Performance Evaluation of Dynamic Scheduling in a Hospital Environment

Daniele Barzazzi Andrea Cester Enrico Siviero

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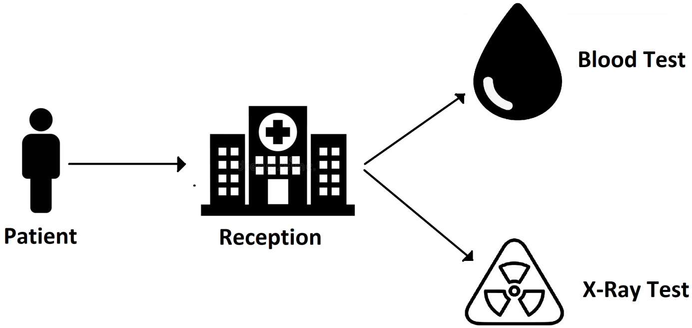
Introduction



Components and actions

Component	Action	Cooperation with
PatientB	arriveB	-
	register	Reception
	blood	Blood
	wait	Wait
PatientX	arriveX	-
	register	Reception
	xray	Xray
	wait	Wait
PatientBX	arriveB	-
	register	Reception
	blood	Blood
	xray	Xray
	wait	Wait

Components and actions

Blood	blood	PatientB PatientX PatientBX
Xray	xray	PatientB PatientX PatientBX
Reception	register	PatientB PatientX PatientBX
DecideB	decideB	PatientBX
DecideX	decideX	PatientBX
Wait	wait	PatientB PatientX PatientBX

Renaming rate

- $r\_blood \longrightarrow \mu_0$ and $r\_xray \longrightarrow \mu_1$
- $r\_arrive\_b \longrightarrow \lambda_0$, $r\_arrive\_x \longrightarrow \lambda_1$ and $r\_arrive\_bx \longrightarrow \lambda_2$
- $r\_register \longrightarrow \beta$
- $r\_wait \longrightarrow \delta$
- $r\_decideB \longrightarrow \gamma_0$ and $r\_decideX \longrightarrow \gamma_1$

PEPA Model

PatientB

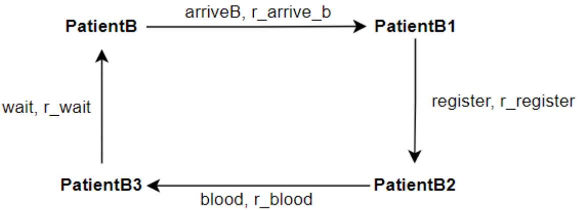
PatientB

$$PatientB \stackrel{\text{def}}{=} (arriveB, r\_arrive\_b).PatientB1$$

$$PatientB1 \stackrel{\text{def}}{=} (register, r\_register).PatientB2$$

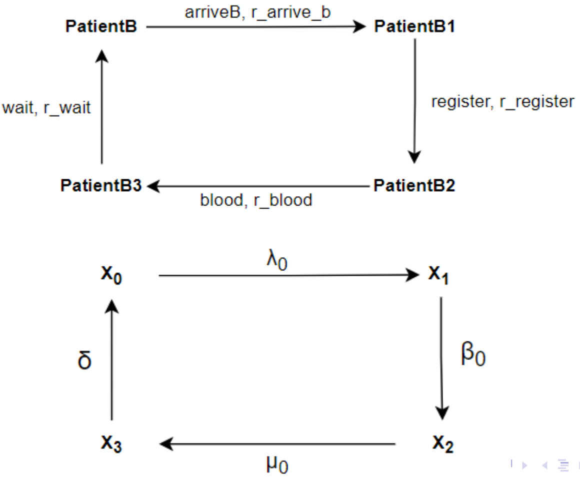
$$PatientB2 \stackrel{\text{def}}{=} (blood, r\_blood).PatientB3$$

$$PatientB3 \stackrel{\text{def}}{=} (wait, r\_wait).PatientB$$



PEPA Model

PatientB - Markov Chains



PEPA Model

PatientB - Markov Chains

$$\mathbf{Q}= \begin{array}{c|cccc} & X_0 & X_1 & X_2 & X_3 \\ X_0 & -\lambda_0 & \lambda_0 & 0 & 0 \\ X_1 & 0 & -\beta & \beta & 0 \\ X_2 & 0 & 0 & -\mu_0 & \mu_0 \\ X_3 & \delta & 0 & 0 & -\delta \end{array}$$

$$Q_N^T = \begin{pmatrix} -\lambda_0 & 0 & 0 & \delta \\ \lambda_0 & -\beta & 0 & 0 \\ 0 & \beta & -\mu_0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

