



**Master Degree in Computer Science**

**Information Retrieval**

# **Evaluation in information retrieval**

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# Goals of evaluation

The goal of the evaluation activity is to **assess the quality of results obtained by an IR system**

The notion of *quality of results* depends on the task at hand, e.g., search, classification, knowledge extraction, etc.

A general issue concerning the evaluation is that it is based on a **ground truth** (or **gold standard**), that is an **annotated corpus** where, for each document, we know if the document is **relevant** with respect to the task

Ground truth may be created by manually annotating documents and/or derived from data with a reference annotation system

# Search evaluation: the notion of quality

Given a corpus **C** and a query **q**, the task of document search is to find the set of documents **A<sub>q,C</sub>** that match **q**

We call **A<sub>q,C</sub>** the **answers** to **q**

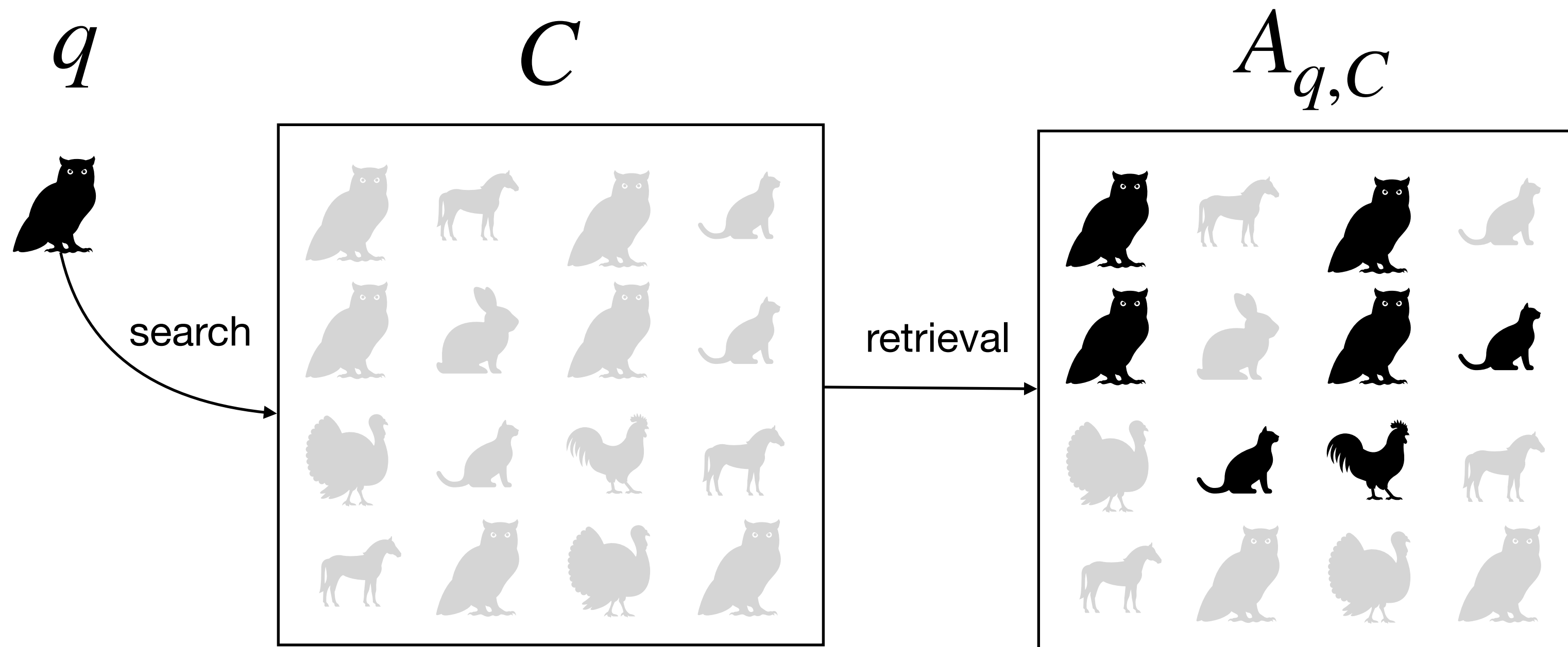
**Question:** when we say that the search answers **A<sub>q,C</sub>** are good?

# Search evaluation: the notion of quality

**Question:** when we say that the search answers  $\mathbf{A}_{q,c}$  are good?

**Definition 1:** *when the documents contained in  $\mathbf{A}_{q,c}$  are relevant to  $q$*

**Remember:** in order to know if a document is actually relevant, we need a **ground truth** (or a **user feedback**)



**We retrieved 7 documents of which 4 are correct**

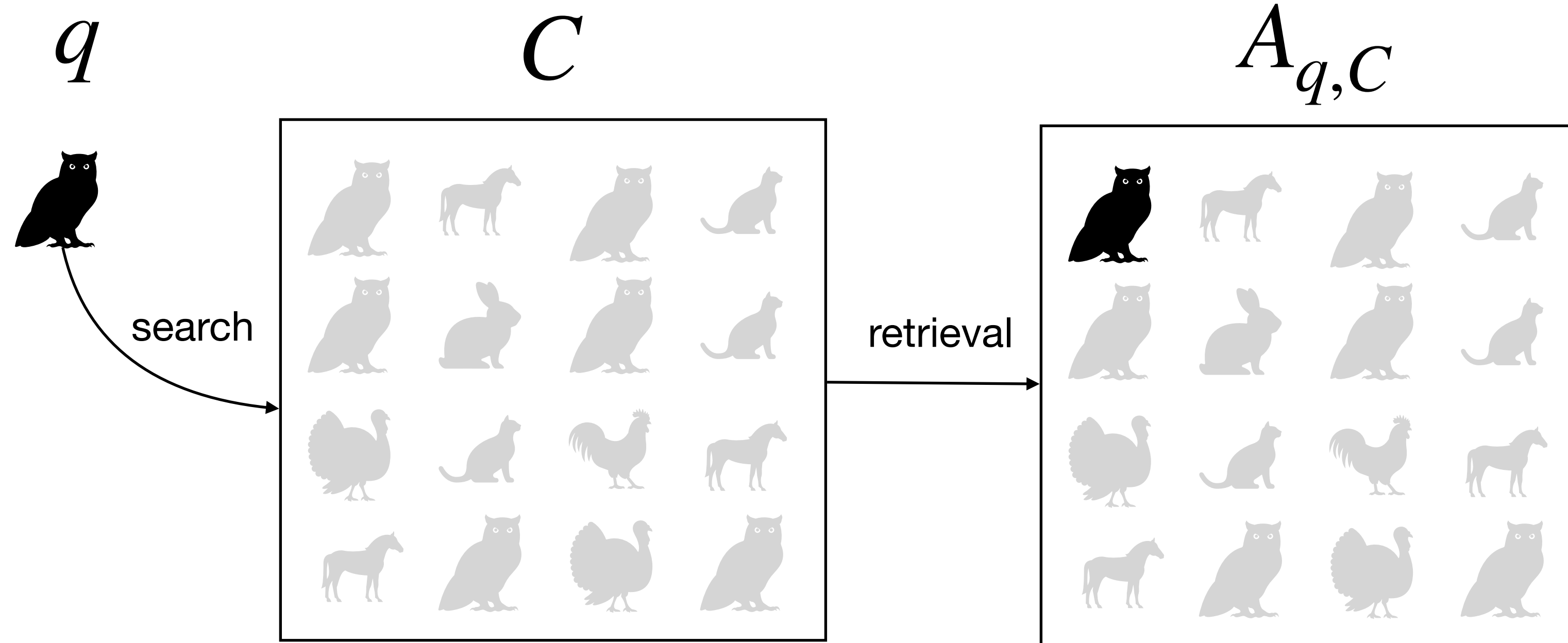
We can measure the quality of our system according to this notion of quality, called **Precision**

