1 McCormick Relaxation

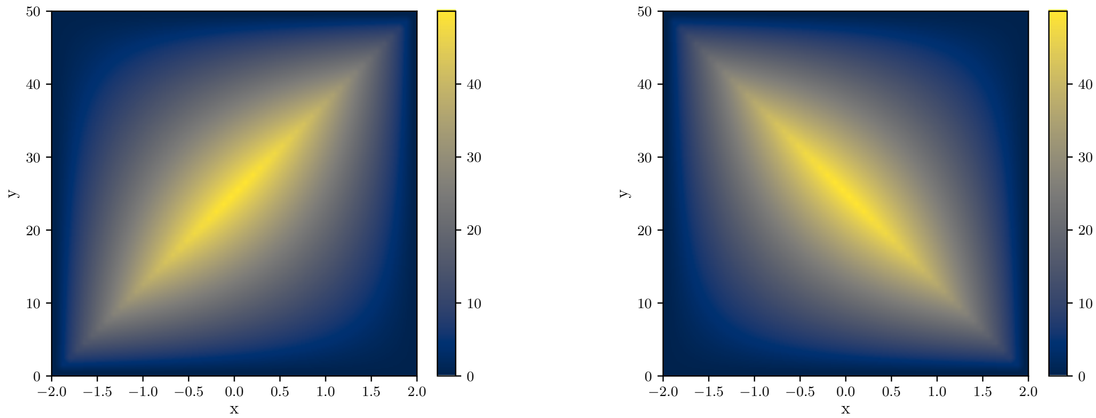
To address bilinear terms of the form $w = xy$, we introduce the following constraints based on the bounds of x and y:

$$x^L \leq x \leq x^U, \qquad y^L \leq y \leq y^U.$$

The resulting McCormick relaxation constraints for w are:

$$\begin{aligned} w &\geq x^L y + x y^L - x^L y^L, \\ w &\geq x^U y + x y^U - x^U y^U, \\ w &\leq x^U y + x y^L - x^U y^L, \\ w &\leq x^L y + x y^U - x^L y^U. \end{aligned}$$

These constraints establish an overestimation and underestimation of the bilinear term w, which can be visualized to assess their accuracy compared to the actual bilinear relationship.

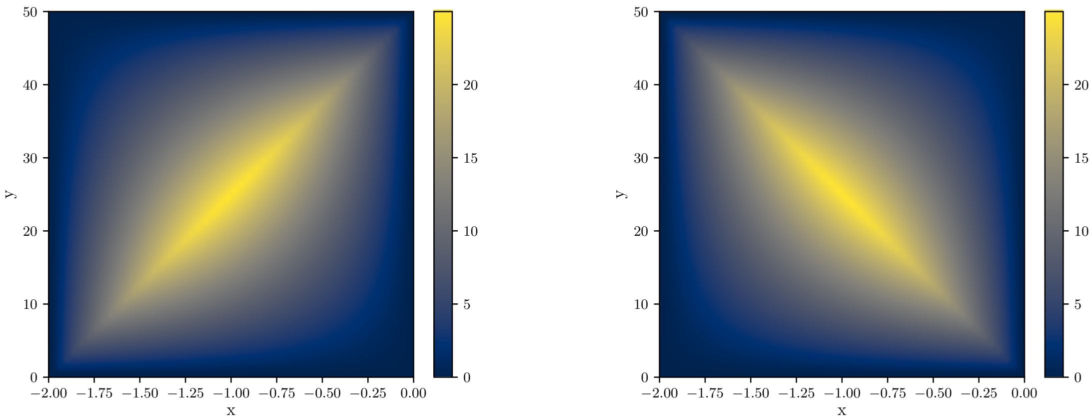


(a) Difference to the upper bound

(b) Difference to the lower bound

Figure 1.1: McCormick relaxation bounds for the bilinear term $w = xy$.

Figure 1.1a illustrates the deviation between the actual bilinear term $w = xy$ and the smallest upper bound provided by the relaxation constraints. Similarly, Figure 1.1b shows the deviation to the greatest lower bound. For the range $-2 \leq x \leq 2$ and $0 \leq y \leq 50$. It is evident that the bounds improve as x and y approach their respective limits.



(a) Difference to the upper bound

(b) Difference to the lower bound

Figure 1.2: McCormick relaxation bounds for the bilinear term $w = xy$ with stricter bounds on x.

