A network characterization of the interbank exposures in Peru☆

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ARTICLE INFO	ABSTRACT
	After the Global Financial Crisis (GFC), systemic risk measurement became crucial for policy makers as well as for academics. We have witnessed an important increase in the number of methodologies proposed. Among such proposals, DebtRank arose as perhaps one of the most relevant in this context, as it resorts to network modeling and captures the all-important aspect of interconnectedness in the financial system. Additionally, within the network modeling approach, there is the multilayer approach, which provides additional insights on the decomposition of systemic risk. In this paper, we apply a multilayer network analysis to study systemic risk in the Peruvian banking system by utilizing DebtRank centrality. The main contributions of this work are as follows: i) It fully characterizes the multilayer exposure network of the Peruvian banking system, and ii) it obtains the systemic risk profile of the banking system according to different types of exposures.

1. Introduction

After the Global Financial Crisis (GFC), several authors suggested that new tools for quantifying and monitoring risk in the financial system were needed and that these tools should account for interconnections and interactions between different financial institutions (see, for instance, Battiston et al., 2016b; Haldane, 2013; Haldane and May, 2011; May et al., 2008). Methodologically, there was an important gap between the risk models developed until then and the models needed to deal with, and understand, the crisis. This was also true of the regulatory framework, as it became evident that a comprehensive approach to financial regulation was badly needed. Moreover, the identification of systemically important banks (SIBs) became a priority in many jurisdictions. The same happened with non-bank financial intermediaries, also known as systemically important financial institutions (SIFIs). Network theory provided the necessary means to study interconnectedness and design novel systemic risk metrics. This work fills a gap in the study of connectivity and contagion risk in Peru's financial system.

The risk of contagion in the interbank market is a key component of systemic risk in any financial system Bank for International Settlements (2014). With this in mind, it is imperative to quantify systemic risk and systemic relevance to design adequate regulations and monitor financial stability. In Peru, the first attempt to define systemic relevance was made by Espino and Rabanal (2011). However, only the size component was considered without taking into account the crucial aspect of interconnectedness. In this

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paper, we present a comprehensive study of the connectivity of Peru's financial system, including a systemic risk analysis utilizing the DebtRank centrality metric. Only a few studies in Latin America and the Caribbean had previously undertaken such a complete examination of the financial system.

In recent years, the study of network structures in economics and finance has grown (see, for instance, the theoretical contributions of Ashcraft and Duffie (2007), Acemoglu et al. (2015), Acemoglu et al. (2011) and Elliott et al. (2014a). See also Caccioli et al. (2017); Young and Glasserman (2016) for recent surveys on the application of network models for systemic risk analysis.)

Regarding the theoretical framework of the relationship between the structure of the financial network and systemic risk, Acemoglu et al. (2015) showed that a more complete network can mitigate losses from smaller negative shocks. This guarantees a more efficient use of excess liquidity for future defaults. If negative shocks are large enough, high connectivity no longer ensures stability. In fact, entities that are less connected to the network are the less affected. Concerning how shocks propagate through the network, Acemoglu et al. (2011) found, on the one hand, that a perfectly diversified pattern of holdings becomes optimal for moderate shocks. On the other hand, perfectly diversified holdings under large shocks become an unfavorable element.

Additionally, for intermediate levels of connectivity, Elliott et al. (2014a) found that integration (i.e., greater dependence per agent or node) and diversification (i.e., more counterparties per agent or node) have different, non-monotonic effects on the extent of cascades. Diversification connects the network initially, permitting cascade transmission; however as it increases further, agents or nodes are better insured against other participants' failures. Integration also faces trade-offs: increased dependence on other organizations versus less sensitivity to personal investments. Similar non-monotonic behaviors had been previously reported by Gai and Kapadia (2010) in the context of counterparty contagion risk and by Watts (2002) for global cascades on networks.

Several empirical studies have also been carried out. In particular recent studies have focused on what Battiston and Martinez-Jaramillo (2018) dubbed the "no-contagion paradox". That is, while during crises, the fear of contagion mounts, many of the early empirical papers show small contagion effects. This was mainly due to the limited set of exposures used in such contagion studies, as well as by some modelling choices such as the maximum entropy algorithm, used to estimate the exposure network from balance sheet data. Furthermore, banks and financial institutions interact in many different markets that can be abstracted into different interacting networks.

This paper examines Perus interbank exposures beyond unsecured lending and estimates systemic risk and systemic relevance for the banking system. We use the DebtRank algorithm to include an estimation of financial contagion effects. The comprehensive data set used for this purpose includes six layers of interbank exposures, all provided by the Central Reserve Bank of Peru (BCRP). Additionally, we perform a complete structural analysis of all interbank exposure networks, comprising a description of the network structure by using some well-known metrics and community identification by identifying the core and peripheral nodes. Furthermore, to disentangle the real level of interconnectedness in Perus banking system, we resort to a structural description of the multiplex defined by such layers of exposures. Multiplex networks (Kivela et al., 2014) arise naturally in many systems beyond the financial world (e.g., transportation and physical infrastructure systems, and social networks and biological systems, to name just a few).

The multilayer network approach has already been used in the context of financial systems, but this is one of the first works in which the structural properties of the aggregate exposure network, as well as its multiplex structure, are described simultaneously.

In this work, Perus interbank exposures are described by different networks, each associated with a specific type of exposure. More specifically, there are networks associated with unsecured short- and long-term interbank loans and deposits, networks arising from derivative exposures, and networks formed by the cross-holding of securities. These exposures define six different networks, which compose the multiplex structure of Perus banking system.

In the second step, all layers were integrated into the single aggregate network of exposures. Importantly, this simple aggregation process is adequate because all of the layers are built around the same concept (i.e., exposure at default). If the layers of the interbank multiplex structure represented different concepts besides exposure at default, this aggregation procedure would not be appropriate.

We perform a systemic risk analysis using the DebtRank algorithm Battiston et al. (2012). Other methods are available in the literature, such as the Furne and Eisenberg-Noe algorithms (Eisenberg and Noe, 2001; Furfine, 2003). However, these algorithms do not account for the propagation of distress before defaults occur (e.g., due to credit quality deterioration). This was shown to lead to weak contagion effects (Glasserman and Young, 2015; Upper, 2011). The DebtRank algorithm was introduced to consider these effects and was shown to be able to capture the buildup of systemic risk prior to the subprime crisis better than cascading failure models could (Battiston et al., 2012).

In addition to the characterization of systemic relevance by applying the DebtRank method to the aggregate exposure network, we identify systemically relevant institutions by using the multiplex structural analysis. This enables us to study systemic relevance and interconnectedness comprehensively.

An advantage of studying multiplex structures instead of single aggregate structures is that with single aggregate structures important aspects of the interaction among layers could be ignored or lost during the aggregation process. In fact, the literature shows that neglecting interactions between different layers can lead to an underestimation of systemic risk (as measured, for instance, in terms of systemic losses in stress-testing exercises; (Poledna et al., 2020)). We will see in Section 5 that this is the case also for Peru and that focusing on individual layers would significantly underestimate the systemic importance of some institutions. Moreover, by using the multilayer approach, it is possible to determine the contribution of each layer to systemic risk and relevance.

The structural analysis indicators used in this work for the single aggregate network include the degree distribution, the clustering coefficient, the reciprocity coefficient, the Herndahl-Hirschman Index, and the core-periphery size and error. For the multiplex structure, we derived the following metrics: the overlapping degree distribution, the multiplex group composition based on stochastic block models, node activity, and the relevance of different layers.

The main results of this work can be summarized as follows: The risk that comes from bank interconnection (contagion risk) is not very high in Peru. This aligns with (Upper, 2011) work, but, in the case of Peru, this is related to the small size of exposures among financial institutions relative to their equity and is not caused by data limitations or algorithmic assumptions. Additionally, we characterize the relative importance of different types of exposures. In particular, we show that, although it is true that the systemic importance of most institutions is dominated by unsecured exposures, neglecting the multiplex structure for some institutions could lead to an underestimation of up to 30% of their impact. Finally, we show that, although there is a correlation between contagion risk and the size of a financial institution, the position of an institution within the exposure network also contributes to its systemic importance. All these results show that it is necessary to complement the assessment of systemic importance with a comprehensive analysis of the connectivity role played by financial institutions.

The remaining part of the paper is organized as follows. Section 2 surveys the literature on network models and systemic risk measurement. Section 3 presents a general description of the data used in this work. Section 4 presents the methodological aspects of this work, from multiplex networks to the DebtRank algorithm. Section 5 presents the results and Section 6 concludes.

2. Literature review

The seminal works on default contagion date back to the beginning of the century (Allen and Gale, 2000; Boss et al., 2004; Eisenberg and Noe, 2001; Upper and Worms, 2004). However, the literature on this topic grew exponentially after the GFC and took a more rigorous approach (Acemoglu et al., 2016; Bardoscia et al., 2015; Elliott et al., 2014b; Gai and Kapadia, 2010; Glasserman and Young, 2015; Upper, 2011).

Most of these papers used only the interbank market layer as a relevant exposure. However, some papers proved the importance of considering many types of exposure among financial institutions. In this regard, Martinez-Jaramillo et al. (2014b) found that payment structures and exposure networks differ in their connectivity. Contagion is not necessarily related to asset size. For banks ranked high in terms of interconnectedness, it is important to determine their systemic importance in financial networks. Landaberry et al. (2021) estimated the contribution of intra-firm exposures to overall systemic risk. They found that ignoring intra-firm exposures results in an important underestimation of systemic risk.

Bargigli et al. (2015) provided a complete analysis of the multiplex structure for Italys interbank network. They found that the multiplex layers have several topological and metric properties that are layer-specific. In particular, overnight unsecured layers lead to the topological properties of the total network. Additionally, they concluded that focusing on one layer as a representative layer could lead to biased estimation of systemic risk because it offers little information about other layers. This could result in a misleading representation of the interlinkages in the interbank market.

Research on multilayered financial networks has been carried out using data from many countries and multiple methodologies. León et al. (2014) explored the main properties of the Colombian multiplex and the interacting financial networks. They examined the financial networks by jointly studying financial institutions and financial market infrastructures within an interacting network. They found that the connections between them are based on dependence links; that is, efficient behavior of financial market infrastructures is important to enable proper functioning of financial institutions and ensure financial stability.

Concerning Mexicos banking system, individual banks create varying systemic risk contributions from different layers, reflecting their different trading strategies. A number of smaller banks have systemic impact on the securities market only. The systemic risk contribution from the interbank (deposits and loans) and derivative markets is clearly smaller than that from the foreign exchange and securities markets. The systemic impact from the combined layers is always larger than the impact from the sum of the layers taken separately (Poledna et al., 2015). In Bolivias banking system, two layers of interaction (payment and interbank exposures) among financial institutions constitute two important sources of connectivity in the financial system (Caceres-Santos et al., 2020). Landaberry et al. (2021) estimated the intra-firm exposures network and its contribution to overall systemic risk. They found that ignoring intra-firm exposures results in an important underestimation of systemic risk.