$$Prec = \frac{\text{relevant retrieved}}{\text{retrieved}}$$

$$Prec_q = \frac{4}{7} = 0.57$$

# Search evaluation: the notion of quality

**Question:** why **Precision** is a **necessary** but **not sufficient** property of a good search system?



We retrieved only 1 document and it is correct

However, many relevant documents are missing

$$Prec = \frac{\text{relevant retrieved}}{\text{retrieved}}$$

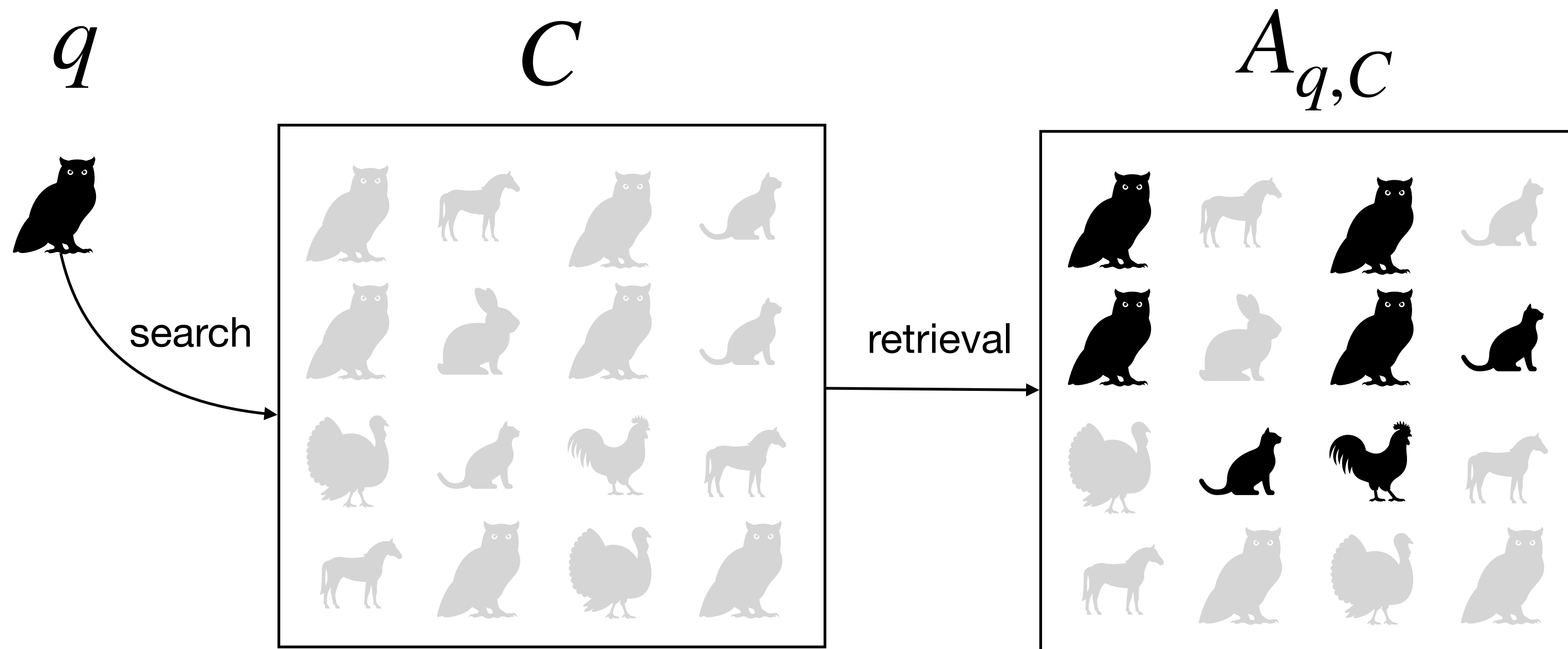
$$Prec_q = \frac{1}{1} = 1$$

# Search evaluation: the notion of quality

**Question:** when we say that the search answers  $A_{q,c}$  are good?

**Definition 2:** *when all the relevant documents contained in  $C$  are retrieved by  $q$*

**Remember:** in order to know if a document is actually relevant, we need a **ground truth** (or a **user feedback**)



**We retrieved 4 relevant documents from a corpus which contains 6 relevant documents**

We can measure the quality of our system according to this notion of quality, called **Recall**