PEPA Model

PatientB - Markov Chains

Global balance equation:

$$\begin{cases} -\lambda_0\pi_0 + \delta\pi_3 = 0 \\ \lambda_0\pi_0 - \beta\pi_1 = 0 \\ \beta\pi_1 - \mu_0\pi_2 = 0 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

Global balance equation:

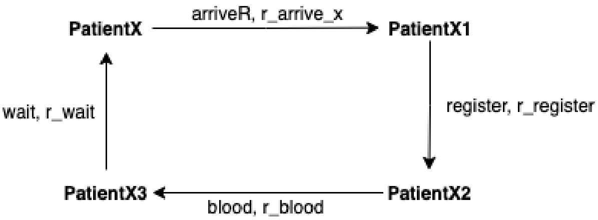
$$\begin{cases} \pi_0 = 0.21126761 \\ \pi_1 = 0.04225352 \\ \pi_2 = 0.70422535 \\ \pi_3 = 0.04225352 \end{cases}$$

PEPA Model

PatientX

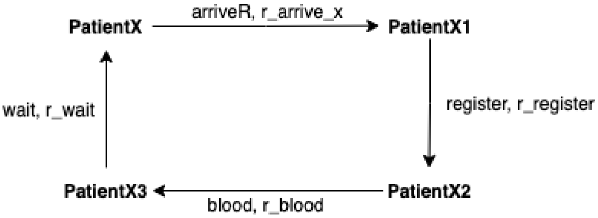
PatientX

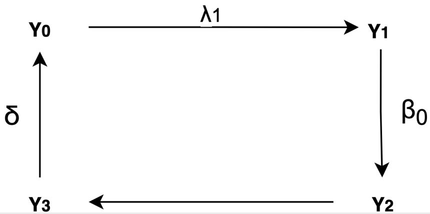
$$PatientX \stackrel{\text{def}}{=} (arriveX, r\_arrive\_x).PatientX1$$
$$PatientX1 \stackrel{\text{def}}{=} (register, r\_register).PatientX2$$
$$PatientX2 \stackrel{\text{def}}{=} (blood, r\_blood).PatientX3$$
$$PatientX3 \stackrel{\text{def}}{=} (wait, r\_wait).PatientX$$



PEPA Model

PatientX - Markov Chains





PEPA Model

PatientX - Markov Chains

$$\mathbf{Q}= \begin{array}{c|cccc} & Y_0 & Y_1 & Y_2 & Y_3 \\ \hline Y_0 & -\lambda_1 & \lambda_1 & 0 & 0 \\ Y_1 & 0 & -\beta & \beta & 0 \\ Y_2 & 0 & 0 & -\mu_1 & \mu_1 \\ Y_3 & \delta & 0 & 0 & -\delta \end{array}$$

$$Q_N^T = \begin{pmatrix} -\lambda_1 & 0 & 0 & \delta \\ \lambda_1 & -\beta & 0 & 0 \\ 0 & \beta & -\mu_1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

PEPA Model

PatientX - Markov Chains

Global balance equation:

$$\begin{cases} -\lambda_1\pi_0 + \delta\pi_3 = 0 \\ \lambda_1\pi_0 - \beta\pi_1 = 0 \\ \beta\pi_1 - \mu_1\pi_2 = 0 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

PEPA Model

PatientX - Markov Chains

Global balance equation:

$$\begin{cases} \pi_0 = 0.15625 \\ \pi_1 = 0.03125 \\ \pi_2 = 0.78125 \\ \pi_3 = 0.03125 \end{cases}$$

PEPA Model

PatientBX

PatientBX

$PatientBX \stackrel{\text{def}}{=} (arriveBX, r\_arrive\_bx).PatientBX1$

$PatientBX1 \stackrel{\text{def}}{=} (register, r\_register).PatientBX2$

$PatientBX2 \stackrel{\text{def}}{=} (decideB, \tau).PatientBX3 +$
$(decideX, \tau).PatientBX4$

