**HW5**
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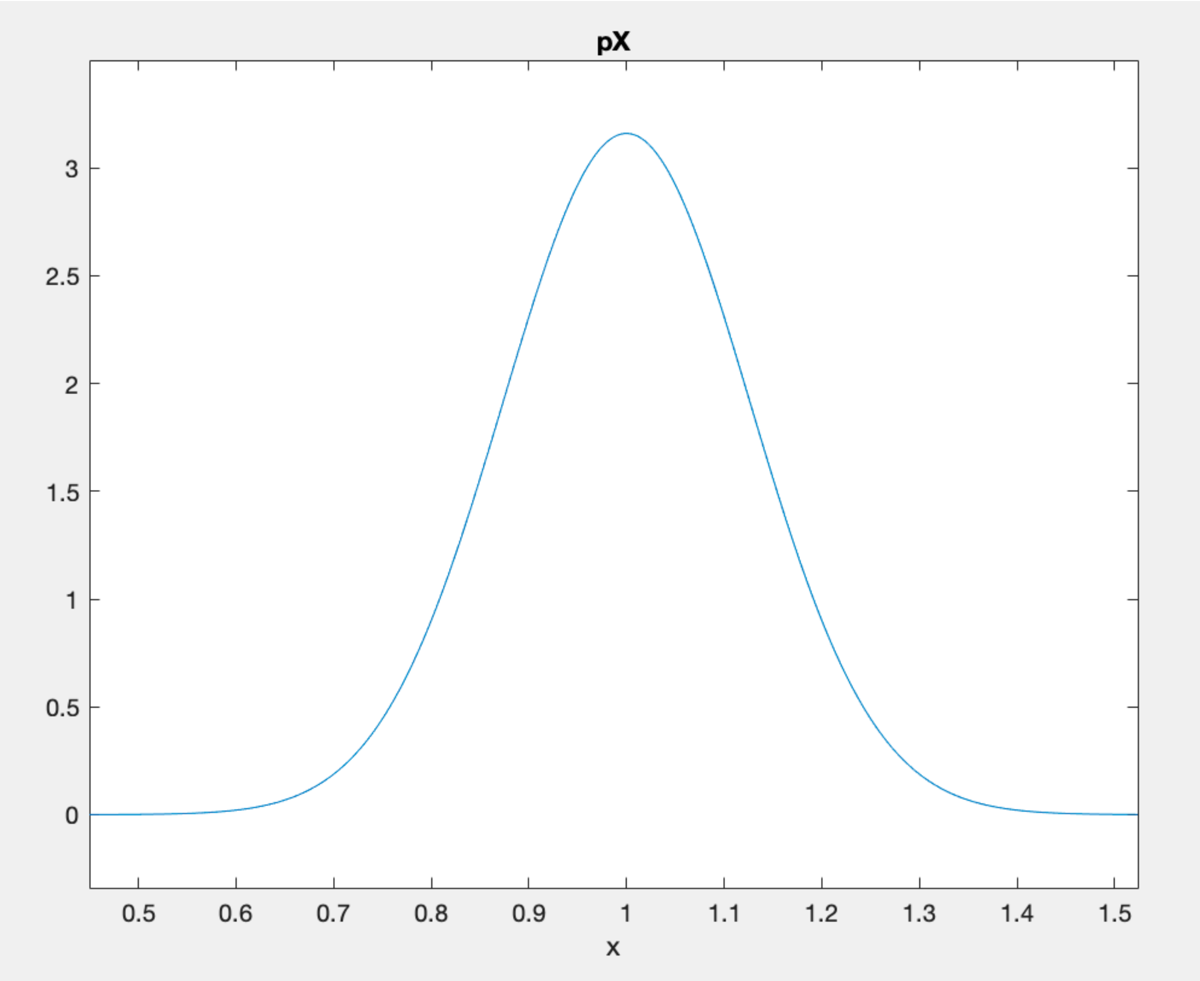
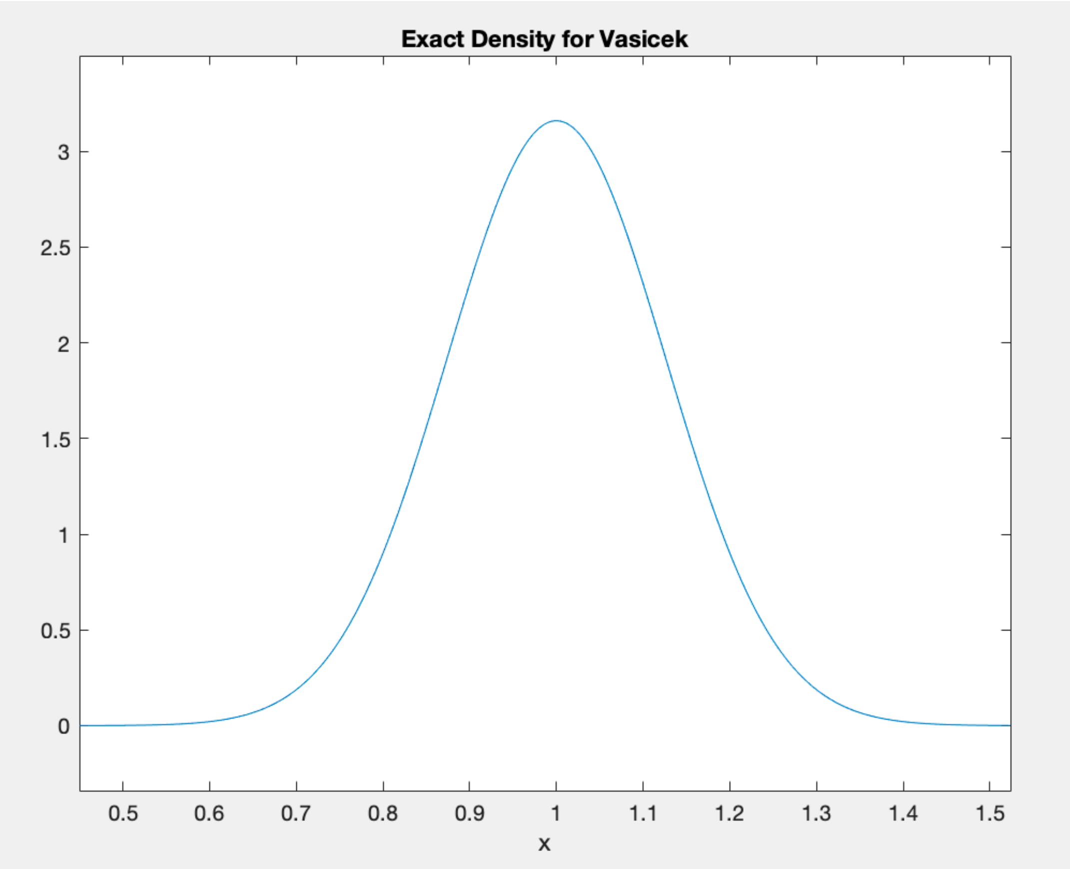
```
%%%% Variables in X(t), Y(t), Z(t)
syms a  b  c
syms xs ys zs
syms x  y  z
syms h t s
K=3;
J=4;

%%%%% Drift and Diffusion for X(t)
% Vasicek
muX=a*(b-x);
sigmaX=c;

%%%%% Transformation X(t) to Y(t)
fX2Y=int(1/sigmaX,x);

fY2X=subs((finverse(fX2Y)), x,y);

%%%%%  Drift and Diffusion for Y(t)
muY_temp=muX/sigmaX-sym('1')/sym('2')*diff(sigmaX,x,1);
muY=subs(muY_temp, x, fY2X);
muY=simplify(muY);

sigmaY=sym('1');

%%%%%%  Transformation Y(t) to Z(t)
fY2Z=h^(-1/2)*(y-ys);

fZ2Y=h^(1/2)*z+ys;

%%%%% Generating Beta
syms Htemp Expectation Beta
clear Beta Htemp Expectation

for n=1:K
     HTemp=subs(Hermite(n), z, fY2Z);
     Expectation=HTemp;

     for k=1:J
       HTemp=muY*diff(HTemp,y,1)+sym('1')/sym('2')*diff(HTemp,