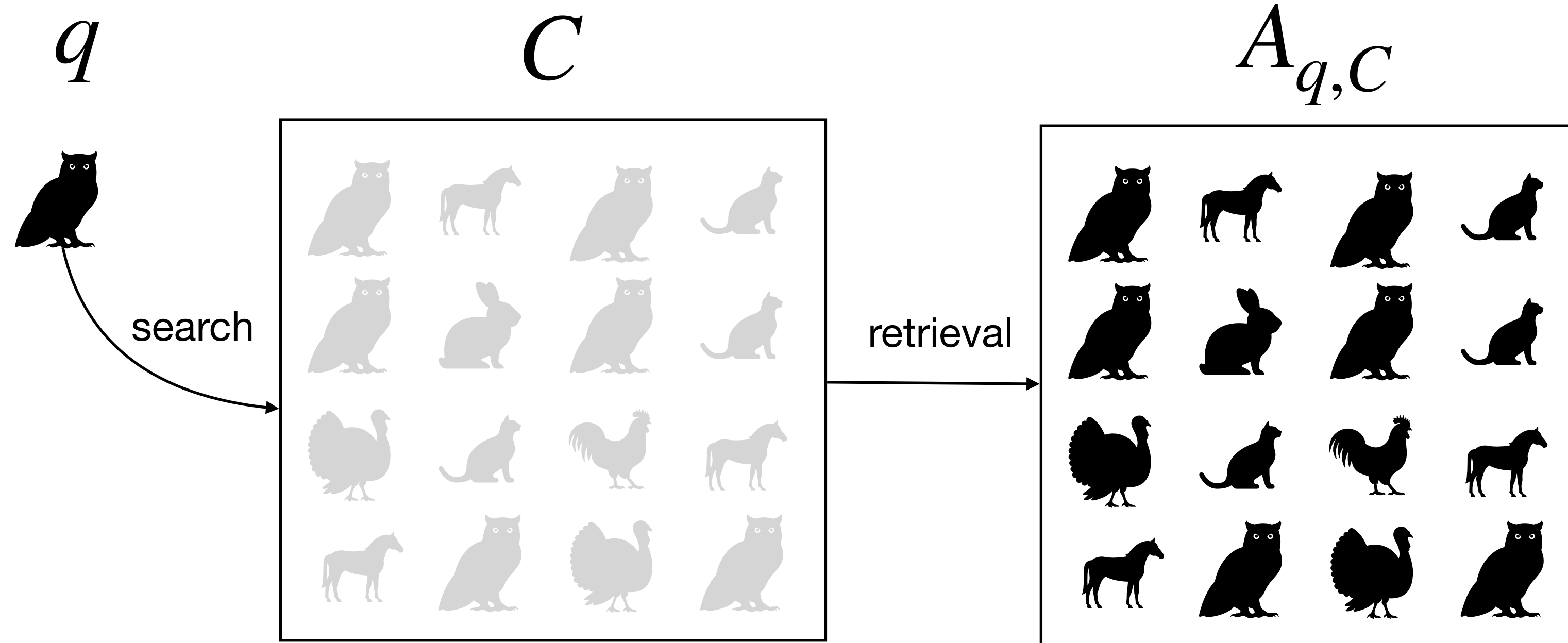
$$Rec = \frac{\text{relevant retrieved}}{\text{relevant}}$$

$$Rec_q = \frac{4}{6} = 0.66$$



# Search evaluation: the notion of quality

**Question:** why **Recall** is a **necessary** but **not sufficient** property of a good search system?



We retrieved all the documents and this means to retrieve all the relevant ones by definition

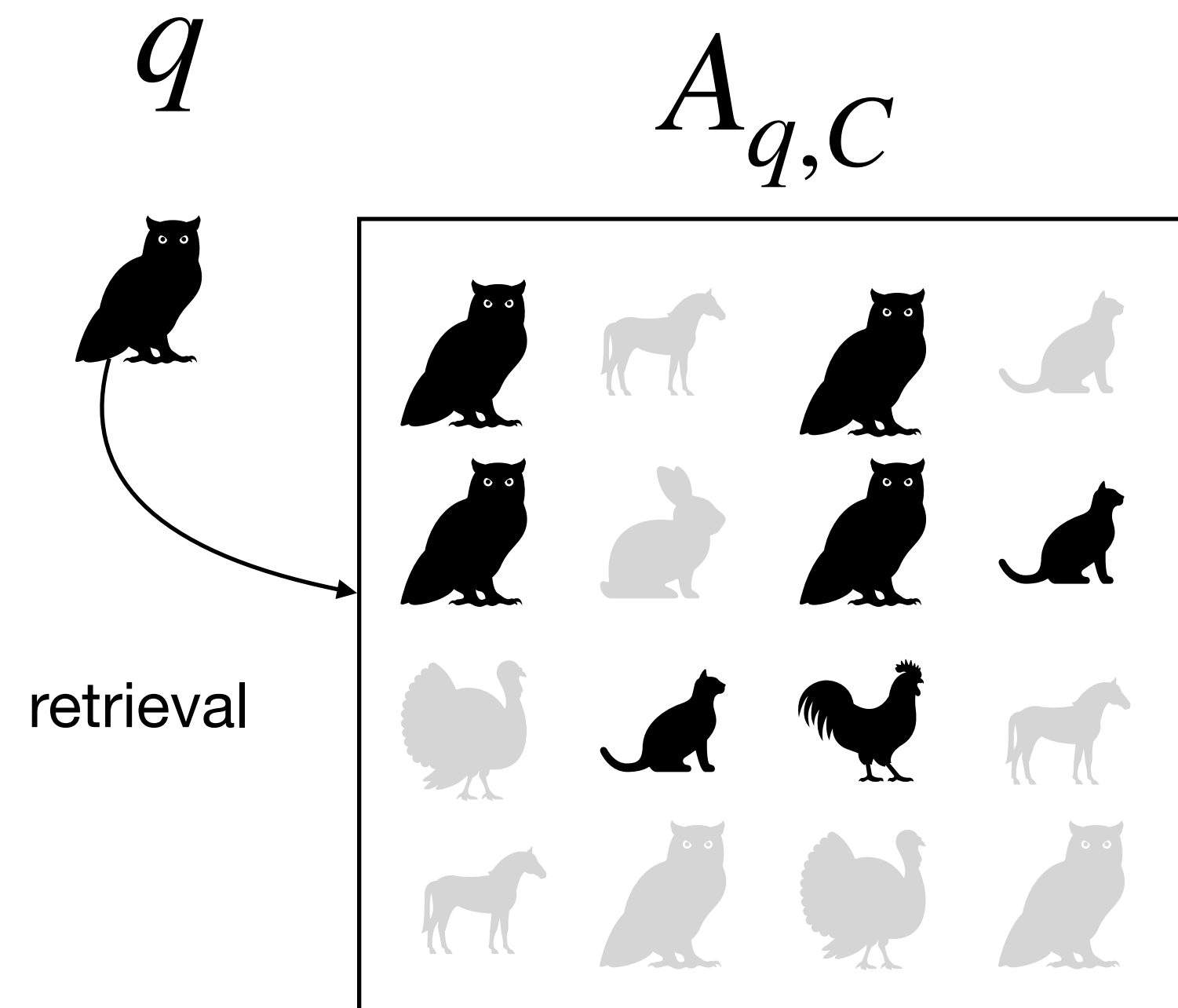
However, there are a lot of wrong results

