

The transformation matrix from the physical coordinate (x, p_x, y, p_y) to the x - y decoupled coordinate (X, P_X, Y, P_Y) is

$$\mathbf{R} = \begin{pmatrix} \mu \mathbf{I} & \mathbf{J} \mathbf{r}^T \mathbf{J} \\ \mathbf{r} & \mu \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mu & . & -\mathbf{R}4 & \mathbf{R}2 \\ . & \mu & \mathbf{R}3 & -\mathbf{R}1 \\ \mathbf{R}1 & \mathbf{R}2 & \mu & . \\ \mathbf{R}3 & \mathbf{R}4 & . & \mu \end{pmatrix}$$

with a submatrix

$$\mathbf{r} = \begin{pmatrix} \mathbf{R}1 & \mathbf{R}2 \\ \mathbf{R}3 & \mathbf{R}4 \end{pmatrix},$$

where

$$\begin{aligned} \mu^2 + \det(\mathbf{r}) &= 1, \\ \mathbf{I} &\equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ \mathbf{J} &\equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned}$$

The inverse of \mathbf{R} is obtained by reversing the sign of \mathbf{r} :

$$\mathbf{R}^{-1} = \begin{pmatrix} \mu \mathbf{I} & -\mathbf{J} \mathbf{r}^T \mathbf{J} \\ -\mathbf{r} & \mu \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mu & . & \mathbf{R}4 & -\mathbf{R}2 \\ . & \mu & -\mathbf{R}3 & \mathbf{R}1 \\ -\mathbf{R}1 & -\mathbf{R}2 & \mu & . \\ -\mathbf{R}3 & -\mathbf{R}4 & . & \mu \end{pmatrix}$$

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The value of the function DETR is equal to $\det(\mathbf{r})$ in this case.

Let \mathbf{T} stand for the physical transfer matrix from location 1 to location 2, then the transformation in the decoupled coordinates is

$$\mathbf{R}_2 \mathbf{T} \mathbf{R}_1^{-1} = \begin{pmatrix} \mathbf{T}_X & 0 \\ 0 & \mathbf{T}_Y \end{pmatrix}.$$

The Twiss parameters are defined for the 2 by 2 matrices \mathbf{T}_X and \mathbf{T}_Y .

If $\det(\mathbf{r}) \geq 1$, the above condition for μ is violated. In such a case, an alternative form of \mathbf{R} is used:

$$\mathbf{R} = \begin{pmatrix} \mathbf{J} \mathbf{r}^T \mathbf{J} & \mu \mathbf{I} \\ \mu \mathbf{I} & \mathbf{r} \end{pmatrix},$$

where $\mu^2 + \det(\mathbf{r}) = 1$. The function DETR shows a number $a - \det(\mathbf{r})$, where $a = 1.375$. thus the alternative form is used