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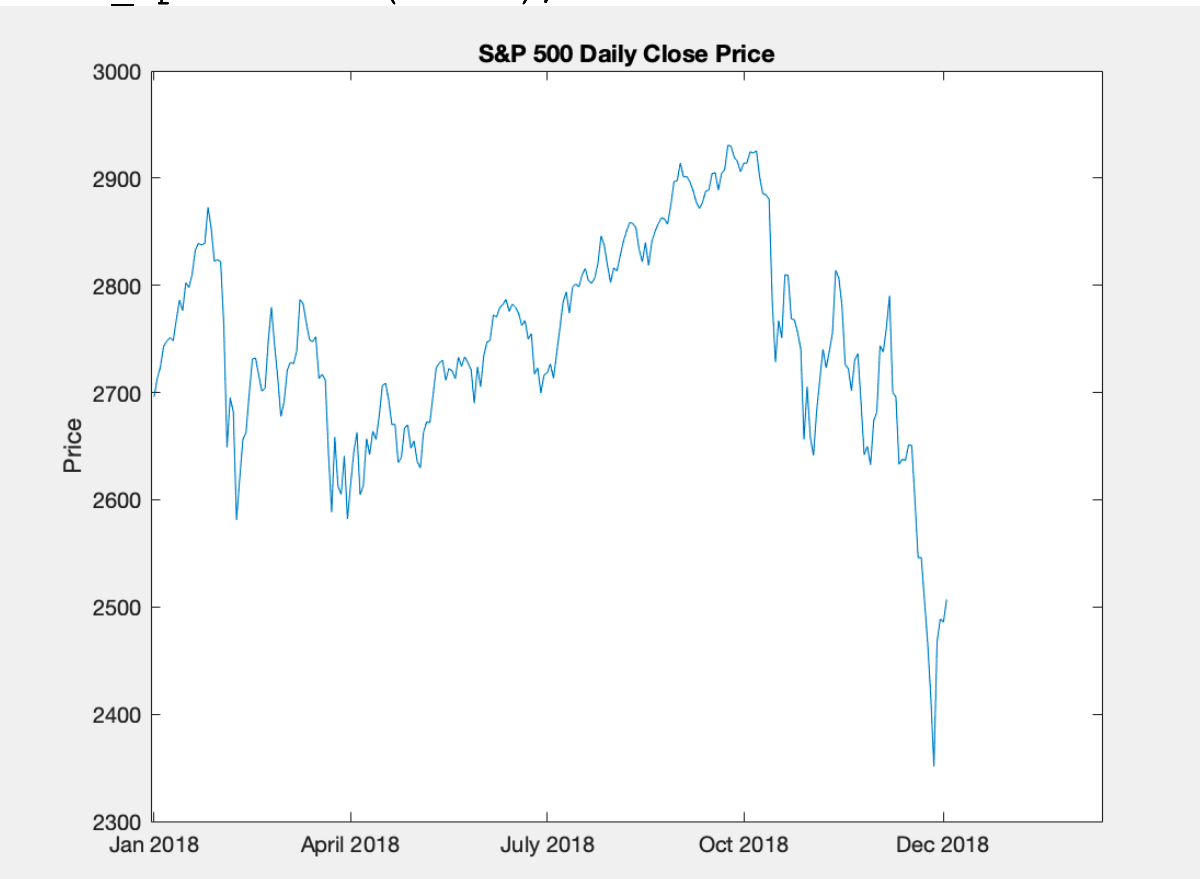
HW 3