y, 2);
       Expectation=Expectation + h^k/factorial(k)*HTemp;
     end
     Beta{n}= sym('1')/factorial(n-1) * subs(Expectation, y,
ys);
end


%%%%% Geberating pZ With Loop
pZ=sym('0');
for m=1:K
```

```
  pZ=pZ+Beta{m}*Hermite(m)
end
findsym(pZ)
%%%%% Generating pY pX
pZ=exp(-z^2/2)/sqrt(2*pi)*pZ;
pY=(h^(-1/2))*subs(pZ, z, fY2Z);
pX=(sigmaX^(-1))*subs(pY, y, fX2Y);
pX=subs(pX, ys, subs(fX2Y, x, xs));
pX=simplify(pX);

%%%%% ploting  pX
g1=subs(pX, {a,b,c,h,xs}, {1,1,2,1/250,1});
ezplot(g1)
title('pX')
```

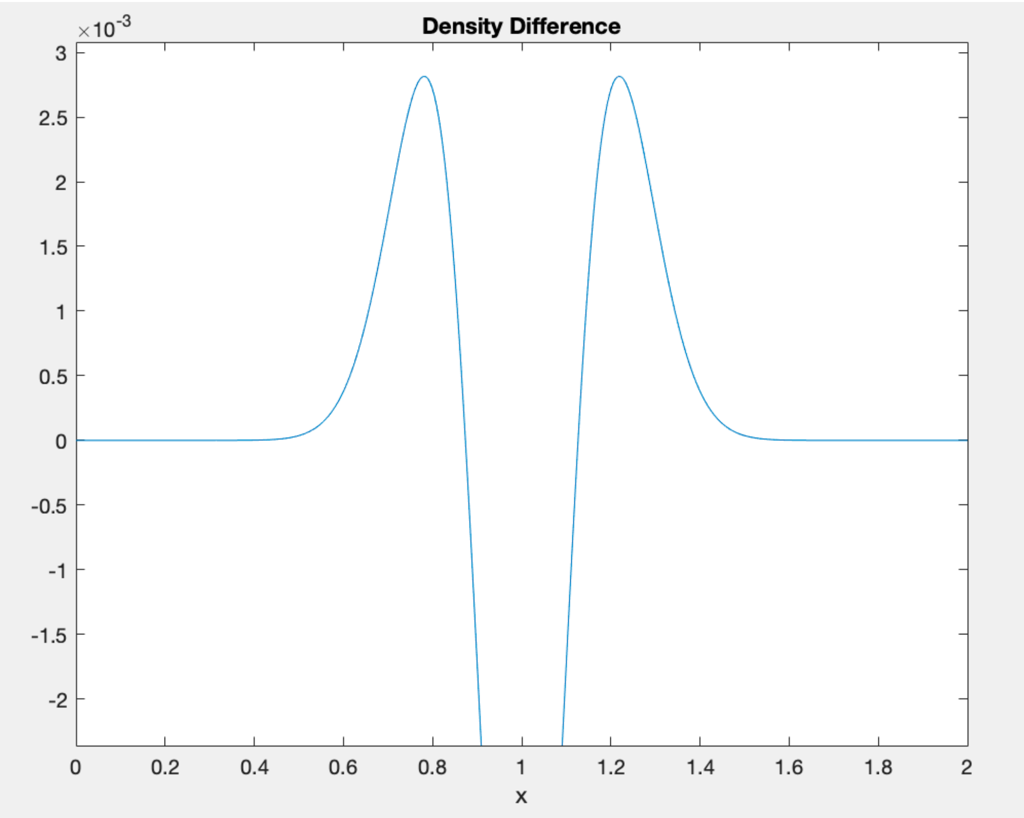



```
%%%%%% Ploting EXact Density for Vasicek
gamm=sigmaX*sqrt(1-exp(-2*a*h));
density=(pi*gamm^2/a)^(-1/2)*exp( -(x-b-(xs-b)*exp(-a*h))^2
*a/(gamm^2));
g2=subs(density, {a,b,c,h,xs},{1,1,2,1/250,1});
g2=simplify(g2);
ezplot(g2)
title('Exact Density for Vasicek')
```

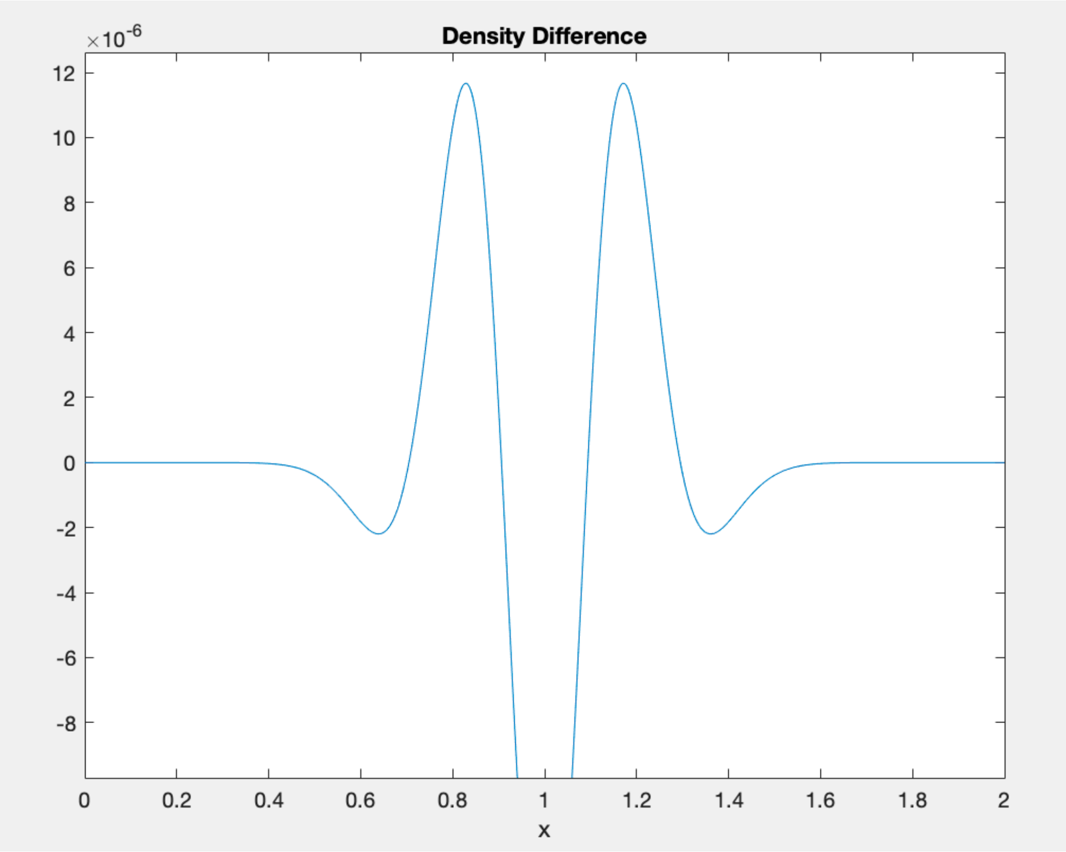