These studies considered not only interconnections within the interbank market but also loans with longer maturities, derivatives, common securities, and stocks from other banks. These works have shown the risk of underestimating systemic risk when considering just one layer of exposure. The effects of financial contagion can be underestimated if the heterogeneity of financial exposures (Korniyenko et al., 2018) is not considered. Hüser et al. (2018) assessed the systemic risk from bail-in operations based on a multi-layered network model. They found no direct contagion to creditor banks. A bail-in significantly reshapes interbank linkages within specific seniority layers. In the same vein, a novel analysis examined how economic growth is influenced by interacting cross-country connections. For future economic growth, it is important to consider multidimensional rather than individual connectivity. This result has important implications; for example, policies will be more beneficial as long as they focus on more than one dimension (Gould et al., 2020).

One part of the literature has focused on studying the structure of financial networks using topological measures. Aldasoro and Alves (2018) performed a multilayer analysis of European bank exposures to identify the similarities between the structures of different layers. One important conclusion is that the layers of exposure are similar in the sense of having a core-periphery structure. A similar result was obtained by Berndsen et al. (2018), who used data from Colombias financial system and found that the general architecture of the different layers is stable. Concha et al. (2018) studied the structure of a multiplex financial network with data from Mexicos financial system. They found that the community structure of the system can be generalized as a core-periphery structure. Boss et al. (2004) explored the Austrian interbank market, in particular the interbank network topology, which shows power law dependencies in the degree distributions. They found a very low degree of separation between any two nodes in the system and a low clustering coefficient in the interbank network.

Another part of the literature has studied the effect of contagion and cascade effects. Within this strand of the literature, Montagna and Kok (2013) carried out an analysis based on the multilayered interbank network for European banks. The layers used were long- and short-term direct lending and common exposures on assets. According to this study, contagion effects, when considering different layers simultaneously, are greater than the sum of the contagion losses generated by each layer individually.

In Peru, the only research about the interconnection between banks was conducted by Espino and Rabanal (2011). The authors used the unsecured interbank market as the only relevant layer, calculating the losses that a bank failure can cause to other financial institutions. According to the authors, the greatest loss caused by the failure of an individual financial institution is 4.6% of equity in the system.

In this paper, we study the interconnection of financial institutions, taking a multilayer approach and using data from Perus financial system. We study the financial network structure of the interbank market and implement a DebtRank algorithm to quantify the systemic risk for each financial institution. Like Poledna et al. (2015), work, this paper adds evidence to the multilayer network studies, presenting an objective data-driven quantification of systemic risk on a national scale. Furthermore, this is the first attempt to analyze the multilayer structure of financial networks in Peru.

3. Data

Perus main financial system regulation authorities are the BCRP and the Superintendence of Banking, Insurance and Pension Funds (SBS). To achieve their goals, both regulatory authorities demand that financial institutions send them reports periodically. The frequencies and the content of these reports are different, as each institution has different goals. The BCRPs main objective is to preserve monetary stability, whereas the SBSs main goal is to supervise and regulate the financial system to preserve the interests of depositors, policyholders, and members of private pension funds.

The information used in this work was obtained from a database constituted by the regulatory reports submitted by financial institutions. The database contains information for 56 financial institutions, including commercial banks, investment banks, second floor multi-sectoral development banks, and microfinance institutions. It is important to note that small institutions are not weighted less than large institutions, because it could be possible that one small institution has several exposures with other financial institutions, making it systemically more important than a larger financial institution.

The data have two frequencies: daily and monthly. The daily data comprise only short-term credit transactions among financial institutions. This data set includes information on the amount and term of short-term credit transactions and which institutions were the borrowers and lenders in each transaction. These daily data are sent to the BCRP, which uses them to calculate the daily average interbank rate. Additionally, the BCRP uses this information to monitor the liquidity of the financial system and establish which institutions require funds to comply with reserve requirements. The BCRP has compiled this information from different sources since 2012. However, because of the growth of money market operations, in December 2017, the BCRP issued legislation to obtain more detailed information on financial institutions operations in the money market. For this reason, we use daily data since January 2018.

The monthly data comprise the outstanding exposures of different financial transactions among financial institutions. The exposures are short-term credit, long-term credit, demand and term deposits, security cross-holdings, and derivatives. These data were obtained from the monthly reports submitted by financial institutions to the SBS. As mentioned before, the SBS requires this information to assess, monitor, and quantify financial institutions risk exposures, such as credit, market, and liquidity risk. On a monthly basis, we consider the only data available (October, November, and December 2019).

Unfortunately, daily data are not available for each exposure. Data of such frequency would provide information about how financial institutions interrelate to each other in different markets. The only daily information available is short-term credit data.

For example, on a monthly basis, banks must maintain an average current account equivalent to reserve requirements. Therefore, at the beginning of the month, some banks are borrowers, whereas at the end of the month they become lenders. This interaction can be captured by daily data. Additionally, because of their accounting and risk policies, some financial institutions do not have credit exposures with other banks at the end of the month, but they do throughout the month. Therefore, considering monthly data may underestimate total exposure and interconnectedness in the financial system.

The daily and monthly data contain transactions in local and foreign currency. In Peru, financial institutions can hold assets and liabilities denominated in foreign currency, as Peru is a partially dollarized economy. Therefore, to calculate the credit risk exposures of each financial institution, we convert the deposits, loans, and cross-security holdings denominated in foreign currency to local currency. Then, we add both exposures (i.e., denominated in foreign and local currency) to calculate the total credit risk exposure of given financial institution. For example, if Bank A has lent USD 10 million and S/ 35 million to Bank B, we sum both exposures to calculate Bank As credit risk exposure to Bank B and express the result in thousands of local currency. In this example, Bank As exposure is S/ 70 million.

A limitation in this study is that we do not consider interconnections associated with indirect exposures from common asset holdings (overlapping portfolios), as the data were not available. Furthermore, monthly and daily data come from different sources, so there could be differences in exposure at the end of the month because of loan cancellations or early repayments.

As there are different types of interactions among entities in the financial system, in this paper, we model such distinct interactions by defining individual layers of different interbank relationships. Here, a layer refers to a particular type of exposure. Following Poledna et al. (2015), we define a multilayer network, $M = (V, \mathbf{W}, \alpha)$, where M has 56 nodes (financial institutions) and $V = \{1, 2, ..., 56\}$; $\alpha = \{1, 2, ..., 6\}$. The six different layers are short-term credit, long-term credit, demand deposits, term deposits, securities, and derivatives. α refers to the set of labels indicating the different layers. $\mathbf{W}$ is a set of matrices containing the set of node pairs in each layer, $\mathbf{W} = W^1, W^2, ..., W^6$. For instance, W_{ij}^1 refers to the exposure of institution i to institution j in layer 1, which represents

short-term credit outstanding exposure. In this paper, we use the multilayer network to capture the complexity of interconnectedness among commercial banks, investment banks, development banks, and microfinance institutions in Perus financial system. In the remainder of this section, we describe in detail how we obtained the different types of financial exposures used in this work.

3.1. Layers description and statistics

One of the most important layers for financial institutions in Peru is short-term credit. In Peru, interbank short-term (mostly overnight) loans between financial institutions are mainly unsecured. From January 2018 to December 2019, overnight credit transactions in the unsecured and secured markets represented 98% and 99% of the total traded in each market, respectively. In these markets, the main participants are banks, which perform transactions in both local and foreign currency. Specifically, the four largest banks are the main participants and concentrate the liquidity of the financial system. Nevertheless, smaller financial institutions usually make loans to institutions related to their financial cluster (usually the largest banks) and therefore can account for a significant counterpart risk.

To account for this layer, we consider the outstanding short-term unsecured loans exposure of each financial institution to its counterparts. Additionally, for the characterization of the interbank credit market and the calculation of the DebtRank algorithm, we use daily and monthly data, respectively.

Another source of risk is long-term credit or, for us, the monthly total debt outstanding (long-term unsecured loans) of each financial institution with its counterparts. Total debt outstanding corresponds to the interbank funding of financial institutions for terms over 90 days. This funding usually occurs between financial institutions that are members of a larger corporate conglomerate. For example, the larger banks benefit from demand deposits of people paid through the financial system. These deposits yield close to 0% interest, whereas the smaller banks need to offer term deposits with higher interest rates to attract funds. Some microfinancial institutions are subsidiaries or strategic commercial partners of the larger banks, so this long-term credit is a strategy to share the benefits of low funding of the larger banks among other institutions.

Demand and term deposits, which are outstanding deposits that each financial institution holds in another financial institution, are another exposure that is worth considering.

In addition, another important source of funding for banks is the issuance of fixed-income instruments in local or international capital markets. In these markets, financial institutions issue fixed-income instruments denominated in local or foreign currency. Credit risk exposure arises when Bank A purchases fixed-income instruments issued by Bank B. Therefore, in the case of the failure of Bank B, Bank A will lose its investment in Bank B.

In Peru, banks send monthly reports containing detailed information of their investments (amount, type of instrument, issuer). To account for this risk, we add the individual exposures of each bank against its counterparties to calculate the total exposure of each bank against other banks. It is worth mentioning that this type of exposure was not included on the early works on financial contagion. However, it is an important component that must be taken into account for calculating systemic risk and systemic importance, as demonstrated in Poledna et al. (2015).

The last layer that we consider is derivatives. As Peru is a partially dollarized economy, households, corporations, pension funds, and foreign investors must hedge against currency risk using currency derivatives. As market makers, banks often take the opposite side of their customers trade.

In Peru, when the local currency depreciates or appreciates, bank customers take one position, causing banks derivative exposure to be negative or positive. For example, when the local currency depreciates, pension funds, foreign investors, households, and corporations take a long position in dollars because they expect the local currency to depreciate. As a result, banks take a short position in dollars, and since most banks are on the same side of the trade, they do not often perform derivative transactions among banks.

It is worth mentioning that derivative operations among financial institutions do not generate relevant exposures, unlike derivative exposure between banks and their customers. Nevertheless, it is important to consider this layer because in the future it may become an important source of systemic risk.

Credit risk exposures arise from the valuation of derivative transactions. For the derivatives layer, we consider forwards and swaps. Banks send biweekly information containing the main characteristics of derivative contracts (amount, spot exchange rate, forward exchange rate, and maturity) and their derivatives valuation with other counterparties. The SBS approves the valuation procedure that each financial institution employs to value its derivative exposures.