# Pre-estimation

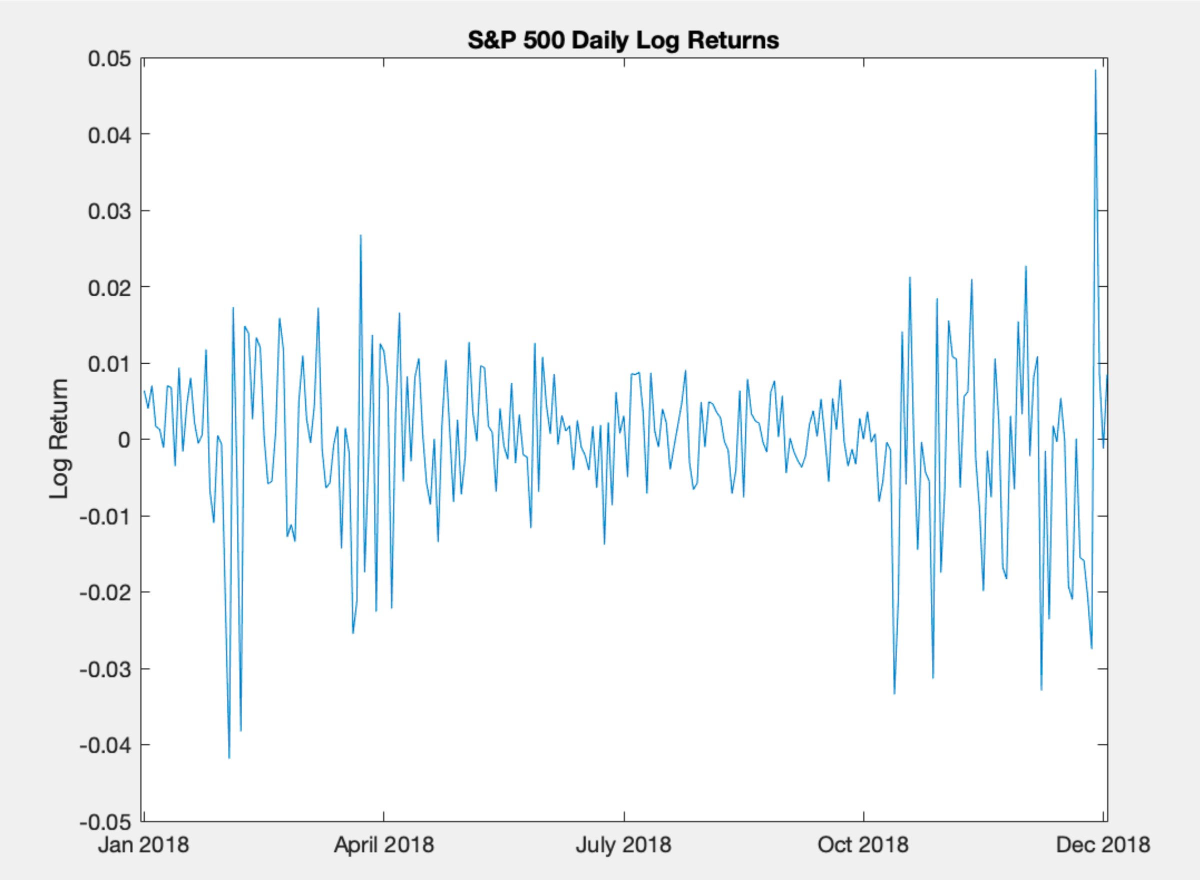
```
Price = csvread('/Users/xuan/Desktop/SP500.csv',1,1);
%draw a plot of the close price of S&P 500 from
2018/1/2 to 2018/12/31
plot(Price);
set(gca,'XTick',[1 63 125 187 250])
set(gca,'XTickLabel',{'Jan 2018' 'April 2018' 'July
2018' 'Oct 2018' 'Dec 2018'})
ylabel('Price')
title('S&P 500 Daily Close Price')
Price_tpls1=Price(2:251);
```



