

The transformation matrix from the physical coordinate (x, p_x, y, p_y) to the x - y decoupled coordinate (X, P_X, Y, P_Y) is written as

$$\mathbf{R} = \begin{pmatrix} \mu\mathbf{I} & \mathbf{J}\mathbf{r}^T\mathbf{J} \\ \mathbf{r} & \mu\mathbf{I} \end{pmatrix} = \begin{pmatrix} \mu & . & -\mathbf{R4} & \mathbf{R2} \\ . & \mu & \mathbf{R3} & -\mathbf{R1} \\ \mathbf{R1} & \mathbf{R2} & \mu & . \\ \mathbf{R3} & \mathbf{R4} & . & \mu \end{pmatrix} \quad (60)$$

with a submatrix

$$\mathbf{r} = \begin{pmatrix} \mathbf{R1} & \mathbf{R2} \\ \mathbf{R3} & \mathbf{R4} \end{pmatrix}, \quad (61)$$

where

$$\mu^2 + \det(\mathbf{r}) = 1, \quad (62)$$

$$\mathbf{I} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (63)$$

$$\mathbf{J} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (64)$$

The inverse of \mathbf{R} is obtained by reversing the sign of \mathbf{r} :

$$\mathbf{R}^{-1} = \begin{pmatrix} \mu\mathbf{I} & -\mathbf{J}\mathbf{r}^T\mathbf{J} \\ -\mathbf{r} & \mu\mathbf{I} \end{pmatrix} = \begin{pmatrix} \mu & . & \mathbf{R4} & -\mathbf{R2} \\ . & \mu & -\mathbf{R3} & \mathbf{R1} \\ -\mathbf{R1} & -\mathbf{R2} & \mu & . \\ -\mathbf{R3} & -\mathbf{R4} & . & \mu \end{pmatrix} \quad (65)$$

The value of the function DETR is equal to $\det(\mathbf{r})$ in this case.

Let \mathbf{T} stand for the physical transfer matrix from location 1 to location 2, then the transformation in the decoupled coordinate is diagonalized as

$$\mathbf{R}_2\mathbf{T}\mathbf{R}_1^{-1} = \begin{pmatrix} \mathbf{T}_X & 0 \\ 0 & \mathbf{T}_Y \end{pmatrix}. \quad (66)$$

The Twiss parameters are defined for the 2 by 2 matrices \mathbf{T}_X and \mathbf{T}_Y .

If $\det(\mathbf{r}) \geq 1$, the above condition for μ is violated. In such a case, an alternative form of \mathbf{R} is used:

$$\mathbf{R} = \begin{pmatrix} \mathbf{J}\mathbf{r}^T\mathbf{J} & \mu\mathbf{I} \\ \mu\mathbf{I} & \mathbf{r} \end{pmatrix}, \quad (67)$$

where $\mu^2 + \det(\mathbf{r}) = 1$. The function DETR shows a number $a - \det(\mathbf{r})$, where $a = 1.375$. thus the alternative form is used when $\det(\mathbf{r}) \geq 0.625$.