

The transformation of a DODECA is given as

$$\begin{aligned} & \exp(: F_{\text{in}} :) \exp(: aL :) \exp(: H_5/2 :) \exp(: bL :) \\ & \times \exp(: V_5 :) \exp(: aL :) \exp(: H_5/2 :) \exp(: bL :) \exp(: F_{\text{out}} :), \end{aligned} \quad (110)$$

where L and H_5 are the Hamiltonians of a drift of length L and a thin dodecapole kick with integrated strength K_5 :

$$H_5 = \frac{K_5}{6!} \Re(x - iy)^6, \quad (111)$$

respectively. The coefficients are $a \equiv 1/2 - 1/\sqrt{12}$ and $b = 1/2 - a$. Terms $\exp(: F_{\text{in}} :)$ and $\exp(: F_{\text{out}} :)$ are transformations for entrance and exit nonlinear fringes. The term $\exp(: V_5 :)$ is a correction to adjust the third-order terms in L :

$$V_5 = \sum_{j=(x,y), k=(x,y)} -\frac{\beta}{2} H_{5,k}^2 + \gamma H_{5,j} H_{5,k} H_{5,j,k}, \quad (112)$$

where i represents the derivative by x or y . We have also introduced two coefficients $\beta \equiv 1/6 - 1/\sqrt{48}$ and $\gamma = 1/40 - 1/(24\sqrt{3})$.