Banks use detailed agreements to specify netting procedures in case a counterparty fails to comply with payment of a derivative. We use the following procedure to calculate the credit risk for this layer: if a derivative contract between Bank A and Bank B has a positive value for Bank A, the latter bears a credit risk, which is equivalent to the value of the derivative contract. If these two banks have multiple derivative contracts, we sum the individual exposures and then assign this sum to the counterparty with a positive net position.

To calculate the DebtRank, we considered the evolution of six variables (short-term loans, long-term loans, demand deposits, term deposits, securities, and derivatives) from October 2019 to December 2019.

Table 1 shows the descriptive statistics of each financial institutions exposures against others in October 2019. The variables are in thousands of local currency (PEN). For example, the average short- and long-term credit exposures that any financial institution had in October 2019 was PEN 34 million (USD 10 million) and PEN 87 million (USD 25 million), respectively. The data is similar for each layer in the last quarter of 2019. The average Herndahl-Hirschman index points to a relatively high concentration in the demand deposits layer among financial institutions than in the rest of the layers. In particular, the securities layer is the least concentrated for

Table 1
Descriptive statistics of exposures IV Quarter 2019.

	Short-term credit			Long-term credit			Demand deposits			Term deposits			Securities			Derivatives		
	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.	Oct.	Nov.	Dec.
Average	34 095	39 571	35 884	87 547	88 430	84 212	1 528	1 749	1 467	23 163	20 573	20 622	12 370	11 919	11 301	1 029	2 192	1 369
Std. Dev.	27 242	43 987	33 177	179 518	180 767	179 041	4 537	6 087	5 141	50 727	44 863	42 049	22 116	22 845	21 590	2 555	4 302	3 625
Median	20 000	30 000	30 000	26 475	23 500	20 000	108	86	90	7 000	7 608	7 233	4 191	4 160	4 167	309	673	314
Percentil 25%	15 000	15 000	12 000	8 293	7 140	6 793	7	5	4	3 000	2 900	3 000	1 936	1 944	1 873	183	319	159
Percentil 75%	49 548	40 000	45 000	70 305	70 461	66 420	1 105	749	789	23 501	19 850	19 141	9 599	8 120	9 101	577	1 241	821
Min. Value	3 000	3 000	1 000	126	124	123	1	1	1	1	1	1	102	261	259	5	15	42
Max. Value	100 000	231 800	149 130	1 085 000	1 045 000	1 085 000	48 925	56 990	57 639	441 547	457 781	427 500	114 910	127 315	124 875	11 690	16 148	15 719
Avg. HHI	0.14	0.19	0.12	0.19	0.21	0.19	0.71	0.72	0.72	0.31	0.33	0.34	0.1	0.09	0.09	0.13	0.09	0.11
Exposures to Capital (%)	4.37	5.07	4.59	11.22	11.33	10.78	0.2	0.22	0.19	2.97	2.64	2.64	1.59	1.53	1.45	0.13	0.28	0.18
Observations	39	49	37	104	104	104	174	176	180	143	152	162	58	54	55	20	19	18

*This table shows the descriptive statistics for the monthly multilayer network. The variables are in thousands of local currency (PEN) expect for the HHI, Exposure to Capital and number of observations.
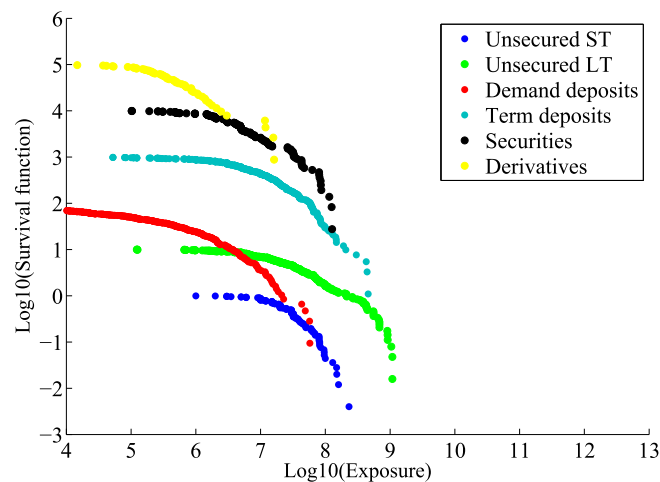
Fig. 1. Zipf plot *Survival function of exposure size (in PEN) for all layers. Aggregate data for the last quarter of 2019. The curves are shifted upwards to avoid overlapping by factors of 10, 100, 1000, 10,000, and 100,000.

this period. The exposures to capital show that the total exposure of all layers is around 20%, with long-term credit being the most important exposure.

The survival function by the size of exposure is shown in Fig. 1. Unsecured long-term exposures are the ones that have the highest values, followed by term-deposit exposures. The observed distributions evidently are not power law, with the shape of the curves more related to an exponential distribution. However, this graph is not sufficient to determine the underlying distribution of the data.

4. Networks and systemic risk metrics

We now provide some basic concepts and definitions about networks. A network is defined in terms of a set of nodes (V) and a set of edges (E), where each edge connects a pair of nodes. Edges can have a direction, in which case we have a directed network, or not, in which case we have an undirected network. Weights can be associated with links. In this case, the network is said to be weighted. Interbank networks are typically weighted directed networks, as links represent, for instance, exposures among institutions.

We characterize the topology of the network using the following standard metrics:[1]

After describing the properties of the network, we will perform a systemic risk analysis of Perus banking system. To this end, we will use the DebtRank algorithm. The details of the method can be found in the appendix. For an intuitive approach, the algorithm implements discrete time dynamics of shock propagation. At each step of the dynamics, an institution suffers losses that depend on those experienced by its counterparties in the previous step. Relative losses are assumed to propagate linearly between institutions.

In the following, we will measure the systemic impact of a bank in terms of its DebtRank, which is computed as the final losses, relative to capital, that would be experienced in the system following the initial default of the bank. The vulnerability of a bank will instead be the average loss the bank would experience following the initial default of another institution.

5. Results

In this section, we present the results of our research. First, we present the evolution of some structural and centrality metrics only for the short-term unsecured interbank layer because it is the only layer for which daily time series for 2018-2019 are available. Then, we describe the multiplex structure using multilayer metrics. Finally, we present the main results of the application of the DebtRank and its relationship with other indicators of systemic relevance.

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[1] A detailed overview of the metrics used can be found in the appendix.

- Completeness (or density) index: number of links in the network divided by the number of node pairs. This index measures the extent to which a network is close to the complete graph.
- Reciprocity: fraction of links for which a link in the opposite direction exists.
- Clustering of a node: number of connections that exist between the node's neighbors divided by the number of neighbors' pairs. It measures the density of triangles in the neighborhood of a node, and it is associated with the counterparty risk externality described by Acharya and Bisin (2014).
- PageRank: this is centrality measure in which the importance of a node depends on the importance of its neighbors in a self-consistent way.
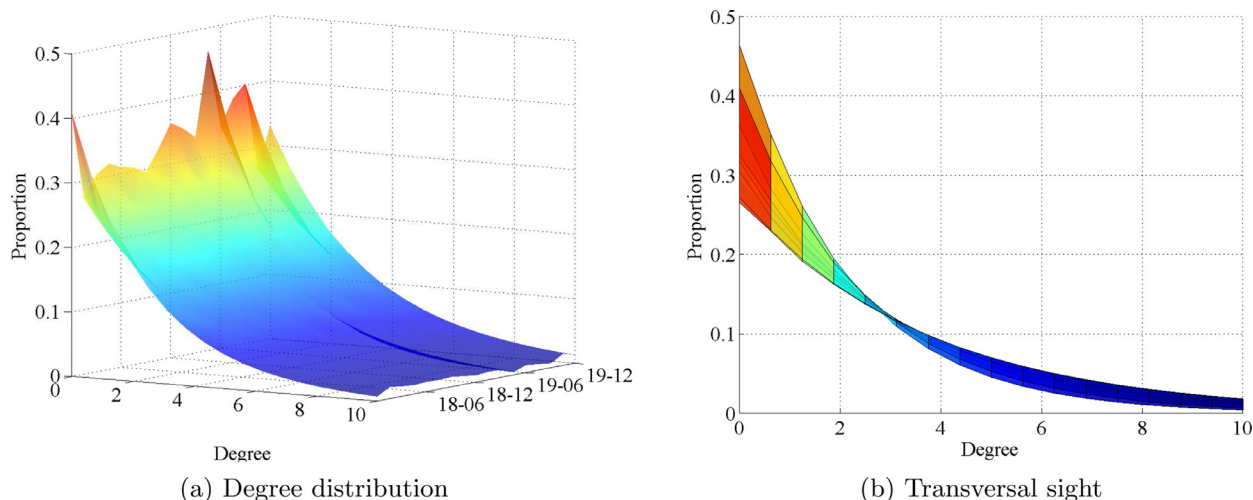- Core-peripheriness: the extent to which a network is close to the ideal core-periphery network.

(a) Degree distribution

(b) Transversal sight

Fig. 2. Indicator for distribution of degrees *This graph uses the daily data of short-term unsecured credits. Panel (a) shows the degree distribution along the sample period for the daily data. Panel (b) shows a cross-sectional view of panel (a). This panel shows that the share of zero-degree institutions ranged from 28% to 45%, because of the difference in activity on the first and last days of each month.

5.1. Structural and centrality metrics

In this section, we report the structural analysis exclusively for the short-term unsecured layer of the multiplex exposure network. The reason behind this decision is that it is the only exposure layer for which daily information is available. We acknowledge that this analysis is incomplete in the sense that it only comprises one layer of the multiplex exposure structure. Nevertheless, the structural properties of such a market have not been documented for Perus banking system before.

The average degree of the network is on average 3 (Appendix C). This means that the network is quite sparse, which appears to be a general property of interbank networks (Anand et al., 2014; Boss et al., 2004; Manna and Iazzetta, 2009). The degrees of the banks show some heterogeneity in our system. For instance, the maximum degree is around 12, and it usually corresponds to one of the four largest banks, although it can vary depending on the day and liquidity distribution.

We can see from the degree distribution that on each day, several institutions did not have linkages with any other (Figure 2a). In the period under analysis, the degree distribution appears quite stable, which is consistent with the fact that there have not been any major changes in the interbank market.

Fig. 2 b shows that the share of zero-degree institutions ranged from 28% to 45%. This happened because Peruvian financial institutions must comply with minimum average reserve requirements each month, so participants seek to fulfill it on the first days of the month. This leads to high activity of the interbank market in the first days of the month, while activity decreases and the share of participants declines in the last days of the month, hence the spikes that appear regularly in the distribution (Fig. 2a).