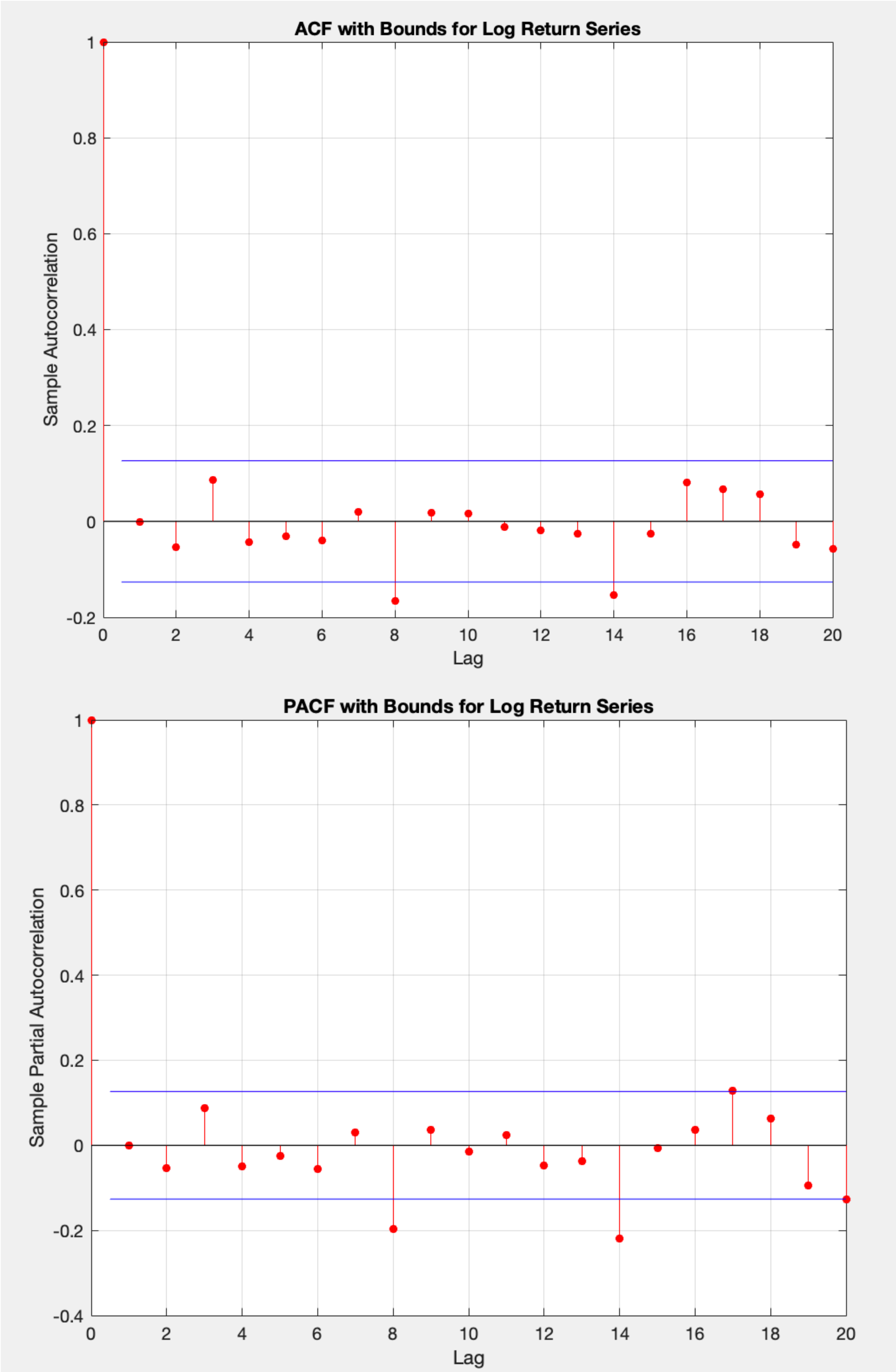

```
%get the log return from the close price of S&P 500
from 2018/1/3 to 2018/12/31
logr = log(Price_tpls1)-log(Price(1:250));

%draw a plot of log returns
```