```
%%%%%% Plot Density Difference
gDiff=g1-g2;
ezplot(gDiff, [0,2])
title('Density Difference')
```

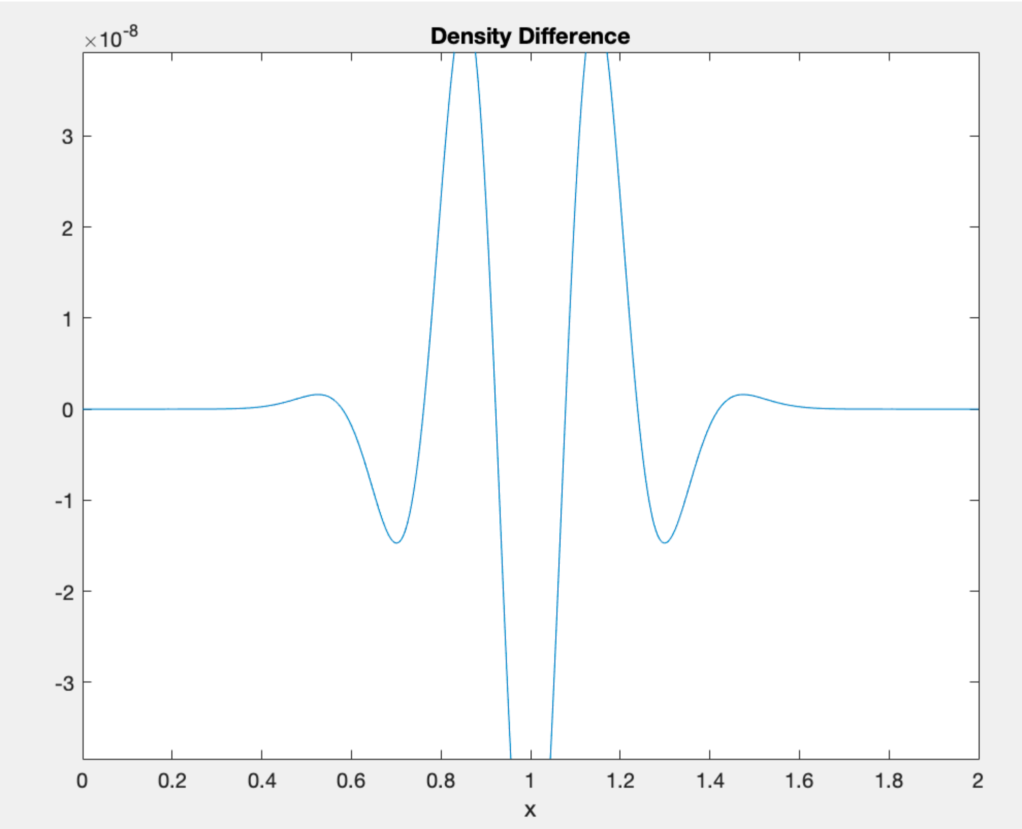
When k=2, J=3



When K=3, J=4




When K=5, J=6



When K=3, J=4, the difference is small enough. So, we use K=3, J=4.

Hence, we get the transition density and likelihood function:

```
function tem=g(x,sigma)
tem=-(2251799813685248*exp(-(x -
sigma(5))^2/(2*sigma(3)^2*sigma(4)))*((((sigma(1)*sigma(4)*(si
gma(1)*sigma(2)^2 - 2*sigma(1)*sigma(2)*sigma(5) - sigma(3)^2
+ sigma(1)*sigma(5)^2))/(2*sigma(3)^2) -
sigma(1)^2*sigma(4)^2*(3*sigma(1)*sigma(2)^2 -
6*sigma(1)*sigma(2) *sigma(5) - 2*sigma(3)^2 +
3*sigma(1)*sigma(5)^2))/(6*sigma(3)^2) +
 (sigma(1)^3*sigma(4)^3*(7*sigma(1)*sigma(2)^2 -
14*sigma(1)*sigma(2)*sigma(5) - 4*sigma(3)^2 +
7*sigma(1)*sigma(5)^2))/(24*sigma(3)^2))*(sigma(4)*sigma(3)^2