$PatientB3 \stackrel{\text{def}}{=} (blood, r\_blood).PatientBX5$

$PatientB5 \stackrel{\text{def}}{=} (xray, r\_xray,).PatientBX6$

$PatientB6 \stackrel{\text{def}}{=} (wait, r\_wait).PatientBX$

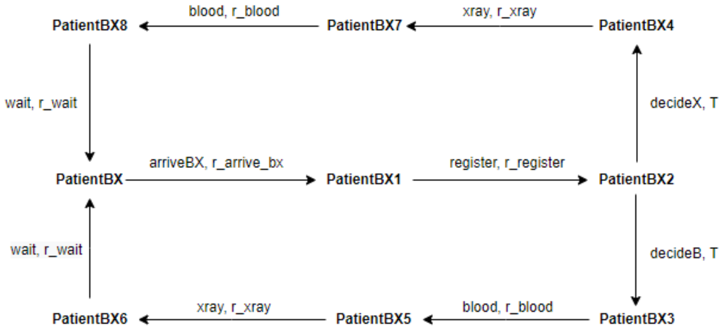
$PatientB4 \stackrel{\text{def}}{=} (xray, r\_xray).PatientBX7$

$PatientB5 \stackrel{\text{def}}{=} (blood, r\_blood).PatientBX8$

$PatientB3 \stackrel{\text{def}}{=} (wait, r\_wait).PatientBX$

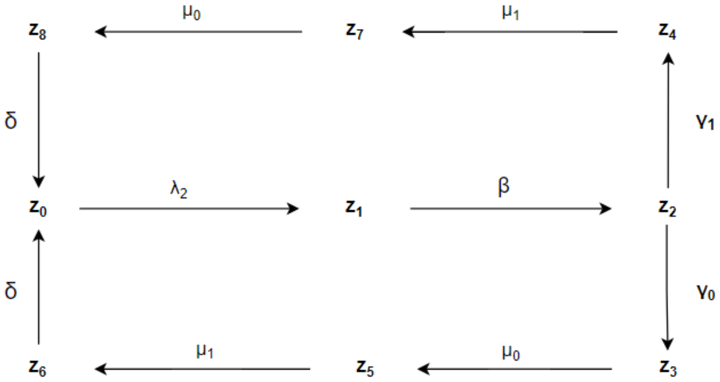
PEPA Model

PatientBX



PEPA Model

PatientBX - Markov Chains



PEPA Model

PatientBX - Markov Chains

$$
\mathbf{Q} = \begin{array}{c|ccccccccc}
 & Z_0 & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 & Z_7 & Z_8 \\
Z_0 & -\lambda_2 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_1 & 0 & -\beta & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\
Z_2 & 0 & 0 & -(\gamma_0 + \gamma_1) & \gamma_0 & \gamma_1 & 0 & 0 & 0 & 0 \\
Z_3 & 0 & 0 & 0 & -\mu_0 & 0 & \mu_0 & 0 & 0 & 0 \\
Z_4 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & \mu_1 & 0 \\
Z_5 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & \mu_1 & 0 & 0 \\
Z_6 & \delta & 0 & 0 & 0 & 0 & 0 & -\delta & 0 & 0 \\
Z_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & \mu_0 \\
Z_8 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta
\end{array}
$$

PEPA Model

PatientX - Markov Chains

$$Q_N^T = \begin{pmatrix} -\lambda_2 & 0 & 0 & 0 & 0 & 0 & \delta & 0 & \delta \\ \lambda_2 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & -(\gamma_0+\gamma_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_0 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_1 & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_1 & -\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & -\mu_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

PEPA Model

PatientX - Markov Chains

Global balance equation:

$$
\begin{cases}
-\lambda_2\pi_0 + \delta\pi_6 + \delta\pi_8 = 0 \\
\lambda_2\pi_0 - \beta\pi_1 = 0 \\
\beta\pi_1 - (\gamma_0 + \gamma_1)\pi_2 = 0 \\
\gamma_0\pi_2 - \mu_0\pi_3 = 0 \\
\gamma_1\pi_2 - \mu_1\pi_4 = 0 \\
\mu_0\pi_3 - \mu_1\pi_5 = 0 \\
\mu_1\pi_5 - \delta\pi_6 = 0 \\
\mu_1\pi_4 - \mu_0\pi_7 = 0 \\
\pi_0 + \pi_1\pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 = 1
\end{cases}
$$