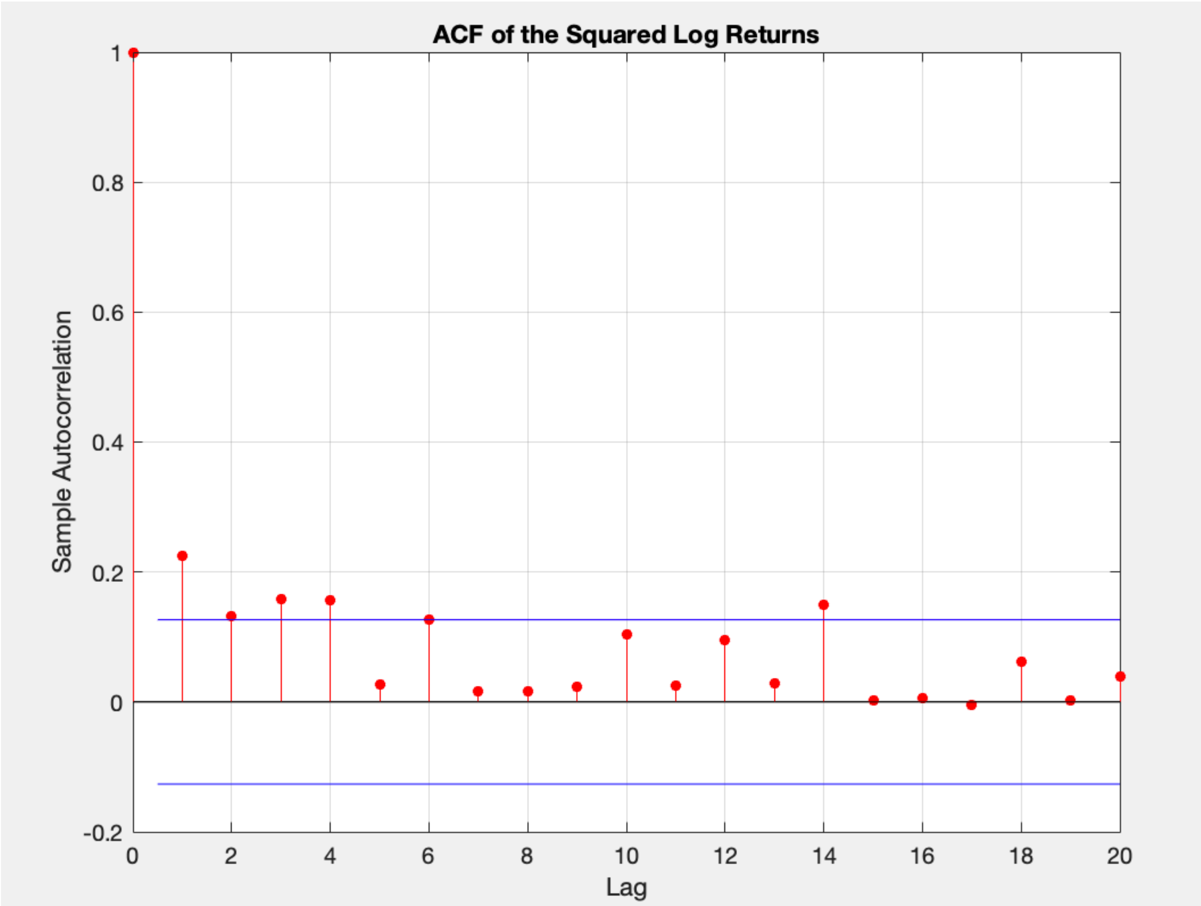
```
plot(logr);
set(gca,'XTick',[1 63 125 187 249])
set(gca,'XTickLabel',{'Jan 2018' 'April 2018' 'July
2018' 'Oct 2018' 'Dec 2018'})
ylabel('Log Return')
title('S&P 500 Daily Log Returns')
```




```
%check for correlation in the log returns series
autocorr(logr)
title('ACF with Bounds for Log Return Series')
parcorr(logr)
title('PACF with Bounds for Log Return Series')
%As is shown in ACF and PACF of log returns, there is
little correlation in the conditional mean
```


ACF with Bounds for Log Return Series
Sample Autocorrelation
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20
Lag
PACF with Bounds for Log Return Series
Sample Partial Autocorrelation
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
12
14
16
18
20
Lag

```
%check for correlation in the squared returns
autocorr(logr.^2)
title('ACF of the Squared Log Returns')
%The variance process exhibits some correlation
```




```
%quantify the correlation
[H,pValue,Stat,CriticalValue] = lbqtest(logr-
mean(logr));
%[H pValue Stat CriticalValue]=[0  0.2684 23.4245
31.4104]
%There is no autocorrelation in the log returns

[H2,pValue2,Stat2,CriticalValue2] = lbqtest((logr-
mean(logr)).^2);
%[H2,pValue2,Stat2,CriticalValue2] = 1.0000    0.0006
47.0571   31.4104
%There is autocorrelation in the squared log returns
```