The clustering coefficient (Appendix C) had an average value around 0.2, with a maximum between 0.4 and 1. This is higher than the average clustering expected in random networks with similar numbers of nodes and links. Sometimes institutions make "triangulations," whereby a participant who has used the entire credit line with a second participant asks for a loan from a third party, which will be funded by the second participant. This explains the oscillatory movement of the average of the clustering coefficient because "triangulations" tend to appear and raise the indicator in periods of high-liquidity shocks.

The completeness index (Appendix C) was below 0.1, indicating that the network was far from being fully connected. This happens because in Peru, the largest banks usually provide short-term credit loans to other large banks and a few smaller institutions, thereby creating hubs in which only certain institutions provide funds to other institutions and not to all institutions in the interbank network.

The lending HHI (Appendix C) has the level that the literature usually defines as concentrated (greater than 0.25), because the four largest banks usually are the most active participants in the market. However, this is not always true e.g., at the end of the month, the institutions that have already fulfilled their monthly average reserve requirements, irrespective of whether they are among the largest or not, tend to lend to institutions that still need to meet this requirement.

When we measure reciprocity, we see that the links that go in both directions are approximately 10% of the total. The largest strongly connected component (LSCC) (Appendix C) consists, on average, of 3-4 nodes, which correspond to the largest 4 banks in Peru. It is important to highlight that the LSCC can be as large as 13. Therefore, in some other periods, other banks also belong to the LSCC.

As a means of describing the global structure of the network, we also apply the core periphery model of Craig and von Peter (2014) to the daily network. Using this model, we can see that, most of the time, the core is composed of two to three financial institutions (Fig. 3). However, we find that the error is high, which provides evidence that the daily network is not compatible with a core-periphery structure.
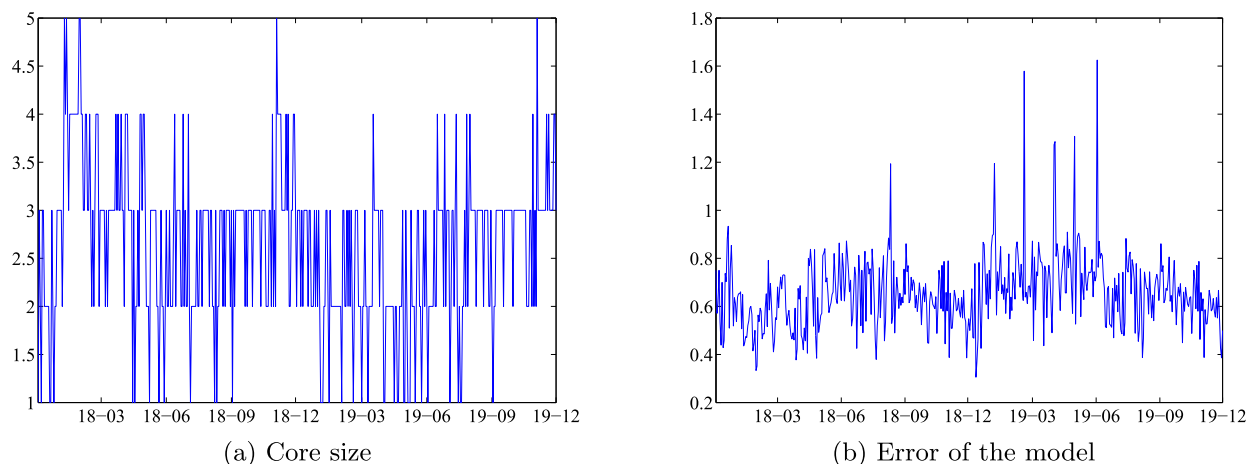
(a) Core size

(b) Error of the model

Fig. 3. Core-Periphery Model *This graph uses the daily data of short-term unsecured credit. We identified the institutions that constituted the core, according to Craig and von Peter (2014) model. Panel (a) shows the number of institutions inside the core. Panel (b) shows the error of the model for each day in the sample. We observe that the error is high, which provides evidence of non-compatibility with a core-periphery structure.
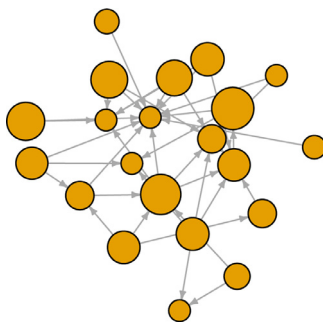

Fig. 4. Daily network *This directed graph corresponds to a random day of the network with unsecured short-term credit. There is no clear sign of a core-periphery structure.

Fig. 4 shows the daily network for a random day in the sample. The nodes are weighted by the share of the funds lent. We can see no evidence of a core-periphery structure: large banks do not tend to form a closely connected core, but they are rather dispersed in the network and connected to different institutions.

In Peru, some financial institutions are more prone to lend to some participants than others. This happens because institutions with better commercial relationships (parent-subsidiary and strategic alliances, among others) have larger credit lines, and loan agreements run faster than in other cases.

Membership in the core is not stable between periods (see Appendix C). In some periods, the cores whole composition changes. This provides evidence that the interbank market does not have a defined structure. Sometimes this happens because some financial institutions have abundant cash in a particular month (because of external shocks, such as large depositors or winning at a deposit auction), and they are prone to establishing multiple connections around them.

PageRank is a centrality measure that can be used to characterize the structure of the network. In our case, it shows that there are different groups of banks (Fig. 5). For instance, the 10 largest participants have a PageRank value between 0.15 and 0.04, whereas the 10 smallest participants have values around 0.04 and 0. Moreover, within the two groups, the relative ranking of a bank can change significantly over time. This shows that, while there is a tendency for larger banks to be more central, the connectivity of a bank and its position in the network affects its centrality.

The analysis also shows that the set of most important institutions is roughly constant over time, so we can say that there is some stability in the relative importance of the financial institutions in the market.

It is important to notice that, while PageRank (and other measures of centrality) can produce a ranking for the systemic importance of institutions, it does not provide a way to estimate the impact in terms of potential losses that the system would experience in case of the default of a given institution. This analysis will be presented later using the DebtRank algorithm (essentially a centrality measure in which the networks adjacency matrix is replaced with the matrix of interbank leverages), whose result has a direct interpretation in terms of financial losses Battiston et al. (2012).
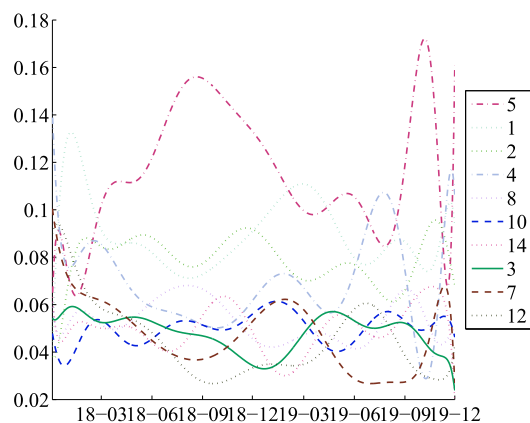
(a) 10 highest centrality.
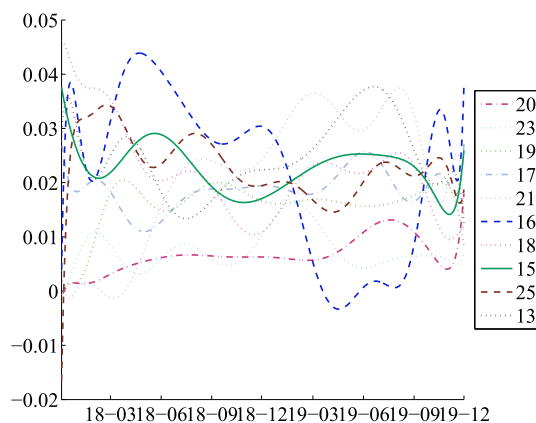
(b) 10 lowest centrality.

Fig. 5. PageRank index *This graph uses the daily data of short-term unsecured credits. The spaghetti plots were made using polynomial functions to smooth the PageRank indices. Panel (a) shows the top 10 institutions with the highest PageRank index, on average, in the sample, while Panel (b) shows the top 10 institutions with the lowest index, on average. This graph shows that PageRank had high variability among the institutions.

Table 2

Layer-importance indicator IV Quarter 2019.

Network	October 2019	November 2019	December 2019
Short-term credit	0.181	0.197	0.152
Long-term credit	0.446	0.422	0.427
Demand deposits	0.565	0.538	0.629
Term deposits	0.442	0.474	0.461
Securities	0.183	0.208	0.183
Derivatives	0.184	0.161	0.147

*This table uses the monthly data of the multilayer network. We calculate the layer importance indicator developed by Zhu and Li (2014). We see that the short-term credit layer is among the least important, while the demand and term deposits have the greatest importance.

5.2. Multilayer metrics

In this section, we compute some metrics adapted to a multilayer network. The first results can be seen in Appendix C. In particular, the overlapping degree of the multilayer network shows heavy tails, while the activity is centered around 3 and 4 layers. From the group of 56 financial institutions analyzed, only 3 are present in just one layer. The latter are micro-financial institutions that have a participation in overall assets of less than 0.1%.

In Table 2 we compute the layer importance indicator developed by Zhu and Li (2014). In the last quarter of 2019, the layer of demand deposits had the highest importance indicator, followed by the term deposits layer. An interesting result is that the short-term credit layer is among the least important, which highlights the importance of using a multilayered approach when analyzing financial interconnections rather than using only the interbank market.

Even though the exposure of the system to derivatives is small compared to the other layers, in October 2019, it had a higher indicator than the short-term credit and securities layer. This is because the indicator of Zhu and Li (2014) measures the importance not in amount, but in correlation. Therefore, in that month the activity of derivatives strongly correlated with the other layers, probably due to shocks in foreign exchange liquidity.

To run the Bernoulli multilayer stochastic block model (SBM), we grouped the six layers into three groups: short- and long-term credit; demand and term deposits; and securities and derivatives.

The model detected two communities. The first community is composed of 15 financial institutions, which is a mix of banking and microfinancial institutions. The network is presented in Fig. 6, where the coral color nodes are the ones belonging to the first community.

