

The transformation of a DODECA is given as

$$\begin{aligned} & \exp(: F_{\text{in}} :) \exp(: aL :) \exp(: H_5/2 :) \exp(: bL :) \\ & \times \exp(: V_5 :) \exp(: aL :) \exp(: H_5/2 :) \exp(: bL :) \exp(: F_{\text{out}} :), \end{aligned}$$

where  $L$  and  $H_5$  are the Hamiltonians of a drift of length  $L$  and a thin dodecapole kick with integrated strength  $K_5$ :

$$H_5 = \frac{K_5}{6!} \Re(x - iy)^6,$$

respectively. The coefficients are  $a \equiv 1/2 - 1/\sqrt{12}$  and  $b = 1/2 - a$ . Terms  $\exp(: F_{\text{in}} :)$  and  $\exp(: F_{\text{out}} :)$  are transformation nonlinear fringes. The term  $\exp(: V_5 :)$  is a correction to adjust the third-order terms in  $L$ :

$$V_5 = \sum_{j=(x,y), k=(x,y)} -\frac{\beta}{2} H_{5,k}^2 + \gamma H_{5,j} H_{5,k} H_{5,j,k},$$

where  $i$  represents the derivative by  $x$  or  $y$ . We have also introduced two coefficients  $\beta \equiv 1/6 - 1/\sqrt{48}$  and  $\gamma = 1/40 - 1$