- x^2 + 2*x*sigma(5) - sigma(5)^2))/(sigma(3)^2*sigma(4)) +
(sigma(1)*(sigma(2)- sigma(5))*(x-
sigma(5))*(sigma(1)^3*sigma(4)^3 - 4*sigma(1)^2*sigma(4)^2 +
12*sigma(1)*sigma(4) - 24))/(24*sigma(3)^2) -
1))/(5644425081792261*sigma(3)*sigma(4)^(1/2));

function like=L(X,sigma)
like=0;
for n = 1:length(X)
    like=like+log(g(X(n),sigma));
end
like=like/length(X);
end
```

Then, use Federal Fund Rate data to fit the model

```
rate=readtable("FRB_H15.csv");
X = table2array(rate(:,1));
lkh=@(sigma)-L(X,sigma);
[MLE,fval,~,~,~,hessian]=fminunc(lkh,[1,1,2,1/250,1]);
%MLE = [1.0696,1.3072,1.7791,0.0250,2.0226]
fval_of_L=-fval;
%fval_of_L=-0.1375
hessian_of_L=inv(-(-hessian)/length(X));
ssbeta=hessian_of_L(1,1);
z_beta=MLE(1)/sqrt(ssbeta/length(X));
p_beta=2*(1-normcdf(abs(z_beta)));
std_beta=sqrt(ssbeta/length(X));
%std_beta=15.8319
%p_beta=0.9461
ssalpha=hessian_of_L(2,2);