PEPA Model

PatientX - Markov Chains

Global balance equation:

$$\begin{cases} \pi_0 = 0.0047679593 \\ \pi_1 = 0.0009535919 \\ \pi_2 = 0.9535918627 \\ \pi_3 = 0.0079465989 \\ \pi_4 = 0.0119198983 \\ \pi_5 = 0.0119198983 \\ \pi_6 = 0.0004767959 \\ \pi_7 = 0.0079465989 \\ \pi_8 = 0.0004767959 \end{cases}$$

PEPA Model

Resources - Blood

Blood

$Blood \stackrel{\text{def}}{=} (blood, r\_blood).Blood$

PEPA Model

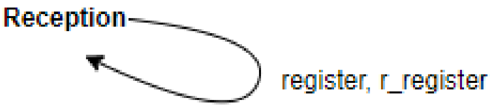
Resources - Xray

Xray

$$Xray \stackrel{\text{def}}{=} (xray, r\_xray).Xray$$

Reception

$$Reception \stackrel{\text{def}}{=} (register, r\_register).Reception$$



PEPA Model

Support components - Decide B

DecideB

$$DecideB \stackrel{\text{def}}{=} (decideB, r\_db).DecideB$$

PEPA Model

Support components - Decide X

DecideX

$$DecideX \stackrel{\text{def}}{=} (decideX, r\_dx).DecideX$$

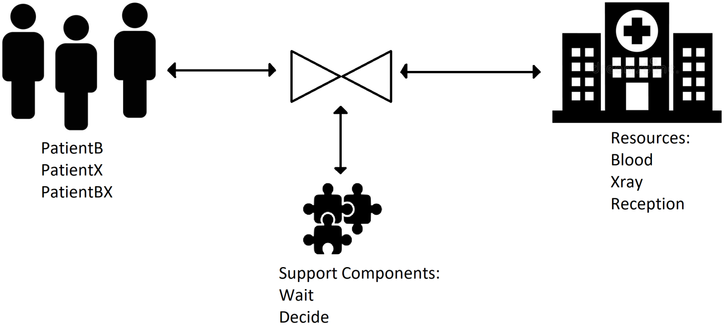
PEPA Model

Support components - Wait

Wait

$Wait \stackrel{\text{def}}{=} (wait, r\_wait).Wait$

Cooperation Model



Cooperation Model

Model from the paper

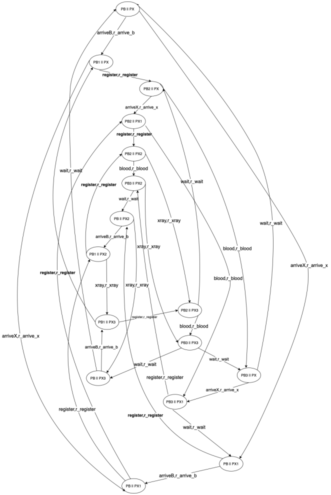
PatientB[n1] || PatientX[n2] || PatientBX[n3] $\bowtie_L$
Blood || Xray || Reception || DecideB || DecideX || Wait
L = {register, blood, xray, decideB, decideX, wait}

Cooperation Model

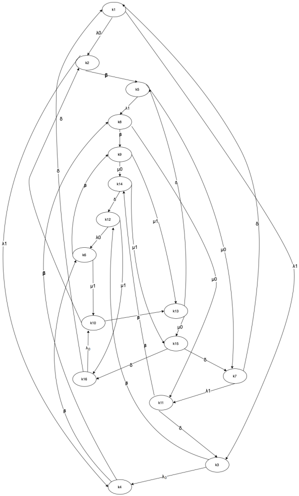
Reduced model

PatientB $||$ PatientX $\bowtie_L$ Blood $||$ Xray $||$ Reception $||$ Wait
L $= \{$register, blood, xray, wait$\}$