$$Rec = \frac{\text{relevant retrieved}}{\text{relevant}}$$

$$Rec_q = \frac{6}{6} = 1$$

# Search evaluation: the notion of quality

**Definition 3:** we aim at a system with a good tradeoff between precision and recall; this can be measured by the **f1-score**



$$F1 = \frac{2 \cdot Prec \cdot Rec}{Prec + Rec} = \frac{2 \cdot 0.57 \cdot 0.66}{0.57 + 0.66} = 0.61$$

**Question:** when the numbers we obtain from these measures are good? Try to perform search by tossing a coin...



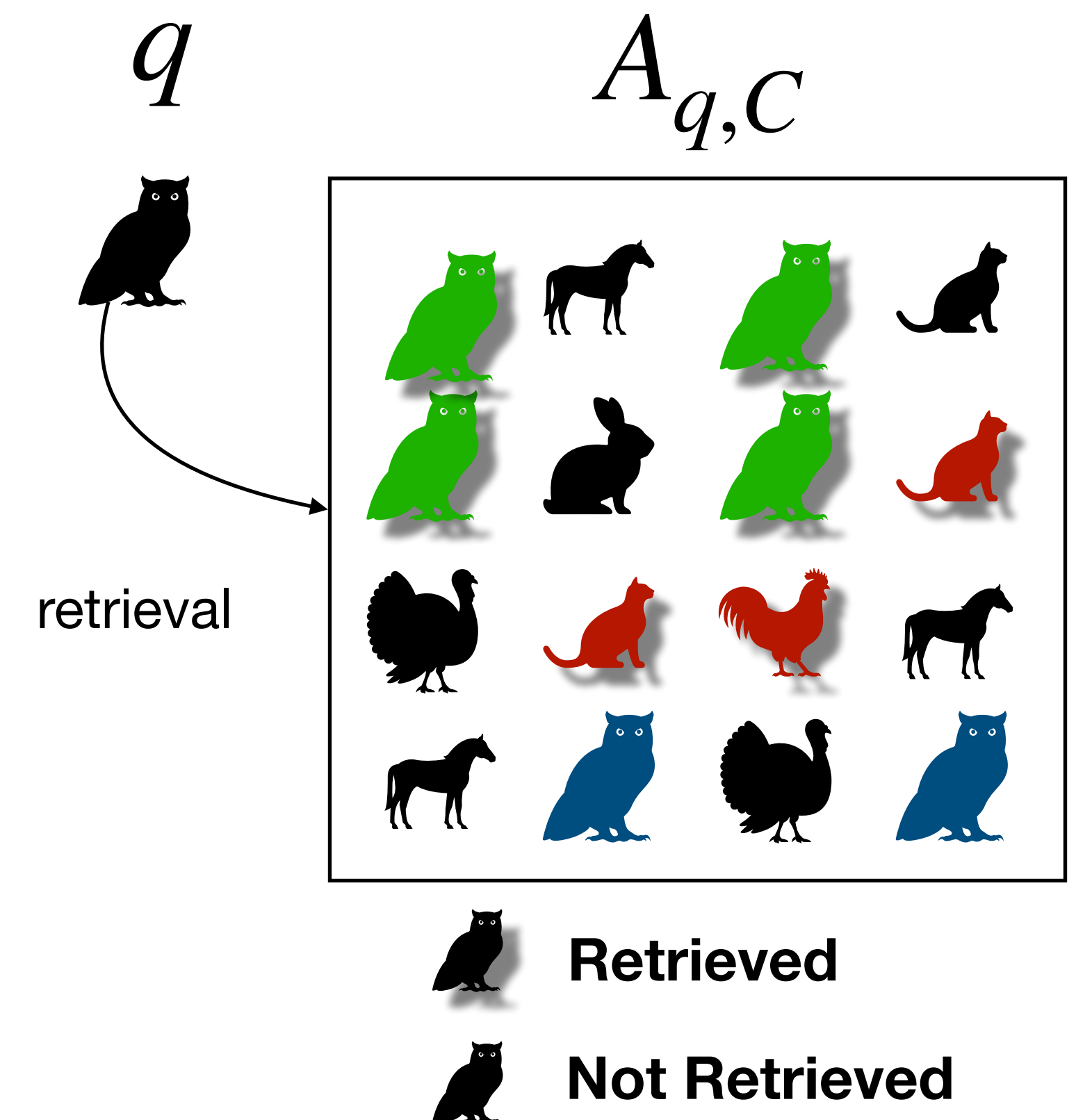
# A more formal definition of Precision and Recall

Given a query  $q$  and a ground truth providing the set  $E_q$  of relevant documents for  $q$ , we denote  $A_q$  the set of query answers provided by the system under evaluation

For each document  $d$ :

