Financial Econometrics

HW4: Estimation of Continuous-time Models in Finance

Due Feb 25, 2019

Important Note: *Please note that the homework projects account for 50% toward your final grade, and needs to be handed in before the deadline. If you need more time to complete it, you can hand in what you have by the due date and "append" your homework within one week of the deadline. However, the highest credit for this late part is 70% of the total credit allocated to it. Be prepared to discuss your results in class.*

Important Note: *The Matlab script I handed out follows AitSahalia's paper (posted on Blackboard), and not my lecture notes. If you want to implement in Matlab the procedures in my lecture notes, please use the probabilist version of Hermite basse.*

This project has two goals:

(1) Implement the transition density $f_X(u,v) \equiv f(X_t = u|X_s = v)$ for the diffusion process

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t.$$

(2) Implement the MLE estimation for the above diffusion by forming the likelihood function using the above expression for the transition density $f(u,v)$. See the Lecture notes for details of how to do this.

Part I. Transition Density for Diffusions.

- Write a Matlab program that computes the transition density $f_X(u,v) \equiv f(X_t = u|X_s = v)$ as a function of the $\mu(x)$ and $\sigma(x)$. The goal is that if you are given any specific $\mu(x)$ and $\sigma(x)$, the program computes the $f_X(u,v)$ accurately. You can refer to my part of my Matlab script handout in class for this purpose –but you need to write you own program.

- Check the correctness of your above program by comparing it with (draw graphs) the diffusions with known transition densities. For example, the Black-Scholes Model, or Vasicek Model, etc. Please also check how many terms you need to keep for β_k and the Hermite expansion by trying different K and J.

- Bonus Question: One important application of the above transition density is for derivatives pricing. Can you use the above transition density to price a Call option through numerical integration and compare it with the value of the Black-Scholes formula ? You should get the same result.

Part II. MLE for SDE and Empirical Applications

- Take the transition density you obtained in Part I and turn it into a numeric function. (Just copy paste the final expression into Notepad. And then Restart MATLAB. You can then define a numeric function $f_X(u, v)$ using the saved function). Don't throw a symbolic function to the optimizer since it will slow the machine too much! Better yet, write a program that can convert a symbolic function into a numeric one [Not required].

- Use the transition density to form the likelihood function and implement the MLE for SDE (you can recycle part of the code in HW1). Make sure your program can output parameter estimation, standard errors, and p–value.

- Use the newly build program to empirically study US federal fund rates dynamics.

  – Download the (daily, OR weekly) Federal Fund Rate data from the Federal Reserve at: `https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15` you can read the data using EXCEL and then load it to MatLab.

  – The models (pick one!) you are want to fit are:

    * The Vasicek Model:

$$dX_t = \beta(\alpha - X_t)dt + \sigma dW_t$$

      where

$$\begin{aligned} \mu(x, \theta) &= \beta(\alpha - x) \\ \sigma(x, \theta) &= \sigma. \end{aligned}$$

    * The Cox-Ingersoll-Ross Model: The model specifies that the short-term interest follows:

$$dX_t = \beta(\alpha - X_t)dt + \sigma\sqrt{X_t}dW_t$$

with

$$\begin{aligned}
\mu(x,\theta) &= \beta(\alpha - x) \\
\sigma(x,\theta) &= \sigma\sqrt{x}.
\end{aligned}$$

NOTE: If you want to examine the robustness of the parameter estimation, you can divide the above period into subpriods and fit the model to each period. Do you see changes in the parameter estimations? For this empirical study, use your creativity and imagination.

Bonus Question: if you have more energy, you can fit the CEV model:

$$dX_t = \beta(\alpha - X_t)dt + \sigma X_t^{\gamma} dW_t$$

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$$\begin{aligned}
\mu(x,\theta) &= \beta(\alpha - x) \\
\sigma(x,\theta) &= \sigma x^{\gamma}.
\end{aligned}$$