When we applied the same technique to the individual layers, the model detected on average four communities for each layer. This is consistent with the fact that institutions behave differently in each layer. For instance, in the long-term credit layer there are groups of institutions where large banks distribute the liquidity to their subsidiaries, while in the short-term credit layer there are groups of institutions that are more keen to lend to each other for commercial reasons. However, it is a remarkable result that,
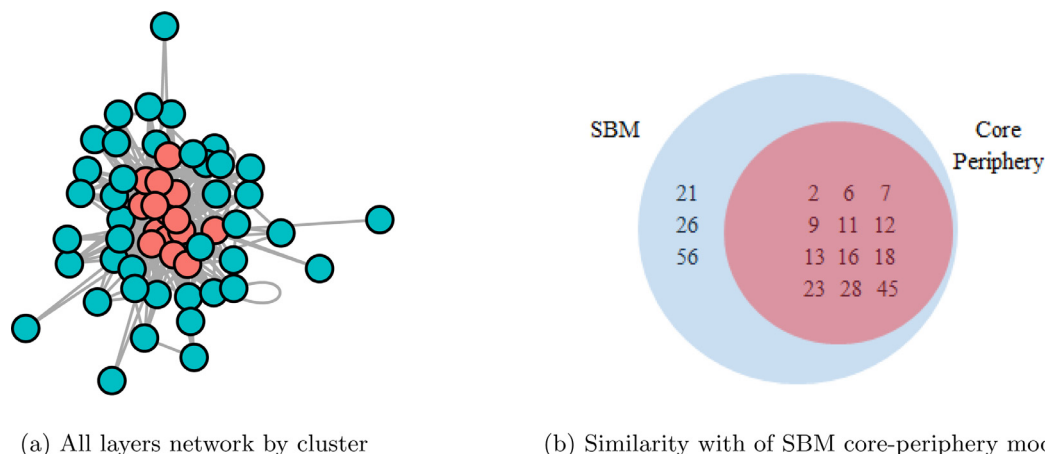
(a) All layers network by cluster

(b) Similarity with of SBM core-periphery model

Fig. 6. Stochastic Block Model *Both panels use monthly December 2019 data. Panel (a) shows the total network, including all types of layers, with the first cluster identified by the SBM at the center in coral color. Panel (b) is a Venn diagram with the IDs of financial institutions as elements and the first cluster of the SBM and the "core" of the core-periphery model as groups. All institutions identified as "core" where classified as part of the first cluster with the SBM.

(a) Overlapping degree vs participation

(b) Degree by layer
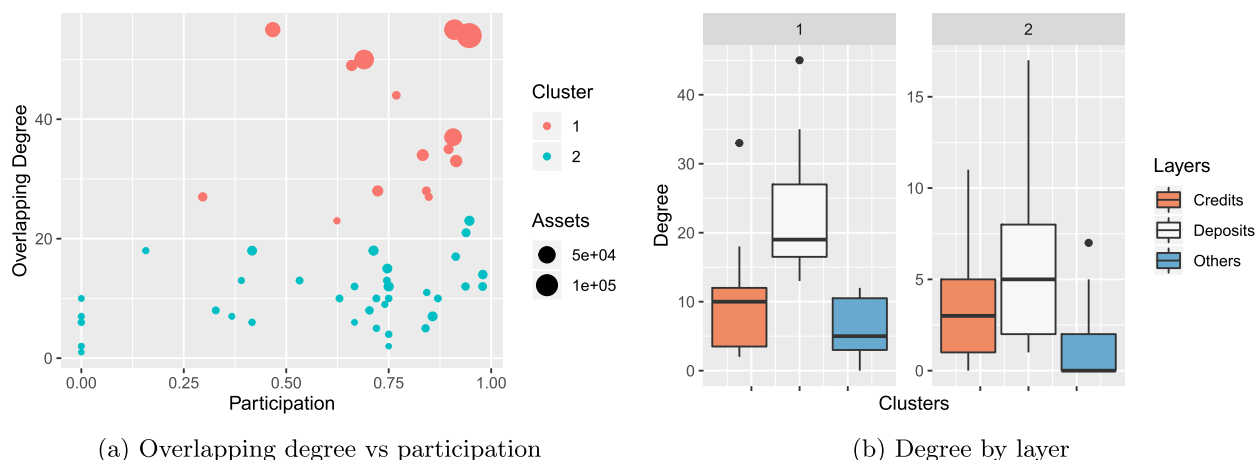
Fig. 7. Clusters characteristics *Panel (a) shows the relation between the overlapping degree and the participation of each financial institution. The cluster that each institution belongs to is indicated by the color of each point (cluster 1: coral and cluster 2: blue), while the size is related to the magnitude of the assets of each institution. Panel (b) consists of two boxplots for the degree of the financial institution. The first and second boxplots correspond to clusters 1 and 2, indicated in panel (a), respectively.

although in each individual layer there are different communities, when we look at the system as a multilayer, there are only two groups, which suggests that the multiplex network as a whole could have a core-periphery structure.

We therefore resort again to the model by Craig and von Peter (2014) using the aggregate network and find out that the algorithm is able to identify as core 12 of the 15 institutions that were detected as the first cluster in the SBM model. Furthermore, the error of the model was 0.31, almost half of the error shown in Fig. 3b, when we used only the short-term daily layer.

Fig. 7 provides some characterization of the two clusters identified with the SBM model. The first cluster is characterized by a high overlapping degree, and most of the nodes have a participation above 0.5 in the multilayer network. Furthermore, the largest banks are included in this cluster. Together, the institutions in this cluster represent 87% of total assets.

Moreover, the first cluster has a higher mean in each of the layers used, especially in the deposit layer (consisting of demand and term deposits), which was found by the Zhu and Li (2014) algorithm to be the two most important layers.

5.3. DebtRank algorithm

In the previous section we described the structure of Perus interbank system. Now we can perform its systemic risk analysis. To this end, as explained earlier, we resort to the so-called DebtRank algorithm (Battiston et al., 2012). The reason for this choice is that, unlike cascading defaults models, the algorithm allows for the propagation of stress before default occurs and, unlike other centrality metrics, it has a direct interpretation in terms of financial losses.
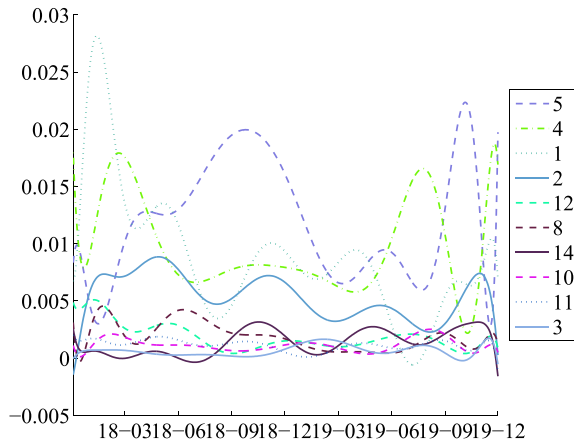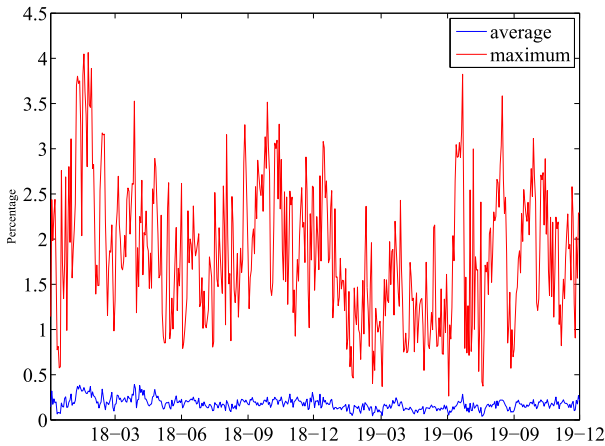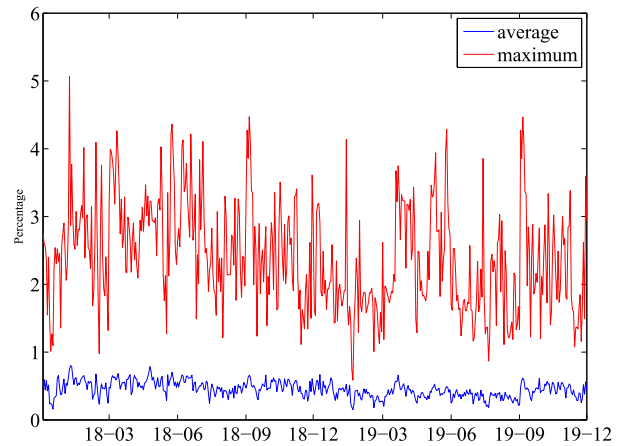
(a) 10 highest DebtRank
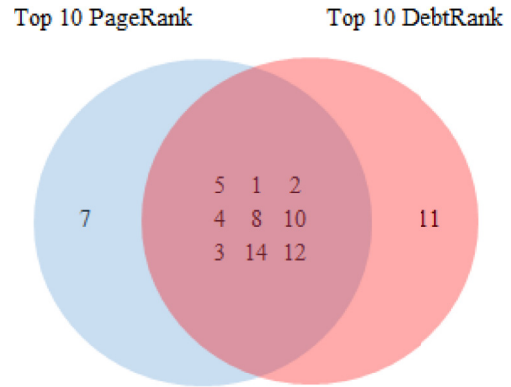
(b) Similarity of DebtRank with PageRank.

Fig. 8. Daily DebtRank *This graph uses the daily data of short-term unsecured credit. Panel (a) shows the daily DebtRank for the institutions with the highest average DebtRank over the sample period. Panel (b) is a Venn diagram with financial institutions IDs as the elements and the top 10 PageRanks and DebtRanks as groups. A majority of the institutions with the highest DebtRank also had the highest PageRank.

(a) Average and max DebtRank

(b) Average and max Vulnerability

Fig. 9. Daily DebtRank and Vulnerability (as percentage of the capital) *This graph uses the daily data of short-term unsecured credit. Panel (a) shows the daily average and maximum DebtRank among the institutions. Panel (b) shows the daily average and maximum vulnerability among the institutions. Both graphs show a large difference between the maximum and the average.

The results for the DebtRank calculated with the information for the daily short-term unsecured interbank exposures show a graph similar to that of PageRank (Fig. 8). In fact, out of the group of institutions in the ranking of PageRank and DebtRank, only institutions 7 and 11 do not appear in both groups. However, this could change in periods of turbulence, because even though PageRank and DebtRank will be influenced by the institutions market participation, DebtRank will take into account the fragility of each institution. Furthermore, the top 10 PageRank and DebtRank classifications do not reveal important structural changes among most central banks.

During the sample period, the daily average DebtRank remained small (Fig. 9). The average vulnerability of banks was higher than their DebtRank, although its magnitude was still very low. This is because many institutions had zero DebtRank. The maximum DebtRank for each day could be as high as 4%, while vulnerability could reach 5%. The maximum DebtRank value had periods of high and low turbulence e.g., the first and last quarters of 2018. However, the maximum vulnerability did not seem to have a clear pattern.

