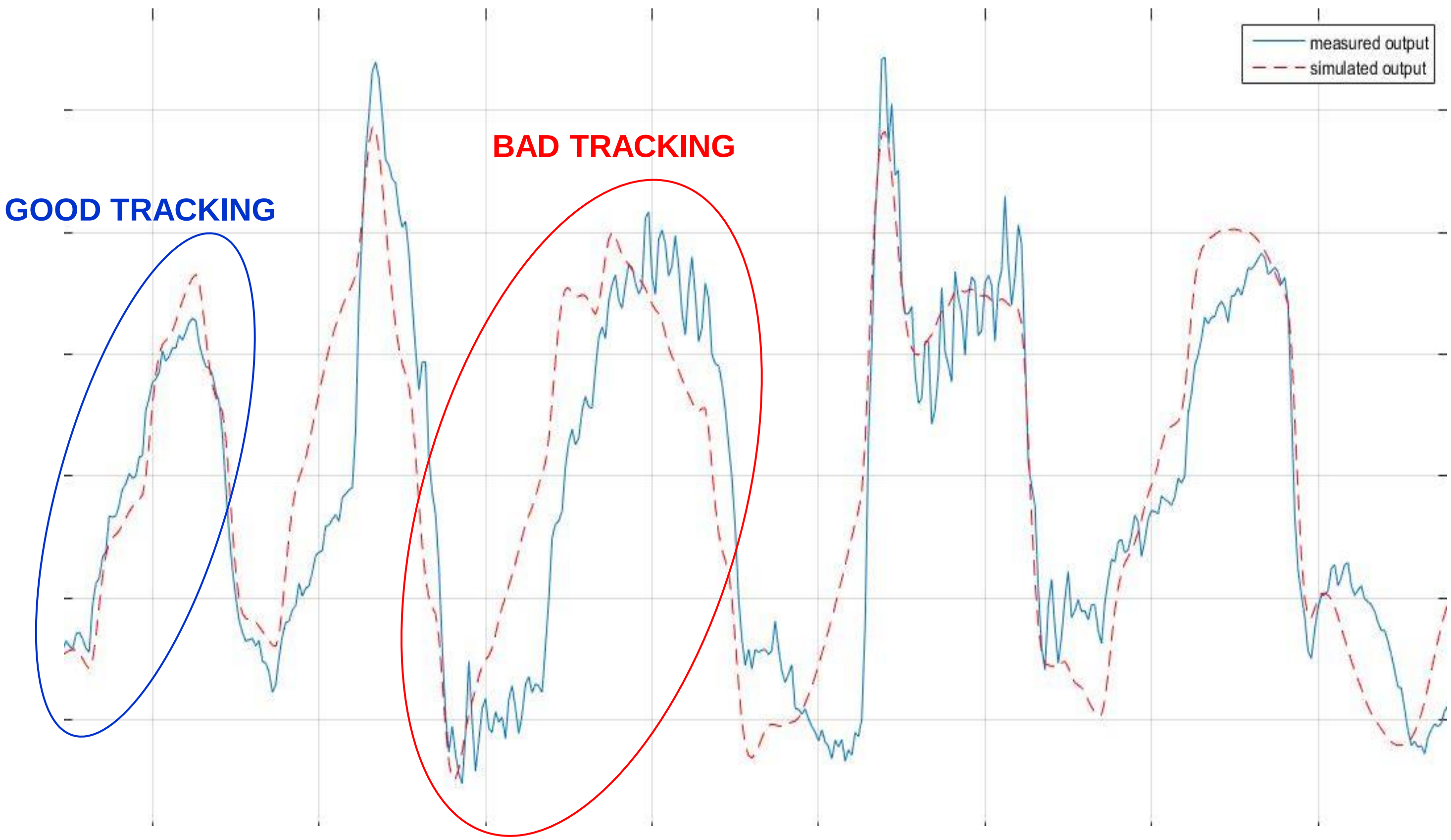
FPE=17.79 MSE=17.63

ADDITIONAL IDENTIFICATION



CONCLUSION

In both approaches the identified models are 4th order system with time-delay of the same order of magnitude.
The models give quite good results in terms of measured output tracking if variations are not too fast as can be seen in the plot below.
This behavior is showed also in the *confidence bounds* graphs.



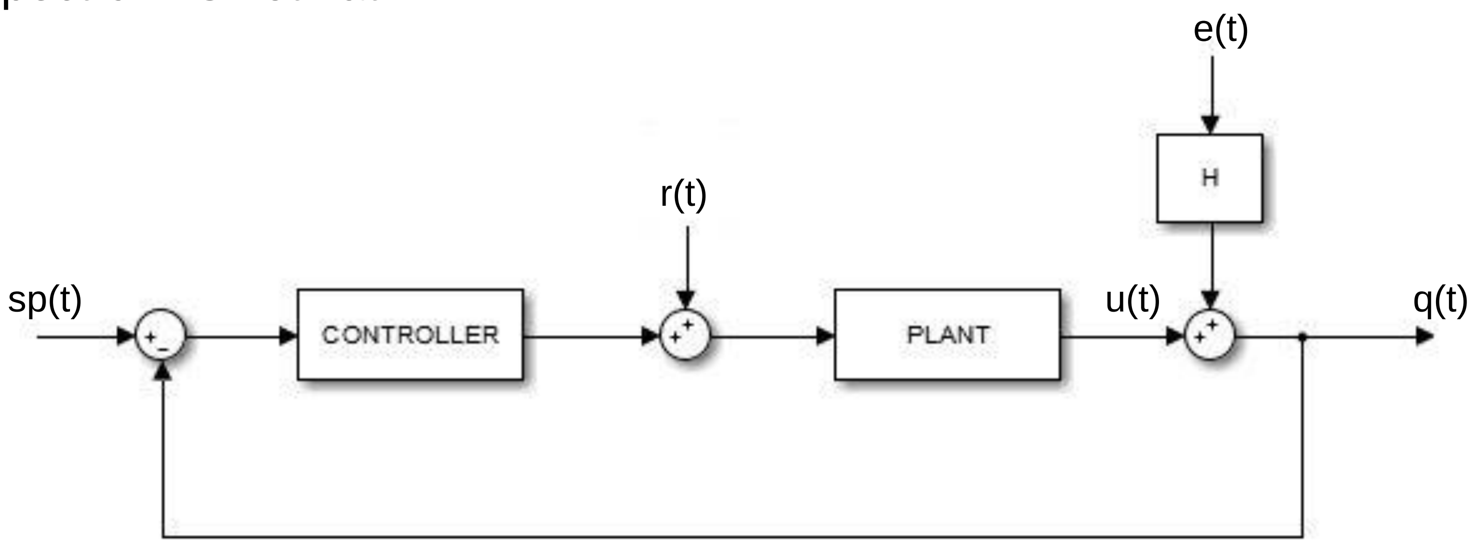
CONCLUSION

The second approach leads to better data fitting with lower FPE and MSE due to the absence of the closed-loop data that have more sudden variations.

The final results are summarized in the following table:

	1st APPROACH	2st APPROACH
FITTING	55.91	68.97
FPE	21.71	17.79
MSE	21.60	17.63
τ	0.0623s	0.0715s

Since the *tdsrivc* algorithm works under the assumption of ***white additive noise***, the obtained results could not be optimal because this constraint could not be suitable for the system being discussed, namely the noise is ***colored*** ($|H(j\omega)| \neq 1$). With colored noise the spectrum is not flat.



Possible solutions can be to estimate the additive noise transfer function and then adding a whitening filter or to do the identification with a Box-Jenkins model.