

The transformation of a OCT is given as

$$\begin{aligned} & \exp(: F_{\text{in}} :) \exp(: aL :) \exp(: H_3/2 :) \exp(: bL :) \\ & \times \exp(: V_3 :) \exp(: aL :) \exp(: H_3/2 :) \exp(: bL :) \exp(: F_{\text{out}} :), \end{aligned}$$

where L and H_3 are the Hamiltonians of a drift of length L and a thin octupole kick with integrated strength K_3 :

$$H_3 = \frac{K_3}{4!} \Re(x - iy)^4,$$

respectively. The coefficients are $a \equiv 1/2 - 1/\sqrt{12}$ and $b = 1/2 - a$. Terms $\exp(: F_{\text{in}} :)$ and $\exp(: F_{\text{out}} :)$ are transformation nonlinear fringes. The term $\exp(: V_3 :)$ is a correction to adjust the third-order terms in L :

$$V_3 = \sum_{j=(x,y), k=(x,y)} -\frac{\beta}{2} H_{3,k}^2 + \gamma H_{3,j} H_{3,k} H_{3,j,k},$$

where i represents the derivative by x or y . We have also introduced two coefficients $\beta \equiv 1/6 - 1/\sqrt{48}$ and $\gamma = 1/40 - 1$