z_alpha=MLE(2)/sqrt(ssalpha/length(X));
p_alpha=2*(1-normcdf(abs(z_alpha)));
std_alpha=sqrt(ssalpha/length(X));
%std_alpha=67.0583
%p_alpha=0.9844
sssigma=hessian_of_L(3,3);
z_sigma=MLE(3)/sqrt(sssigma/length(X));
```

```
p_sigma=2*(1-normcdf(abs(z_sigma)));
std_sigma=sqrt(sssigma/length(X));
%std_sigma=0+12.7991i
%p_sigma=0.8894

%To test the robustness of the parameter estimation,
we divide the data into two parts
X1 = X(1:26);
lkh=@(sigma)-L(X1,sigma);
[MLE,fval,~,~,~,hessian]=fminunc(lkh,[1,1,2,1/250,1]);
%MLE = [1.0606,1.1760,1.7744,0.0067,1.7785]
fval_of_L=-fval;
%fval_of_L=0.5118
hessian_of_L=inv(-(-hessian)/length(X1));
ssbeta=hessian_of_L(1,1);
z_beta=MLE(1)/sqrt(ssbeta/length(X1));
p_beta=2*(1-normcdf(abs(z_beta)));
std_beta=sqrt(ssbeta/length(X));
%p_beta=0.9754
ssalpha=hessian_of_L(2,2);
z_alpha=MLE(2)/sqrt(ssalpha/length(X1));
p_alpha=2*(1-normcdf(abs(z_alpha)));