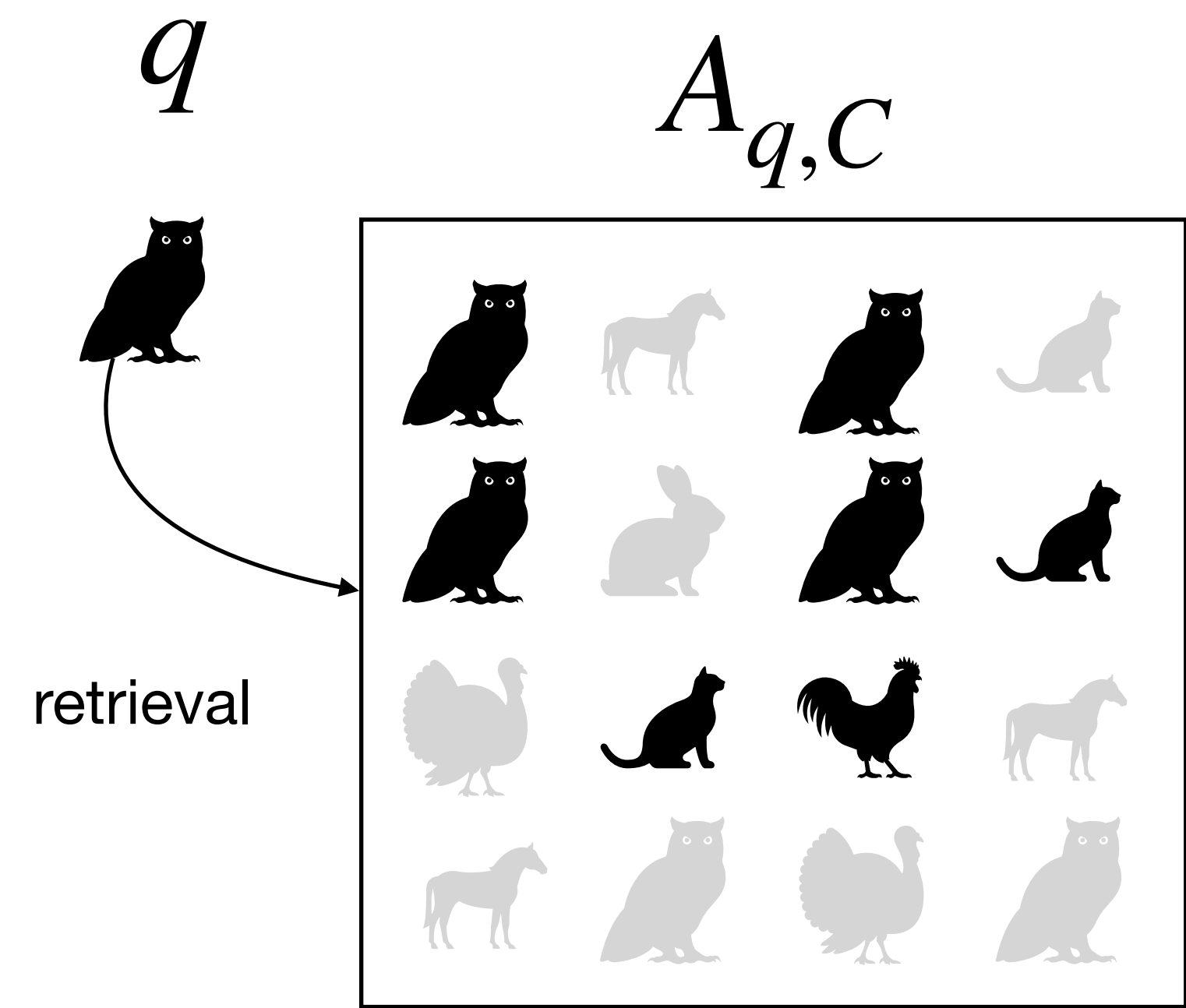
	$d \in E_q$	$d \notin E_q$	
$d \in A_q$	<b>TP</b> True Positive	<b>FP</b> False Positive	<b>Retrieved</b>
$d \notin A_q$	<b>FN</b> False Negative	<b>TN</b> True Negative	<b>Not retrieved</b>
	<b>Relevant</b>	<b>Not Relevant</b>	

$$Prec = \frac{TP}{TP + FP}; \quad Rec = \frac{TP}{TP + FN}; \quad F1 = \frac{TP}{TP + \frac{1}{2}(FP + FN)}$$



# Confusion Matrix

When evaluating a search system, it is important to understand in which cases we have errors: it is more the **retrieval of wrong documents (FP)** or instead the fact that we **miss many documents (FN)**



		Ground truth		
Predicted values	$q$	$R = 1$	$R = 0$	
	$R = 1$	$TP = 4$	$FP = 3$	7
	$R = 0$	$FN = 2$	$TN = 7$	9
		6	10	16

R is the variable that represent the document relevance to the query

# Search evaluation: confusion matrix and other measures

specificity	negative predictive value	miss rate	fall-out	false discovery rate	false omission rate	critical success index
$TNR = \frac{TN}{TN + FP}$	$NPV = \frac{TN}{TN + FN}$	$FNR = \frac{FN}{FN + TP}$	$FPR = \frac{FP}{FP + TN}$	$FDR = \frac{FP}{FP + TP}$	$FOR = \frac{FN}{FN + TN}$	$TS = \frac{TP}{TP + FN + FP}$

**Prevalence threshold**  

$$PT = \frac{\sqrt{Rec(1 - TNR)} + TNR - 1}{Rec + TNR - 1}$$

**Accuracy**  

$$ACC = \frac{TP + TN}{TP + TN + FP + FN}$$

**Balanced accuracy**  

$$BA = \frac{Rec + TNR}{2}$$

**Informedness**  

$$BM = Rec + TNR - 1$$

**Markedness**  

$$MK = Prec + NPV - 1$$

**Matthews correlation coefficient**  

$$MCC = \frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}$$

**Fowlkes-Mallows index**  

$$FM = \sqrt{\frac{2TP}{(TP + FP)(TP + FN)}}$$

**Quick summary:** [https://en.wikipedia.org/wiki/Evaluation\\_measures\\_\(information\\_retrieval\)](https://en.wikipedia.org/wiki/Evaluation_measures_(information_retrieval))

