The Hamiltonian H can be expanded up to the second order of the transverse coordinates as:

$$H = H_2 + \mathit{\Delta} H\,, \tag{18}$$

$$H_2 = -\left(p - \frac{(p_x - a_{x1})^2}{2p} - \frac{(p_y - a_{y1})^2}{2p} + a_{s2}\right)\left(1 + \frac{x}{\rho}\right) + \frac{E}{p_0 v_0}\,, \tag{19}$$

where a_{x1} and a_{y1} are the transverse vector potentials up to the first order, and a_{s2} longitudinal one up to the second order of x and y, respectively. Note that $a_{x,y} = 0$ where $1/\rho \neq 0$. Thus H_2 represents up to quadrupole fields overlapped with a uniform solenoid. It is known that H_2 has an *analytic solution*, if a_{s2} is time-independent. Unless H itself is solvable, SAD uses such an analytic solution of H_2, then apply the residual $\mathit{\Delta} H$ by slicing.