# Parameter Estimation

```
%Estimate the Model Parameters; alpha and beta are
statistically significant from zero
Mdl = garch('GARCHLags',1,'ARCHLags',1);
EstMdl = estimate(Mdl,logr);
```

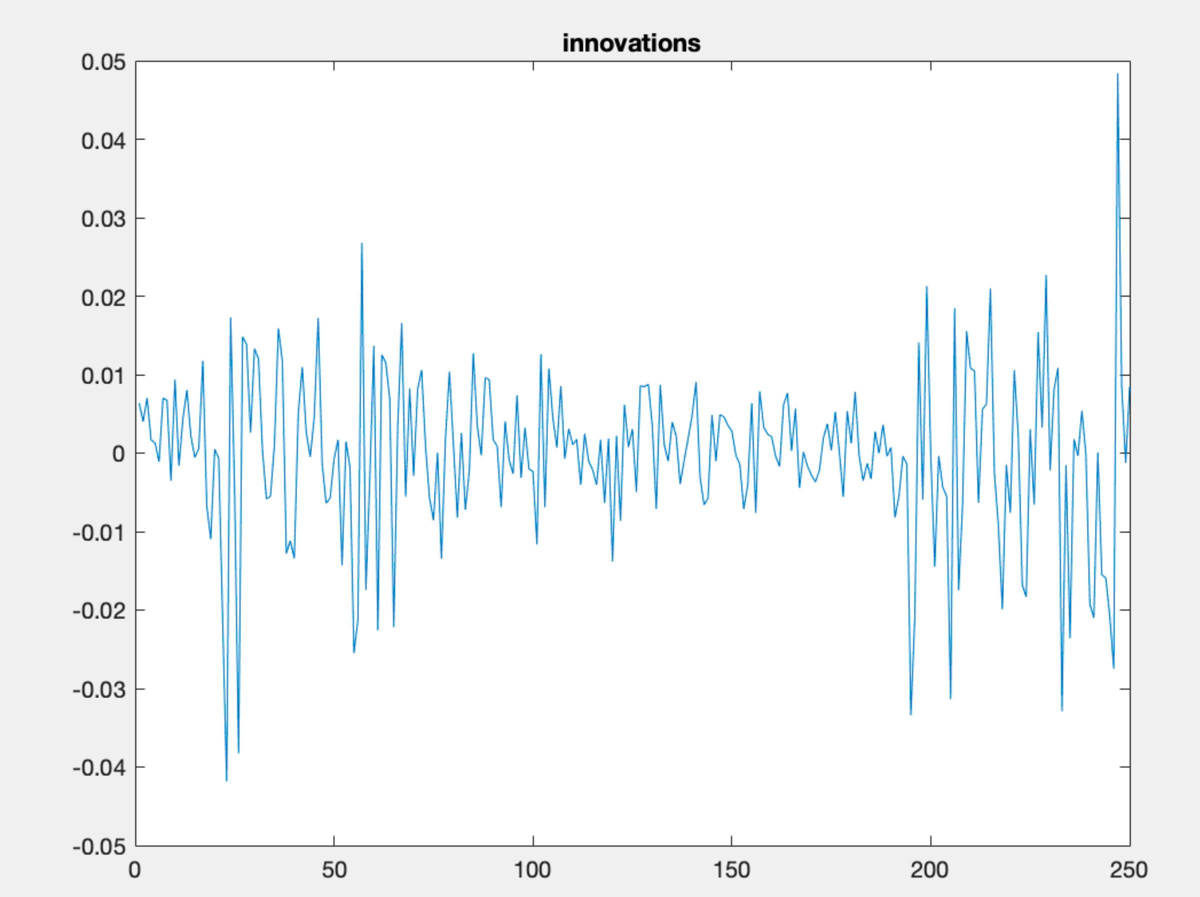
```
  GARCH(1,1) Conditional Variance Model (Gaussian Distribution):
```