# Evaluation of ranking systems

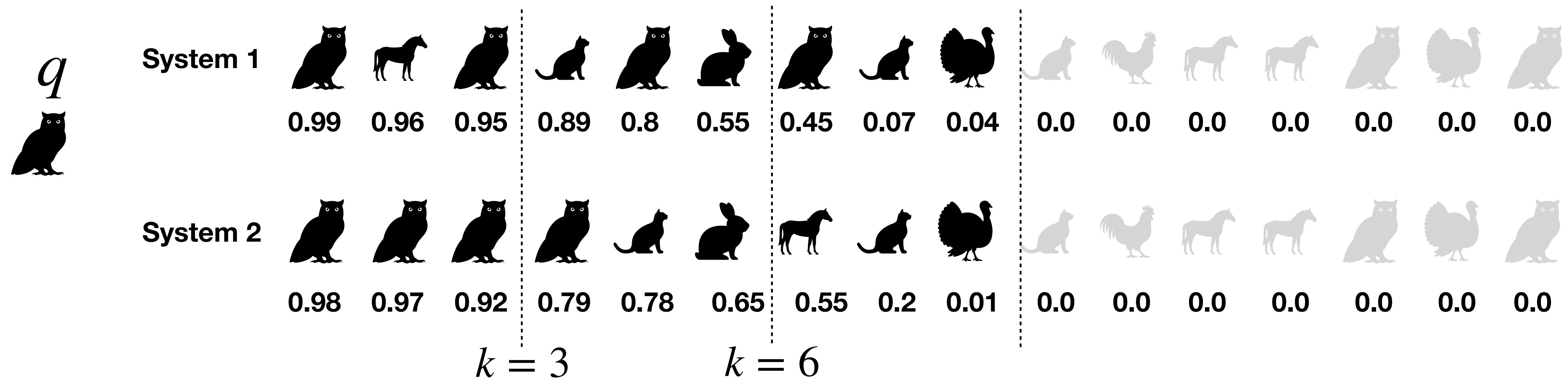
The answer of a non boolean search system is not a set of retrieved documents, rather a rank of documents with a relevance score (that is typically the cosine similarity between the query and the document). **How do we evaluate the system in this case?**



Note that the two systems achieve the **same Precision and Recall**. However, the second system is better.

# Evaluation of ranking systems

**Solution 1:** we could set a threshold too select the top-k results and use them to evaluate precision and recall.  
But where should we put the threshold?



**Precision at k**      **System 1:**  $Prec_{k=3} = \frac{2}{3}$ ;  $Prec_{k=6} = \frac{1}{2}$

**System 2:**  $Prec_{k=3} = 1$ ;  $Prec_{k=6} = \frac{2}{3}$

Still the order of results is not completely taken into account

# Evaluation of ranking systems

**Solution 2: Discounted cumulative gain:** we discount the relevance of each document according to its position in the ranking

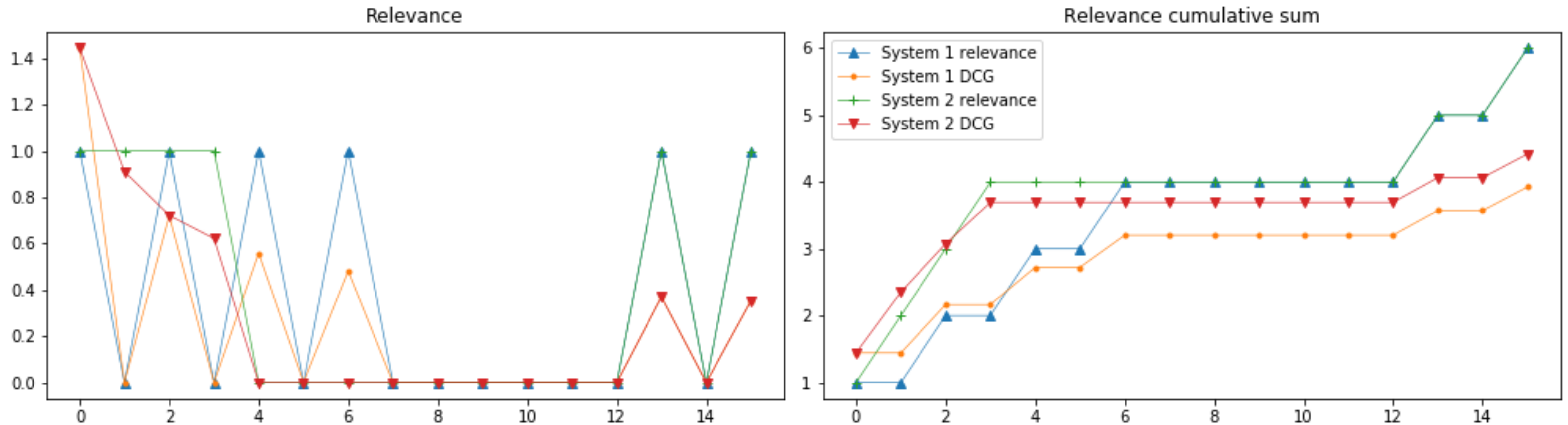


**Discounted cumulative gain:**  $DCG = \sum_{i=1}^n \frac{R_i}{\log(i+1)}$



# Evaluation of ranking systems

**Solution 2: Discounted cumulative gain:** we discount the relevance of each document according to its position in the ranking



















**System 1: 3.93 System 2: 4.42**

















# Evaluation of ranking systems

**Solution 3:** Precision Vs Recall. When moving along the ranking from top to bottom, the **Recall** increases by definition. We can just measure the **Precision** scored for different levels of recall.

