Financial Econometrics II

Homework #2 MATLAB for Markov & Dependent Data

Due in class on January 29

NOTE: *The homework needs to be typped and only hard copies are accepted. Please keep a copy for yourself since HW won't be returned.*

After HW1, you have implemented MLE for i.i.d. data. This HW is to extend your work to Markov and Dependent Data.

Problem 1. Suppose that we have Markov data $X_1, X_2, ..., X_n$, which has conditional density $f(X_{i+1} = y|X_i = x) = g(y, x, \theta)$ where θ is the unknown parameter and g function is given. Write a MATLAB program that implements the MLE estimation $\hat{\theta}$ defined by

$$\hat{\theta} = \arg\max_{\theta} L(\theta). \tag{1}$$

Here the likelihood function $L(\theta)$ is defined by

$$L(\theta) = \sum_{i=1}^{n} \log f(X_{i+1}|X_i, \theta) = \sum_{i=1}^{n} \log g(X_{i+1}, X_i, \theta).$$

Your program should be able able to out put the MLE $\hat{\theta}$, $L(\hat{\theta})$, the condifence interval, and the p-value for testing $H_0 : \theta = \theta_0$.

Note 1: Your program should cover the case where θ is K dimensional vector. See my lecture notes on how to construct confidence interval for each component of θ.

Note 2: You may want simulated data to test your program. This can be done as follows: Let

$$X_{i+1} = \alpha + \beta X_i + \varepsilon_{i+1}, \text{ for } \varepsilon_i \text{ i.i.d. standard normal random variable.}$$

with intitial vlue

$$X_1 = 2$$

and parameter values

$$\alpha = 1, \beta = 0.6.$$

The above equation allows you to generate all X_i (you can choose sample size to be 1000). For this model conditional density is (Note the true model is for $\alpha_0 = 1$, $\beta_0 = 0.6$)

$$f(X_{i+1} = y|X_i = x) = g(y, x, \theta) = \text{Normal}(1+0.6x, 1) \text{ Density} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{[y - (1 + 0.6x)]^2}{2}\right),$$

here $\theta = (\alpha, \beta)$