Cooperation Model



Cooperation Model



Cooperation Model

		k1	k2	k3	k4	k5	k6	k7	k8	k9	10	k11	k12	k13	k14	k15	k16
	k1	$-(\lambda_0+\lambda_1)$	λ_0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0
	k2	0	$-(\lambda_1+\beta)$	0	λ_1	β	0	0	0	0	0	0	0	0	0	0	0
	k3	0	0	$-(\lambda_0+\beta)$	λ_0	0	0	0	0	0	0	0	β	0	0	0	0
	k4	0	0	0	-2β	0	β	0	β	0	0	0	0	0	0	0	0
	k5	0	0	0	0	$-(\mu_0+\lambda_1)$	0	μ_0	λ_1	0	0	0	0	0	0	0	0
	k6	0	0	0	0	0	$-(\beta+\mu_1)$	0	0	β	μ_1	0	0	0	0	0	0
	k7	δ	0	0	0	0	0	$-(\delta+\lambda_1)$	0	0	0	λ_1	0	0	0	0	0
Q	k8	0	0	0	0	0	0	0	$-(\beta+\mu_0)$	β	0	μ_0	0	0	0	0	0
	k9	0	0	0	0	0	0	0	0	$-(\mu_1+\mu_0)$	0	0	0	μ_1	μ_0	0	0
	k10	0	δ	0	0	0	0	0	0	0	$-(\delta+\beta)$	0	0	β	0	0	0
	k11	0	0	δ	0	0	0	0	0	0	0	$-(\delta+\beta)$	0	0	β	0	0
	k12	0	0	0	0	0	λ_0	0	0	0	0	0	$-(\lambda_0+\mu_1)$	0	0	0	μ_1
	k13	0	0	0	0	δ	0	0	0	0	0	0	0	$-(\delta+\mu_0)$	0	μ_0	0
	k14	0	0	0	0	0	0	0	0	0	0	0	δ	0	$-(\delta+\mu_1)$	μ_1	0
	k15	0	0	0	0	0	0	δ	0	0	0	0	0	0	0	-2δ	δ
	k16	δ	0	0	0	0	0	0	0	0	λ_0	0	0	0	0	0	$-(\delta+\lambda_0)$

Cooperation Model

$$
Q_N^T \quad
\begin{array}{c|cccccccccccccccc}
 & \pi_1 & \pi_2 & \pi_3 & \pi_4 & \pi_5 & \pi_6 & \pi_7 & \pi_8 & \pi_9 & \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & \pi_{15} & \pi_{16} \\
\hline
\pi_1 & -(\lambda_0+\lambda_1) & 0 & 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta \\
\pi_2 & \lambda_0 & -(\lambda_1+\beta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi_3 & \lambda_1 & 0 & -(\lambda_0+\beta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 \\
\pi_4 & 0 & \lambda_1 & \lambda_0 & -2\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi_5 & 0 & \beta & 0 & 0 & -(\mu_0+\lambda_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta & 0 & 0 & 0 \\
\pi_6 & 0 & 0 & 0 & \beta & 0 & -(\beta+\mu_1) & 0 & 0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 \\
\pi_7 & 0 & 0 & 0 & 0 & \mu_0 & 0 & -(\delta+\lambda_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta & 0 \\
\pi_8 & 0 & 0 & 0 & \beta & \lambda_1 & 0 & 0 & -(\beta+\mu_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi_9 & 0 & 0 & 0 & 0 & 0 & \beta & 0 & \beta & -(\mu_1+\mu_0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi_{10} & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & -(\delta+\beta) & 0 & 0 & 0 & 0 & 0 & \lambda_0 \\
\pi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & \mu_0 & 0 & 0 & -(\delta+\beta) & 0 & 0 & 0 & 0 & 0 \\
\pi_{12} & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda_0+\mu_1) & 0 & \delta & 0 & 0 \\
\pi_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & \beta & 0 & 0 & -(\delta+\mu_0) & 0 & 0 & 0 \\
\pi_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_0 & 0 & \beta & 0 & 0 & -(\delta+\mu_1) & 0 & 0 \\
\pi_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_0 & \mu_1 & -2\delta & 0 \\
\pi_{16} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
$$