Fig. 10 the result of running the DebtRank algorithm on the multilayer network as well as each layers contribution to contagion losses for three different months (only the 15 institutions with the highest DebtRank are shown). Fig. 10 is constructed as proposed in Poledna et al. (2015). A layer was built by considering only one type of exposure e.g., the cyan-colored contribution corresponds to the network of unsecured long-term lending. Then, the DebtRank was computed for each bank in each individual layer, and the process was repeated for all layers (exposure types). Finally, all exposures were aggregated into a single network, and the DebtRank
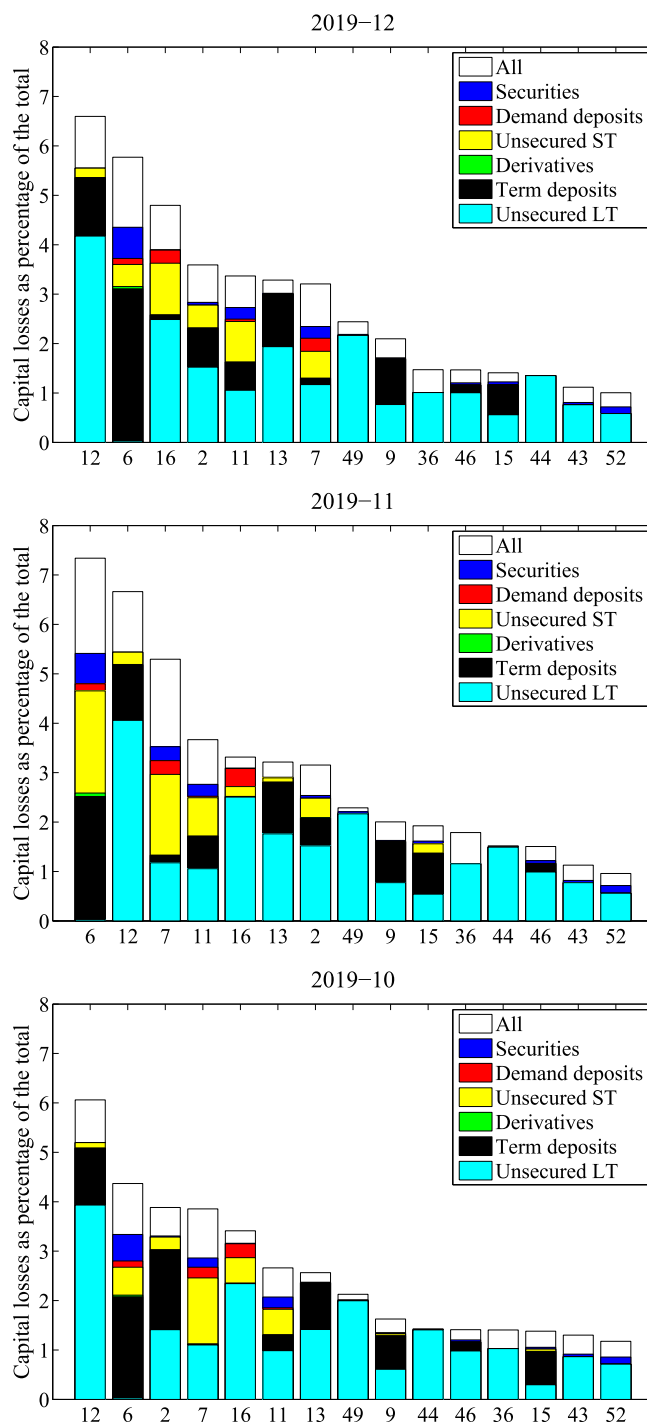
Fig. 10. Multilayer DebtRank for multiple dates *This graph uses the monthly data of the multilayer network. We show the top 15 institutions with the highest DebtRank and the contribution of each layer.

was computed. The aggregate exposure network can be calculated simply by adding all of the exposures between each pair of nodes because a common notion underlies the computation of each individual layer of exposures: the concept of exposure at default.

From the calculation of DebtRank, it transpires that the total DebtRank of the combined exposure network is higher than the sum DebtRank of the individual layers (securities, deposits, unsecured credit, and derivatives) i.e., the sum of individual DebtRanks underestimates the total DebtRank and therefore the systemic risk. Additionally, the source of contagion differs different for each
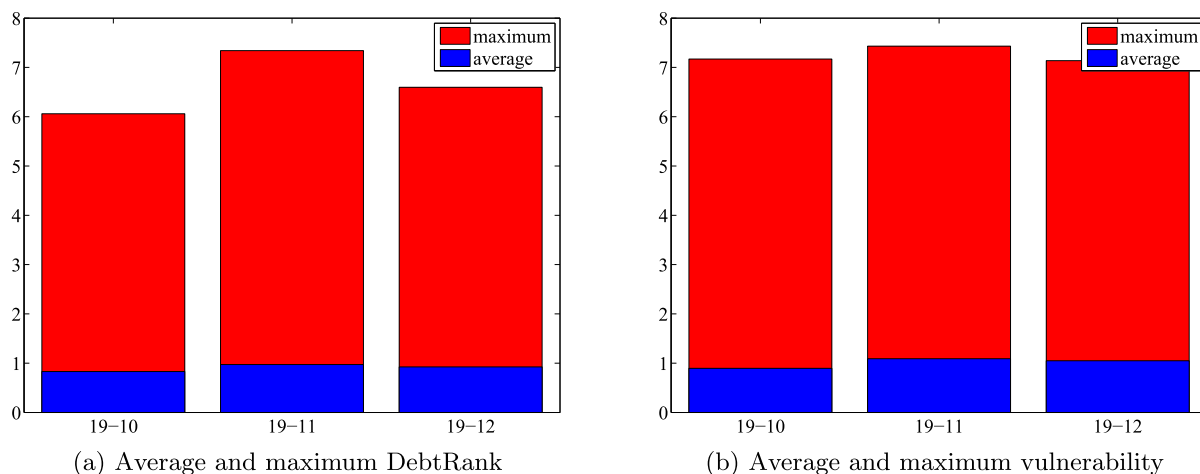
(a) Average and maximum DebtRank

(b) Average and maximum vulnerability

Fig. 11. Monthly DebtRank and Vulnerability - Dec. 2019 (as percentage of the capital) *This graph uses the monthly data of the multilayer network for December 2019. Panel (a) shows the daily average and maximum DebtRank among the institutions. Panel (b) shows the daily average and maximum vulnerability among the institutions. We can see a large difference between the maximum and the average in both graphs, as seen with the daily data of the short-term unsecured credit.
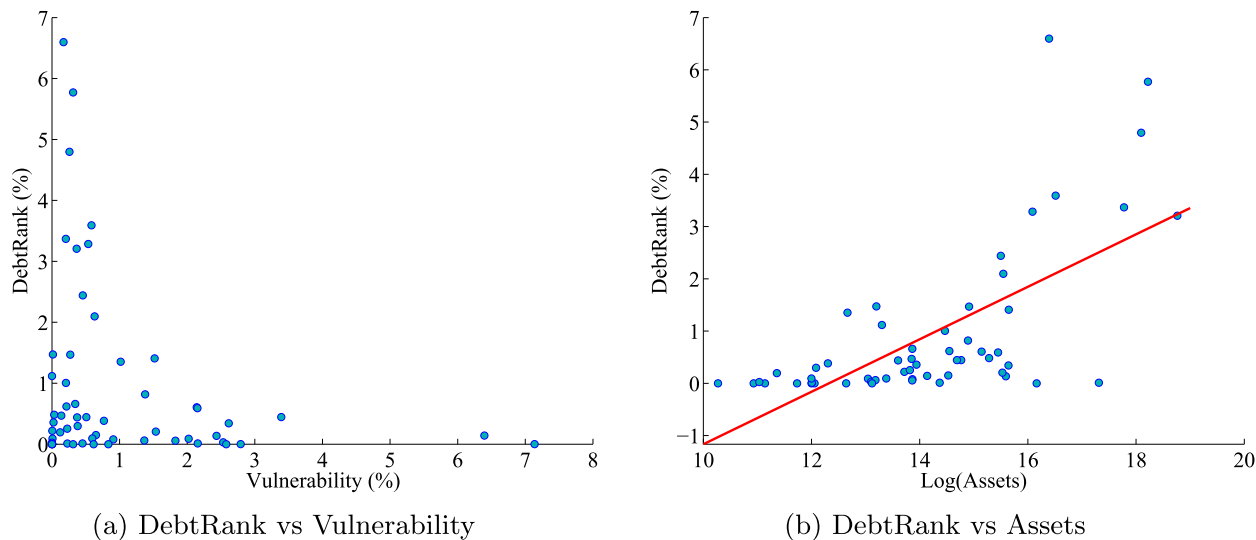

(a) DebtRank vs Vulnerability

(b) DebtRank vs Assets

Fig. 12. DebtRank Vulnerability and Assets - Dec. 2019 *This graph uses the monthly data of the multilayer network for December 2019. Panel (a) shows the relationship between the DebtRank and vulnerability index. Panel (b) shows the relationship between the DebtRank and logarithm of total assets of each institution. There is no relationship between DebtRank and Vulnerability.

bank, and its magnitude varies across time. Nevertheless, the layer that matters the most for contagion risk is unsecured long-term lending. As mentioned above, this lending usually occurs between two related parties. For this reason, it is not collateralized.

The sixth institution had the second-highest DebtRank in October and December 2019 but the highest in November 2019, implying that even if the unsecured long-term layer is the most important variable, term deposits and unsecured short-term layers make this institution the most relevant to the financial system. Therefore, institutions in charge of assessing the financial system should monitor DebtRanks continuously to quantify systemic risk and determine which institutions are the riskiest. The results of the DebtRank analysis offer an overall positive assessment of Perus financial system. According to the model, the largest losses were equivalent to only 7% of the financial systems equity, equivalent to US$873 million. These losses do not represent a significant risk to Perus financial stability.

The average DebtRank of the system was small because many institutions had little participation in the multilayer network (Fig. 11). The average vulnerability was higher but still very low, with values around 0.01%. However, as in the case of the daily DebtRank, the maximum indicator in each period was much higher. The maximum DebtRank was around 6.5%, and vulnerability was around 7%. As in the case of the daily data, vulnerability was higher.

Fig. 12 a shows that banks with a higher DebtRank had lower vulnerability, and vice versa. This result shows that banks that could generate the largest losses to the financial system were the least vulnerable. For example, the bank with a vulnerability of 7.1% had a DebtRank of zero, which is desirable in terms of systemic stability because this bank was the most exposed to interconnection risk, but its own losses would not impact other financial institutions.

