

The transformation of a OCT is given as

$$\begin{aligned} & \exp(: F_{\text{in}} :) \exp(: aL :) \exp(: H_3/2 :) \exp(: bL :) \\ & \times \exp(: V_3 :) \exp(: aL :) \exp(: H_3/2 :) \exp(: bL :) \exp(: F_{\text{out}} :), \end{aligned} \quad (160)$$

where  $L$  and  $H_3$  are the Hamiltonians of a drift of length  $L$  and a thin octupole kick with integrated strength  $K_3$ :

$$H_3 = \frac{K_3}{4!} \Re(x - iy)^4, \quad (161)$$

respectively. The coefficients are  $a \equiv 1/2 - 1/\sqrt{12}$  and  $b = 1/2 - a$ . Terms  $\exp(: F_{\text{in}} :)$  and  $\exp(: F_{\text{out}} :)$  are transformations for entrance and exit nonlinear fringes. The term  $\exp(: V_3 :)$  is a correction to adjust the third-order terms in  $L$ :

$$V_3 = \sum_{j=(x,y), k=(x,y)} -\frac{\beta}{2} H_{3,k}^2 + \gamma H_{3,j} H_{3,k} H_{3,j,k}, \quad (162)$$

where  $i$  represents the derivative by  $x$  or  $y$ . We have also introduced two coefficients  $\beta \equiv 1/6 - 1/\sqrt{48}$  and  $\gamma = 1/40 - 1/(24\sqrt{3})$ .