1 Mapping Controls and States

Given: A car model which has state:

$$x_{car} = \begin{bmatrix} p_x \\ p_y \\ \delta \\ v \\ \psi \\ \dot{\psi} \\ \beta \end{bmatrix}$$

p_x Global Position, p_y Global Postion, δ Steering Angle, v Velocity, ψ Orientation, $\dot{\psi}$ Yaw Rate, β Slip Angle.
and control inputs:

$$u_{car} = \begin{bmatrix} \dot{\delta} \\ a \end{bmatrix}$$

$\dot{\delta}$ Steering Angle Rate, a Longitudinal Acceleration

The used planning model has its own state definition $x_{planning}$ and control inputs $u_{planning}$.

A planning model has to provide a mapping from the car states to its states:

$$x_{car} \mapsto x_{planning}$$

and a mapping from its control inputs to the control inputs of the car:

$$u_{planning} \mapsto u_{car}$$

1.1 Single Track Planning Model

$$x_{planning} = \begin{bmatrix} s \\ n \\ \xi \\ v \\ \delta \end{bmatrix}$$

$(s,\ n)$ Frenet Frame Coordinates along the road, ξ alignment error to the road, v velocity, δ steering angl.

$$u_{planning} = \begin{bmatrix} a \\ \dot{\delta} \end{bmatrix}$$

a Longitudinal Acceleration, $\dot{\delta}$ Steering Angle Rate

$$x_{car} = \begin{bmatrix} p_x \\ p_y \\ \delta \\ v \\ \psi \\ \dot{\psi} \\ \beta \end{bmatrix} \mapsto \begin{bmatrix} self.road.get\_road\_position(x, y)[0] \\ self.road.get\_road\_position(x, y)[1] \\ \psi - \theta(s) \\ v \\ \delta \end{bmatrix}$$

$$u_{planning} = \begin{bmatrix} a \\ \dot{\delta} \end{bmatrix} \mapsto \begin{bmatrix} \dot{\delta} \\ a \end{bmatrix} = \begin{bmatrix} \dot{\delta} \\ a \end{bmatrix}$$

1.2 Double Integrator Planning Model

$$x_{planning} = \begin{bmatrix} s \\ n \\ \dot{s} \\ \dot{n} \end{bmatrix}$$

$(s,\ n)$ Frenet Frame Coordinates along the road, $(\dot{s},\ \dot{n})$ Change of the Frenet Frame Coordinates along the road,

$$u_{planning} = \begin{bmatrix} u_t \\ u_n \end{bmatrix}$$

u_t, u_n artificial control inputs, which can be mapped (a_s, a_n) Acceleration of the Frenet Frame Coordinates along the road using g.

$$x_{car} = \begin{bmatrix} p_x \\ p_y \\ \delta \\ v \\ \psi \\ \dot{\psi} \\ \beta \end{bmatrix} \mapsto \begin{bmatrix} self.road.get\_road\_position(x, y)[0] \\ self.road.get\_road\_position(x, y)[1] \\ v\cos(\psi - \theta(s)) \\ v\sin(\psi - \theta(s)) \end{bmatrix}$$

For control input we additional require the current state $x = [s, n, \dot{s}, \dot{n}]^T$ of the planning model and we need to store the current steering angle of the car δ_{cur}:

1.2.1 Approach 1

$$\xi = \arctan(\frac{\dot{n}}{\dot{s}(1 - nC(s))})$$

$$v = \sqrt{(\dot{s}(1 - nC(s)))^2 + \dot{n}^2}$$

$$\dot{\psi} = \frac{u_n - \tan(\xi)u_t}{v(\tan(\xi)\sin(\xi) + \cos(\xi))}$$

$$a = \frac{u_t + v\dot{\psi}\sin(\xi)}{\cos(\xi)}$$

$$\dot{C} = \dot{C}(s) = C'(s)\dot{s} \approx \frac{C(s + \dot{s}\Delta t) - C(s)}{\Delta t}$$

$$\dot{\xi} = \frac{1}{1 + (\frac{\dot{n}}{\dot{s}(1-nC(s))})^2} \frac{u_n\dot{s}(1 - nC(s)) - \dot{n}(u_t - C(s)(u_t n + \dot{s}\dot{n}) - \dot{C}\dot{s}n)}{(\dot{s}(1 - nC(s)))^2}$$

$$\delta = \arctan((\dot{\xi} + C(s)\dot{s})\frac{l_{wb}}{v})$$

$$v_\delta = \max(\min(\frac{\delta - \delta_{cur}}{\Delta t}, \overline{v_\delta}), \underline{v_\delta})$$

$$u_{planning}, x_{planning} \mapsto \begin{bmatrix} v_\delta \\ a \end{bmatrix} = \begin{bmatrix} \dot{\delta} \\ a \end{bmatrix}$$

1.2.2 Approach 2

$$[s_0, n_0, \dot{s}_0, \dot{n}_0]^T := [s, n, \dot{s}, \dot{n}]^T$$

$$[s_1, n_1, \dot{s}_1, \dot{n}_1]^T := \begin{bmatrix} s_0 \\ n_0 \\ \dot{s}_0 \\ \dot{n}_0 \end{bmatrix} + \Delta t \begin{bmatrix} \dot{s}_0 \\ \dot{n}_0 \\ u_n \\ u_t \end{bmatrix}$$

$$v_0 = \sqrt{(\dot{s}_0^2 + \dot{n}_0^2)}$$

$$v_1 = \sqrt{(\dot{s}_1^2 + \dot{n}_1^2)}$$

$$a = \frac{v_1 - v_0}{\Delta t}$$

$$v_\delta = \arctan(l_{wb}(planned_psi_2 - (cur_psi + 1/l_wb * v_0 * np.tan(cur_steering_angle) * dt))/(v_1 * dt))/dt$$

$$x_1 = \begin{bmatrix} s_1 \\ n_1 \\ \dot{s}_1 \\ \dot{n}_1 \end{bmatrix} = x_0 + \begin{bmatrix} \dot{s}_0 \\ \dot{n}_0 \\ u_t \\ u_n \end{bmatrix} dt$$

$$\begin{bmatrix} a_s \\ a_n \end{bmatrix} = g(u_t, u_n)$$

$$\xi_0 = \arctan\left(\frac{\dot{n}_0}{\dot{s}_0}\right)$$
$$\xi_1 = \arctan\left(\frac{\dot{n}_1}{\dot{s}_1}\right)$$

$$v_0 = \sqrt{\dot{s}_0^2 + \dot{n}_0^2}$$
$$v_1 = \sqrt{\dot{s}_1^2 + \dot{n}_1^2}$$

$$\psi_0 = \xi_0 + \theta(s_0)$$
$$\psi_1 = \xi_1 + \theta(s_1)$$

$$\dot{\psi} = \frac{\psi_1 - \psi_0}{dt}$$

$$u_{planning}, x_{planning} \mapsto \frac{1}{dt} \begin{bmatrix} \arctan(l_{wb}\dfrac{\dot{\psi}}{v_1}) - \delta_{cur} \\ v_1 - v_0 \end{bmatrix} = \begin{bmatrix} \dot{\delta} \\ a \end{bmatrix}$$