The Lagrangean L defines the *canonical* momenta as

$$
\begin{aligned}
p_x =& \frac{\partial L}{\partial x'} = \frac{mcx'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}} + a_x\,, && (6)\\
p_y =& \frac{\partial L}{\partial y'} = \frac{mcy'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}} + a_y\,, && (7)\\
p_t =& \frac{\partial L}{\partial t'} = -\frac{mc^3t'}{p_0\sqrt{c^2t'^2 - x'^2 - y'^2 - (1+x/\rho)^2}}\,, && (8)
\end{aligned}
$$

which derives the Hamiltonian as

$$
\begin{aligned}
H_t =& x'p_x + y'p_y + t'p_t - L && (9)\\
=& -\left(\sqrt{-c^2m^2/p_0^2 + p_t^2/c^2 - (p_x-a_x)^2 + (p_y-a_y)^2} + a_s\right)\left(1+\frac{x}{\rho}\right). && (10)
\end{aligned}
$$

Instead of the canonical variables (t, p_t), SAD uses another set (z, p), The variable z means the logitudinal postion, and p the total momentum, which is more convenient than p_t especially in a low-energy case, ie., $\gamma \sim 1$. The canonical variables (z, p) as well as the Hamiltonian H are obtained using a mother function

$$
\begin{aligned}
G =& G(p_t, z) = \frac{z}{c}\sqrt{p_t^2 - m^2c^4/p_0^2} - t_0(s)\,, && (11)\\
p =& \frac{\partial G}{\partial z} = \frac{\sqrt{p_t^2p_0^2 - m^2c^4}}{p_0}\,, && (12)\\
t =& \frac{\partial G}{\partial p_t} = -z\frac{\sqrt{p^2p_0^2 - m^2c^2}}{cpp_0} + t_0(s)\,, && (13)\\
H =& H_t - \frac{\partial G}{\partial s} && (14)\\
=& -\left(\sqrt{p^2 - (p_x-a_x)^2 - (p_y-a_y)^2} + a_s\right)\left(1+\frac{x}{\rho}\right) + \frac{E}{p_0v_0}\,, && (15)
\end{aligned}
$$

where $t_0(s)$ is the *design arrival time* at location s, $E = \sqrt{m^2c^4 + p_0^2p^2}$ the energy of the particle, and $v_0 = 1/t_0'(s)$ the design velocity. The longitudinal position z is written as

$$
z = -v\,(t - t_0(s))\;, \tag{16}
$$

where v is the total velocity of the particle. Note that $z > 0$ for the head of a bunch.

Thus the canonical variables in SAD are:

$$
(x, p_x, y, p_y, z, \delta \equiv p - 1)\,. \tag{17}
$$