Cooperation Model

Global balance equation:

$$
\begin{cases}
-(\lambda_0 + \lambda_1)\pi_1 + \delta\pi_7 + \delta\pi_{16} = 0 \\
\lambda_0\pi_1 + -(\lambda_1 + \beta)\pi_2 + \delta\pi_{10} = 0 \\
\lambda_1\pi_1 - (\lambda_0 + \beta)\pi_3 + \delta\pi_{11} = 0 \\
\lambda_1\pi_2 + \lambda_0\pi_3 - 2\beta\pi_4 = 0 \\
\beta\pi_2 - (\mu_0 + \lambda_1)\pi_5 + \delta\pi_{13} = 0 \\
\beta\pi_4 - (\beta + \mu_1)\pi_6 + \lambda_0\pi_{12} = 0 \\
\mu_0\pi_5 - (\delta + \lambda_1)\pi_7 + \delta\pi_{15} = 0 \\
\beta\pi_4 + \lambda_1\pi_5 - (\beta + \mu_0)\pi_8 = 0 \\
\beta\pi_6 + \beta\pi_8 - (\mu_1 + \mu_0)\pi_9 = 0 \\
\mu_1\pi_6 - (\delta + \beta)\pi_{10} + \lambda_0\pi_{16} = 0 \\
\lambda_1\pi_7 + \mu_0\pi_8 - (\delta + \beta)\pi_{11} = 0 \\
\beta\pi_3 - (\lambda_0 + \mu_1)\pi_{12} + \delta\pi_{14} = 0 \\
\mu_1\pi_9 + \beta\pi_{10} - (\delta + \mu_0)\pi_{13} = 0 \\
\mu_0\pi_9 + \beta\pi_{11} - (\delta + \mu_1)\pi_{14} = 0 \\
\mu_0\pi_{13} + \mu_1\pi_{14} - 2\delta\pi_{15} = 0 \\
\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 + \pi_8 + \pi_9 + \pi_{10} + \pi_{11} + \pi_{12} + \pi_{13} + \pi_{14} + \pi_{15} + \pi_{16} = 1
\end{cases}
$$

Cooperation Model

Result of the global balance equation:

$$
\begin{cases}
\pi_1 = 0.033010563 \\
\pi_2 = 0.006602113 \\
\pi_3 = 0.006602113 \\
\pi_4 = 0.001320423 \\
\pi_5 = 0.110035211 \\
\pi_6 = 0.033010563 \\
\pi_7 = 0.006602113 \\
\pi_8 = 0.022007042 \\
\pi_9 = 0.550176056 \\
\pi_{10} = 0.001320423 \\
\pi_{11} = 0.001320423 \\
\pi_{12} = 0.165052817 \\
\pi_{13} = 0.022007042 \\
\pi_{14} = 0.033010563 \\
\pi_{15} = 0.001320423 \\
\pi_{16} = 0.006602113
\end{cases}
$$

Steady State Probability

Utilisation

$$\mathrm{U}_{Blood} = \pi_5 + \pi_8 + \pi_9 + \pi_{13} = 70\%$$

$$\mathrm{U}_{Xray} = \pi_9 + \pi_{12} + \pi_{14} = 64\%$$

$$\mathrm{U}_{Reception} = \pi_2 + \pi_3 + \pi_4 + \pi_6 + \pi_8 + \pi_{10} + \pi_{11} = 7\%$$

$$\mathrm{U}_{Wait} = \pi_6 + \pi_{10} + \pi_{11} + \pi_{13} + \pi_{14} + \pi_{15} + \pi_{16} = 7\%$$

Steady State Probability

Throughput

$$\mathrm{T}_{\mathit{Blood}} = \mu_0 \cdot (\pi_5 + \pi_8 + \pi_9 + \pi_{13}) = 84.507 \sim 0.012 \text{ msec}$$

$$\mathrm{T}_{\mathit{Xray}} = \mu_1 \cdot (\pi_6 + \pi_9 + \pi_{12} + \pi_{14}) = 62.499 \sim 0.016 \text{ msec}$$

$$\mathrm{T}_{\mathit{Reception}} = \beta \cdot (\pi_2 + \pi_3 + \pi_4 + \pi_6 + \pi_8 + \pi_{10} + \pi_{11}) = 144.366 \sim 0.007 \text{ msec}$$

$$\mathrm{T}_{\mathit{Wait}} = \delta \cdot (\pi_6 + \pi_{10} + \pi_{11} + \pi_{13} + \pi_{14} + \pi_{15} + \pi_{16}) = 144.366 \sim 0.007 \text{ msec}$$

