

The transformation of a DECA is given as

$$\exp(:F_{\text{in}}:)\exp(:aL:)\exp(:H_4/2:)\exp(:bL:)\\ \times \exp(:V_4:)\exp(:aL:)\exp(:H_4/2:)\exp(:bL:)\exp(:F_{\text{out}}:),$$

where  $L$  and  $H_4$  are the Hamiltonians of a drift of length  $L$  and a thin decapole kick with integrated strength  $K_4$ :

$$H_4 = \frac{K_4}{5!} \Re(x - iy)^5,$$

respectively. The coeffients are  $a \equiv 1/2 - 1/\sqrt{12}$  and  $b = 1/2 - a$ . Terms  $\exp(:F_{\text{in}}:)$  and  $\exp(:F_{\text{out}}:)$  are transformation nonlinear fringes. The term  $\exp(:V_4:)$  is a correction to adjust the third-order terms in  $L$ :

$$V_4 = \sum_{j=(x,y),k=(x,y)} -\frac{\beta}{2} H_{4,k}^2 + \gamma H_{4,j} H_{4,k} H_{4,j,k},$$

where  $i$  represents the derivative by  $x$  or  $y$ . We have also introduced two coefficients  $\beta \equiv 1/6 - 1/\sqrt{48}$  and  $\gamma = 1/40 - 1$