Realistic Optimization-based Driving Using a Constrained Double-Integrator Model

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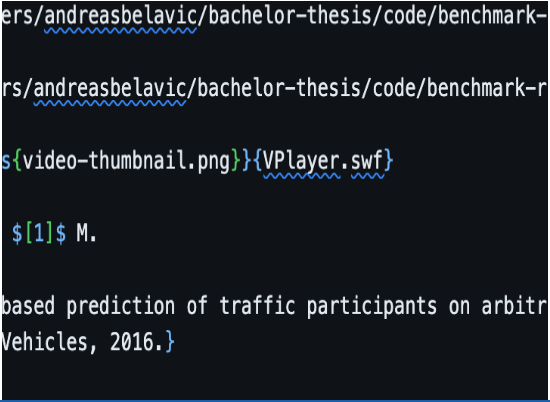
Motivation & Problem Statement

- Autonomous driving demands safe, real-time trajectory planning.
- Balancing model accuracy with computational efficiency is critical.
- Our approach uses convex optimization with a constrained double-integrator model.

[Placeholder: Schematic diagram of motion planning challenges]



Overview of the Approach

- Reformulates trajectory planning as a convex optimization problem.
- Simplifies vehicle dynamics via the double-integrator model.
- Applies feedback linearization and convex inner approximations.

[Placeholder: Block diagram of the planning framework]

[2] A. Belavic, Bachelor's Thesis, TUM, 2025.

Vehicle Modeling: Double Integrator Model

- Simplified dynamics: considers only position and velocity.
- Ignores orientation for computational efficiency.
- Leads to a convex formulation ideal for real-time planning.

[Placeholder: Diagram of double integrator dynamics]

See Chapter 2 of the thesis.

Vehicle Modeling: Kinematic Bicycle Model

- Captures vehicle orientation and steering dynamics.
- More realistic than the double integrator but computationally heavier.
- Often used for validation against simplified models.

[Placeholder: Diagram of the bicycle model representation]

Refer to Chapter 2 for model comparisons.

Coordinate Transformation: Frenet Frame

- Transforms global coordinates into a path-following system.
- Simplifies handling of road curvature and lateral deviations.

[Placeholder: Illustration of the Frenet frame]

Adapted from the thesis discussion in Section 2.3.

Motion Planning Framework Architecture

- Multi-layered approach: global route, behavioral layer, local trajectory optimization.
- Our focus: the local planner using convex optimization.

[Placeholder: Block diagram of planning layers]

Inspired by early work on motion planning architectures [1].

Convex Optimization in Motion Planning

- Ensures smooth and dynamically feasible trajectories.
- Allows efficient solution of sub-problems to global optimality.
- Supports real-time implementation.

See related discussions in Chapters 3 and 4.

Feedback Linearization Technique

- Linearizes nonlinear vehicle dynamics via state feedback.
- Decouples acceleration and steering commands.
- Simplifies controller design.

[Placeholder: Schematic showing feedback linearization]

Technique detailed in Section 3.1.4 of the thesis.

Constraint Handling in the Framework

- Incorporates physical limits: acceleration, velocity, and road boundaries.
- Uses convex inner approximations for non-convex constraints.
- Maintains safety and feasibility.

[Placeholder: Diagram of constraint mapping]

Constraint formulations are discussed in Section 3.1.5.

Quantifier Elimination & Convex Approximations

- Two approaches: interval fitting and Cylindrical Algebraic Decomposition (CAD).
- Derives an inscribed polytope for the feasible set.
- Balances computational efficiency with accuracy.

[Placeholder: Graphical comparison of interval fitting and CAD]

For details, see Sections 3.1.6 of the thesis.

Simulation Setup: Implementation Details

- Road segments, planner configurations, and soft constraints are defined.
- Simulation scenarios mimic realistic driving conditions.
- Ensures the evaluation is comprehensive.

[Placeholder: Table summarizing simulation parameters]

Implementation details from Section 4.1.

Evaluation Metrics and Scenarios

- Metrics: trajectory feasibility, computational time, and safety margins.
- Multiple scenarios: straight roads, curved segments, and dynamic obstacles.

[Placeholder: Diagram or chart of evaluation metrics]

Refer to Section 4.2 of the thesis.

Numerical Experiment: Scenario 1

- Scenario: Moderate-speed driving on a straight road.
- Result: Smooth trajectory with rapid solver convergence.

[Placeholder: Simulation snapshot of Scenario 1]

Simulation results are detailed in Chapter 4.

Numerical Experiment: Scenario 2

- Scenario: Handling a sharp curve.
- Result: Effective management of lateral deviations.

[Placeholder: Simulation snapshot of Scenario 2]

See performance analysis in Chapter 4.

Performance: Computational Efficiency

- Optimization runs in real time under dynamic conditions.
- Solver times validate the framework's efficiency.

[Placeholder: Plot of solver runtime vs. scenario complexity]

Performance evaluation from Section 4.2.

Comparison: Double Integrator vs. Bicycle Model

- Double integrator: Fast and efficient, less detailed.
- Bicycle model: Higher fidelity but increased computational load.
- Trade-offs guide model selection based on application needs.

[Placeholder: Comparative table/graph]

Comparison drawn from Chapter 2 and related literature.

Advantages of the Proposed Framework

- Convex formulation ensures global optimality within sub-problems.
- Robust to dynamic environmental changes.
- Provides a foundation for scalable autonomous driving solutions.

Advantages summarized in the thesis conclusion.

Limitations and Challenges

- Trade-off: Simplified models may sacrifice some accuracy.
- Conservative approximations can limit optimality.
- Extension to highly dynamic scenarios remains challenging.

[Placeholder: Diagram illustrating limitations]

Challenges discussed in Chapter 5.

Future Work and Extensions

- Incorporate full dynamic models for enhanced accuracy.
- Integrate with sensor data for real-world validation.
- Develop adaptive control strategies to improve performance.

[Placeholder: Flowchart of future research directions]

Future work is outlined in the thesis conclusion.

Key Findings and Contributions

- Proposed a convex optimization framework for motion planning.
- Demonstrated realistic, safe trajectories with a constrained double-integrator.
- Validated through extensive simulations and performance evaluations.

Key contributions summarized in Chapter 5.

Impact on Autonomous Driving

- Enhances real-time trajectory planning capabilities.
- Improves safety and efficiency in autonomous vehicle operations.
- Lays groundwork for further advancements in motion planning.

Impact discussed in the thesis discussion section.

Discussion and Q&A Preparation

- How does convex approximation affect optimality?
- What are the trade-offs of using a double-integrator model?
- How scalable is the approach for complex, dynamic scenarios?

[Placeholder: List of potential discussion questions]

Questions based on open issues in Chapters 4 and 5.

Summary and Final Thoughts

- Reviewed the modeling, optimization, and simulation of our framework.
- Highlighted benefits, limitations, and future research directions.
- Emphasized the importance of real-time feasibility and safety.

[Placeholder: Summary diagram of the presentation]

Summary adapted from the thesis conclusion.

Conclusions

- Presented a comprehensive, convex optimization framework for motion planning.
- Balanced computational efficiency with realistic vehicle dynamics.
- Contributed to advancing motion planning in autonomous driving.

[Placeholder: Final summary graphic]

Conclusions drawn from the overall thesis work.

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