Homework #6

Part I. Analytical work [100 pts]:

- [20pts] Starting from the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-d)S\frac{\partial V}{\partial S} - rV = 0$$

  Make the following change of variable: $x = \ln\frac{S}{S_0}$, $\tau = T - t$, and $\mu = r - d - \frac{1}{2}\sigma^2$. Derive the resulting PDE in terms of these new variables. **Show the intermediate steps** in your work!

- [80pts] Using Crank-Nicolson finite difference approximation (google for Crank-Nicolson if you don't know what it is)
  i. Derive the resulting linear equation systems of $V_{n,j}$ defined on the (x_n, τ_j) grid points. **Show the intermediate steps** in your work clearly!

  ii. Express the above linear equation system in matrix and vector form

Scan your derivation and submit your work online

Happy deriving!

Part II. Programming work [100pts]:

- [60pts] Design and write a python class to price European/American options using the Crank-Nicolson approximation. It should also produce (1) delta, (2) gamma, and (3) theta Greeks for the same option.
- [40pts] Use your program to price the following options, plot the intermediate option values (as shown in lecture):
    i. European put option with the following parameters:
    Spot= 110, Strike=100, vol=30%, Maturity=1yr, r=0.05%, d=0%
        - Calculate the option premium
        - Calculate Delta, Gamma, and Theta
        - Check the above results against the analytical solution you implemented before
    ii. European call option with the following parameters:
    Spot= 90, Strike=100, vol=30%, Maturity=1yr, r=0.01%, d=0.15%
        - Calculate the option premium
        - Calculate Delta, Gamma, and Theta
        - Check the above results against the analytical solution you implemented before

    Once you are confident your program is working correctly, price the following options:

    iii. American put option with the following parameters:
    Spot= 110, Strike=100, vol=30%, Maturity=1yr, r=0.05%, d=0%
        - Calculate the option premium
        - Calculate Delta, Gamma, and Theta
    iv. American call option with the following parameters:
    Spot= 90, Strike=100, vol=30%, Maturity=1yr, r=0.01%, d=0.15%
        - Calculate the option premium
        - Calculate Delta, Gamma, and Theta

Submit your source program as well as the results online

Happy coding!