PEPA Evaluation

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PEPA Evaluation

```
r_arrive_b=400.0;
r_arrive_X=400.0;
r_register=2000.0;

r_wait=2000.0;
r_blood=120.0;
r_xray=80.0;

PatientB=(arriveB,r_arrive_b).PatientB1;
PatientB1=(register,r_register).PatientB2;
PatientB2=(blood,r_blood).PatientB3;
PatientB3=(wait,r_wait).PatientB;

PatientX=(arriveX,r_arrive_X).PatientX1;
PatientX1=(register,r_register).PatientX2;
PatientX2=(xray,r_xray).PatientX3;
PatientX3=(wait,r_wait).PatientX;

Blood=(blood,r_blood).Blood;
Xray=(xray,r_xray).Xray;
Reception=(register,r_register).Reception;

Wait=(wait,r_wait).Wait;

(PatientB || PatientX) <register,blood,xray,wait> (Blood||Xray||Reception||Wait)
```

PEPA Evaluation

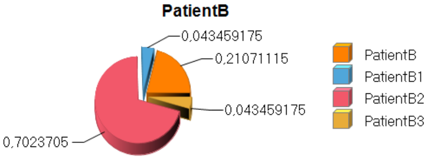
Steady state probability

1	PatientB	PatientX	Blood	Xray	Reception	Wait	0.03292361720807725
2	PatientB1	PatientX	Blood	Xray	Reception	Wait	0.006584723441615451
3	PatientB	PatientX1	Blood	Xray	Reception	Wait	0.006584723441615451
4	PatientB2	PatientX	Blood	Xray	Reception	Wait	0.10974539069359084
5	PatientB1	PatientX1	Blood	Xray	Reception	Wait	0.0026338893766461808
6	PatientB	PatientX2	Blood	Xray	Reception	Wait	0.16461808604038627
7	PatientB3	PatientX	Blood	Xray	Reception	Wait	0.0065847234416154515
8	PatientB2	PatientX1	Blood	Xray	Reception	Wait	0.021949078138718173
9	PatientB1	PatientX2	Blood	Xray	Reception	Wait	0.03292361720807725
10	PatientB	PatientX3	Blood	Xray	Reception	Wait	0.006584723441615451
11	PatientB3	PatientX1	Blood	Xray	Reception	Wait	0.0013169446883230904
12	PatientB2	PatientX2	Blood	Xray	Reception	Wait	0.5487269534679543
13	PatientB1	PatientX3	Blood	Xray	Reception	Wait	0.0013169446883230902
14	PatientB3	PatientX2	Blood	Xray	Reception	Wait	0.03292361720807725
15	PatientB2	PatientX3	Blood	Xray	Reception	Wait	0.02194907813871817
16	PatientB3	PatientX3	Blood	Xray	Reception	Wait	0.002633889376646181

PEPA Evaluation

PatientB - PatientX - Steady state distribution

$$
\begin{cases}
\pi_0 = 0.21126761 \\
\pi_1 = 0.04225352 \\
\pi_2 = 0.70422535 \\
\pi_3 = 0.04225352
\end{cases}
$$



PatientB (4 local states)

PatientB = 0.21071115013169442

PatientB1 = 0.04345917471466198

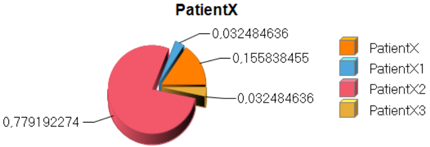
PatientB2 = 0.7023705004389815

PatientB3 = 0.04345917471466197

PEPA Evaluation

PatientX - Steady state distribution

$$
\begin{cases}
\pi_0 = 0.15625 \\
\pi_1 = 0.03125 \\
\pi_2 = 0.78125 \\
\pi_3 = 0.03125
\end{cases}
$$



PatientX (4 local states)

PatientX = 0.155838454784899

PatientX1 = 0.0324846356453029

PatientX2 = 0.779192273924951

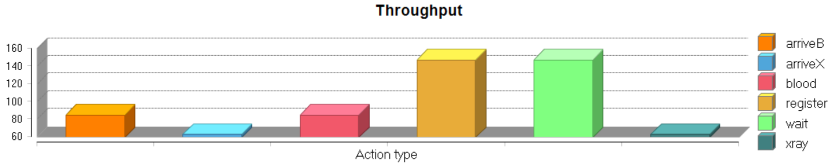
PatientX3 = 0.0324846356453028 9

PEPA Evaluation

Throughput

$$T_{Blood} = 84.50704212$$
$$T_{Xray} = 62.49999992$$
$$T_{Reception} = 144.3662$$
$$T_{Wait} = 144.3662$$

Action	Throughput
arriveB	84.28446005267777
arriveX	62.335381913959594
blood	84.28446005267779
register	146.61984196663735
wait	146.61984196663735
xray	62.33538191395961



Lumpability

Let X(t) be a CTMC with state space S and $\sim$ be an equivalence relation over S.

