

Let  $V$  denote the matrix to define the normal mode, i.e.,

$$\mathbf{U} = V\mathbf{u}, \quad (68)$$

where  $\mathbf{U} = (X, P_x, Y, P_y, Z, P_z)$  and  $\mathbf{u} = (x, p_x, y, p_y, z, \delta \equiv p-1)$  are the normal and physical coordinates, respectively. The matrix  $V$  can be expressed as

$$V = PBR_6H, \quad (69)$$

where

$$H = \begin{pmatrix} \left(1 - \frac{\det H_x}{1+a}\right)I & \frac{H_x J_2 H_y^T J_2}{1+a} & -H_x \\ \frac{H_y J_2 H_x^T J_2}{1+a} & \left(1 - \frac{\det H_y}{1+a}\right)I & -H_y \\ -J_2 H_x^T J_2 & -J_2 H_y^T J_2 & aI \end{pmatrix}, \quad (70)$$

$$R_6 = \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & I \end{pmatrix} = \begin{pmatrix} bI & J_2 r^T J_2 & 0 \\ r & bI & 0 \\ 0 & 0 & I \end{pmatrix}, \quad (71)$$

$$PB = \begin{pmatrix} P_x B_x & 0 & 0 \\ 0 & P_y B_y & 0 \\ 0 & 0 & P_z B_z \end{pmatrix}, \quad (72)$$

with

$$a^2 + \det H_x + \det H_y = 1, \quad (73)$$

$$b^2 + \det R = 1. \quad (74)$$

Symbols  $I, J_2, H_{x,y}, r, B_{x,y,z}, P_{x,y,z}$  above are 2 by 2 matrices:

$$I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (75)$$

$$J_2 \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (76)$$

$$r \equiv \begin{pmatrix} R1 & R2 \\ R3 & R4 \end{pmatrix}, \quad (77)$$

$$B_{x,y} \equiv \begin{pmatrix} \frac{1}{\sqrt{\beta_{x,y}}} & 0 \\ \frac{\alpha_{x,y}}{\sqrt{\beta_{x,y}}} & \sqrt{\beta_{x,y}} \end{pmatrix}, \quad (78)$$

$$P_{x,y,z} \equiv \begin{pmatrix} \cos \psi_{x,y,z} & \sin \psi_{x,y,z} \\ -\sin \psi_{x,y,z} & \cos \psi_{x,y,z} \end{pmatrix}. \quad (79)$$

Matrices  $H_{x,y}$  define dispersions as

$$\begin{pmatrix} ZX & EX \\ ZPX & EPX \\ ZY & EY \\ ZPY & EPY \end{pmatrix} \equiv R \begin{pmatrix} H_x \\ H_y \end{pmatrix}. \quad (80)$$