We also compare the DebtRank observed for each bank to its assets. Fig. 12b shows that the size of financial institutions can explain to some extent the variance of DebtRank (66% of correlation), the rest being due to their interconnectivity. with the rest being due to their interconnectivity. In fact, we observe that the largest banks were not necessarily the most systemically important. The sixth-largest bank in terms of size was the most important in terms of interconnection. Additionally, the fourth-largest bank did not pose any interconnection risks. These results can provide financial regulators with a measure of the interconnection risk that each bank contributes to the system and, together with the banks size, help them to determine which of them are the most systemically important.

6. Conclusions and further work

We have performed a comprehensive study of the multiplex structure of interbank exposures within Perus banking system. Our analysis includes several structural and centrality metrics as well as the DebtRank systemic-risk indicator for quantifying the systemic importance of financial institutions, which takes into account interconnectedness in the banking system.

First, we analyzed the network of short-term unsecured exposures, for which daily data were available. The structural and centrality metrics suggest that the interbank exposure network in Peru shows high concentration, although this cannot be always attributed to the same institutions; that, on average, each financial institution maintains relations with three other institutions; and that this layer does not display a core-periphery structure.

We then analyzed the multilayer structure of the Peruvian banking system by considering different layers of exposure between the institutions. In particular, we considered short- and long-term credit, demand deposits, term deposits, cross-holdings of securities, and derivatives, for which we had access to monthly data from the last quarter of 2019. Using the stochastic block model, we were able to identify two communities of banks, corresponding to institutions with relatively high/low degrees of overlapping, and we showed that the aggregate monthly network can be approximated more closely as a core-periphery network than as a daily short-term layer.

We then performed a contagion analysis on the aggregate network of exposures using the DebtRank algorithm. The analysis showed that, according to DebtRank, the risk of contagion in the financial system was not large. More importantly, we show that there was not necessarily a direct relationship between a financial institutions size and the risk that it bears through its bilateral exposure against other financial institutions. In this way, we can identify too-interconnected-to-fail financial institutions. Furthermore, the systemic importance of individual banks, as measured by the DebtRank, changes by time period. This is important for monitoring financial stability.

By comparing the results obtained for the whole multilayer network vs. individual layers, we show that total contagion losses were larger than the sum of the losses associated with the individual layers. This is because of the interactions between the different layers.

We were also able to measure the contribution of each layer to the impact of each bank, thus providing a complete description of the channels through which an institution can propagate shocks. This can be useful for a targeted response during the development of a crisis. Overall, we found that unsecured long-term loans contributed the most to banks impact, but short-term unsecured loans and term deposits contributed significantly to the impact of some institutions.

Our findings imply that studying the banking system as a multilayer network provides more information with respect to the characterization of individual layers. For instance, to quantify systemic risk correctly, it is insufficient to aggregate the estimated losses computed for individual layers in isolation. Similarly, to identify systemically important banks correctly, it is insufficient to identify banks that are systemically important for individual layers. In fact, a bank that is not of particular importance in any of the individual layers may be important from a multilayer perspective because of how it connects different layers.

We also show that the impact and vulnerability of banks, as measured by DebtRank, are inversely related, with high-impact banks being the least vulnerable in the dataset considered. This is an important finding regarding systemic stability because we can consider impact and vulnerability as two different dimensions of systemic risk. If the most vulnerable banks were also those creating the highest impact, then the endogenous amplification of exogenous shocks would be larger. The observed negative correlation between the two instead suggests a higher robustness of the system.

All in all, our findings suggest that multilayer analysis of banking networks is a more comprehensive method for assessing systemic risk with respect to single-layer analyses and that it can complement existing methodologies for identifying systemically important institutions.

This work can be extended in two direct ways to study interconnectedness and systemic risk in Perus financial system: i) including indirect exposures from common asset holdings and ii) including additional financial intermediaries like funds and insurance companies.

Appendix A. Network metrics

We used conventional measures of centrality to characterize the network and identify the nodes (banks) that are more central. Silva et al. (2016) divided the structural measures in three categories: local, quasi-local, and global. Local measures refer to and describe only a node's properties. They do not take into account either its neighborhood or the rest of the nodes in the network.

Quasi-local measures are also associated with nodes, but in describing a node they take into account the information provided by its neighbors. Global measures use global information from a network and are useful for identifying the characteristics of the network as a whole, not only as a collection of individual nodes.

Centrality is a useful tool for identifying institutions that are more relevant to financial stability and systemic risk monitoring. Martinez-Jaramillo et al. (2014) measured and monitored systemic risk through topological metrics for payment systems and interbank networks. Additionally, the authors suggest non-topological measures for describing banks' individual behavior in both networks. They found that the structures of payment and exposure networks differ regarding their connectivity.

The greater a node's centrality, the greater the node's in a network. Examples of centrality measures include degree, strength, betweenness, closeness, eigenvector, DebtRank, and PageRank.

Degree centrality highlights nodes that are important because they have many neighbors, and as such they can affect many nodes in the network. The centrality degree of a node is defined as the degree of the node.

$$C_d(i) = d_i$$

The out-degree centrality of a node is the number of nodes to which its outgoing edges are connected. The in-degree centrality of a node is the number of nodes to which its incoming edges are connected.

In general $a_{ij} \neq a_{ji}$ in a directed graph. Reciprocity refers to the fraction of arcs in any direction for which there is an arc in the opposite direction, defined as follows:

$$r = \frac{\sum_{i \in V} \sum_{j \in N_{(i)}} a_{ij} a_{ji}}{\sum_{i \in V} \sum_{j \in N_{(i)}} a_{ij}}, \tag{1}$$

The completeness index is defined as a measure of how close a graph is to the complete graph. The closer the index is to 1 (the complete graph has an index of 1), the closer the graph is to being fully connected. The index is defined as follows:

$$C(G) = \frac{\sum_i \sum_j a_{ij}}{n(n-1)}, \tag{2}$$

The core-periphery model refers to a separation of the nodes into two different sets: the core and the periphery. Nodes in the core are highly interconnected among each other, while nodes in the periphery are connected to the core but rarely interact with each other. We use the implementation of Craig and von Peter (2014).[2]

Strength centrality refers to the sum of a node's interbank assets and liabilities. Inner strength is the sum of its interbank assets, while the outer strength is the sum of its interbank liabilities. This is very important for to determining which bank is lending (borrowing) the most in the network.

$$C_s(i) = s_i$$

Betweenness centrality is important in the payment-systems network because a node with high betweenness centrality can have an important influence on other nodes because it can stop or distort the information that passes through it.

Let $\sigma_{ij} = \sigma_{ji}$ denote the total number of shortest paths between i and j, and let $\sigma_{ij}(v)$ be the total number of shortest paths between i and j that pass through vertex v. Then:

$$C_B(k) = \sum_{i \neq k \neq j \in N} \frac{\sigma_{ij}(k)}{\sigma_{ij}}. \tag{3}$$

Additionally, closeness centrality can be associated with a node's capacity to spread contagion, as such a node is close to the rest of the network. This measure is defined as follows:

$$C_C(i) = \sum_{j \in N \setminus \{i\}} \frac{1}{\delta_{ij}}. \tag{4}$$

Eigenvector centrality considers the neighbors' centrality to compute a node's centrality, and by doing so it takes into account both direct and indirect connections. The eigenvector centrality of node i is defined as the i-th entry of the eigenvector e associated with the largest eigenvalue λ of the adjacency matrix A

$$\lambda e = Ae$$

PageRank centrality considers the relevance of neighbors to determine a node's relevance in the network. Its calculation does not take into account the weights, but this aspect is important in the context of interbank exposures and payment systems, because it provides relevant information. Therefore, this measure of node i should be multiplied by the dominant weight.

Martinez-Jaramillo et al. (2014) applied these metrics to provide an empirical analysis of interbank exposures and payment system flows for Mexico's banking system. They show the robustness of centrality metrics for characterizing the individuals participating in the financial network. Also noteworthy is that the contagion metric was not related to asset size for all banks. Some banks ranked

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[2] We thank the authors for sharing the code to fit their model to Peru's interbank exposure networks.

very high in terms of interconnectedness (contagion potential) were important for determining systemic importance in the banking systems.

Appendix B. DebtRank Algorithm

DebtRank (Battiston et al., 2012) is a recursive method to quantify the systemic importance of financial institutions in terms of the position they occupy in the network of interbank exposures. The method was introduced by Battiston et al. (2012), but here we consider the generalization by Bardoscia et al. (2015), which accounts for the effect of loops in the network.

The method is described in terms of the dynamics that evolves in discrete time. At time $t = 0$ the system is initialized, and at time $t = 1$ a shock is applied to the system. A dynamic rule is then given for how the losses that banks experience at time t are propagated to their counterparties. The assumption of DebtRank is that this propagation is linear: the relative devaluation of the interbank assets that correspond to exposures towards a given bank i is equal to the relative loss of equity of i.

Following Bardoscia et al. (2015), we denote by $W^+(0)$ the matrix of interbank exposures, with $W_{ij}^+(0)$ representing the exposure of i toward j, by $E_i(0)$ the equity (tier 1 capital) of bank i, and by $h_i(t)$ the relative loss of equity experience by bank i between time 0 and time t, i.e. $h_i(t) = (E_i(0) - E_i(t))/E_i(0)$. Liabilities and non-interbank assets are assumed to be constant throughout the dynamics. By requiring that the balance sheet identity is satisfied at each time, the DebtRank dynamic can be written as follows:

$$h_i(t) = \min\left\{1, \sum_j \frac{W_{ij}^+(0)}{E_i(0)} h_j(t-1) + h_i(1)\right\}. \tag{5}$$

In this equation the losses experienced by i at time t are the sum of a contribution ($h_i(1)$) due to the exogenous shock and a contribution due to the losses experienced by the counterparties of i up to time $t - 1$. Note that the contribution of i's counterparties to the loss of i are weighted by the factor $W_{ij}^+(0)/E_i(0)$, which is the amount of i's leverage associated with its exposure toward j. The matrix Λ with elements $W_{ij}^+(0)/E_i(0)$ is called the matrix of interbank leverage (Battiston et al., 2016a); and its largest eigenvalue determines the amplification of shocks due to the structure of the network (Bardoscia et al., 2015; 2017). The cap at 1 is required to avoid creditors to loose more than the nominal value of their exposures.

DebtRank can be used to determine the impact and vulnerability of individual institutions. To measure the impact of bank i, we imagine an initial shock in which i defaults at time $t = 1$; i.e., $h_i(1) = 1$ and $h_j(1) = 0$ for all $j \neq i$. We then iterate (5) until convergence to a fixed point, and we measure the fraction of equity lost in the system due to contagion only (i.e., without considering the initial shock).

$$R_i = \frac{\sum_{j \neq i} h_j^* E_j(0)}{\sum_j E_j(0)}, \tag{6}$$

where h_j^* is the relative loss of equity of j at the fixed point of (5).