Figures 1.2a and 1.2b present the results when x is more tightly bounded, specifically $-2 \leq x \leq 0$. One can observe that the maximum deviation is considerably reduced compared to the previous scenario, indicating that tighter bounds yield a more accurate relaxation.

To illustrate the application of these relaxations in practice, consider a path-planning scenario with $v_{min} = 1$, $v_{max} = 4$, and $v_{start} = 1$. In this scenario, the bilinear term $v\xi$, which appears in the equation of motion for $\dot{n} = v \sin \xi \approx v\xi$, is approximated using McCormick relaxations.

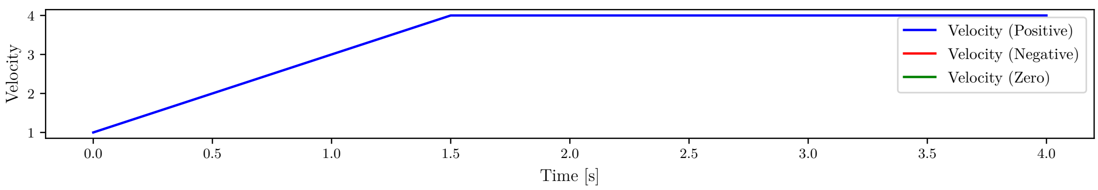


Figure 1.3: Planned velocity profile.

Figure 1.3 shows the planned velocity profile, which quickly reaches its upper limit.

Figure 1.4 depicts the alignment error ξ at each planned time point. Here, ξ is bounded within $-45° \leq \xi \leq 45°$. It is noteworthy that ξ does not reach these bounds.

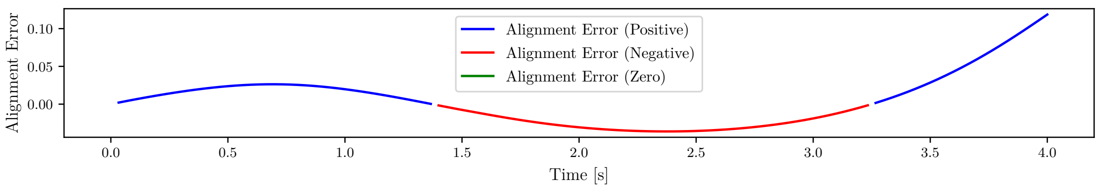


Figure 1.4: Alignment error ξ over time.

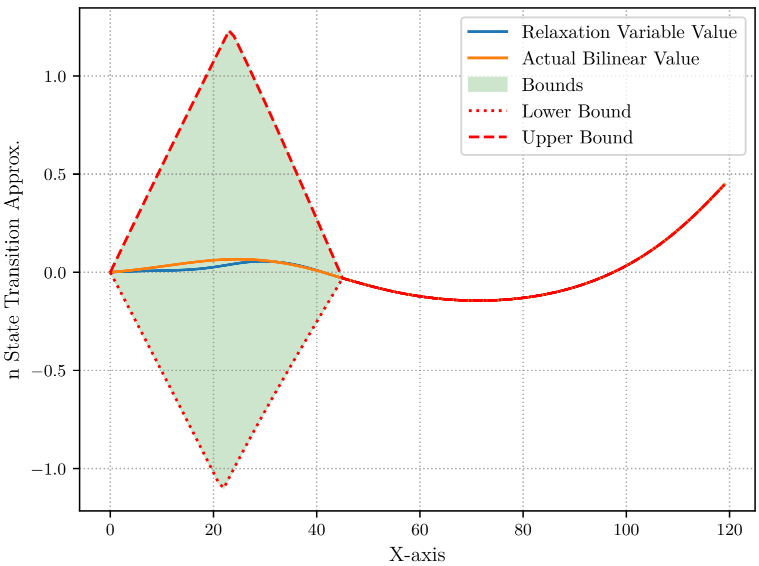


Figure 1.5: State transition approximation for n using bilinear term relaxation.

Figure 1.5 compares the actual bilinear value $v\xi$ with the relaxation variable w introduced via McCormick envelopes. This comparison highlights the accuracy of the relaxation approach in approximating the bilinear interaction and its effect on the state transition of n. Once the velocity reaches its limit, the approximation becomes increasingly accurate.

Note: The x-axis does not represent time directly but instead shows discrete time points. Here, 30 time points per second were chosen, resulting in a range from 0 to 120 for this figure, whereas the other figures range from 0 to 4.