

The primary position variables are  $(x, y, s)$ , where  $x$  and  $y$  are the displacements along the normal and binormal vectors,  $\mathbf{n}$  and  $\mathbf{b}$ , respectively. Let  $\mathbf{t}$  denote the tangential vector along  $s$ , then  $\mathbf{n}$ ,  $\mathbf{b}$ ,  $\mathbf{t}$  consist a right-handed system. The action in  $t$  is expressed by

$$S = \int L_t c dt, \quad (2)$$

$$L_t = -\frac{mc}{p_0} \sqrt{1 - \dot{x}^2 + \dot{y}^2 + (1 + x/\rho)^2 \dot{s}^2} + a_x \dot{x} + a_y \dot{y} + (1 + x/\rho) a_s \dot{s}, \quad (3)$$

where  $p_0$  and  $(a_x, a_y, a_z) = e(A_x, A_y, A_z)/p_0$  are the design momentum and the normalized vector potentials, respectively, and  $\dot{\phantom{x}}$  denotes the derivative by  $ct$ . SAD's coordinate only has the radius of curvature  $\rho$  in the local  $x$ - $s$  plane. Note that  $\rho$  is the curvature of the coordinate system, not that of the orbit. The transverse vector potentials  $(a_x, a_y)$  are non-zero only in the solenoid region, where  $1/\rho$  is zero.

Currently SAD does not handle the electrostatic potential.

As SAD uses  $s$  for the independent variable instead of  $t$ , the Lagrangean  $L$  for  $s$  is written as

$$L = L_t \frac{dct}{ds}, \quad (4)$$

$$= -\frac{mc}{p_0} \sqrt{c^2 t'^2 - x'^2 + y'^2 + (1 + x/\rho)^2} + a_x x' + a_y y' + (1 + x/\rho) a_s, \quad (5)$$

where  $'$  is the derivative by  $s$ .