System 1

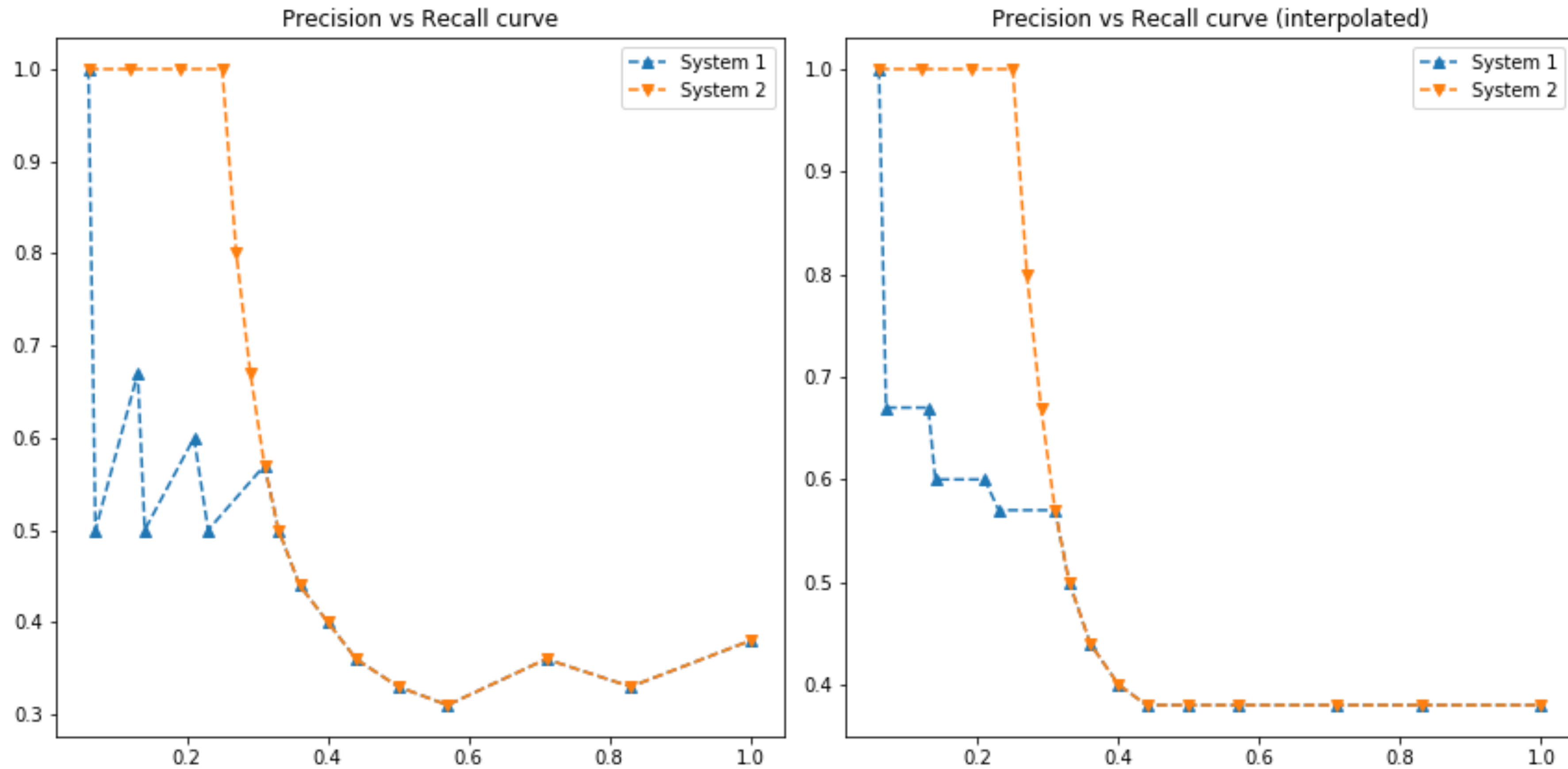
																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
TP	1.00	1.00	2.00	2.00	3.00	3.00	4.00	4.00	4.00	4.0	4.00	4.00	4.00	5.00	5.00	6.00
FP	0.00	1.00	1.00	2.00	2.00	3.00	3.00	4.00	5.00	6.0	7.00	8.00	9.00	9.00	10.00	10.00
FN	15.00	14.00	13.00	12.00	11.00	10.00	9.00	8.00	7.00	6.0	5.00	4.00	3.00	2.00	1.00	0.00
P	1.00	0.50	0.67	0.50	0.60	0.50	0.57	0.50	0.44	0.4	0.36	0.33	0.31	0.36	0.33	0.38
R	0.06	0.07	0.13	0.14	0.21	0.23	0.31	0.33	0.36	0.4	0.44	0.50	0.57	0.71	0.83	1.00

System 2

																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
TP	1.00	2.00	3.00	4.00	4.00	4.00	4.00	4.00	4.00	4.0	4.00	4.00	4.00	5.00	5.00	6.00
FP	0.00	0.00	0.00	0.00	1.00	2.00	3.00	4.00	5.00	6.0	7.00	8.00	9.00	9.00	10.00	10.00
FN	15.00	14.00	13.00	12.00	11.00	10.00	9.00	8.00	7.00	6.0	5.00	4.00	3.00	2.00	1.00	0.00
P	1.00	1.00	1.00	1.00	0.80	0.67	0.57	0.50	0.44	0.4	0.36	0.33	0.31	0.36	0.33	0.38
R	0.06	0.12	0.19	0.25	0.27	0.29	0.31	0.33	0.36	0.4	0.44	0.50	0.57	0.71	0.83	1.00

# Precision vs Recall curve

$$AvgP = \int_0^1 P(R) dR \approx \sum_{k=1}^n P(k) \Delta R(k), \text{ where } \Delta R(k) \text{ is the change in } R \text{ from } k-1 \text{ to } k$$

