The Lagrangean L defines the *canonical* momenta as

$$p_x = \frac{\partial L}{\partial x'} = \frac{mcx'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}} + a_x \,,$$

$$p_y = \frac{\partial L}{\partial y'} = \frac{mcy'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}} + a_y \,,$$

$$p_t = \frac{\partial L}{\partial t'} = -\frac{mc^3 t'}{p_0 \sqrt{c^2 t'^2 - x'^2 - y'^2 - (1 + x/\rho)^2}} \,,$$

which derives the Hamiltonian as

$$H_t = x' p_x + y' p_y + t' p_t - L$$
$$= -\left(\sqrt{-c^2 m^2 / p_0^2 + p_t^2 / c^2 - (p_x - a_x)^2 + (p_y - a_y)^2} + a_s \right)\left(1 + \frac{x}{\rho} \right) \,.$$

Instead of the canonical variables (t, p_t), SAD uses another set (z, p), The variable z means the logitudinal postion, and p
which is more convenient than p_t especially in a low-energy case, ie., $\gamma \sim 1$. The canonical variables (z, p) as well as t
obtained using a mother function

$$G = G(p_t, z) = \frac{z}{c} \sqrt{p_t^2 - m^2 c^4 / p_0^2} - t_0(s) \,,$$

$$p = \frac{\partial G}{\partial z} = \frac{\sqrt{p_t^2 p_0^2 - m^2 c^4}}{p_0} \,,$$

$$t = \frac{\partial G}{\partial p_t} = -z \frac{\sqrt{p^2 p_0^2 - m^2 c^2}}{c p p_0} + t_0(s) \,,$$

$$H = H_t - \frac{\partial G}{\partial s}$$
$$= -\left(\sqrt{p^2 - (p_x - a_x)^2 - (p_y - a_y)^2} + a_s \right)\left(1 + \frac{x}{\rho} \right) + \frac{E}{p_0 v_0} \,,$$

where $t_0(s)$ is the *design arrival time* at location s, $E = \sqrt{m^2 c^4 + p_0^2 p^2}$ the energy of the particle, and $v_0 = 1/t_0'(s)$ the
longitudinal position z is written as

$$z = -v(t - t_0(s)) \,,$$

where v is the total velocity of the particle. Note that $z > 0$ for the head of a bunch.

Thus the canonical variables in SAD are:

$$(x, p_x, y, p_y, z, \delta \equiv p - 1) \,.$$