

The Lagrangean  $L$  defines the *canonical* momenta as

$$p_x = \frac{\partial L}{\partial x'} = \frac{mcx'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}} + a_x, \quad (6)$$

$$p_y = \frac{\partial L}{\partial y'} = \frac{mcy'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}} + a_y, \quad (7)$$

$$p_t = \frac{\partial L}{\partial t'} = -\frac{mc^3 t'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}}, \quad (8)$$

which derives the Hamiltonian as

$$H_t = x' p_x + y' p_y + t' p_t - L \quad (9)$$

$$= - \left( \sqrt{-c^2 m^2 / p_0^2 + p_t^2 / c^2 - (p_x - a_x)^2 + (p_y - a_y)^2 + a_s} \right) \left( 1 + \frac{x}{\rho} \right). \quad (10)$$

Instead of the canonical variables  $(t, p_t)$ , SAD uses another set  $(z, p)$ , The variable  $z$  means the logitudinal postion, and  $p$  the total momentum, which is more convenient than  $p_t$  especially in a low-energy case, ie.,  $\gamma \sim 1$ . The canonical variables  $(z, p)$  as well as the Hamiltonian  $H$  are obtained using a mother function

$$G = G(p_t, z) = \frac{z}{c} \sqrt{p_t^2 - m^2 c^4 / p_0^2} - t_0(s), \quad (11)$$

$$p = \frac{\partial G}{\partial z} = \frac{\sqrt{p_t^2 p_0^2 - m^2 c^4}}{p_0}, \quad (12)$$

$$t = \frac{\partial G}{\partial p_t} = -z \frac{\sqrt{p^2 p_0^2 - m^2 c^2}}{c p p_0} + t_0(s), \quad (13)$$

$$H = H_t - \frac{\partial G}{\partial s} \quad (14)$$

$$= - \left( \sqrt{p^2 - (p_x - a_x)^2 - (p_y - a_y)^2 + a_s} \right) \left( 1 + \frac{x}{\rho} \right) + \frac{E}{p_0 v_0}, \quad (15)$$

where  $t_0(s)$  is the *design arrival time* at location  $s$ ,  $E = \sqrt{m^2 c^4 + p_0^2 p^2}$  the energy of the particle, and  $v_0 = 1/t'_0(s)$  the design velocity. The longitudinal position  $z$  is written as

$$z = -v(t - t_0(s)), \quad (16)$$

where  $v$  is the total velocity of the particle. Note that  $z > 0$  for the head of a bunch.

Thus the canonical variables in SAD are:

$$(x, p_x, y, p_y, z, \delta \equiv p - 1). \quad (17)$$