std_alpha=sqrt(ssalpha/length(X));
%p_alpha=0.9811
sssigma=hessian_of_L(3,3);
z_sigma=MLE(3)/sqrt(sssigma/length(X1));
p_sigma=2*(1-normcdf(abs(z_sigma)));
std_sigma=sqrt(sssigma/length(X));
%p_sigma=0.7884

X2 = X(27:51);
lkh=@(sigma)-L(X2,sigma);
[MLE,fval,~,~,~,hessian]=fminunc(lkh,[1,1,2,1/250,1]);
%MLE = [1.2207,1.4833,1.45985939996535,0.0113,2.2529]
fval_of_L=-fval;
%fval_of_L=0.4530
hessian_of_L=inv(-(-hessian)/length(X2));
ssbeta=hessian_of_L(1,1);
z_beta=MLE(1)/sqrt(ssbeta/length(X2));
p_beta=2*(1-normcdf(abs(z_beta)));
%p_beta=9783
ssalpha=hessian_of_L(2,2);
z_alpha=MLE(2)/sqrt(ssalpha/length(X2));
p_alpha=2*(1-normcdf(abs(z_alpha)));
%p_alpha=0.9658
sssigma=hessian_of_L(3,3);
z_sigma=MLE(3)/sqrt(sssigma/length(X2));
p_sigma=2*(1-normcdf(abs(z_sigma)));
%p_sigma=0.8408

%There is little change in the estimation of the
parameters; the model is robust.
```