

The transformation of a DECA is given as

$$\exp(:F_{\text{in}}:)\exp(:aL:)\exp(:H_4/2:)\exp(:bL:)\\ \times \exp(:V_4:)\exp(:aL:)\exp(:H_4/2:)\exp(:bL:)\exp(:F_{\text{out}}:),$$

where L and H_4 are the Hamiltonians of a drift of length L and a thin decapole kick with integrated strength K_4 :

$$H_4 = \frac{K_4}{5!} \Re(x - iy)^5,$$

respectively. The coeffients are $a \equiv 1/2 - 1/\sqrt{12}$ and $b = 1/2 - a$. Terms $\exp(:F_{\text{in}}:)$ and $\exp(:F_{\text{out}}:)$ are transformation nonlinear fringes. The term $\exp(:V_4:)$ is a correction to adjust the third-order terms in L :

$$V_4 = \sum_{j=(x,y),k=(x,y)} -\frac{\beta}{2} H_{4,k}^2 + \gamma H_{4,j} H_{4,k} H_{4,j,k},$$

where i represents the derivative by x or y . We have also introduced two coefficients $\beta \equiv 1/6 - 1/\sqrt{48}$ and $\gamma = 1/40 - 1$