We say that X(t) is **strongly lumpable** with respect to if for any $[k] \neq [l]$ and $i, j \in [l]$, it holds that:

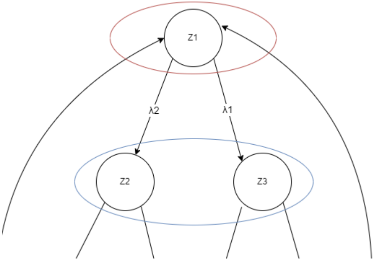
$$q_{i[k]} = q_{j[k]}, i.e. \sum_{h \in [k]} q_{ih} = \sum_{h \in [k]} q_{jh}$$

Also, we say that X(t) is exactly lumpable with respect to if for any $[k], [l] \in S/ \ andi, j \in [l]$, it holds that:

$$q_{k[i]} = q_{k[j]}, i.e. \sum_{h \in [k]} q_{hi} = \sum_{h \in [k]} q_{hj}$$

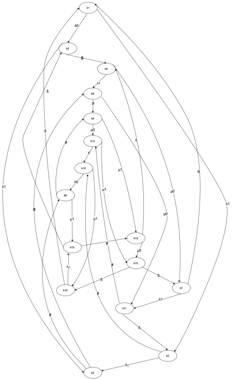
Lumpability Example

Take in consideration the Markov chain graph:
$[l] = 1, [k] = z2, z3$
If we test the definition of strongly lumpability we have:
$q_{z1,[2]} = \lambda 1 \neq \lambda 2 = q_{z4,[z5,z7]}$
This first try of strong lumpability does not respect the propriety.



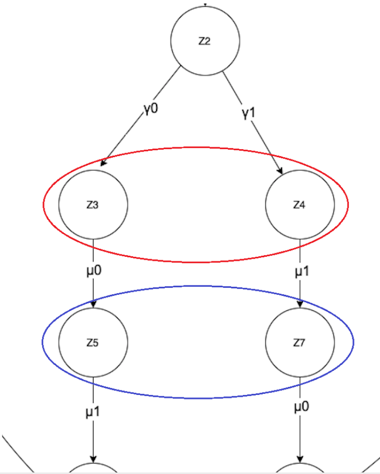
Lumpability Example

We can say that the markov chain graph can have few simplifications since most of the graph does not respect the properties of strong and exact lumbability.



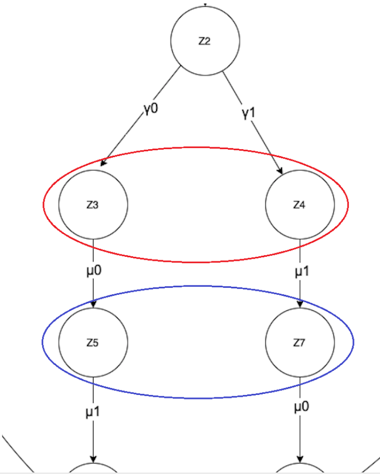
Lumpability Example

Instead of the main graph we choose to take in consideration the components PBX as example:
$[l] = z3, z4, [k] = z5, z7$
If we test the definition of strongly lumpability we have:
$q_{z3,[z5,z7]} = q_{z4,[z5,z7]} = \mu_0 + 0$



Lumpability Example

Take in consideration the same components as example:
$[l] = z3, z4, [k] = z5, z7$
If we test the definition of exact lumpability we have:
$q_{[z3,z4],z5} = q_{[z3,z4],z7} = \mu_0$
$q_{[z3,z4],z3} = q_{[z3,z4],z4} = 0$



Lumpability Example

Strong lumpability:

$q_{z2,[z3,z4]} = \gamma_0 + \gamma 1$

$q_{z3,[z5,z7]} = \mu_0 = q_{z4,[z5,z7]}$

$q_{z5,[z6,z8]} = \mu_0 = q_{z7,[z6,z8]}$

$q_{z6,[z1]} = q_{z8,[z1]} = 2\delta$

Exact Lumpability

Lumpability Example