Similarly, we can measure the vulnerability v_i of bank i as the average loss that i would experience when the initial shock is the default of a bank other than i.

The methodology introduced in this section can be used to study the effect of both single and multiple layers of exposure. In the case of a single layer, W_{ij}^+ would represent the network of exposures on that specific layer. Let us imagine now that banks interact through multiple layers of exposures, and let us denote by $W_{ij}^{+(\ell)}$ the exposures associated with layer ℓ, with $\ell = 1, 2, \ldots, q$. To quantify losses in this case one would simply replace $W_{ij}^+(0)$ in (5) with the aggregate exposures across all layers $W_{ij}^{+\text{agg}}(0) = \sum_{\ell=1}^q W_{ij}^{+(\ell)}(0)$.

Notably, the iterating (5) using aggregate exposures leads to different results, as compared with summing the losses that one would obtain by iterating the map on the individual layers. This is because in the former case one would account for interactions between the different layers, which sometimes can become substantial (Poledna et al., 2020).

This can be understood easily as follows: Imagine the case in which no defaults occur, so that we can consider the simple linear dynamics

$$h_i(t) = \sum_j \frac{W_{ij}^+(0)}{E_i(0)} h_j(t-1) + h_i(1). \tag{7}$$

The fixed point of this set of equations is

$$\vec{h}^* = (\mathbb{I} - \Lambda)^{-1} \vec{h}(1), \tag{8}$$

where we denote by $\vec{h}$ the vector of components h_i with $i = 1, \ldots, N$, and by Λ the matrix with elements $\Lambda_{ij} = \frac{W_{ij}^+(0)}{E_i(0)}$. Since

$$\left(\mathbb{I} - \sum_{\ell=1}^q \Lambda^\ell\right)^{-1} \neq \sum_{\ell=1}^q \left(\mathbb{I} - \Lambda^\ell\right)^{-1},$$

it is clear from (8) that the losses obtained running a stress test on the multilayer network are different than the sum of the losses obtained by running a stress test in each individual layer. We will see that this is the case in section 5.2.
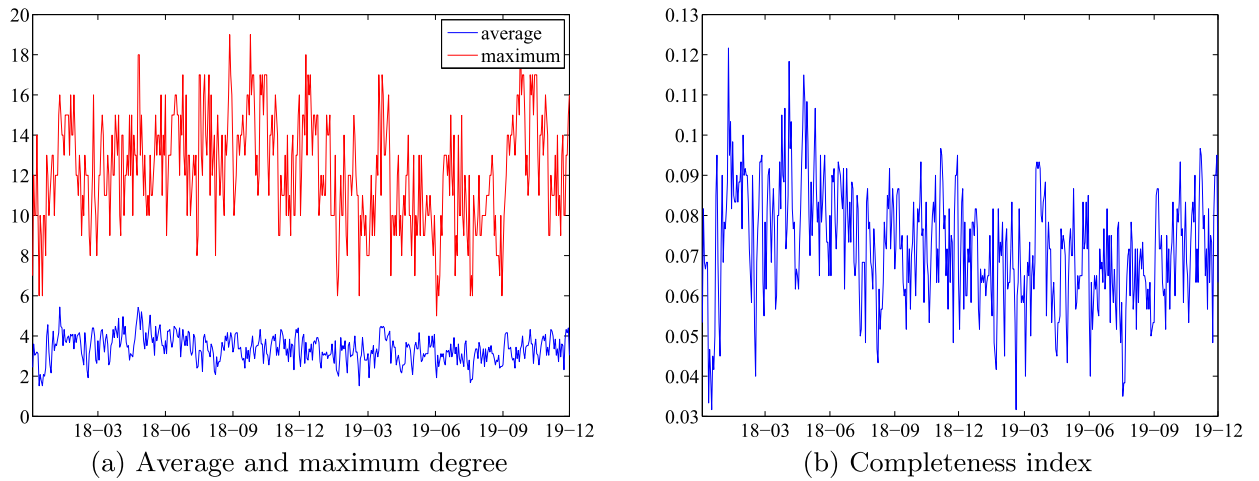
(a) Average and maximum degree

(b) Completeness index

Fig. 13. Daily network exposure *This graph uses the daily data of short-term unsecured credit. Panel (a) shows the daily average and maximum degree among the institutions. Panel (b) shows the network's completeness index.
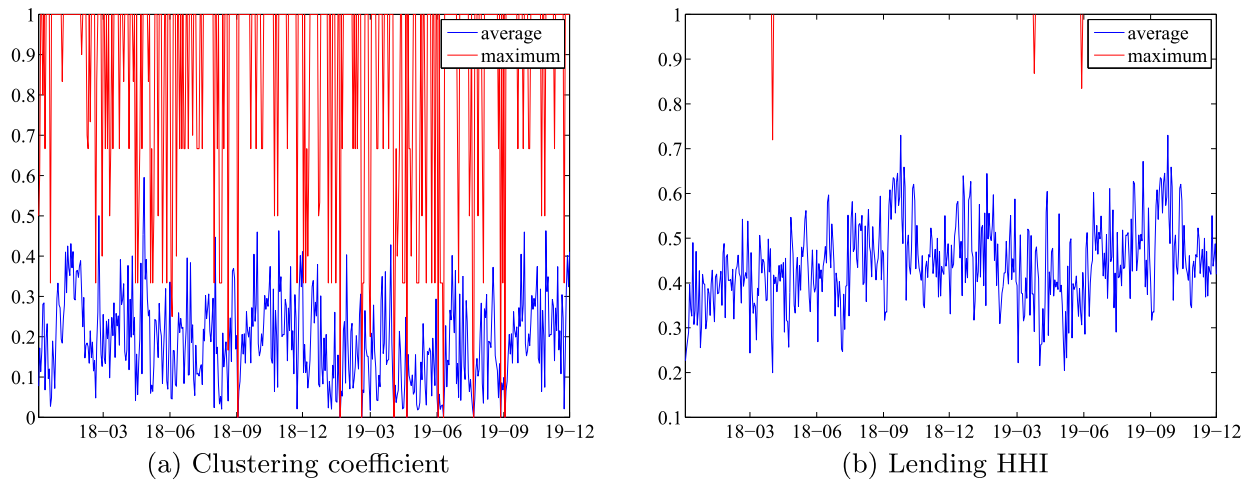

(a) Clustering coefficient

(b) Lending HHI

Fig. 14. Clustering and HHI indicators *This graph uses the daily data of short-term unsecured credits. Panel (a) shows the daily average and maximum clustering coefficient. Panel (b) shows the network's daily average and maximum lending HHI.
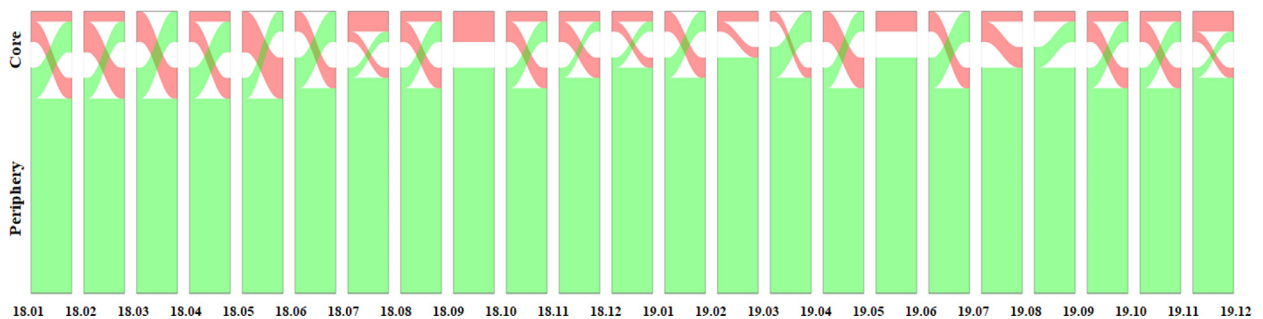

Fig. 15. Core-periphery alluvial graph *This graph uses the daily data of short-term unsecured credit. We used the information from the last day of each month. We identified the core and periphery members using Craig and von Peters (2014) model. Within each month were many reclassifications between the core and periphery.
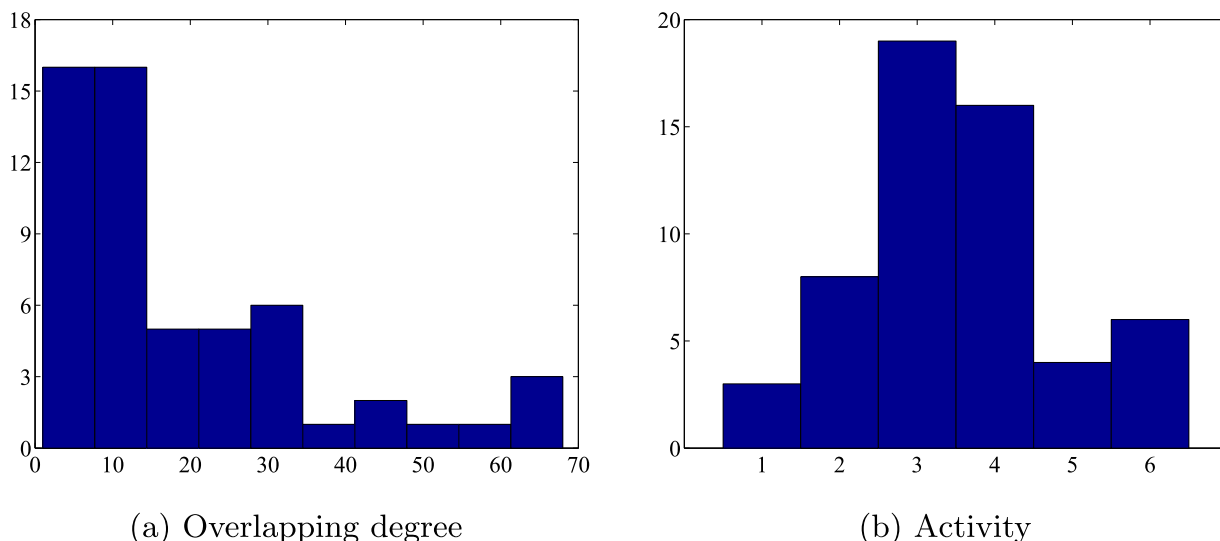
(a) Overlapping degree   (b) Activity

Fig. 16. Overlapping degree and activity *This graph uses the monthly data of the multilayer network for December 2019. Panel (a) shows the networks degree of overlapping. Panel (b) shows the networks activity indicator.

Appendix C. Additional Results

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