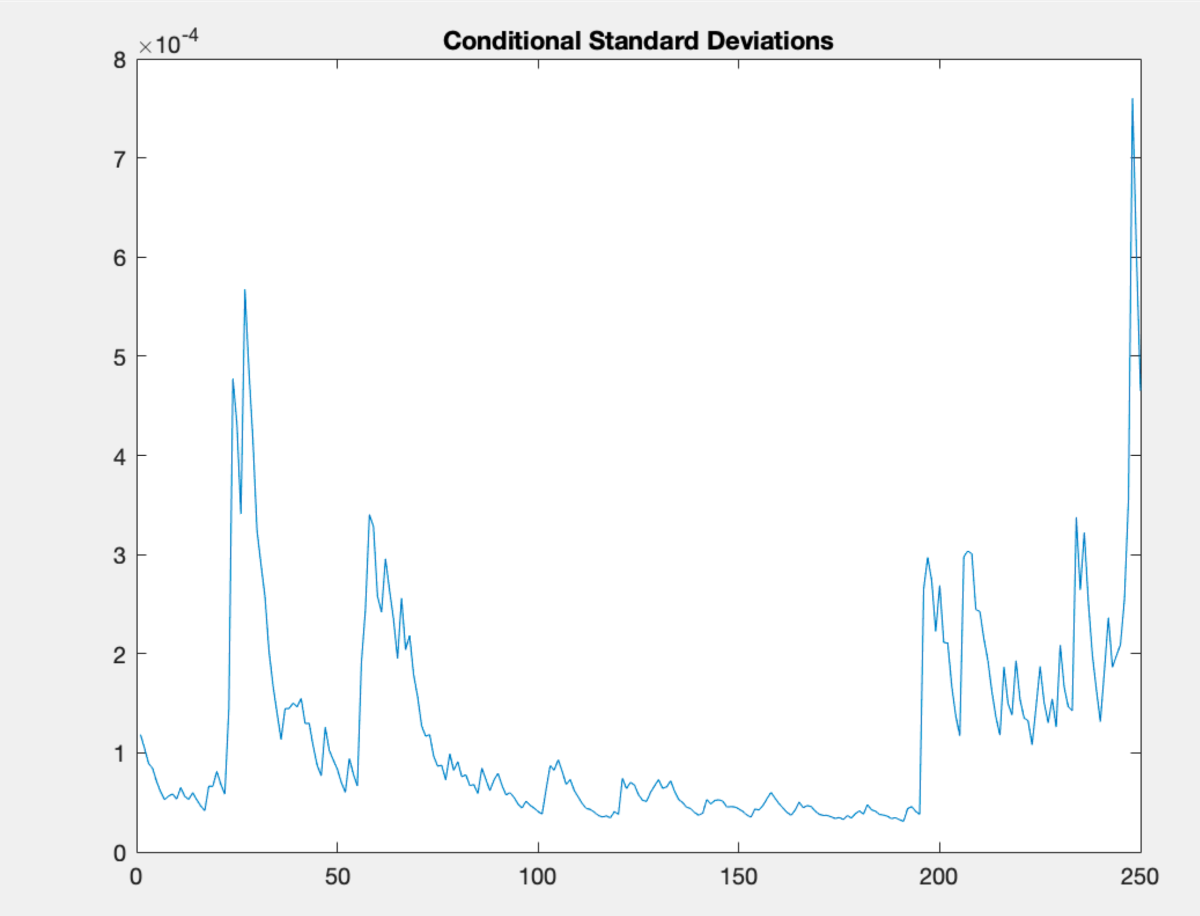
| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| **Constant** | 6.1187e-06 | 2.2873e-06 | 2.6751 | 0.0074713 |
| **GARCH{1}** | 0.76316 | 0.050537 | 15.101 | 1.5991e-51 |
| **ARCH{1}** | 0.20585 | 0.045615 | 4.5127 | 6.4007e-06 |