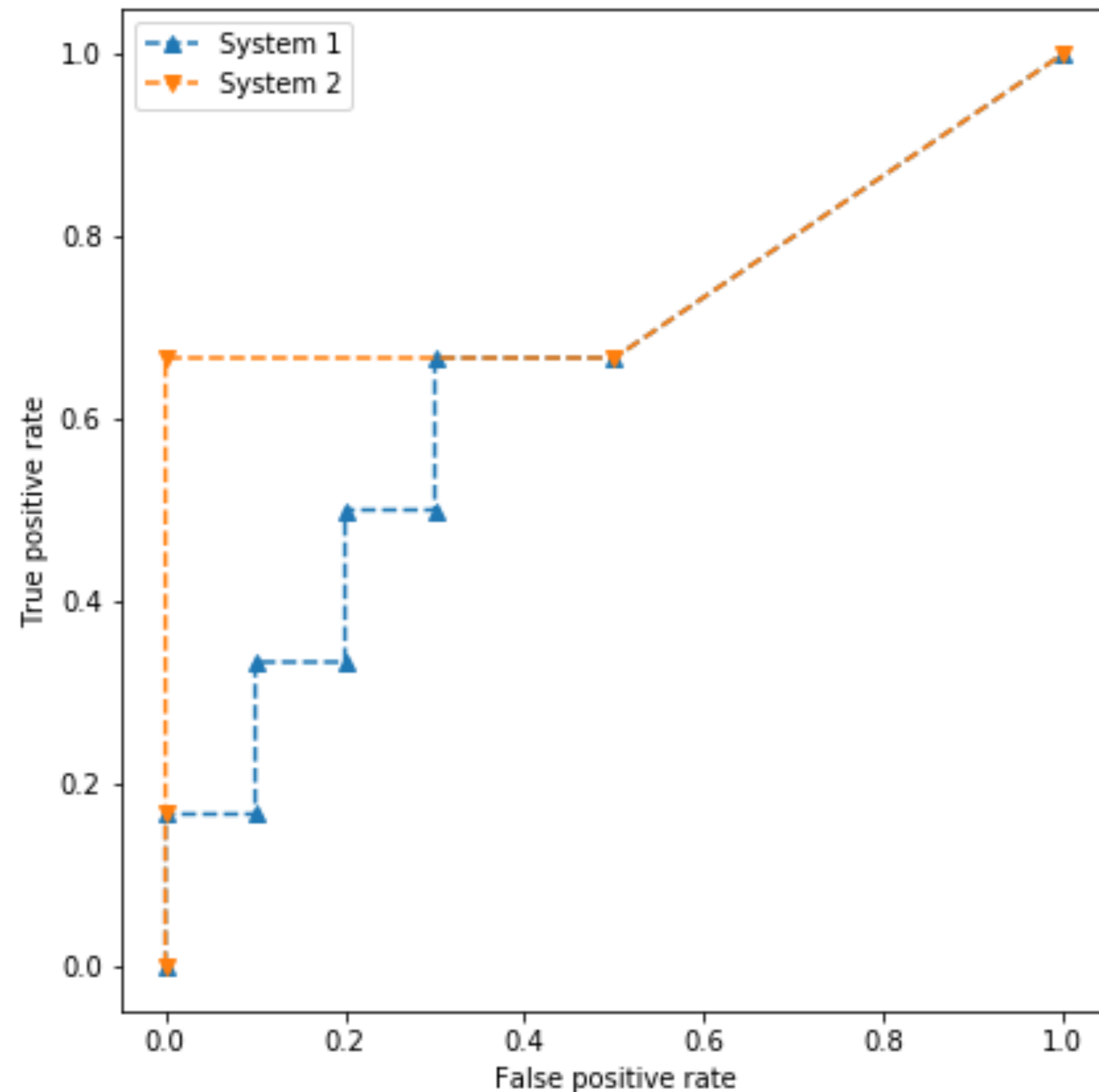


$$Prec_{interpolated}(Rec_i) = \max_{j \geq i} Prec(Rec_j)$$

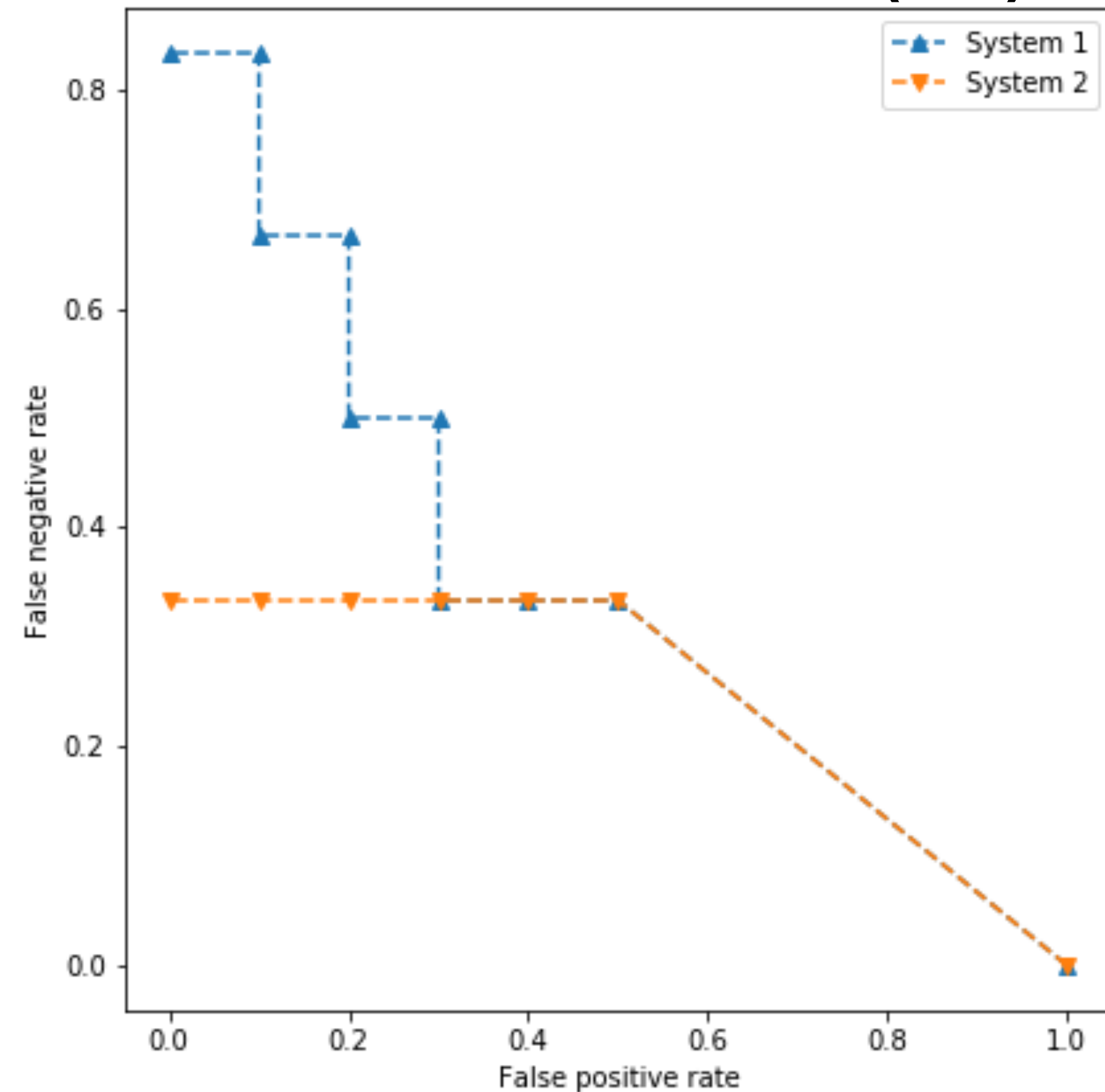
# Other curves

The **ROC (receiver operating characteristic)** curve is created by plotting the **true positive rate (TPR) (Recall)** against the **false positive rate (FPR) ( $FP / (FP + TN)$ )** at various threshold settings.

**ROC (receiver operating characteristic)**



**Detection Error Tradeoff (DET)**



The **Detection Error Tradeoff (DET)** curve is created by plotting the **false negative rate (FNR) ( $FN / (FN + TP)$ )** against the **false positive rate (FPR) ( $FP / (FP + TN)$ )** at various threshold settings.