Financial Econometrics II HW2 Part II

GARCH Models and Its Application

Due in class Feb 12, 2019

Important Note: *Note that this homework usually takes more time to finish than the first homework, so don't wait until the last minute! Please note that the homework needs to be handed in by the deadline. If you need more time to complete it, you can hand in what you have by the due date and "append" your homework within one week of the deadline. However, the highest credit for this late part is 70% of the total credit allocated to it. Be prepared to discuss your results in class.*

Implement the maximum likelihood estimation procedure for GARCH$(1,1)$ model by adapting your program in HW#1. Assuming that your HW#1 was done properly, then you will only need to code the likelihood function for the GARCH(1,1) models, and then use your implementation of the generic MLE in HW#1. Note that the GARCH MLE need the choice of the intitial value of the σ_t's (see the lecture notes). Your program should be able to do the following: (1). Calculate the MLE for the estimates of the parameters in GARCH(1,1) model; (2). Produce the p-value for testing the null hypothesis $H_0 : \theta = \theta_0$ for a prespecified θ_0 [note this part is more general than what you ha done in HW1, in which you only tested $H_0 : \theta = 0$]; (3). Calculate the confidence interval for a significant level α for each parameter.

After you have programmed the MLE for GARCH$(1,1)$ above, you need to verify that your program is correct. Because for the GARCH$(1,1)$ model, there is no closed form solution for the MLE, you will need to implement the following size and power analysis to verify your implementation indeed does what is it supposed to do:

Size/Power Analysis of a statistical estimator/test.

1. Choose a particular GARCH(1,1) model with known parameters $\theta_0 = (\alpha_0, \alpha_1, \beta_1)$. Note that you need to choose sensible parameters because some parameters are not allowed.

2. Generate data $X_1, ..., X_T$ from the above GARCH(1,1) [Write a program to so, or use the builtin garchsim function].

3. Run your MLE for on the above generated data to get the parameter estimates, p-values, and confidence intervals for a particular significance level (say 5%). Record your

inference if you will reject the null hypothesis $H_0 : \theta = \theta_0$ at 5%.significance level according to the p-value or confidence interval.

4. Repeat the above step 1-3 for 10,000 times (or a large feasible number) and calculate the percentage of times that you rejected the null hypothesis $H_0 : \theta = \theta_0$. If the MLE theory and your program are both correct, then you should see about 5% rejections (why?).

To conduct the power analysis, process is basically the same except you generate the data from a different model (called alternative model) with a different θ_0 (say θ_0^*). You run the same steps 1-4 again and count the percentages you rejected the null $H_0 : \theta = \theta_0$ using the p-value and confidence interval. A good estimation and testing theory should give you high percentage of rejection, since the data is not from the null model $H_0 : \theta = \theta_0$ any more. But with finite data sample, this happens only if the alternative model is very different from θ_0.

Another less desirable method is to compare the estimator/p-value/confidence interval with the results produced by the MATLAB builtin function. But this requires the existence of such programs.

Bonus Questionn(!!!): Can you extend your program to $GARCH(p, q)$?