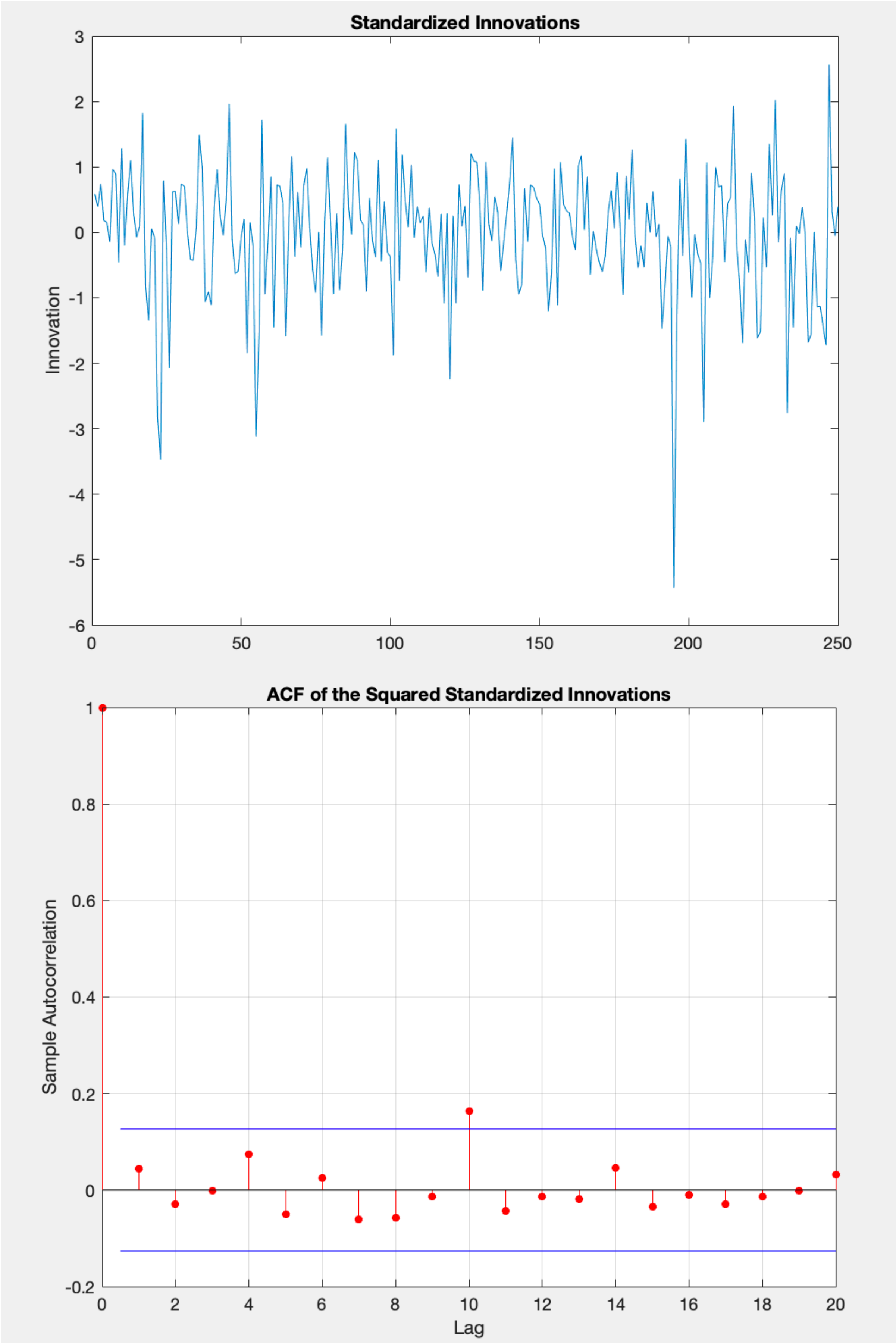
# Post Estimation

```
%innovations
T=length(logr);
inno=logr-EstMdl.Offset;
plot(inno)
xlim([0,T])
title('innovations')
%Conditional Standard Deviations
v = infer(EstMdl,logr);
sd=sqrt(v);
figure
plot(v)
xlim([0,T])
title('Conditional Standard Deviations')
%plot standardized innovations
plot(inno./sd)
ylabel('Innovation')
title('Standardized Innovations')
%the ACF of the squared standardized innovations show
no correlation.
autocorr((inno./sd).^2)
title('ACF of the Squared Standardized Innovations')
```

Both innovations and conditional standard deviations exhibit volatility clustering. However, the standardized innovations appear generally stable with little clustering. The comparison between the ACF of the squared standardized innovations to the ACF of the squared log returns shows that the GARCH(1,1) model sufficiently explains the heteroscedasticity in the log returns.


innovations
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
50
100
150
200
250


Conditional Standard Deviations
×10-4
8
7
6
5
4
3
2
1
0
0
50
100
150
200
250



Standardized Innovations
Innovation
3
2
1
0
-1
-2
-3
-4
-5
-6
0
50
100
150
200
250
ACF of the Squared Standardized Innovations
Sample Autocorrelation
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20
Lag

```
%Quantify and Compare Correlation of the Standardized Innovations
[H3,pValue3,Stat3,CriticalValue3] = lbqtest((inno./sd).^2);
%[H3,pValue3,Stat3,CriticalValue3] = 0  0.8384 13.8433 31.4104
```

With highly significant P-Value, we cannot reject the null hypotheses. Thus, there is no correlation between standardized innovations and the GARCH(1,1) model has explanatory power.

```
[H4,pValue4,Stat4,CriticalValue4]=archtest(inno./sd);
%[H4,pValue4,Stat4,CriticalValue4]=0  0.4829  0.4922  3.8415
```

There is no obvious improvement using GARCH(1,2) or GARCH(2,1) models. Thus GARCH(1,1) sufficiently describes the data.

```
%fit GARCH(1,2)
Mdl12 = garch('GARCHLags',1,'ARCHLags',2);
EstMdl12 = estimate(Mdl12,logr);
%fit GARCH(2,1)
Mdl21 = garch('GARCHLags',2,'ARCHLags',1);
EstMdl21 = estimate(Mdl21,logr);
```

GARCH(1,2) Conditional Variance Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| **Constant** | 8.4658e-06 | 2.0428e-06 | 4.1441 | 3.4113e-05 |
| **GARCH{1}** | 0.77081 | 0.06969 | 11.06 | 1.9511e-28 |
| **ARCH{2}** | 0.18293 | 0.071475 | 2.5593 | 0.010487 |

GARCH(2,1) Conditional Variance Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| **Constant** | 1.6845e-05 | 3.9603e-06 | 4.2534 | 2.1057e-05 |
| **GARCH{2}** | 0.57204 | 0.084245 | 6.7902 | 1.1197e-11 |
| **ARCH{1}** | 0.33565 | 0.080652 | 4.1617 | 3.1586e-05 |

The final GARCH(1,1) model is

$$y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^2 = 6.1187 \times 10^{-6} + 0.76316 * \sigma_{t-1}^2 + 0.20585 \times \varepsilon_{t-1}^2$$

```
%Forcast
fc = forecast(EstMdl,1);
%fc = 1.9739e-04
```

# Robustness test

```
%break the log returns into to sub-samples and refit
the GARCH(1,1) model
y1=logr(1:125);
y2=logr(126:250);
EstMdlsub1 = estimate(Mdl,y1);
EstMdlsub2 = estimate(Mdl,y2);
```

```
EstMdlsub1 = estimate(Mdl,y1);
EstMdlsub2 = estimate(Mdl,y2);
```

GARCH(1,1) Conditional Variance Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| **Constant** | 1.245e-05 | 6.7716e-06 | 1.8385 | 0.065984 |
| **GARCH{1}** | 0.64433 | 0.11325 | 5.6894 | 1.2746e-08 |
| **ARCH{1}** | 0.24211 | 0.068815 | 3.5182 | 0.00043441 |

GARCH(1,1) Conditional Variance Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| **Constant** | 4.113e-06 | 4.0216e-06 | 1.0227 | 0.30644 |
| **GARCH{1}** | 0.8257 | 0.10379 | 7.9558 | 1.7798e-15 |
| **ARCH{1}** | 0.1743 | 0.10148 | 1.7176 | 0.085878 |

The parameters are slightly different from the ones we got using the whole sample data. But in all the three estimated models, alpha plus beta approximately equals 1.